Semesterarbeit
Spin echo measurements in a superconducting qubit

Lars Steffen
lsteffen@phys.ethz.ch

Semester work
at the Quantum Device Lab
Laboratory for Solid State Physics, ETH Zürich
Professor: Prof. Andreas Wallraff
Supervisor: Johannes Fink

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Abstract

The aim of this work was to analyse the dephasing of a superconducting quantum bit and the effects of additional π pulses ("spin echo" pulses) on it. The dephasing time T_2 can be determined by observing the decay of Ramsey oscillations. Using the spin echo technique, it is possible to enhance this dephasing time.

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1 Introduction

1.1 Quantum bits

A quantum bit ("qubit") is a unit of quantum information. It is a quantum system in which the Boolean states "0" and "1" are represented by a pair of normalized and orthogonal quantum states labeled as $\{|0\rangle, |1\rangle\}$ (denoted as "ground state" and "excited state"). Unlike a classical bit which can be either "0" or "1", a qubit can be in any superposition state $\alpha|0\rangle + \beta|1\rangle$ for some α and β such that $|\alpha|^2 + |\beta|^2 = 1$.

A common visualization of a qubit state is the so called Bloch sphere representation. A (pure) qubit state $|\psi\rangle$ can be written as

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

with $0 \leq \theta \leq \pi$ and $0 \leq \varphi \leq 2\pi$. The two variables θ and φ define a point on the surface of a three-dimensional unit sphere. This is the Bloch sphere.

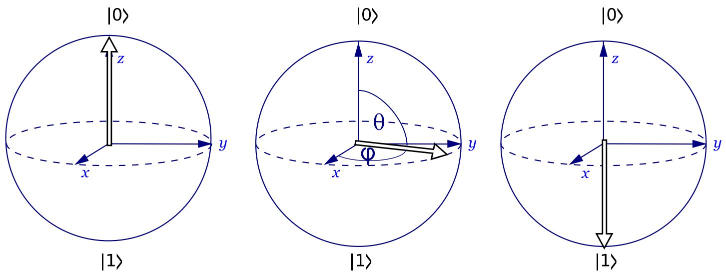


Figure 1: Bloch sphere representation of three different qubit states: Ground state, superposition state and excited state.

In the experiments described in this work, one can only measure the qubit state along the z axis of the Bloch sphere. So the expression "population" means the population of the excited state, which is the ratio of measured excited states to the total number of measurements.

1.2 Realization of the qubit

There are various ways of implementing a qubit. The measurements in this work are performed on a superconducting qubit embedded into a microwave resonator [1, 2, 3]. The qubit is realised with a Cooper pair box [4] which is capacitiveley coupled to the electromagnetic field of a superconducting coplanar waveguide resonator [5] (cf. Fig. 2). The cooper pair box acts as an artificial atom in which the two lowest energy levels represent the two different states of the qubit.

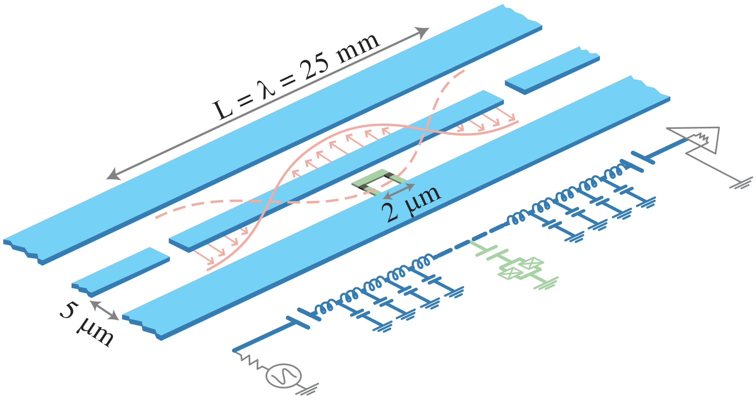


Figure 2: Schematic representation of the cooper pair box (green) and the coplanar waveguide resonator (blue) [3].

Figure 3 shows a simplified circuit diagram of the measurement setup. The transmission line resonator with the Cooper pair box is in a dilution refrigerator at a temperature of ~ 10 mK, whereas the microwaves are generated at room temperature.

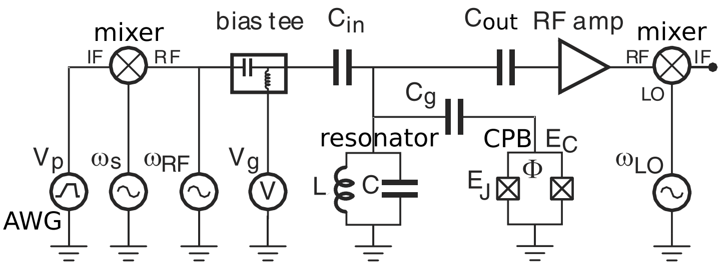


Figure 3: Schematic representation of the experimental setup [1].

The sample which is used is the same as in [1] but three years older. So most of the parameters are similar except the Josephson energy $E_J \approx h \times 3.71$ GHz which has decreased due to the oxidation of the Josephson contacts. The charging energy is $E_C \approx h \times 5.1$ GHz and the coupling $g \approx 2\pi \times 20$ GHz. The resonator has a quality factor of $Q \sim 6500$, which corresponds to a photon decay rate $\kappa/2\pi = 0.83$ MHz.

The qubit state is manipulated with microwave pulses at a frequency ω_s, which is nearly resonant with the qubit transition frequency $\omega_{01} \approx 2\pi \times 3.7$ GHz. Although ω_s is strongly detuned from the resonator frequency $\omega_r \approx 2\pi \times 5.4$ GHz, the resonator can be populated with some drive photons which induce Rabi oscillations in the qubit. The shapes of the pulses are generated with an arbitrary waveform generator (AWG) which is capable of generating pulses of 1 ns resolution. The AWG is connected via an upconversion two quadrature (I,Q) mixer with the microwave generator. Depending on which of the two inputs of the mixer a voltage is applied, the microwave is either just transmitted with an amplitude proportional to the pulse height or gets an extra phase of 90 degrees. The first case corresponds to a pulse which induces a rotation of the qubit around the x axis in the Bloch sphere, the latter case corresponds to rotation around the y axis.

The qubit state is measured by determining the phase and the amplitude of a coherent microwave beam transmitted through the resonator at a frequency ω_{RF}, which is nearly resonant with the resonator frequency ω_{r}. The transmitted microwave is amplified several times (at low and at room temperature) before it is mixed to low frequency with a local oscillator (LO). The signal is then digitally acquired and post-processed by a computer using LabView software [6, 1].

1.3 Software for the simulation

The following paragraphs describe the theoretical backgroud of the software used for the simulation and the implementation in Mathematica which was written by Alexandre Blais [7, 8]. The simulation is used to simulate the effects of the microwave pulses on the excited state population. So one can see what population has to be expected when one uses different pulse sequences, and one can examine the consequences of different pulse lengths or detuning frequencies.

1.3.1 Theoretical background

To describe the time evolution of an isolated system, represented by a density matrix, one uses the Schrödinger equation in the operator representation (von Neumann equation):

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho].$$

However, in case of a nonunitary (dissipative) evolution one uses the master equation in the Lindblad form,

$$\dot{\rho} = \mathcal{L}[\rho], \tag{1}$$

where $\mathcal{L}$ is the Lindbladian (or in analogy to classical mechanics also called "Liouvillian"). In a general case, $\mathcal{L}$ looks like this:

$$\mathcal{L}[\rho] = -\frac{i}{\hbar}[H, \rho] + \sum_{\mu} \gamma_\mu \underbrace{\left(L_\mu \rho L_\mu^\dagger - \frac{1}{2} L_\mu^\dagger L_\mu \rho - \frac{1}{2} \rho L_\mu^\dagger L_\mu \right)}_{\mathcal{D}[L_\mu]\rho}.$$

The first term in $\mathcal{L}[\rho]$ is the usual Schrï¿½dinger term. The other terms describe the possible transitions that the system may undergo due to interactios with the environment. The L_μ operators describe decoherece processes (e.g. energy relaxation) which happen at constant rates γ_μ. The $L_\mu \rho L_\mu^\dagger$ terms induces these possible transitions, whereas the other terms are needed to normalize properly the case in which no decoherence processes occur.

In the experiment considered in this work, the following form of $\mathcal{L}$ is used:

$$\mathcal{L}[\rho] = -\frac{i}{\hbar}[H, \rho] - \gamma_1 \mathcal{D}[\sigma_-]\rho - \frac{\gamma_\phi}{2}\mathcal{D}[\sigma_z]\rho,$$

where γ_1 is the energy relaxation rate and γ_ϕ the pure dephasing rate (see section 2). The Hamiltonian is assumed as

$$H = \hbar \left(\frac{\Delta}{2}\sigma_z + \frac{\Omega_x}{2}\sigma_x + \frac{\Omega_y}{2}\sigma_y \right),$$

where Δ is the detuning frequency between the qubit transition frequency and the microwave angular frequency and $\Omega_{x,y}$ is the angular frequency around the x, y axis. The frequencies $\Omega_{x,y}$ are generated due to the microwave pulses which induce a rotation of the qubit around an axis of the Bloch sphere. That means, that during free evolution $\Omega_{x,y}$ is set to 0, and during a pulse around the x or y axis, $\Omega_{x,y}$ is set to π/t_π, where t_π is the time for a π pulse (in the experiment, t_π is set by the power of the microwave pulse). The formal solution of Eq. 1 is

$$\rho(t) = e^{\mathcal{L}t}\rho_0.$$

To simulate a sequence of pulses and free evolution one has just to apply the different exponentiated Lindbladians to the initial density matrix, e.g. for a sequence of two $\pi/2$ pulses with a delay t in between, the calculation looks like this:

$$\rho(t) = e^{\mathcal{L}_x t_\pi/2} e^{\mathcal{L}_f t} e^{\mathcal{L}_x t_\pi/2} \rho_0,$$

where $\mathcal{L}_f$ is the Lindbladian for the free evolution (i.e. $\Omega_{x,y} = 0$), $\mathcal{L}_x$ is the Lindbladian for the rotation around the x axis and t_π is the time needed for a π pulse. The expectation value of an operator $\mathcal{O}$ is then

$$\langle \mathcal{O} \rangle (t) = \mathrm{tr}\left\{ \mathcal{O}\rho(t) \right\}.$$

So, to calculate the expected value for the polarization along the z axis, one uses

$$\langle \sigma_z \rangle (t) = \mathrm{tr}\left\{ \sigma_z \rho(t) \right\}.$$

An implementation of this formalism as a Mathematica code can be found in appendix A.

2 Dephasing time T_2

2.1 Description of dephasing

Dephasing is caused by processes that randomly modify the effective transition frequency of the qubit, which cause the qubit to accumulate a random phase. In this experiment, the main reason for dephasing is charge noise in the environment of the Cooper pair box which couples to the electrostatic part of the Hamiltonian.

If the noise has a flat spectral density (white noise), the dephasing rate $\Gamma_2 = T_2^{-1}$ can be considered as the following combination of the energy relaxation $\Gamma_1 = T_1^{-1}$ (in this experiment is $T_1 \approx 8 - 10\ \mu$s) and the so called pure dephasing rate Γ_ϕ (Bloch-Redfield approach) [9, 10]:

$$\Gamma_2 = \frac{1}{2}\Gamma_1 + \Gamma_\phi$$

In more general situation, especially if the pure dephasing is dominated by a noise singular near $\omega \approx 0$, e.g. $1/f$ noise, the exponential decay law $e^{-\Gamma_2 t}$ is replaced by $f(t)e^{-\Gamma_1/2t}$, where $f(t)$ is a non-exponential decay function determined by the particular characteristics of the noise realized in an experiment [9].

2.2 Determination of the T_2 time by measurements of the decay of Ramsey oscillations

Experimentally, the dephasing time T_2 of a qubit is measured by observing the temporal decay of the average transverse polarization of its effective spin. The time T_2 is defined by a decay by a factor of $1/e$, also if the decay is nonexponential.

This works in the following way: By applying a $\pi/2$ pulse to a qubit in the ground state $|0\rangle$, one can align it along the $-y$ axis of the Bloch sphere. During a time Δt the qubit will then evolve freely and accumulate a phase depending on a detuning between the qubit drive and its larmor frequency. Because in this experiment we can only measure the qubit state along the z axis, we have to apply another $\pi/2$ pulse to convert the aquired phase into a polarization along the z axis (cf. Fig. 4). These two $\pi/2$ pulses form the Ramsey sequence, which gives an oscillation of the z polarization with Δt at the detuning frequency $\Delta\omega/2\pi = (\omega_{01} - \omega_s)/2\pi$, where ω_{01} is the qubit transition frequency and ω_s the microwave angular frequency .

To perform the experiment, a series of pulse sequences with different Δt has to be applied to the qubit and after each sequence, the porarization of the qubit along the z axis has to be measured. The sequences are designed in Mathematica and stored in files ("pattern files") which are loaded into the memory of the AWG.

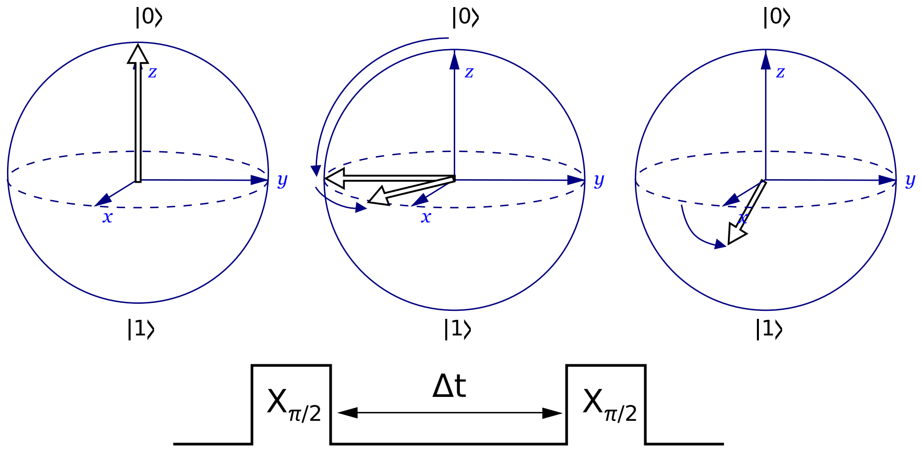


Figure 4: Top: Schematic representation of Ramsey oscillations: Initially, the qubit is in the ground state. By a $\pi/2$ pulse rotation around the x axis it is aligned along the $-y$ axis. During free precession it accumumlates a phase with respect to the rotating frame set by ω_s. This phase is then converted into polarization along the z axis. Bottom: Pulse scheme of the Ramsey sequence.

After the last pulse of each sequence, the averaged polarization along the z axis is measured continuously until the qubit has relaxed to the ground state, by repeating the experiment 10^4 times. The time resolution of the measurment is 1 ns, and the measurment time is $5 - 15$ μs, depending on the experiment.

The data for each sequence is then given by a series of points where one can see a decay due to the energy relaxation. To determine the excited state population, each series is compared to a theoretical prediction [1]. Fig. 5 shows an example for such a measurement. The red line is the theoretically predicted measurement response for a qubit in the excited state, the blue line is the measured data. The maximum of the theoretical prediction is not at the beginning although the excited state population has its maximum at the beginning of the measurement period. This is due to the fact that the measurment beam starts not until the last $\pi/2$ pulse has finished. It needs some time then, until the photon number in the resonator (and therefore also the number of outcoming photons) has reached its maximum.

The results of such a Ramsey measurement are presented in Fig. 6. The power of the spectroscopy pulse was adjusted such that a 10 ns pulse corresponds to a π pulse and the frequency of the spectroscopy pulse was $\omega_{\mu w} = 3.704$ GHz. To extract the decay time, a exponentially damped sinusoid was fitted to the data which gives the value $T_2 \approx 650$ ns. The fitted frequency is the detuning frequency which gives $\Delta\omega/2\pi \approx 3.5$ MHz. This value for the T_2 time is similar to previous measurements on this qubit [1]

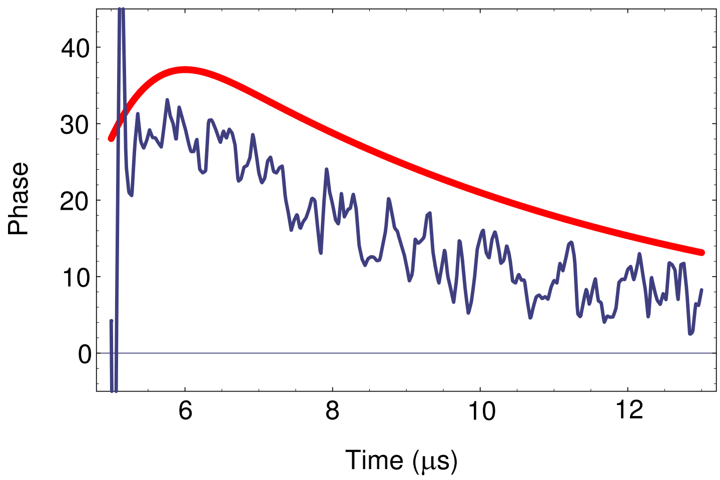


Figure 5: Red: Theoretical expected measurement response for a π pulse. Blue: Example for experimental data. To get the population, the experimental data is normalized with the theoretical calculation.

and is one of the highest value for charge qubits measured so far.

Although the measurment gives a good value for the dephasing time, one can see that the contrast is not optimal. At zero pulse separation the population of the excited state should be one, and for long times the population should go to 0.5 (mixed state). In the experiment, the highest population is ≈ 0.7 and for long times it goes to ≈ 0.35. The reason for this low contrast is not yet clear. The finite detuning and pulse length prevent the qubit to go exactly to the excited state. But this cannot be the reason for the low visibility, because simulations with parameters similar to those in the experiment showed, that the visibility should still be larger than 99%. Another reason could be, that the π and $\pi/2$ pulses were not ideal, either too long or too short. Simulations showed, that if the pulses are too short (long), it can really look as if the contrast would be decreased, but for long times the population will go to less (more) than the half of the population at minimal pulse separation. For the visibility in the experiment, this would mean that the π pulses were more than 30% too short (or too less amplitude), which seems unrealistic due to the pulse calibration which were performed before each mesurement (see also appendix B). The fact that in the experiment the population decays to the half of the initial population could also mean that there is something wrong with the normalization with the theoretical prediction.

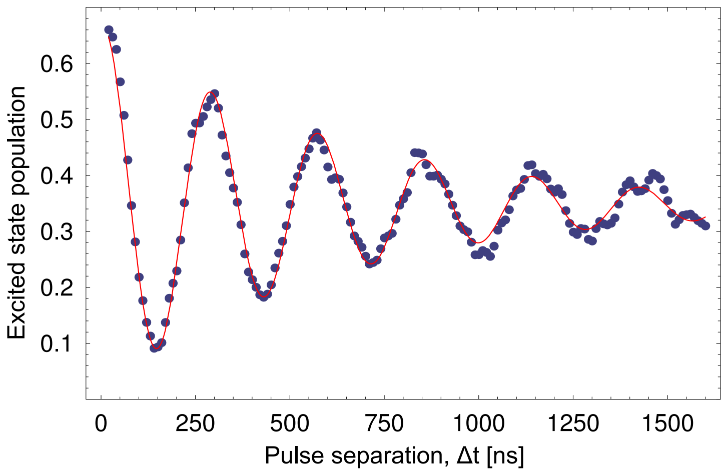


Figure 6: Ramsey experiment to obtain the T_2 time. Dots are experimental data, solid line is an exponentially damped sinusoid fitted to the experimental results. The T_2 time obtained by the fit gives a value of ≈ 650 ns

3 Spin echoes

The spin echo technique is known form NMR (nuclear magnetic resonance), where it is used to cancel an inhomogeneous broadening of the spin resonance lines, e.g. due to the spatial inhomogenity of the magnetic field. In this experiment, the spin echo technique is used to cancel the low frequency noise in the phase of a qubit state which is accumulated during the free precession in the equatorial plane of the Bloch sphere. This leads to a more intrinsic dephasing time which is larger than the T_2 inferred from the Ramsey fringes [9].

3.1 Principle of spin echoes

The spin echo sequence is a modified Ramsey sequence with an additional π pulse placed symmetrically between the two $\pi/2$ pulses. The random phases accumulated before and after the π pulse compensate exactly if the frequency does not change during the sequence. The additional π rotation can be a rotation around the x axis or around the y axis. In the first case, the qubit is expected to go always (ideally) to the ground state, in the latter one, the qubit will go to the excited state (cf. Fig. 7).

As calculations show [9], the spin echo method can increase the decay time if the noise is not white. So the observation of an improved decay time will also provide some information about the spectral properties of the noise.

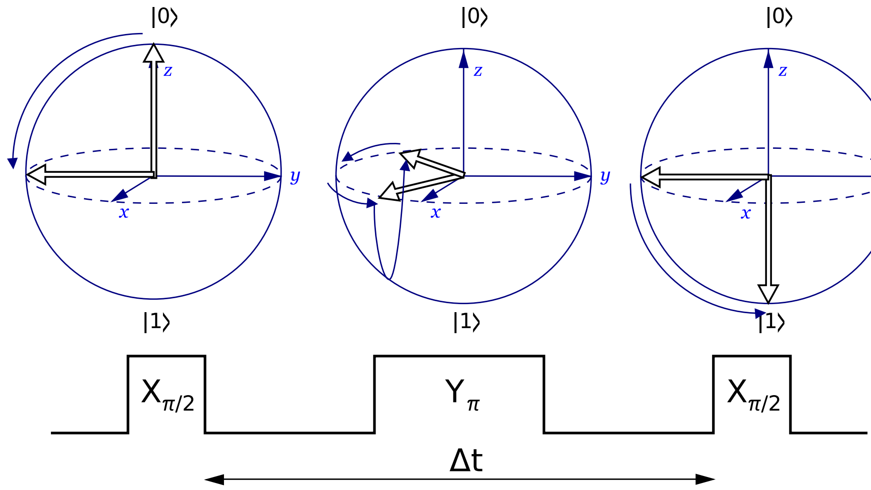


Figure 7: Top: Schematic representation of the spin echo technique: An extra π pulse around the y axis between the two $\pi/2$ pulses brings the qubit in a symmetric position with respect to the y axis. During the second free precession, the qubit can refocus. Bottom: Pulse scheme of the spin echo sequence.

3.2 Simulation

The simulation is done with the same code as for the Ramsey experiment just with additional exponential operators for the extra pulse and free precession time. In this simulation we can not see any improvement of the decay time. But this is clear, because the simulation only assumes white noise that can be characterized by a simple exponential rate. The spin echo technique is only effective to cancel low frequency non-white noise.

3.3 Measurements

If one uses the pattern described in figure 7, i.e. where the π pulse is a rotation around the y axis and the two $\pi/2$ pulses are a rotation around the x axis, one can observe directly the decay by measuring the qubit state just after the last $\pi/2$ pulse as a function of the time between the two Ramsey pulses. Fig. 8 shows the result of such a measurement. An exponential fit to this data gives a spin echo decay time $T_{2,E} \approx 1.9\ \mu$s. Repeated experiments all show values of $T_{2,E} \approx 1.4 - 1.9\ \mu$s. This result shows, that it is possible to enhance the dephasing time with an additional π pulse up to a factor of 3. Compared to other superconducting charge qubits (e.g. in [9] they measured $T_2 = 300$ ns and $T_{2,E} = 550$ ns) these are probably the longest dephasing times measured so far. In other types of superconducting qubit, such as flux qubits, spin echo decay times up to 4 μs have been measured [12].

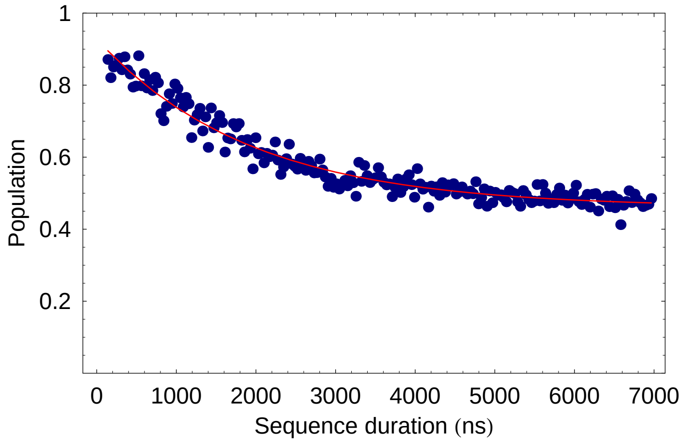


Figure 8: Spin echo experiment, where the π pulse is around the y axis, such that a direct plot of the decay is yielded. The dots are the experimental data, the solid line is an expontential fit. The T_2 time obtained by the fit gives a value of $\approx 1.9\ \mu$s

4 Multiple π pulses

4.1 Two π pulses

Of course, one can also apply more than one π pulse between the two Ramsey pulses. As calculations show [6], more π pulses can echo away noise at higher frequencies. If one wants to apply two pulses, there are two different ways to do this. To obtain a plot where it is possible to directly read out the decay time (similar to the case with just one pulse), the time between the two π pulses has to be twice the time between a $\pi/2$ and a π pulse. If one chooses equally spaced pulses, one obtains fringes at about a third of the detuning frequency. Fig. 9 shows these two different pulse sequences and simulations for the expected measurements. In the left simulation, one can see some small oscillations. These are due to the fact that the qubit precesses also during the application of the pulse, that means that a higher detuning or longer pulse times (slower angular frequency around the x, y axis) increase such additional oscillations.

Due to the very low frequency 1/f noise, the qubit transition frequency is not stable and can vary from one experiment to another. To be able to compare the case with one π pulse and the case with two π pulses, one has to ensure, that one measures in both cases the same qubit. To do that, a pattern file was used, in which alternately a sequence with one and two π pulses was applied. The results of such a measurement can be found in Fig. 10. In almost all of the experiments performed in that way, the $T_{2,E}$ time was typically $10-30\%$ bigger when one applied two pulses than when one applied just one. This shows that another π pulse will cancel out more

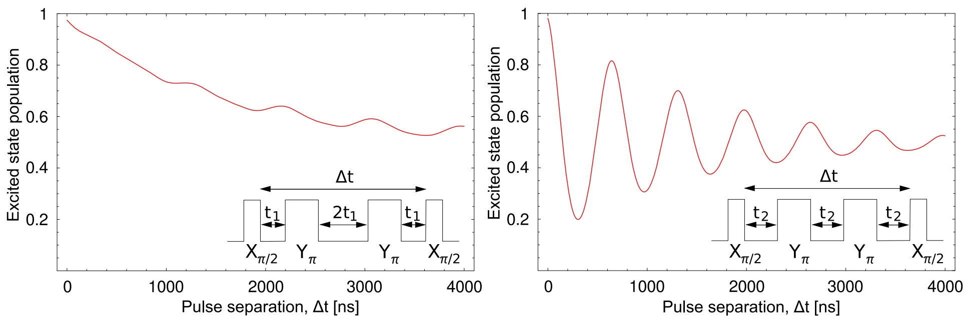


Figure 9: Pulse sequences and expected measurement outcomes with two π pulses with a length of 10 ns at a detuning of 4.5 MHz.

noise than just one pulse (i.e. noise at higher frequencies), but the additional gain is much smaller than that from the first pulse.

Here again, one can see a very low visibility, even lower than in the Ramsey experiments. But here one can definitely exclude the too short or too long pulses as reason, see appendix B.

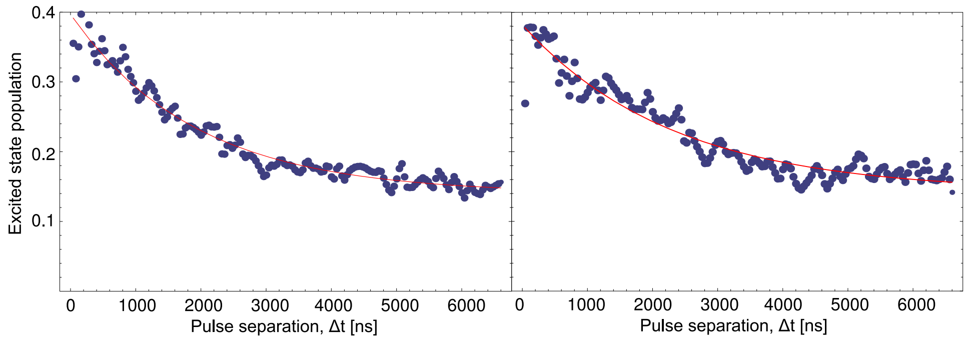


Figure 10: Spin echo experiment with one (left) and two (right) π pulses around the y axis. The dots are the experimental data, the solid lines are expontential fits, which gives decay times $T_{2,E} \approx 1.8$ μs (left) and $T_{2,E} \approx 2.1$ μs (right).

In the next experiment, the pattern with two π pulses which generates a sort of Ramsey fringes (see above) was used. As expected, the frequency is about a third of the normal detuning frequency measured in a Ramsey experiment, and the dephasing time $T_{2,E}$ is similar to the one measured in other spin echo experiments (c.f. Fig. 11).

4.2 Many π pulses

To check whether it is useful to apply even more pulses, lets have a look at the simulations. Due to the finite pulse length, the qubit precesses also

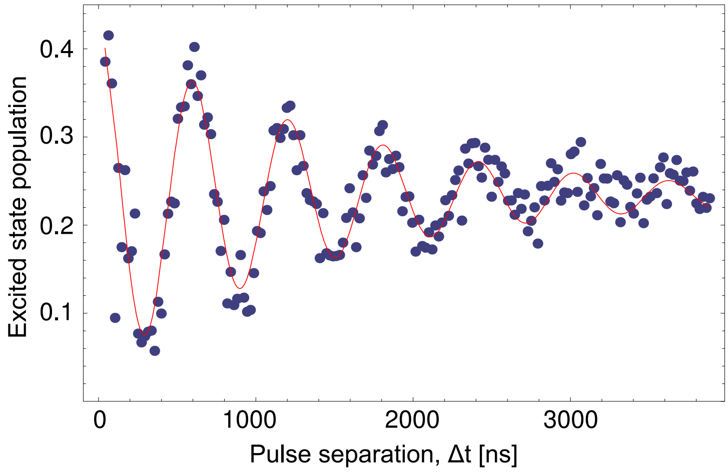


Figure 11: Spin echo experiment with two π pulses with equal time of free evolution between all the pulses. One obtains a sort of Ramsey fringes (c.f. Fig. 9). Dots are experimental data, solid line is an exponentially damped sinusoid fitted to the experimental results. The $T_{2,E}$ time obtained by the fit gives a value of ≈ 1.5 μs.

during the application of the pulse. This means, that the more pulses one applies, the more sensitive the measurement reacts to the detuning. This leads to more and more extra-fringes, the more pulses one applies. Because it is difficult to do measurements at the exact resonnance frequency, it gets more difficult to get useful results by applying more pulses. The simulation for a series of three pulses of a length of 10 ns with a detuning of 5 MHz is shown on the left side of Fig. 12. As one can see, there are now a lot of extra fringes and its difficult to fit an adequate curve. In the measurement which is shown on the left side of Fig. 12, one can see the characteristic patterns which appear in the simulation.

5 Moving π pulse

Another experiment which can be performed is the "moving π pulse" experiment. In this experiment, the pulse separation Δt is left constant while the position of the π pulse is varied between the two $\pi/2$ pulses. In this experiment one expects to see oscillations at time scales much larger than the decay time of the Ramsey fringes [11].

The reason for performing such an experiment is to ensure that the π pulse is applied exactly in the middle between the two $\pi/2$ pulses. If one would not have such precise waveform generators, one could although get a spin echo plot to infer the decay time $T_{2,E}$ by repeating this experiment for many different pulse separations Δt, where each experiment would con-

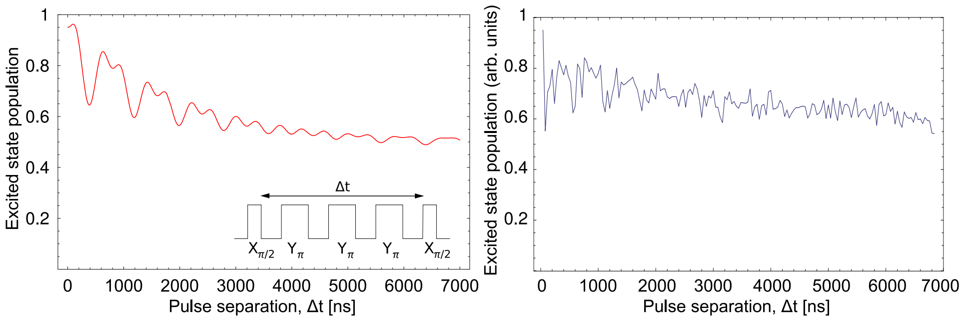


Figure 12: Experiment with three π pulses. Left: Simulation for a detuning of 5 MHz. Right: Experimental data. The solid line connects the data points such that one can see the analogy to the simulation. An exponential fit to this data gives no reasonable information.

tribute just one data point (the maximum near the middle) to the final spin echo plot.

In a Ramsey experiment, the signal varies as $\cos(\Delta\omega\Delta t)$, where in this experiment, the signal varies as $\cos\left(\Delta\omega(t_2 - t_1)\right) = \cos\left(\Delta\omega(\Delta t - \tau - 2t_1)\right)$. So by plotting the signal against the time t_1, one expects to observe an oscillation with twice the frequency of the Ramsey fringes. When $t_1 = t_2$, i.e. when the π pulse is exactly in the middle of the sequence, one expects a maximum.

The result of a "moving π pulse" experiment can be seen in Fig. 13 for two different pulse separations Δt. It shows on the one hand, that if the π pulse is not exactly in the middle one loses population contrast, on the other hand it shows that the waveform generators used in this experiment are really precise because the maximum in each plot is exactly in the middle.

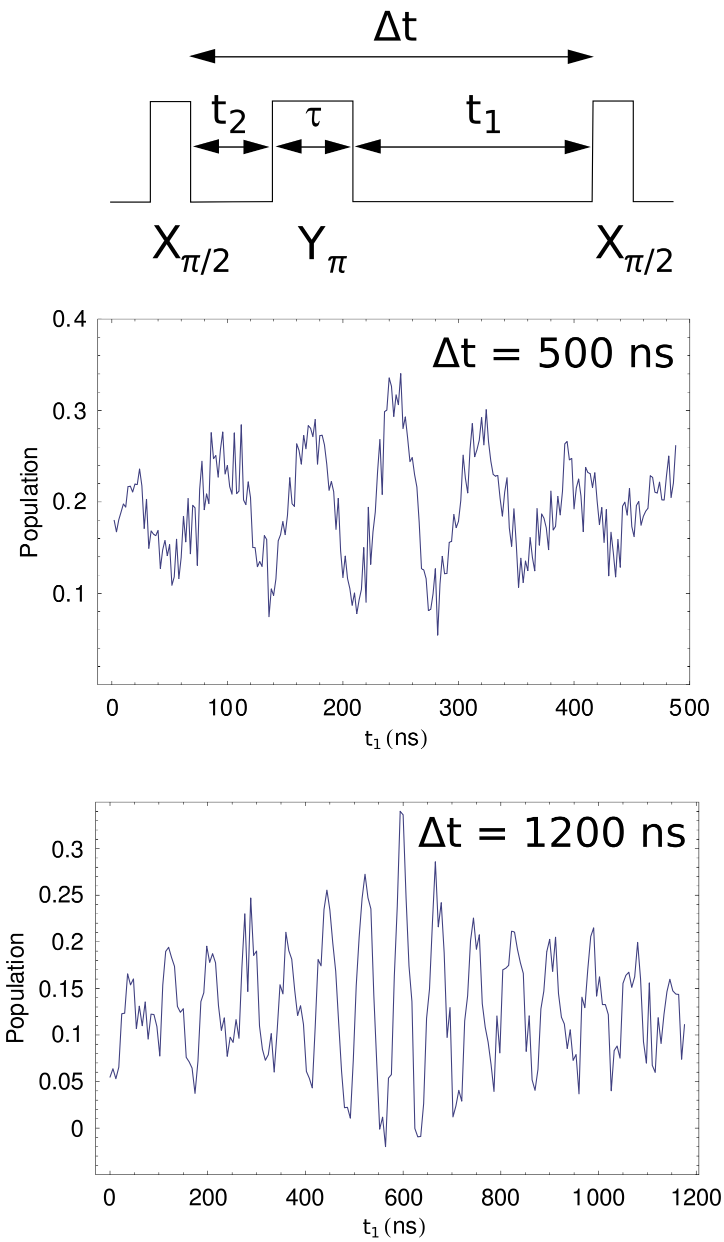


Figure 13: Top: Pulse sequence used in this experiment. Below: Experimental data for a pulse separation Δt of 500 ns (middle) and 1200 ns (bottom).

6 Conclusion

The dephasing time inferred from the decay of Ramsey fringes is $T_2 \approx 500 - 650$ ns. The experiments showed, that it is possible to enhance the dephasing time by applying additional π pulses. With a single π pulse the decay time could be enhanced to a value of $T_{2,E} \approx 1.4 - 1.9$ μs. Compared to other experiments (e.g. in reference [9] they measured $T_2 = 300$ ns and $T_{2,E} = 550$ ns) these are probably the longest dephasing times measured so far in a charge qubit.

Although the measurements give nice values for the dephasing time, there is sometimes a visibility of just $\approx 40\%$. The reason for this very low visibility is not clear yet. Some reasons could already be excluded. E.g. the finite detuning and the finite pulse length prevent the qubit to go exactly to the excited state, but simulations show that nevertheless the visibility should be larger than 99% for parameters similar to those in the experiments. Also the non-ideal pulses cannot have such a large influence, see also appendix B.

Maybe the reason for this low visibility is related to the pulesd measurement which uses high measurement amplitudes (in contrast to a weak continous measurement, cf. also [1]).

Another problem that arose was that the transition frequency varied slightly, such that it was very difficult to set the detuning to zero. But at nonzero detuning the simulations and experiments show that more and more π pulses lead to no useful results.

The main task for the future is to understand the reasons of the low visibility and find ways to enhance it.

A Mathematica Notebook for the simulation

The following code is an extract of the Mathematica notebook written by Alexandre Blais (with some small extentsions by myself) to simulate the Ramsey- and spin echo experiments. At first, the initialisation and definition of the functions are shown and afterwards, an example of the application is given [7].

Including T1 and T2

We will need the following definitions:

Standard SU(2) operations:

```
σx = {{0, 1}, {1, 0}};
σy = {{0, −i}, {i, 0}};
σz = {{1, 0}, {0, −1}};
σ+ = (σx + iσy)/2;
σ− = (σx − iσy)/2;
id = IdentityMatrix[2];

GHz = 10^9; ns = 10^−9; MHz = 10^6; us = 10^−6;

<< LinearAlgebra`MatrixManipulation`

(*
TensorProduct[a_?SquareMatrixQ, b_?SquareMatrixQ]:=
SparseArray[BlockMatrix[Outer[Times, a, b]]]

ident[dim_]:=SparseArray[{{i_, i_}->1}, {dim, dim}]
*)

TensorProduct[A_, B_]:= (* For square matrices only *)
Module[{temp, dima, dimb, i, j, r, s},
(
        Clear[temp, i, j];
        dima = Length[A[[1]]];
        dimb = Length[B[[1]]];
        temp = IdentityMatrix[dima ∗ dimb];
        For[i = 1, i<=dima, i++, (
            For[j = 1, j<=dima, j++,
                For[r = 1, r<=dimb, r++, (
                    For[s = 1, s<=dimb, s++,
temp[[r + dimb ∗ (i − 1), s + dimb ∗ (j − 1)]] = A[[i, j]] ∗ B[[r, s]]];
                )]; (*endfor r*)
            ]; (*endfor j*)
```

```
            )]; (*endfor i*)
      temp)]

ident[dim_]:=IdentityMatrix[dim];

spre[A_, dim_]:=TensorProduct[ident[dim], A]

spost[A_, dim_]:=TensorProduct[Transpose[A], ident[dim]]

flat[A_]:=Flatten[Transpose[A]]

unflat[a_, dim_]:=Transpose[Partition[a, dim]]
```

Here is the usual Hamiltonian:

$$H = \frac{\Delta}{2}\sigma z + \frac{\Omega x}{2}\sigma x + \frac{\Omega y}{2}\sigma y;$$

This is the master equation $\partial_t \rho = \mathrm{L}\,\rho$ where L is the Liouvillian. The operator L multiplies the density matrix ρ from the left and the right. It is more useful to rewrite the master equation in such a way that the Liouvillian only operates of the left of ρ. This is done at the cost enlarging the size of the matrices we need to deal with. The big advantage however is that we can do all the calculations using basic linear algebra. Here is the Liouvillian in this unusual form:

$$L = -i(\mathrm{spre}[H,2] - \mathrm{spost}[H,2]) +$$
$$\gamma 1\,(\mathrm{spre}\,[\sigma_-,2]\,.\mathrm{spost}\,[\sigma_+,2] - \mathrm{spre}\,[\sigma_+.\sigma_-,2]/\,2 - \mathrm{spost}\,[\sigma_+.\sigma_-,2]/\,2) +$$
$$\gamma\phi(\mathrm{spre}[\sigma z,2].\mathrm{spost}[\sigma z,2] - \mathrm{spre}[\sigma z.\sigma z,2]/2 - \mathrm{spost}[\sigma z.\sigma z,2]/2)/2;$$

Let's now move to the Ramsey sequence. Without dissipation, we calculated the time-evolved state of the system by exponentiating the Hamiltonian. In the presence of dissipation, we will simply need to exponentiate the Liouvillian, there are no other changes. This is the advantage of using this funny representation of L.

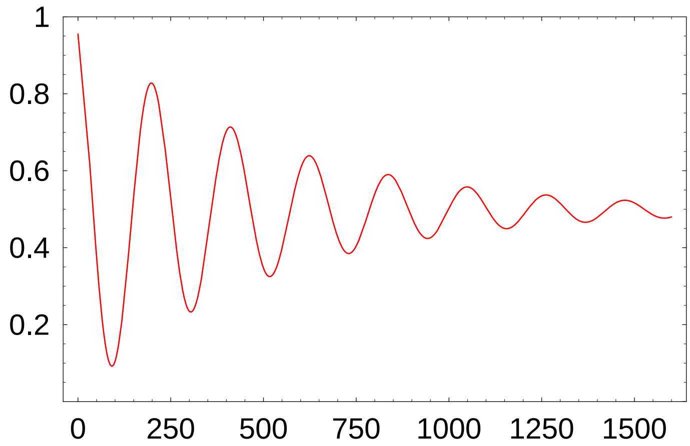
Note that we go from the regular to the funny representation of L by using the commands flat and unflat defined above. No need to worry about these. Just use them as I do below.

Start in the ground state:

$$\rho 0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix};$$

Building the Ramsey sequence:

```
(* Dephasing and relaxation always present *)

γ1 = 1/(5000ns);
γϕ =  1/(500ns) − 1/(2 ∗ 5000ns);

(* The detuning *)

Δ = 4.7 MHz ∗ 2π;

(* The π/2 pulses takes a time tπ *)

Ωx = π/(2 ∗ 10ns);
Ωy = 0;
tπ = π/(2Ωx);
U1 = Chop[N[MatrixExp[L ∗ tπ]]];

(* Wait for some time tz *)

Ωx = 0;
U2[tz_]:=Chop[N[MatrixExp[L ∗ tz]]];

(* Calculating the qubit population *)

plot1 = Plot[Chop[(1 + Tr[σz.unflat[Chop[U1.U2[tzns].U1.flat[ρ0]], 2]])/2],
{tz, 0, 1600}, PlotRange → {0, 1}, PlotStyle → Hue[0], Frame → True]

Clear[Δ, Ωx, Ωy, tπ, tz, U1, U2, γ1, γϕ]
```



B Simulations with non-ideal π pulses

B.1 Ramsey experiments

The simulation for the measurement outcome of a Ramsey experiment with a detuning of 4.7 MHz but with 40% too short and 40% too long π and $\pi/2$ pulses is shown in Fig. 14. One can see that for short times it seems just like a normal Ramsey experiment but with lower contrast. But for long times one can see that the population goes below (above) the half of the population at the beginning for pulses shorter (longer) than the ideal ones.

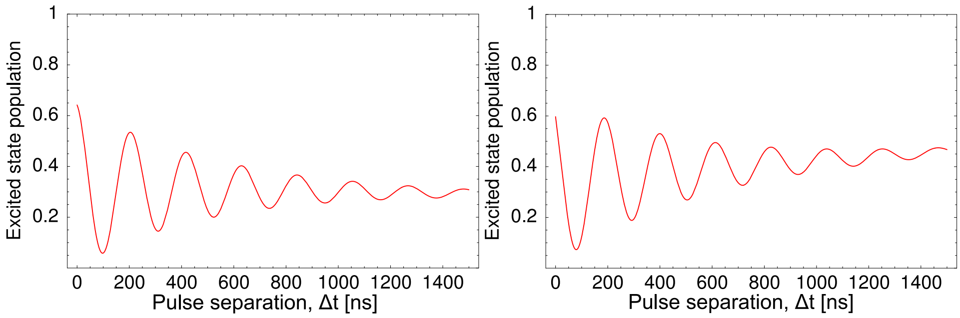


Figure 14: Left: Simulation of a Ramsey experiment with 40% too short π and $\pi/2$ pulses. Right: Same simulation with 40% too long π and $\pi/2$ pulses.

B.2 Spin echo experiment

The simulations for a spin echo experiment for the same conditions as above are shown in Fig. 15. The effect of the too short or too long pulses is completely different than in the case for the Ramsey experiment. Especially the effect that one has the impression of measuring an ideal experiment but with lower contrast cannot be seen in the spin echo experiment. This shows that there must be another reason for the low contrast in the real experiments.

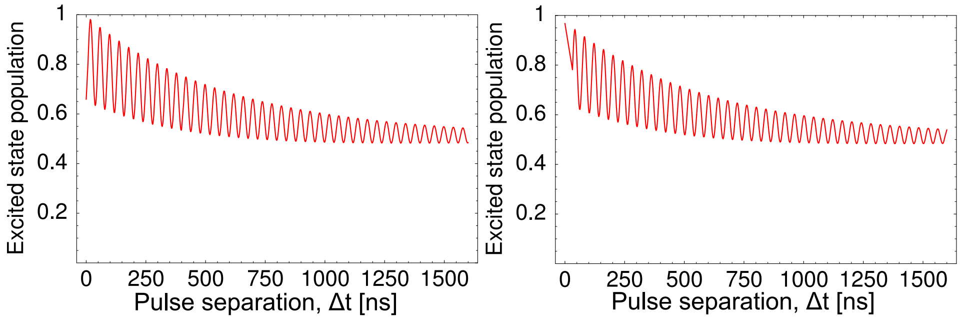


Figure 15: Left: Simulation of a spin echo experiment with 40% too short π and $\pi/2$ pulses. Right: Same simulation with 40% too long π and $\pi/2$ pulses.

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