ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Characterization of Lumped-Element Resonators with Periodic Boundary Conditions

Semester thesis

Kyounghun Oh

February 29th, 2016

Supervisor: Dr. Anton Potočnik
Group Leader: Prof. Dr. Andreas Wallraff
Group: Quantum Device Lab, ETH Zürich

Characterization of Lumped-Element Resonators with Periodic Boundary Conditions

Kyounghun Oh
Dept. of Physics, ETH Zurich, Switzerland
(Dated: 18 Feburary 2016)

In this thesis we measured the scattering matrix data of coupled lumped-element resonators with periodic boundary condition by performing dipstick measurement. We fitted scattering matrix data with Mathematica model function to understand how each capacitances of circuit capacitors varies as a function of design parameters. We found the recipe to fabricate capacitor with certain value of capacitance, which is very important while designing any kind of quantum simulation experiment using superconducting circuits. Furthermore, we observed asymmetry of shunt capacitances. The reason of asymmetry is presence of a transmission line and geometry difference among shunt capacitors. In conclusion, we understood how capacitor design parameters affect the capacitance. This is important because we want to control design parameters to fabricate a specific circuit that can be used as quantum simulators.

ACKNOWLEDGEMENT

As an exchange student from other university, performing physics experiment at ETH and writing thesis was extremely valuable experience. I would like to thank Prof. Andreas Wallraff who gave me the chance to work in Quntum Device Labaratory. Also, I would like to thank Dr. Anton Potocnik, supervisor of my project at ETH. He taught me procedure of dipstick measurement and gave me valuable feedback regarding data analysis. I also want to thank Mr. Samuel Gyger. He gave me lot of advice about model function fitting.

INTRODUCTION

Quantum mechanics is a mathematical framework for the construction of physical theories that describes a microscopic scale world. The rules of quantum mechanics are simple but quite counter-intuitive since we live in macroscopic world. Scietists of past centuries studied how to apply quantum mechanical effects to real microscopic world. The scientists are trying to take advantage of characteristics of quantum mechanics such as entanglement and superposition in order to design quantum computers can solve much complicated problems compare to classical computers. This domain of knowledge is called quantum information theory [1].

Simulating quantum mechanics of large system is a difficult task. Instead, simulating partial quantum mechanical system is less difficult [4]. Thus, recently physicists are designing simple quantum simulators with lumped element resonators consist of circuit elements such as capacitors and nonlinear inductors [2]. Understanding how design parameters of the lumped elements affects its physical quantity precisely is important task to build quantum devices. In this thesis we will focus on characterizing capacitors, especially capacitors of circuits with periodic boundary condition. Previously, the research of characterizing linear array resonators is already done [2]. In this thesis, we will measure the capacitances of capacitors following similar way. We will perform dipstick measurement to obtain scattering matrix elements and analyze the results with theory [3].

THEORY

Lumped-element Resonators

In our experiment, we used circular circuit of lumped-element resonators. The lumped-element resonators we have used consist of resistor with resistance R, and inductor with inductance L, and capacitor with capacitance C. Each resonator contains mentioned three components which are connected in parallel [6]. In our circuit resonators are connected leading to a circular circuit structure. Resonators are coupled with coupling capacitances. These capacitors are called coupling capacitor and denoted by C_J while capacitor in each resonators are called shunt capacitor. Furthermore, we used transmission lines to measure reflection and transmission coefficients of the structure. Each transmission lines contains coupling capacitor which is denoted by C_κ. The impedence of transmission line is Z_C. Fig. 1 shows the circuit representation of our system with four resonators. This circuit is called quatromer. In general, the circuit may contain N resonators. Since there are a number of resonators in our circuit, we denoted each shunt capacitor as C_i. i is an integer from 1 to N counting resonators in counterclockwise manner. Each inductors and resistors are also denoted as L_i and C_i respectively with lowercase index i. By our construction the circuit also contains N coupling capacitors. We denoted each coupling capacitor between ith capacitor and $i+1$th capacitor as C_{Ji}. The coupling capacitor between Nth capacitor and 1st capacitor would be C_{JN}. The coupling capacitors of transmission lines would be denoted in the same way $C_{\kappa i}$.

Lagrangian and Hamiltonian Formalism

We can employ circuit quantumelectrodynamics to analyze lumped-element circuits with several resonators. Let us assume there are N resonators and coupling capacitors in circuit without any transmission lines. Then Lagrangian of the system would be given as

$$L = \frac{1}{2}\dot{\Phi}^T C\dot{\Phi} - \frac{1}{2}\Phi^T L^{-1}\Phi \tag{1}$$

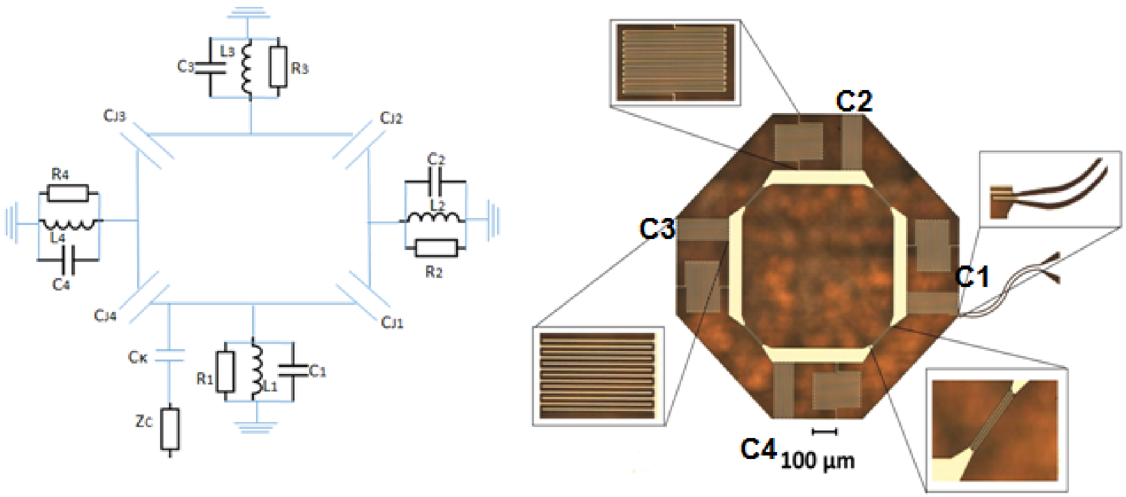

FIG. 1. Right picture is a optical micrograph of circular quatromer and left picture is its circuit diagram.

and Hamiltonian of the system would be given as

$$H = \frac{1}{2}\dot{Q}^T C^{-1}\dot{Q} + \frac{1}{2}\Phi^T L^{-1}\Phi \tag{2}$$

where Φ is a vector of flux nodes, L is a square diagonal matrix of inductance, C is a square matrix of shunt capacitors and coupling capacitors. Q is a vector of charges [7]. We can evaluate Hamiltonian of the system with canonical commutation relation $[\hat{q}_i, \hat{\phi}_j] = -i\hbar\delta_{ij}$ and introducing creation and annihilation operators $\hat{a_i}^\dagger$ and $\hat{a}_i$. Then our Hamiltonian becomes

$$H = \sum_{i=1}^{N} \hbar\omega_i \hat{a_i}^\dagger \hat{a}_i + \sum_{<i,j>} \hbar J_{ij}(\hat{a_i}^\dagger \hat{a}_j + \hat{a}_i \hat{a_j}^\dagger) \tag{3}$$

form where $\omega_i = \sqrt{(C^{-1})_{ii}(L^{-1})_{ii}}$ and $J_{ij} = \frac{(C^{-1})_{ij}\sqrt{(\omega)_i(\omega)_j}}{2\sqrt{(C^{-1})_{ii}(C^{-1})_{jj}}}$. Furthermore, if we apply input-output theory, we can theoretically derive scattering matrix parameters [8].

Degeneracies and Dark Modes

If one measure the reflection coefficients of circular quatromer at extremely low temperature, one would expect to see four resonance frequencies because there are four resonators in the system circuit. One can simply calculate resonance frequencies by plotting absolute value of reflection coefficients as a function of frequency. Absolute value of reflection coefficients are about one, but at resonance frequency, this value becomes smaller. Interestingly we observe only three resonances. For instance, if one sets all four values of shunt capacitances, inductances, coupling capacitances and resistances to be the same, one would observe only three resonances.

This can be seen either by theoretical calculation or by performing a dipstick measurement. This phenomenon occurs because of degeneracy of two modes. One can calculate eigenstates and eigenvalues of circular quatromer system by solving eigenvalue problem of Hamiltonian. Since a matrix representation of Hamiltonian is 4 by 4 matrix, there exist 4 eigenvalues and eigenvectors. For aforementioned perfectly symmetric system, two eigenvalues are the same.

However, what have been described above is not the only case with three resonances. By theoretical calculation, one can find out if only diagonally pairwise symmetry is kept, a disappearence of one resonance can be still observed. Let us assume a transmission line is connected to C_1 capacitor only and all inductance L, resistance R and coupling capacitance C_{Ji} are same. Then diagonally pairwise symmetery means $C_1 = C_3$ and $C_2 = C_4$. Since the symmetry is broken, there are no degenerate eigenvalues. However one resonance would still be missing in the reflection type measurement. We call this mode a dark mode. The reason why we can not observe dark mode can be explained by calculating eigenstates of Hamiltonian. When there exists a diagonal symmetry, dark mode has zero current amplitude at L, C resonator which is coupled to transmission line [9]. To be more specific, we do not need diagonally pairwise symmetry to obtain a dark mode. What we need is merely $C_2 = C_4$ conditon where transmission line connected to C_1. Value of C_1 and C_3 will not affect required folding symmetry. That is why we can still observe dark mode in the presence of transmission line. The presence of transmission line changes local geometry which brakes the symmetry..

EXPERIMENT

Dipstick Measurement

We performed dipstick measurement in order to measure the scattering matrix parameters. We placed the sample to the bottom of the sample carrying stick at room temperature. This stick is connected to the VNA, vector network analyzer. VNA device measures the scattering matrix parameters. We performed measurement at liquid helium temperture where the sample is superconducting. To achieve liquid helium temperature sample is submerged to liquid helium dewar.

First of all, we need to calibrate the VNA. Calibration procedure was done using Rosenberg calibration kit. This calibration kit contains four componenets, short, load, open and through connectors.

Once the sample and cables are connected to the stick, we opened the helium dewar and slowly push the dipstick into the dewar. We had to move the stick very slowly to maintain maximal pressure inside the helium dewar. While pushing the stick we carefully observed the VNA monitor. When superconductivity occurs, we submerged the stick deeper to place our sample in the center of the dewar. Then we saved the data of scattering matrix parameters. After saving, we pulled the stick out. When superconductivity vanishes, we save the data of scattering matrix parameters again to correct the measurements later.

Model Function Fitting

We analyzed our experiment data with Mathematica program. We load the data of the scattering matrix elements. Then we use the model function that we programmed using ABCD matrix formalism. We analyzed each lumped element with ABCD matrix. We converted parallel ABCD matrices to Y matrices and added them, then reconverted Y matrix to ABCD matrix to obtain total ABCD matrix that describes whole circular quatromer circuit. For example, we can calculate total ABCD matrix of parallel connected circuit with following equation [10].

$$M_{1||2} = M(Y(M_1) + Y(M_2)) \tag{4}$$

where M_1 and M_2 are ABCD matrices of each circuit parts, and two circuit parts are connected parallelly. Y function converts ABCD matrix to Y matrix and M function converts Y matrix to ABCD matrix. On the other hand, we can calculate total ABCD matrix of series connected circuit by simply multiplying two ABCD matrices.

We double checked the validity of the model function with AWR circuit simulator. In order to extract capacitances of coupling capacitors and shunt capacitors, our goal was to find out best combination of capacitors that generates the same scattering matrix parameter function as experiment results. We assumed all inductances are fixed to the same value. First, we tried to find the proper set of capacitances by trial and error method. At this stage we tried to make our model function graph and experiment graph to be similar as possible by changing capacitance parameter by hand. Then we used "NonLinearFit" Mathematica function to extract the capacitances by minimizing the error between the model function result and the experiment result with least square method. As shown in Fig. 2, blue dots are experiment data and a red line is Mathematica fitting function. Experiment data and model function are in good agreement.

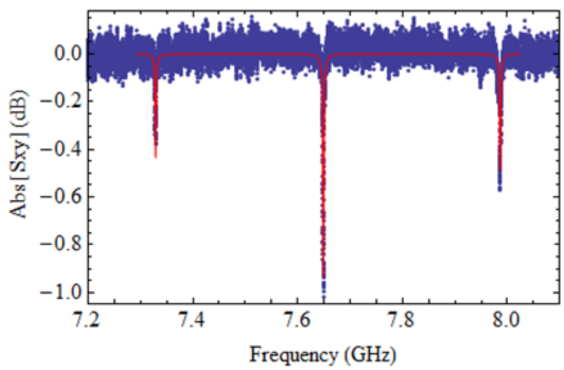

FIG. 2. A graph of a scattering matrix parameter (reflection coefficient). Blue dots are experimental data and a red line is the fitting function.

RESULTS AND ANALYSIS

CQJ

A CQJ is a sample with 6 cirular quatromers with all the same design parameters except for the coupling capacitance C_J. We varied the number of fingers of coupling capacitance from 1 to 9. We measured the reflection coefficient via a transmission line. We successfully obtained capacitances of shunt capacitors and coupling capacitances with mathematica model function method. We found out there exists folding symmetry of shunt capacitors since we observe only three resonances instead of four. This indicates $C_2 = C_4$ even without fitting. And from the fitting result, we also observed that $C_1 \neq C_3$. For example, the results of one circular quatromer shunt capacitance extraction are shown in Table I for 19 fingers capacitor. Note that all capacitors were designed identical. There are two reasons for this asymmetry of experimental result.

TABLE I. Design parameter extraction result of CQ.

C_1	230.7 fF
C_2	234.5 fF
C_3	232.1 fF
C_4	234.5 fF
C_J	10.9 fF
L	1.7 nH
R	131.6 kΩ

There are two kinds of shunt capacitors in circular quatromer. The fingers of C_1 and C_3

are alligned in vertical direction. So we will call this capacitors as vertical capacitors. C_2 and C_4 are horizontal capacitors as shown in Fig. 1. We zoomed in each capacitors and found out finger-gap ratio, the ratio of width of capacitor finger and capacitor gap, of the vertical capacitors and horizontal capacitors are different. This is not an intentional effect but origined from a problem at a fabrication step. With the theoretical capacitance formula using elliptical integral, we could calculate how the finger-gap ratio affects capacitace. We found out there are 5% difference, which is about 12 fF. Here I used the former experimental result of shunt capacitor with linear array of lumped elements which is $C = 55 + 10\times$(the number of fingers) (fF). Thus expected value of each capacitors is about 245 fF, which gives similar value as fitting results. The second reason is a presence of transmission line. Since transmission line is connected to C_1 only, it may affected the capacitance of C_1. We confirmed this fact with Maxwell program simulation. Shunt capacitance with no transmission line was 238.6 fF and shunt capacitance with transmission line with ratio 0.2 was 220.7 fF. The ratio 0.2 means the length ratio of the finger of coupling capacitor C_κ to one finger of shunt capacitor is 0.2. These are two reasons why $C_1 \neq C_3$. However, there is still folding symmetry ($C_2 = C_4$) so we can still observe dark mode.

We also extracted coupling capacitance C_J as a function of the number of fingers of coupling capacitor. The formula is $C_J = 2.9+4.4\times$(the number of fingers-1) (fF). The previous formula obtained from linear array of lumped elements was $C = 4.6 + 4.7\times$(the number of fingers-1) (fF) [2]. They are in agreement.

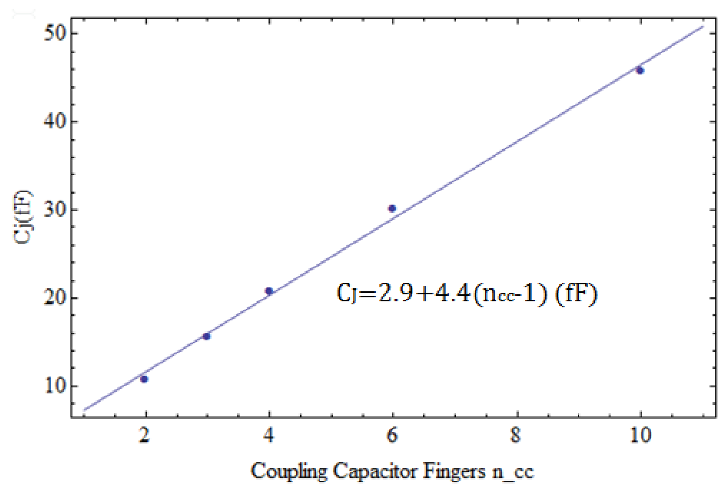

FIG. 3. Coupling capacitance C_J as a function of finger number. Blue line is a linear fit to the data which is $C_J = 2.9 + 4.4\times$(the number of fingers-1) (fF).

CQK

A CQK is a sample with six circular quatromers with all design parameters the same except for the coupling capacitance C_κ. We varied the coupling capacitance by changing the length of the coupling capacitor finger relative to a shunt capacitor finger length. We varied this ratio from 0.01 to 2 and with dipstick measurements we we observed a linear behavior of a coupling capacitor as a function of the relative length of C_κ shown in Fig. 4. The formula was $C_\kappa = 13.1\times$(ratio) (fF). Numerical simulation using Maxwell program give $C_\kappa = 12.8\times$(ratio) (fF) which is in fairly good agreement. For strongly coupled circuler quatromers such as ratio=0.75, 1, 2 circuits of CQK, we observed unexpected additional resonance peaks. For instance, we observed five resonances instead of three for the sample with ratio 2. We verified that this is not a measurement problem by repeating the measurements and adding wirebonds. Finally, we built the same sample again and repeated the measurement, however we still could not get rid of unexpected resonanances. Thus we conclude that it is an intrinsic problem of the sample itself. So we will have to solve this problem by investigating the circuits further with laser scanning microscopy.

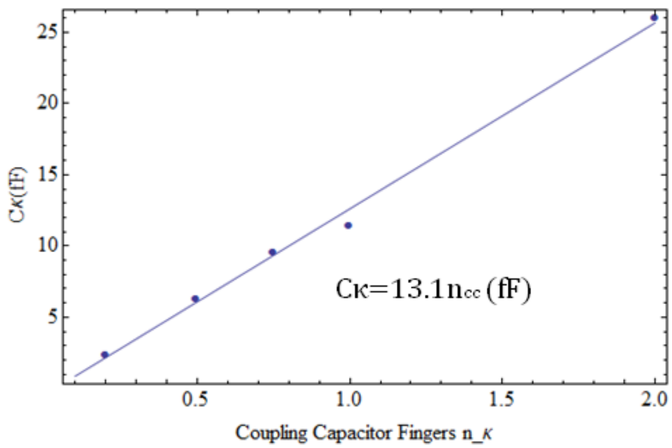

FIG. 4. Coupling capacitance C_κ as a function of finger number. Blue line is a linear fit to the data yielding $C_\kappa = 13.1\times$(the number of fingers) (fF).

CQC

A CQC is a sample with six circular quatromers with all the same design parameters except the shunt capacitance C_i (i=1, 2, 3, 4). We varied the shunt capacitance by changing the number of shunt capacitors fingers. Using dipstick measurement and fitting the data we obtained shunt capacitance as a function of the number of fingers of C_i. The linear fit to the data gives $C_1 = 46 + 9.3\times$(the number of fingers) (fF), $C_2 = C_4 = 62 + 8.7\times$(the number of fingers) (fF), $C_3 = 62 + 8.5\times$(the number of fingers) (fF). The formula of C_2 and C_3 were very similar and a capacitance of C_1 was slightly smaller than the capacitance of C_2, C_3, C_4, which indicates that presence of a tranmission line decreased the capacitance of C_1. The previous formula obtained from linear array of lumped elements was $C = 55 + 10\times$(the number of fingers) (fF). Since the order of standard error is about 10 fF, the previous exprememental results and new experimental results are in agreement.

CQL

A CQL is a sample of circular quatromer with four transmission lines connected to each resonators instead of one transmission line as was the case for previous samples. Sample figure is shown in Fig. 7. We obtained nine graphs of scattering matrix elements (four reflection coefficients and five transmission coefficients) as shown in Fig. 6. We found the combination of capacitances that fit all nine scattering parameters as a function of frequency. The result is shown in Table II.

We found out angular dependency of the coupling capacitor C_J also exists, so that capacitances of the up-right direction coupling capacitance (C_{J1}) and the up-left direction coupling capacitance (C_{J2}) were different. Also, we are surely confident that this extraction results are very accurate because these results explains all nine graphs. The result is unexpected because all four shunt capacitance should be the same in principle, or at least C_1 and C_3 should be same but there is no diagonally pairwise symmetry. All four shunt capacitances are different. A plausible explanation is related to assymmetry of transmission lines placement. As one can recognize from the figure, the geometry of transmissinon lines

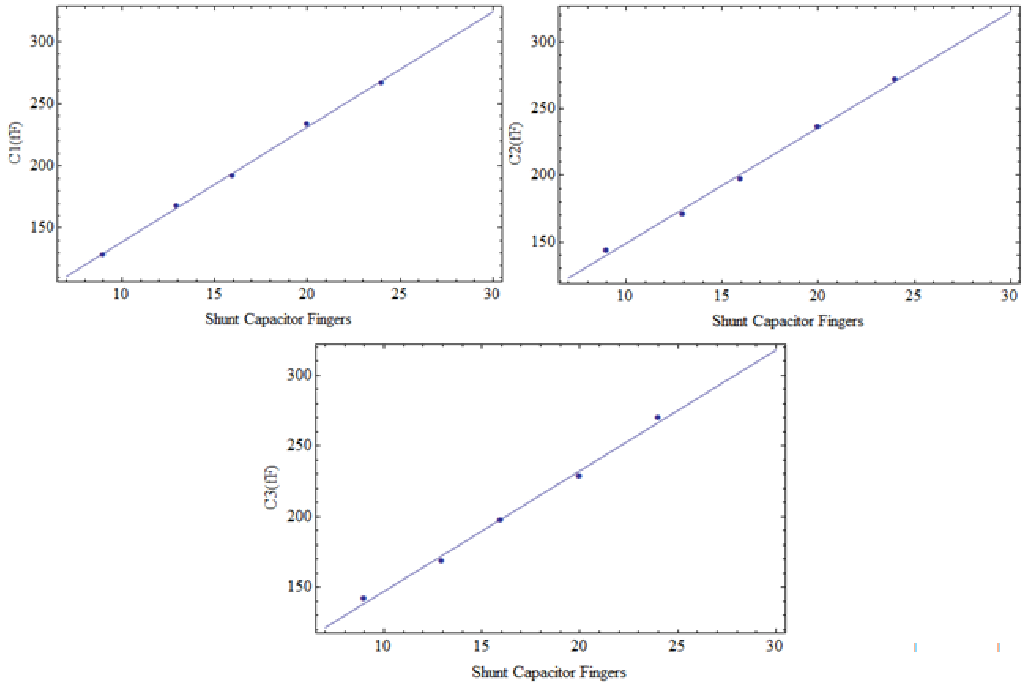

FIG. 5. Shunt capacitance C_i (i=1, 2, 3, 4) as a function of finger number. Blue line is a linear fit to the data.

TABLE II. Design parameter extraction result of CQL.

C_1	237.7 fF
C_2	248.0 fF
C_3	246.4 fF
C_4	217.5 fF
C_{J1}	13.4 fF
C_{J2}	16.0 fF
L	1.7 nH
R	131.6 kΩ

are assymmetric. Hence in order to get a good symmetry, we should design a new sample with exactly same shape and geometry of all four transmission lines.

CTL

A CTL is a sample of circular trimer with four transmission lines connected to each resonators instead of one transmission line. We obtained six graphs of scattering matrix elements (three reflection coefficients and three transmission coefficients) and found the combination of capacitances that could fit all six graphs. The result is shown in Table III.

TABLE III. Shunt capacitance and coupling capacitor extraction result of CTL.

C_1	229.1 fF
C_2	232.2 fF
C_3	232.2 fF
C_J	16.2 fF
L	1.7 nH
R	131.6 kΩ

We observe that all three shunt capacitances are again not the same, which is not a surprising

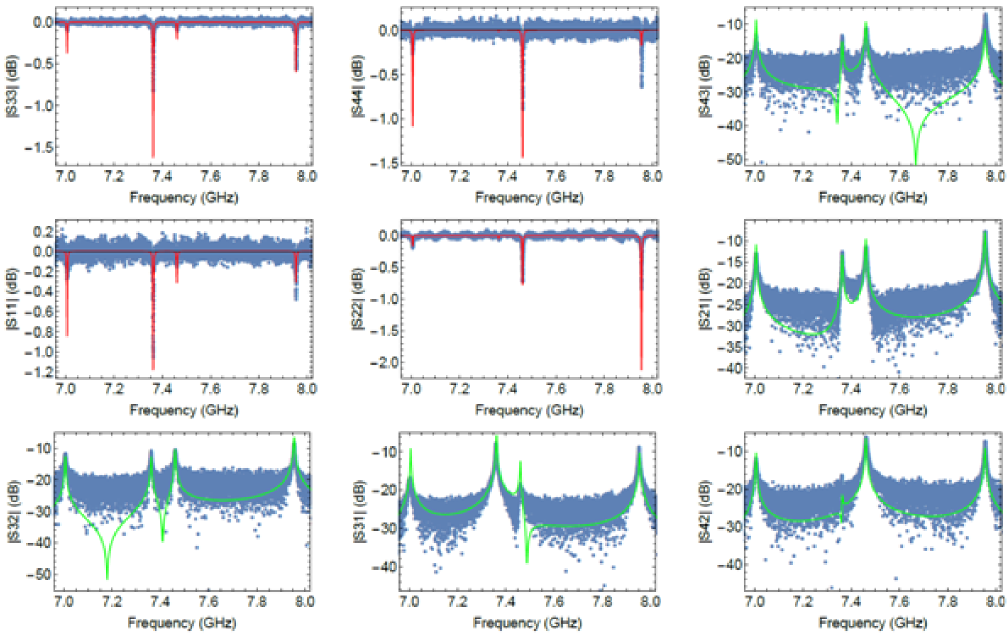

FIG. 6. Nine graphs of scattering matrix elements (four reflection coefficients and five transmission coefficients). Blue dots are experiment data, green and red lines are fitting function.

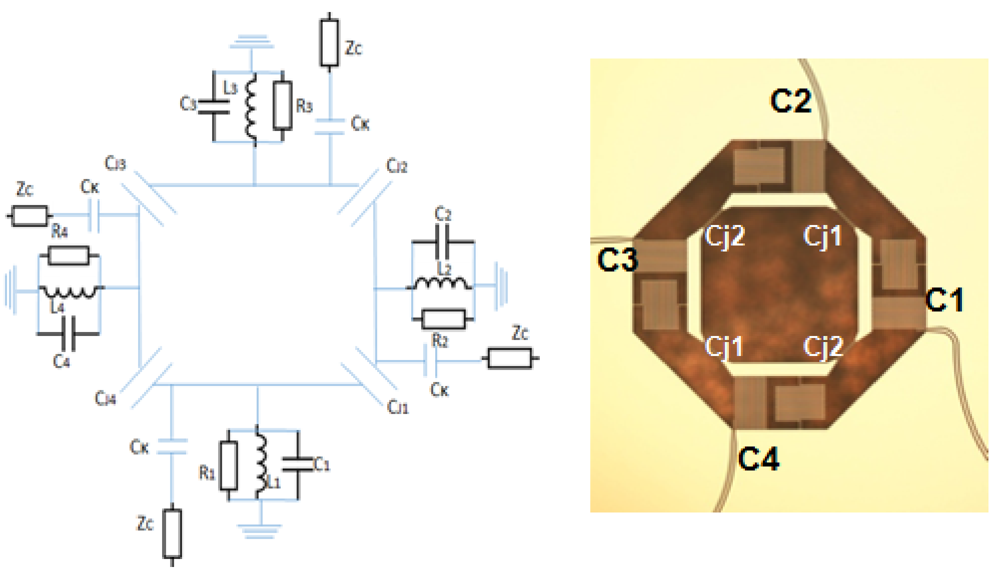

FIG. 7. Right picture is a optical micrograph of the CQL sample and left picture is its circuit diagram.

result because of different capacitor orientation and transmission lines geometries. However, we observed that C_2 and C_3 were same. This is because geometry of C_2 and C_3 are symmetric as shown in Fig. 8. Based on these measurements we can draw the same conclusions as for previous samples.

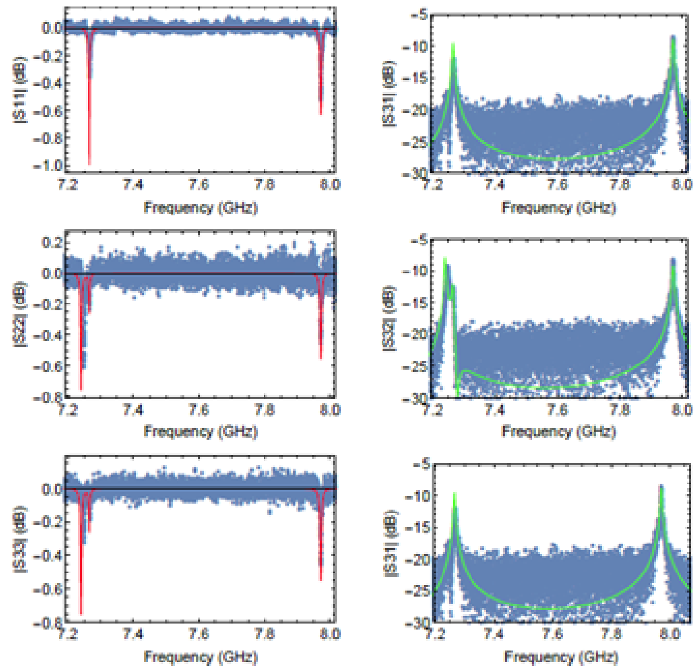

FIG. 8. Six graphs of scattering matrix elements (three reflection coefficients and three transmission coefficients). Blue dots are experiment data, green and red lines are fitting function.

CONCLUSION

In this thesis we studied lumped element resonators with periodic boundary conditions. We performed dipstick measurement to obtain scattering matrix parameter data. We extracted capacitances of circuit capacitors by model fitting method. Most of experimental results of circular quatromer show only three resonances with the fourth being dark.

For the CQJ samples, we found a dependency of coupling capacitances as a function of coupling capacitor finger number. The previous formula obtained from linear array of lumped elements and new formula are in good agreement.

For the CQK samples, we found a dependency of coupling capacitances of transmission lines as a function of coupling capacitor finger number. The formula obtained by MXWL simulation and new formula are in good agreement.

For the CQC samples, we found a dependency of shunt capacitances. The previous formulae obtained from linear array of lumped elements and new formulae are in agree.

For the CQL and CTL samples, we observed asymmetry since the arrangement of four transmission lines are not fully symmetric. In order to get a good symmetry of shunt capacitors, we should design a new sample with exactly the same shape and geometry of four transmission lines.

To sum up, we successed to extract capacitances from scattering matrix parameters and we obtained the formulae to calculate capacitances. Capacitances C_J, C_κ and C_i are linear function of designed parameters, which agrees well with simulation or previous experimental results. However, there is mode splitting for stronger coupling circuits of the CQK sample. We will have to investigate further to explain those phenomena. We also found designing full symmetry of circular quatromer is extremely difficult. Thus we have to develope accurate tool to build capacitors in order to study exotic quantum physics. Eventually, we will be able to apply quantum simulator to study condenced matter physics, quantum chemistry, cosmology or even biology [5].

BIBLIOGRAPHY

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*. Cambridge ; New York: Cambridge University Press, 1 ed., 2000.

[2] Antonio Rubio Abadal, *Josephson Parametric Amplifiers with Lumped element Coupled Resonators*. Master Thesis, ETH Zurich, Apr. 2015.

[3] Arno Bargerbos, *Coupled Lumped-element Resonators with Periodic Boundary Conditions*. Semester Thesis, ETH Zurich, Oct. 2015.

[4] Georgescu, I. M., Ashhab, S., and Nori, F. (2014). Quantum simulation. *Reviews of Modern Physics*, 86(1), 153.

[5] Devoret, M. H., and Schoelkopf, R. J. (2013). Superconducting circuits for quantum information: an outlook. *Science*, 339(6124), 1169-1174.

[6] Walker, J., Halliday, D., Resnick, R., Resnick, R., and Resnick, R. (2008). *Fundamentals of physics*. New York: Wiley.

[7] Russer, J. A., and Russer, P. (2012, February). Lagrangian and Hamiltonian formulations for classical and quantum circuits,". In *Proc. 7th Vienna Conference on Mathematical Modelling, MATHMOD, Vienna*.

[8] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Photons and Atoms: Introduction to Quantum Electrodynamics*. Physics textbook, Weinheim: Wiley, nachdr. ed., 2004.

[9] D. J. Griffiths, *Introduction to Quantum Mechanics*. Upper Saddle River, NJ: Pearson Prentice Hall, 2nd ed ed., 2005.

[10] Pozar, D. M. (2009). *Microwave engineering*. John Wiley and Sons.

ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Declaration of originality

The signed declaration of originality is a component of every semester paper, Bachelor's thesis, Master's thesis and any other degree paper undertaken during the course of studies, including the respective electronic versions.

Lecturers may also require a declaration of originality for other written papers compiled for their courses.

I hereby confirm that I am the sole author of the written work here enclosed and that I have compiled it in my own words. Parts excepted are corrections of form and content by the supervisor.

Title of work (in block letters):

Characterization of Lumped-Element Resonators with Periodic Boundary Conditions.

Authored by (in block letters):

For papers written by groups the names of all authors are required.

Name(s):	**First name(s):**
Oh	Kyoung Hun

With my signature I confirm that

- I have committed none of the forms of plagiarism described in the 'Citation etiquette' information sheet.
- I have documented all methods, data and processes truthfully.
- I have not manipulated any data.
- I have mentioned all persons who were significant facilitators of the work.

I am aware that the work may be screened electronically for plagiarism.

Place, date	**Signature(s)**
Seoul, South Korea / 18-02-2016	오경훈

For papers written by groups the names of all authors are required. Their signatures collectively guarantee the entire content of the written paper.