Physik IV - Lösungen - Serie 5

18. März 2011

1 a) $\Psi_{detector} = \frac{1}{\sqrt{2}}(\Psi_1 + \Psi_2) =$

$= \frac{1}{2}\left(e^{-i(kL-\omega t)+i\pi/2} + e^{-i(kL_1-\omega t)+i\pi/2+i\phi}\right)$

Here: L - is optical path

$\pi/2$ - is the phase aquired by reflection from the beam splitter

ϕ - is the phase aquired at the phase shifter

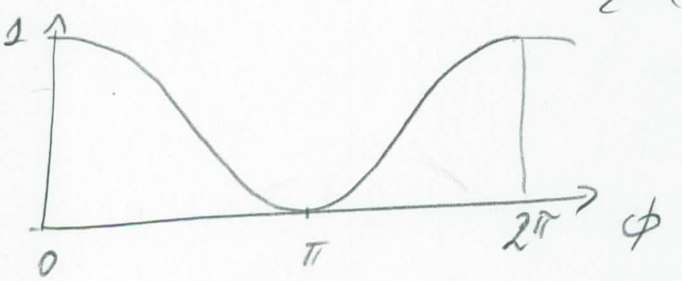
Probability to detect the particle:

$P = |\Psi_{detector}|^2 = \frac{1}{4}|\Psi_1 + \Psi_2|^2 =$

$= \frac{1}{4}(|\Psi_1|^2 + |\Psi_2|^2 + 2|\Psi_1||\Psi_2|\cos\phi) =$

$|\Psi_1|^2 = |\Psi_2|^2$

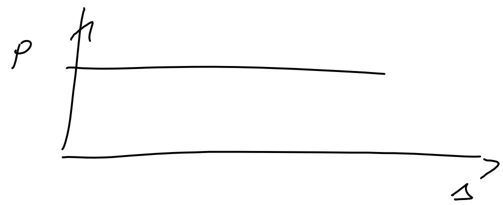
$= \frac{1}{2}(1 + \cos\phi)$



For $\phi = \pi$ or $\Delta = \lambda/2$ the probability to measure a particle is zero.

The particle will fly in another exit of the interferometer. The probability to find a particle in another interferometer will $\pi/2$ shifted due to an extra reflection at the last splitter of the first metawave.

b) Wenn die Wellenlänge der Teilchen nicht genau genug bestimmt ist, kommt es entweder zu konstruktiver oder destruktiver Interferenz. Im Mittel wird das Interferenzmuster verschwinden:



Teilchen werden dann mit 50% Wahrscheinlichkeit den Detektor erreichen.

2. Rutherford Scattering

$$Z(Au) = 79$$
$$Z(Ag) = 47.$$

(a) $$\frac{N(\theta)}{N_i} = \frac{n d z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 E_k^2 \sin^4\frac{\theta}{2}}$$

$\frac{N(\theta)}{N_i} \propto z^2 \Rightarrow$ If 10^3 particles collected for Au,

$$\frac{47^2}{79^2} \times 10^3 = 354 //$$

(b) $N(\theta) = 1$ $\qquad N(\theta) \propto \frac{1}{\sin^4\frac{\theta}{2}}$

$$\frac{\sin^4 \theta_1/2}{\sin^4 \theta_{100}/2} = 1000.$$

$$\sin \theta_1/2 = \sqrt[4]{1000} \sin \theta_{1000}/2 = 0.49$$

$\Rightarrow \theta_1 = 58.7° //$ Au.

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$$\sin \theta_1/2 = \sqrt[4]{354} \sin \theta_{1000}/2 = 0.38$$

$\Rightarrow \theta_1 = 44.4° //$ Ag.

(c) Electrons as lighter particles are easier to deflect by the outer shell electrons of the atoms. Therefore they need larger energies to scatter in accordance with the Rutherford formula.
Protons are similar to alpha particle and should be described well by Rutherford formula. For very large energies electron, proton and alpha-particles will have enough energy to approach the nuclei. In this case the size of the nuclei is not negligible and the scattering cross-section will deviate from the Rutherfod formula.
Neutrons are neutral and will not deflect by Coulomb interaction -> not Rutherford scattering

(d) Kinetic energy = potential energy @ nuclear radius

$$E_k = \frac{e^2 \times Z}{4\pi \varepsilon_0 r_0}$$ ← proton charge × nuclear charge

$$= 1.40 \times 10^{-11}\ J = 87\ MeV$$ //.

3) $r = \frac{\hbar^2}{\mu^*} \frac{4\pi\varepsilon}{e^2}$

μ^*... reduzierte effektive Masse
ε... Dielektrizitätskonstante
(Abschirmung durch Umgebung im Festkörper modifiziert)

$\mu^* = \frac{m_e^* m^*}{m_e^* + m^*} = 0.028\, m_e$ m_e ... Elektronenmasse

$\varepsilon = \varepsilon_r \cdot \varepsilon_0 = 15.8\,\varepsilon_0$

$\Rightarrow r = \frac{\hbar^2 4\pi\varepsilon_0}{e^2 m_e} \frac{15.8}{0.028} \sim 567\, a_0 \sim 30\text{ nm}$

$E = -\frac{\mu^* e^4}{32\pi^2\hbar^2 c^2 \varepsilon_0^2} = -\frac{m_e e^4}{32\pi^2\hbar^2\varepsilon_0} \frac{0.028}{15.8^2} = 1.1\, 10^{-4} E_1$ $E_1 = -13.6\text{ eV}$

$= -0.0015\text{ eV}$

(4)

Rydberg ENERGY $= 2.18 \times 10^{-18}$ J.

$$\frac{hc}{\lambda} \approx \frac{\mu}{m_e} R \left(\frac{1}{n^2} - \frac{1}{m^2}\right) \qquad \mu = \frac{m_e m}{m_e + m}$$

(a) $m = m_p$ for Hydrogen.

$n=3$ to $n=2$ transition

$$\lambda = \frac{1}{R_\infty} \frac{m_e + m}{m} \left(\frac{1}{n^2} - \frac{1}{m^2}\right)^{-1}$$

$\lambda = 656.47$ nm for $m = m_p$ Hydrogen.

$\lambda = 656.29$ nm for $m = m_n + m_p$ Deuterium.

(This was the discovery of Deuterium)

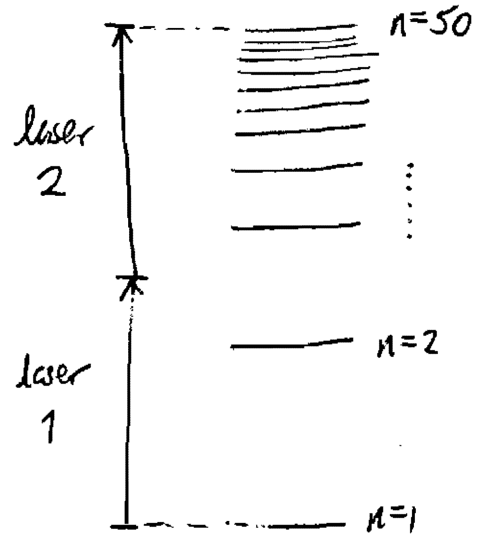
(b) Magnesium has 12 protons & 12 neutrons.

$m' = 24\, m_u \Rightarrow$ mass correction quite small. Paschen Series since e^- is in $n=3$.

Lowest energy transition is $n=4$ to $n=3$ since $n=1$ and $n=2$ are already filled with electrons.

$$\Rightarrow \lambda = \frac{1}{R_\infty} \underbrace{\frac{m_e + m'}{m'}}_{\approx 1} \left(\frac{1}{9} - \frac{1}{16}\right)^{-1} = 1.875\ \mu m$$

(5) Rydberg Atoms.



$$E_1 = 11.5\,eV = 11.5 \times 1.6 \times 10^{-19}\,J$$

$$\Delta E_{1\to n} = -R_\infty hc\left(\frac{1}{n^2} - 1\right)$$

$$= E_1 + E_2$$

$$= E_1 + \frac{hc}{\lambda_2}.$$

$$\lambda_2 = \left(\Delta E_{1\to n} - E_1\right)^{-1} \cdot hc$$

n	λ_2 / nm	r_n / nm	E_I / meV.
20	598.47	21	34
30	593.06	48	15
40	591.19	85	8.5
50	590.33	130	5.4

Closest spaced energy levels are $n=50, 51$

$$\Delta E = -R_\infty hc\left(\frac{1}{50^2} - \frac{1}{51^2}\right) = 0.21\,meV.$$

Linewidth in frequency is:

$$\Delta f = \frac{\Delta E}{h} = 51\,GHz.$$ Compare to 600nm laser,

$$f = \frac{c}{\lambda} = 500\,THz\;;\; E = 2.1\,eV.$$

Wasserstoffähnliche Atome

a)

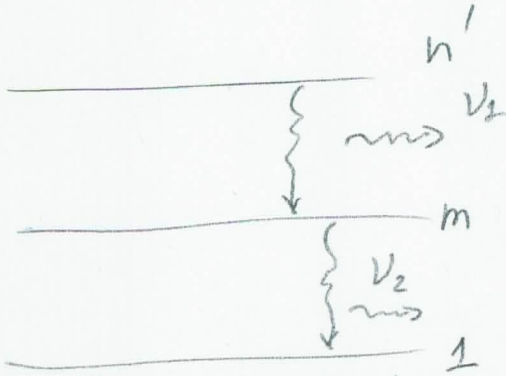
$$E_n = -\frac{Z^2 m_e e^4}{8\epsilon_0^2 h^2}\frac{1}{n^2} = -13.59\,eV\,Z^2\frac{1}{n^2}$$

where 13.59eV is ionization energy of Hydrogen atom

Ionisation energy equals to E_1

$E_1 = +13.59\,eV\cdot 2^2 = 54.4\,eV$ icreased 4 times relative to that of Hydrogen

b)



$$\begin{cases} \nu_1 = \dfrac{c}{\lambda_1} = -Z^2R\cdot c\left(\dfrac{1}{n'^2} - \dfrac{1}{m^2}\right) \\ \nu_2 = \dfrac{c}{\lambda_2} = -Z^2R\cdot c\left(\dfrac{1}{m^2} - 1\right) \end{cases}$$

where $R \cong 1.1\cdot 10^7\,m^{-1}$ is the Rydberg constant.

add two equations =>

$$\cancel{c}\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right) = -Z^2R\cdot\cancel{c}\left(\frac{1}{n'^2} - 1\right) \Rightarrow$$

$$n' = \left(1 - \frac{1}{Z^2R}\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)\right)^{-1/2} = 5$$