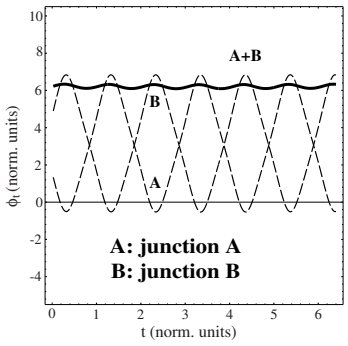
## Out–Of–Phase Mode

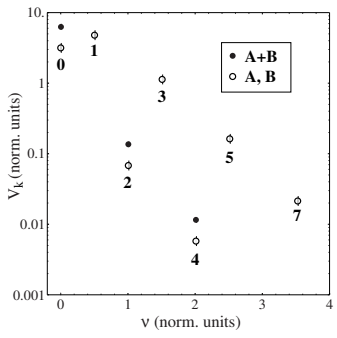
### Equal Junction Parameters

**c₋ mode**

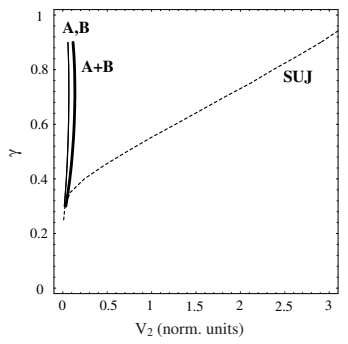
- suppression of even harmonics by magnetic images
- reduced frequency
- performance of single uncoupled junction is better than that of a stack in c- mode




The time dependent voltages $V^A(x_0,t), V^B(x_0,t)$ and $V^\Sigma(x_0,t)$ at the arbitarily chosen coordinate $x_0$ in a stack with equal junction parameters at $\gamma = 0.5$.



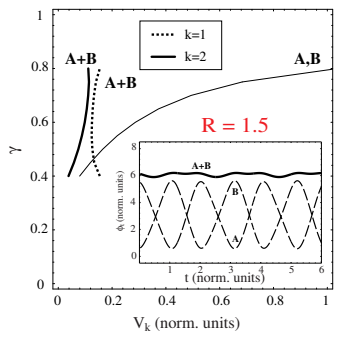

Spectrum of the voltage oscillations with harmonic amplitudes shown by open circles ($V_k^{A,B}$) and small solid circles (total voltage $V_k^\Sigma$) at $\gamma = 0.7$. Numbers indicate the harmonic index k.




The amplitudes of the second harmonic $V_2$ *vs.* bias current $\gamma$ of the stack of identical junctions synchronized in out–of–phase mode. The dashed line shows $V_2$ of the single uncoupled junction (SUJ) for comparison.

### Spread in Junction Parameters

**Spread in resistances: R = $R^A/R^B$**



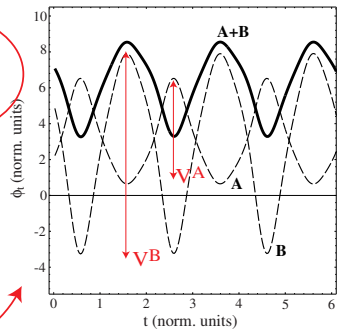

- phase shifts in harmonics occur
- amplitudes remain constant
- even harmonics almost cancel
- odd harmonics grow

also see frame on **Harmonic Amplitudes**

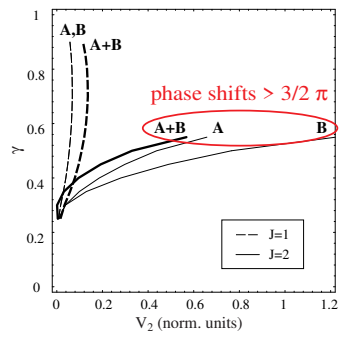
- amplitudes change $V^A/V^B \approx 1/J$
- phase shift $\delta\varphi_k$ appears

Solid lines show $V_2^A, V_2^B$ (thin line) and $V_2^\Sigma$ (thick line) as a function of $\gamma$. $V_1^\Sigma$ (dotted line) is plotted for reference. The voltage profiles $V^A(x_0,t), V^B(x_0,t)$ and $V^\Sigma(x_0,t)$ at $\gamma = 0.5$ are shown in the inset.

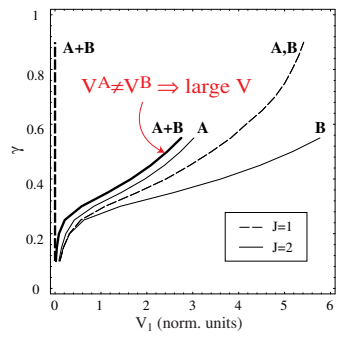
**Spread in critical currents: J = $j^A/j^B \neq 1$**




The evolution of $V^{A,B}(x_0,t)$ and $V^\Sigma(x_0,t)$ in time for $J = j_c^A/j_c^B = 2$ at $\gamma = 0.5$.



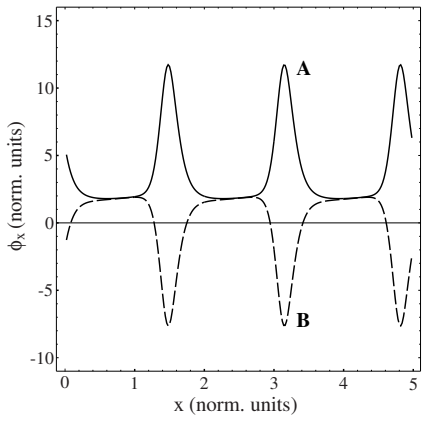

The second harmonic amplitudes $V_2^{A,B}$ and $V_2^\Sigma$ plotted *vs.* $\gamma$ for $J = 1$ (dashed lines) and $J = 2$ (solid lines).




The first harmonic amplitudes $V_1^{A,B}$ and $V_1^\Sigma$ plotted *vs.* $\gamma$ for $J = 2$ (solid lines) and $J = 1$ (dashed lines).

## Magnetic Images

**a) Out–of–phase, $N^A = 3$, $N^B = 0$, S = -0.5**




The strong supression of even harmonic amplitudes in the out–of–phase mode can be explained by considering the interaction of the junctions via the magnetic coupling. Fluxons moving in $JJ^A$, with magnetic field profile $\bar{\phi}_x^A(x)$, create the image profile $\tilde{\phi}_x^A(x)$ in $JJ^B$, as can be seen in plot a). The image profile in good approximation is given by

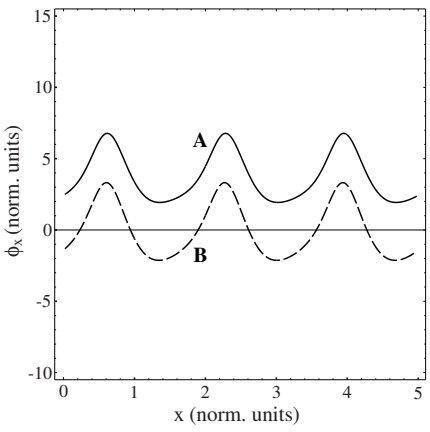
$$\tilde{\phi}_x^A(x) \approx \text{const} - \epsilon\bar{\phi}_x^A(x)\,, \qquad (1)$$

where $\epsilon$ is a constant close unity, for the values of $S$ examined. Supposing existance of fluxons in $JJ^B$ we get

$$\phi_x^B(x) \approx \bar{\phi}_x^B(x) + \tilde{\phi}_x^A(x) \approx \bar{\phi}_x^B(x) - \epsilon\bar{\phi}_x^A(x) + \text{const} \qquad (2)$$

For the out–of–phase mode $\bar{\phi}_x^B(x) = \bar{\phi}_x^A(x - T/2)$. Fourier expansion of $\phi_x^B(x)$ shows that all odd (even) harmonics of $\phi_x^B(x)$ increase (decrease) by a factor of $1 + \epsilon$ $(1 - \epsilon)$. Considering the addition of $\phi_x^B(x)$ and $\phi_x^A(x)$ with a phase shift of half a period results in vanishing odd and considerably reduced even harmonics as observed in the simulations of the out–of–phase mode.

**b) In–phase, $N^A = 3$, $N^B = 0$, S = -0.5**

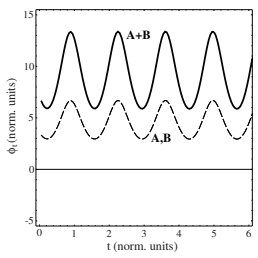



## In–Phase Mode

### Equal Junction Parameters

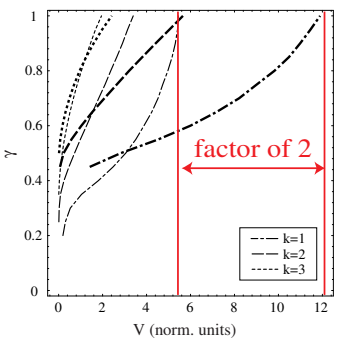
**$c_+$ mode**

Local voltages at γ = 0.5




- gain in amplitude of $V_1$ is larger than 2
- increase of frequency

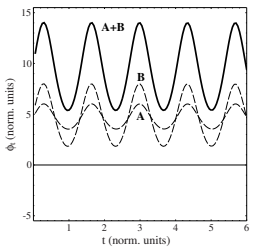
Harmonic amplitudes $V_1$, $V_2$, $V_3$ of the stack (thick lines) and of the single uncoupled junction



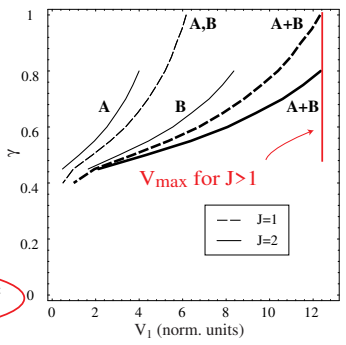

### Spread in Junction Parameters

**Spread in critical currents**

Voltage profiles at γ = 0.5



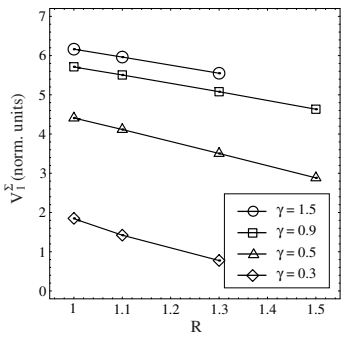

Harmonic amplitude $V_1$




- almost constant first harmonic amplitudes for J > 1

**Spread in loss parameters**

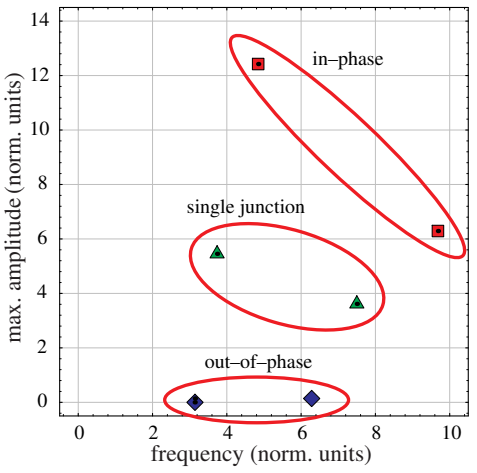
Harmonic amplitudes $V_1$ at γ = const.




- linear decrease of $V_1$ with R

## Conlusions

- harmonic amplitudes are determined by magnetic images induced by the inductive coupling
- spread in junction parameters has strong influence on the harmonic amplitudes
- Feasibility of modes
  - In–phase
    - increase of power in low harmonics in comparison to single uncoupled junction
    - gain of more than a factor of 4 in the power of first harmonic
    - increased frequency
    - less stable with respect to parameter spread
  - Out–of–phase
    - less power in all harmonics than single uncoupled junction
    - reduced frequency

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