Digital quantum simulation with trapped ions

Tobias Thiele, Damian Berger

presented papers

2008: **Towards fault-tolerant quantum computing with trapped ions**

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2011: **Universal Digital Quantum Simulation with Trapped Ions**

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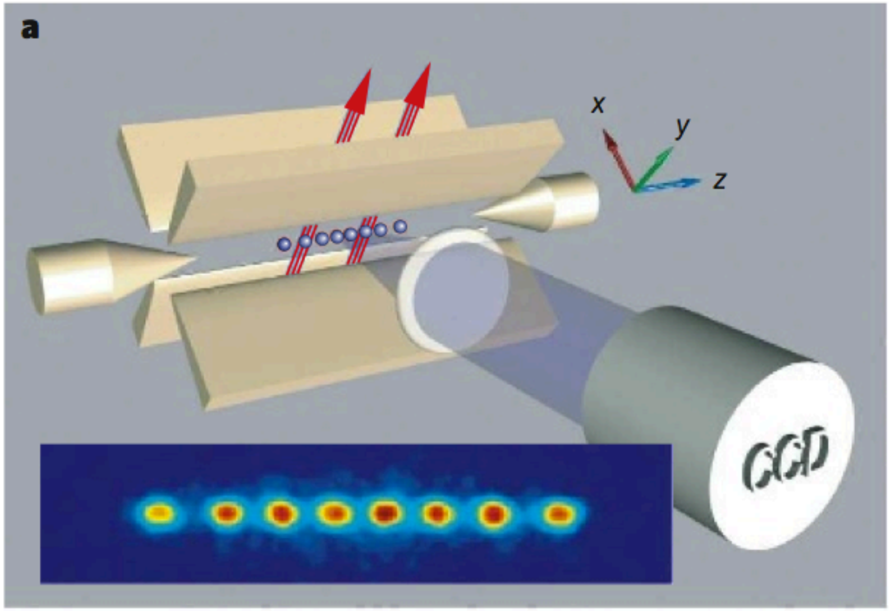
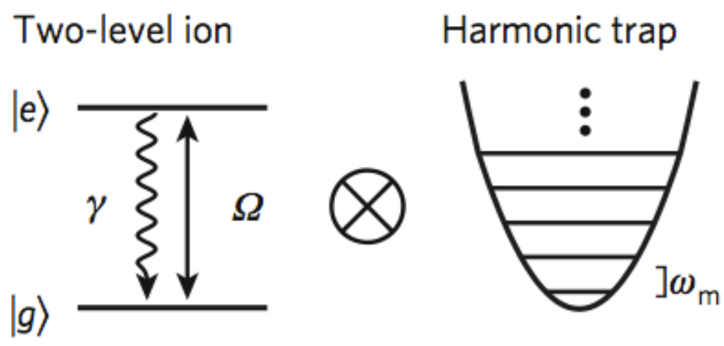
P. Schindler,[2] J. T. Barreiro,[2] M. Rambach,[1,2] G. Kirchmair,[1,2] M. Hennrich,[2] P. Zoller,[1,3]
R. Blatt,[1,2] C. F. Roos[1,2]

Overview

- Description of the Ion Trap:
  -Atomic Energy Levels and Motional States
  -Measurement
- Operations and Gates
  -Single Qubit Rotation, Cirac Gate, Mølmer–Sørensen Gate
- Quantum Simulation
- Digital Quantum Simulation with Ions

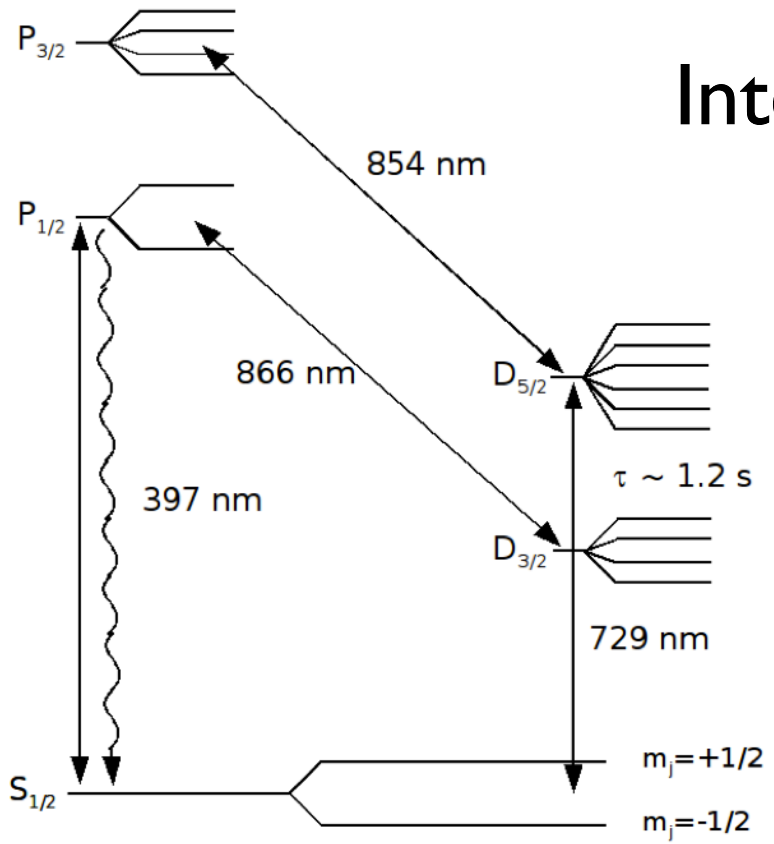




pictures from „Entangled states of trapped atomic ions“ R.Blatt & D.Wineland, Nature, 2008

- The qubit is represented by two internal states of the ion.
- Motional States mediate qubit-qubit interactions

Internal States of $^{40}Ca^+$



- The qubits are encoded in the $S_{1/2}$ and the $D_{3/2}$ level.
- The probability of the qubit being in the $S_{1/2}$ state is determined by measuring the scattered light of the $S_{1/2}$ - $P_{3/2}$ transition.

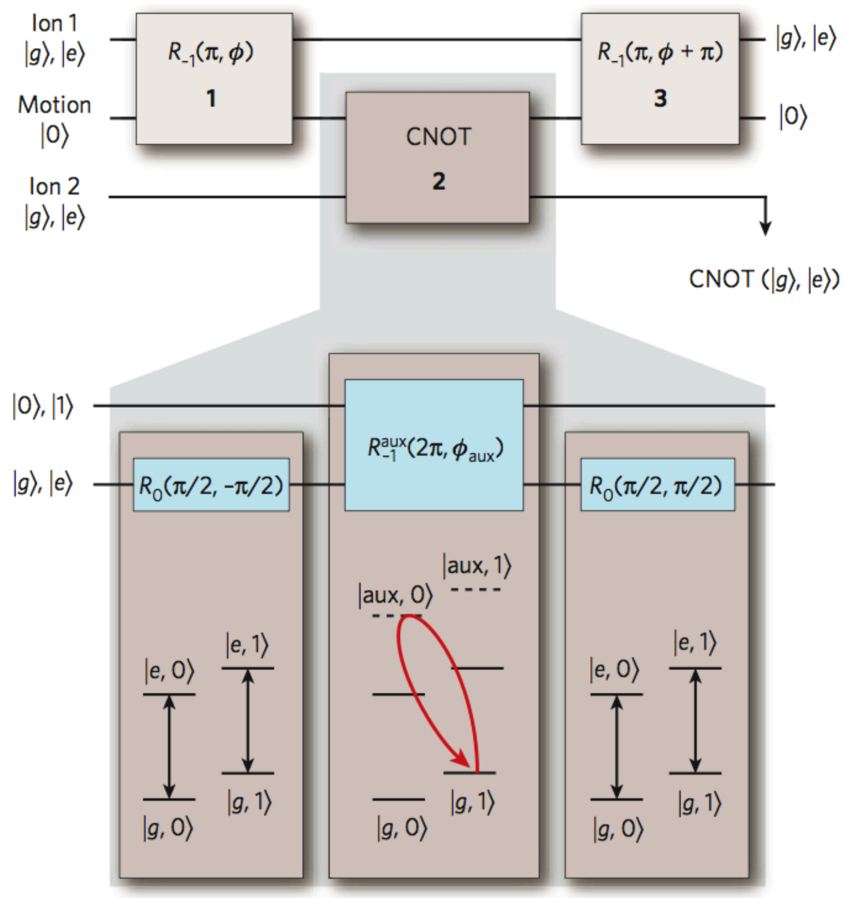
Single Qubit Operations

- Laser Light can do all single qubit operations
- Laser frequency w_0 and transition frequency w have to be equal
- Internal Hamiltonian of the two level system:
  $H = \hbar\Omega(\sigma_+ e^{i\varphi} + \sigma_- e^{-i\varphi})$
  (Assumptions: w0 = w, classical described laser field)
- By choosing $\varphi = 0$ or $\varphi = \pi/2$ this allows rotation arount x and y axis of the bloch sphere
- Rotations around z-axis can be done by composition of x and y rotations or by <u>detuning</u> the laser light

The Cirac Zoller CNOT Gate

- narrow Laser light to adress only one ion
- tuning the laser light to ‚sideband' frequencies $w_{ion} \pm w_m$ drives transitions $|g,n\rangle \leftrightarrow |e,n\pm 1\rangle$

The Cirac Zoller CNOT Gate



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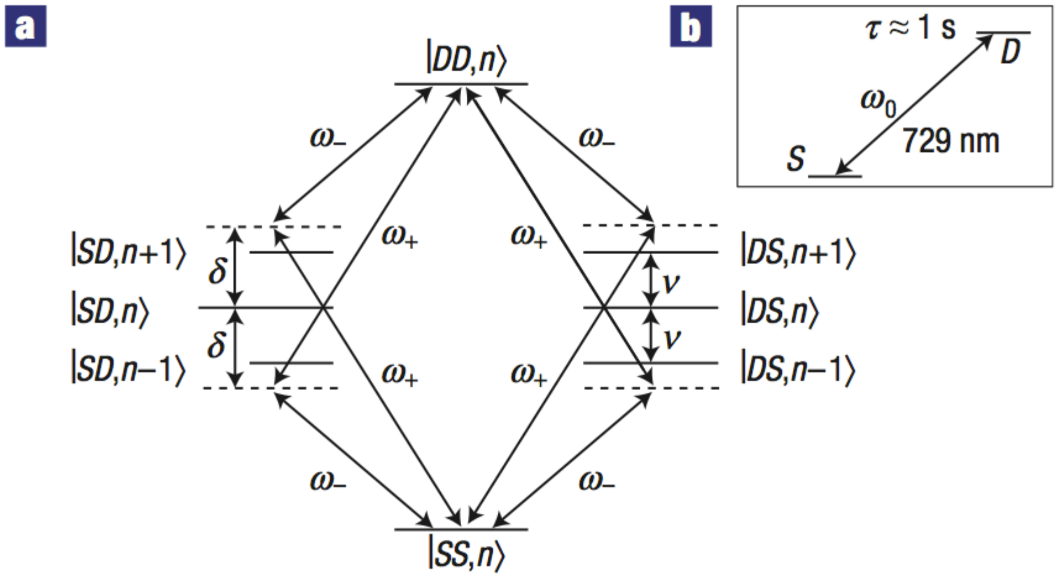
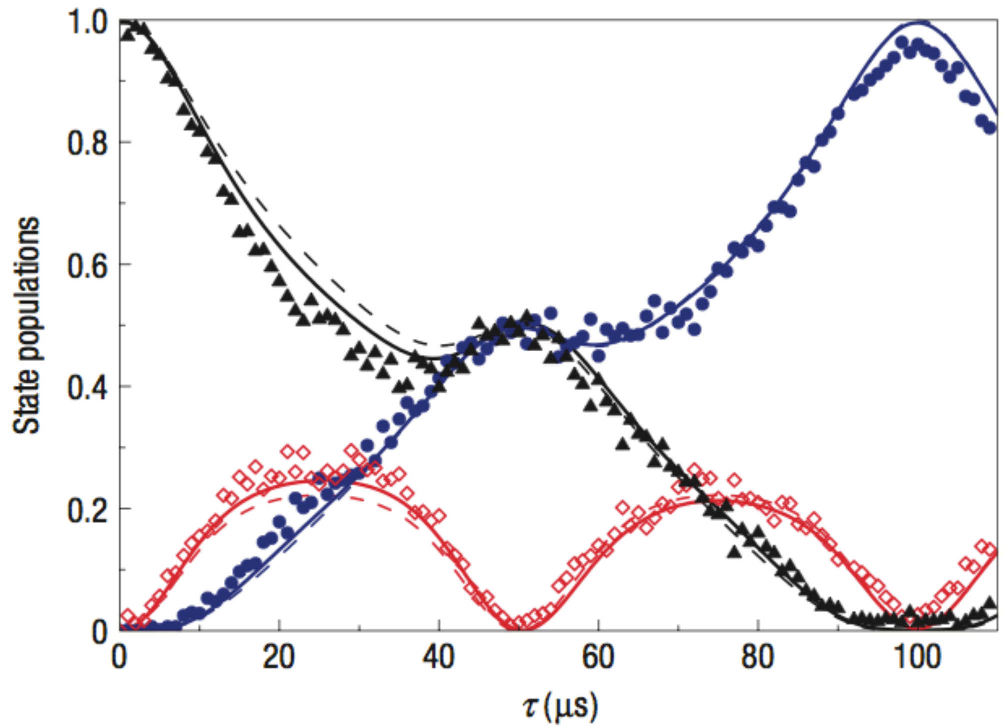
Mølmer–Sørensen Gate



from „Towards fault-tolerant quan...“ by R.Blatt et al. , Nat. Phys. 4, 463 (2008)

-both qubits in bichromatic laser field with frequency $\omega_0 \pm \delta$
-vibrational degrees of freedom only enter virtually

$$|SS\rangle \xrightarrow{\tau_{\text{gate}}} \underbrace{|SS\rangle + i|DD\rangle}_{\Psi_1} \xrightarrow{\tau_{\text{gate}}} |DD\rangle$$



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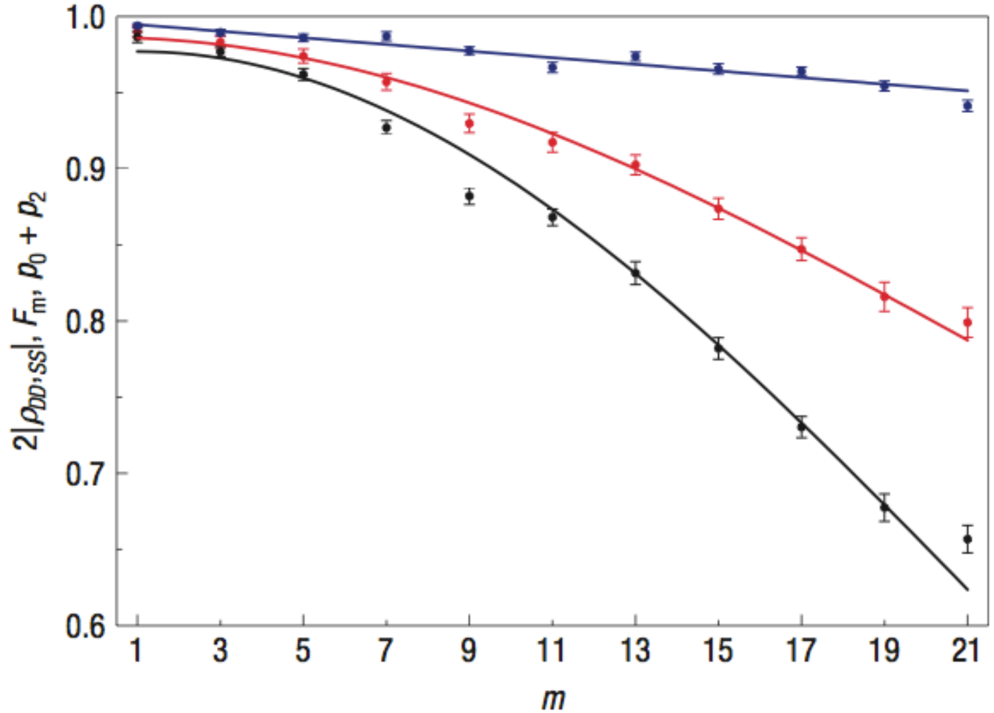

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Red Line: Fidelity of the state |SS⟩+i|DD⟩ after m gate operations

Content

- Quantum Simulation
  - Analog Quantum Simulation
- Digital Quantum Simulation
  - Ising Model
- Digital Quantum Simulation with Ions
  - More complex system/Results

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Quantum Simulation

- Problem:
  - Quantum System with Hamiltonian H
    - Ground state?
    - Phase transitions?
    - Time evolution?
    - Correlations?
  - Not solvable with classical methods/computers

Quantum Simulation

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    - Ground state?
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- Solution:
  - Use quantum system to simulate H
    - „measure result“

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Analog Quantum Simulation

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  - Build a system that implements only H

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- Famous system: Bose-Hubbard Hamiltonian
  - $H = -J \sum_{\langle i,j \rangle} \hat{a}_i^t \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1) + \sum_i \varepsilon_i \hat{n}_i$

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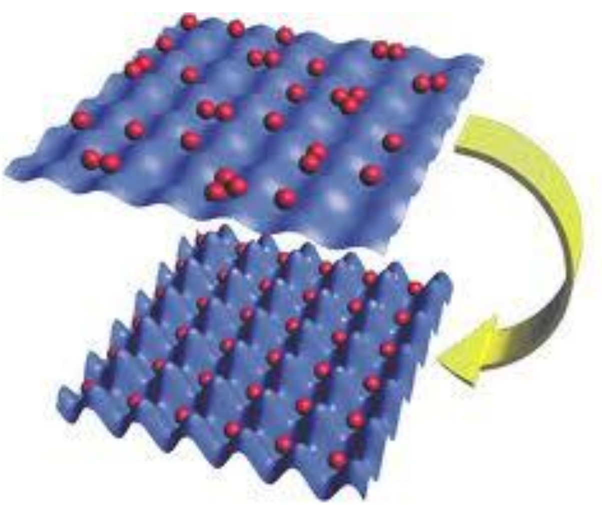
  - Neutral atoms in optical lattice
  - Changing detuning of laser
    $\Longrightarrow$ change $\frac{J}{U}$



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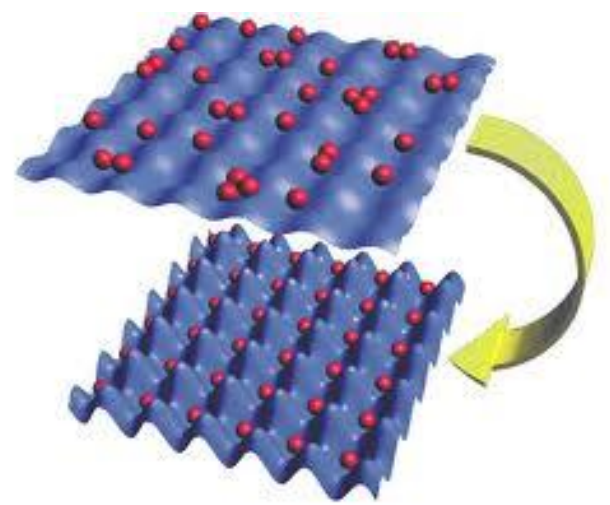
    $\Longrightarrow$ change $\frac{J}{U}$
  - Superfluid to Mott Insulator



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  - i.e. 2 Spin Ising model:

$$H_X = B\sum_i \sigma_z^i + J\sum_{\langle i,j\rangle} \sigma_x^i \sigma_x^j = B(\sigma_z^1 + \sigma_z^2) + J\sigma_x^1\sigma_x^2$$

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  - Goal: Determine time evolution of system

Digital Quantum Simulation

$$H_X = B(\sigma_z^1 + \sigma_z^2) + J\sigma_x^1\sigma_x^2$$

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- Time evolution of Ψ:
  - $\Psi(t) = U(t)\Psi(0) = \exp\left(-\frac{i}{\hbar}Ht\right)\Psi(0)$

Digital Quantum Simulation

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- Time evolution of Ψ:
  - $\Psi(t) = U(t)\Psi(0) = \exp\left(-\frac{i}{\hbar}Ht\right)\Psi(0)$

- Strategy:
  - Divide Hamiltonian into operations that can be applied at once

$$H_X = \boxed{B(\sigma_z^1 + \sigma_z^2)} + \boxed{J\sigma_x^1\sigma_x^2} = \boxed{H_1} + \boxed{H_2}$$

Single Qubit Operations   Multi Qubit Operations

Digital Quantum Simulation

$$H_X = \boxed{B(\sigma_z^1 + \sigma_z^2)} + \boxed{J\sigma_x^1\sigma_x^2} = \boxed{H_1} + \boxed{H_2}$$

Single Qubit Operations

Multi Qubit Operations

- Unitary Time evolution:
  - $U(t) = \exp\left(-\frac{i}{\hbar}(H_1 + H_2)t\right)$

Digital Quantum Simulation

$$H_X = B(\sigma_z^1 + \sigma_z^2) + J\sigma_x^1\sigma_x^2 = H_1 + H_2$$

Single Qubit Operations

Multi Qubit Operations

- Unitary Time evolution:
  - $U(t) = \exp\left(-\frac{i}{\hbar}(H_1 + H_2)t\right)$
- In experiment only possible to apply:
  - $U_i(t) = \exp\left(-\frac{i}{\hbar}H_i t\right)$

Digital Quantum Simulation

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Multi Qubit Operations

- Unitary Time evolution:
  - $U(t) = \exp\left(-\frac{i}{\hbar}(H_1 + H_2)t\right)$
- In experiment only possible to apply:
  - $U_i(t) = \exp\left(-\frac{i}{\hbar}H_i t\right)$
- And: $\exp\left(-\frac{i}{\hbar}(H_1 + H_2)t\right) \neq \exp\left(-\frac{i}{\hbar}H_1 t\right)\exp\left(-\frac{i}{\hbar}H_2 t\right)$
  - since $[H_1, H_2] \neq 0$

Digital Quantum Simulation

$$H_X = B(\sigma_z^1 + \sigma_z^2) + J\sigma_x^1\sigma_x^2 = H_1 + H_2$$

Single Qubit Operations

Multi Qubit Operations

- Trotter formula

$$\exp\left(-\frac{i}{\hbar}(H_1 + H_2)t\right) = \lim_{n\to\infty}\left[\exp\left(-\frac{i}{\hbar}H_1\frac{t}{n}\right)\exp\left(-\frac{i}{\hbar}H_2\frac{t}{n}\right)\right]^n$$

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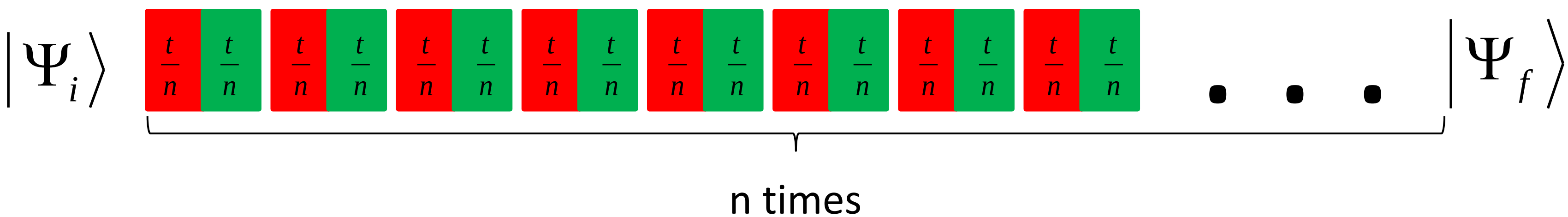
- Therefore: planned sequence



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DQS with Ions

$$H_X = B(\sigma_z^1 + \sigma_z^2) + J\sigma_x^1\sigma_x^2 = H_1 + H_2$$

Single Qubit Operations

Multi Qubit Operations

- We have:

DQS with Ions

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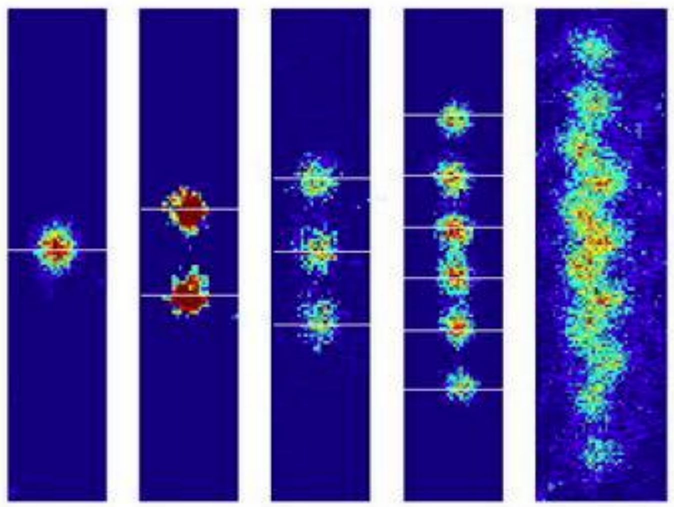
Single Qubit Operations

Multi Qubit Operations

- We have:
  - A string of ions/qubits (Ca^+)



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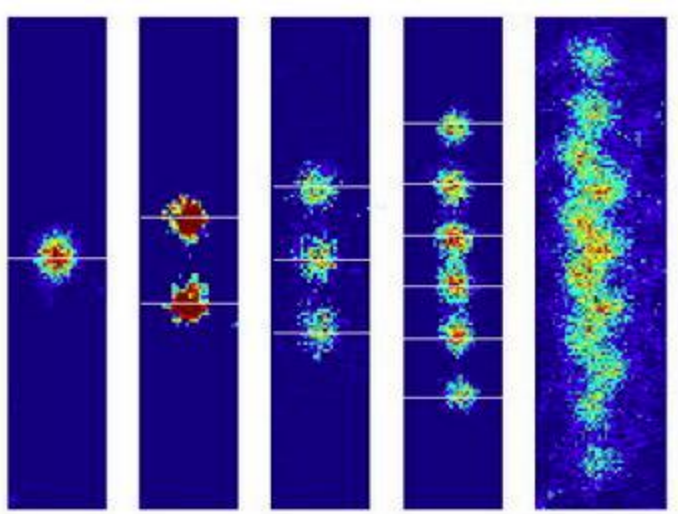
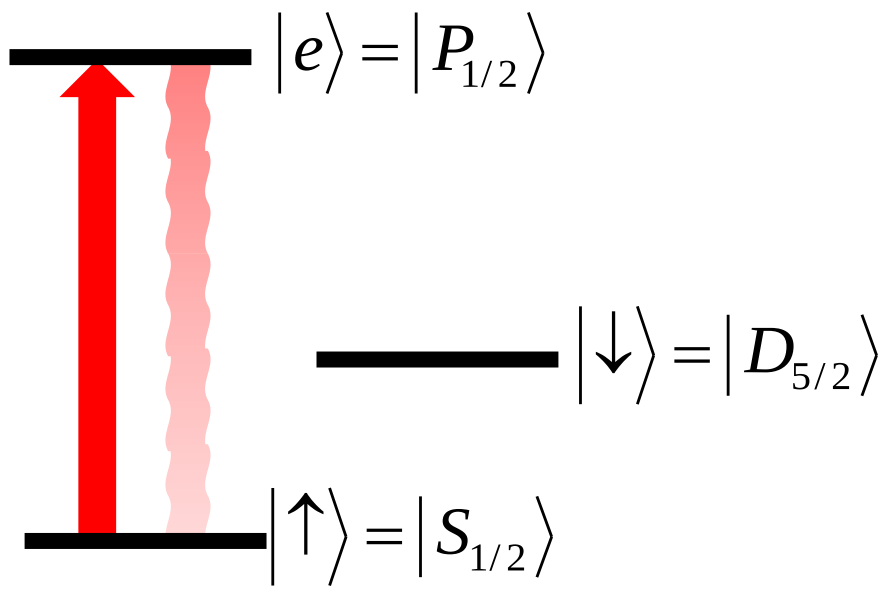
Single Qubit Operations

Multi Qubit Operations

- We have:
  - A string of ions/qubits (Ca^+) with measurement





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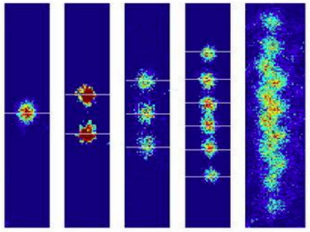
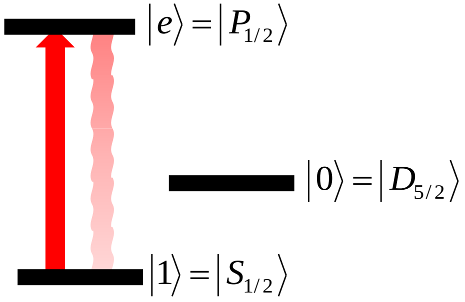
Single Qubit Operations

Multi Qubit Operations

- We have:
  - A string of ions/qubits
  - Measurement
  - A universal set of operations





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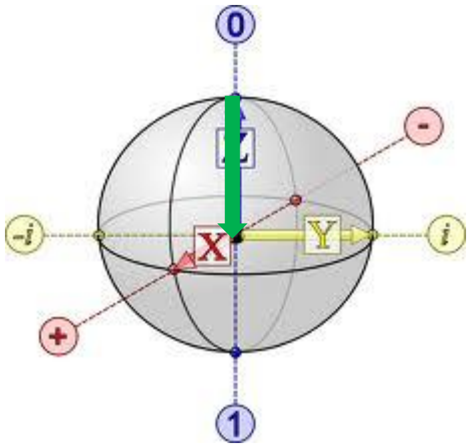
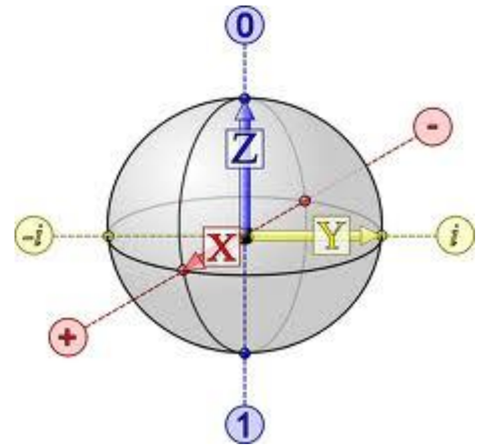
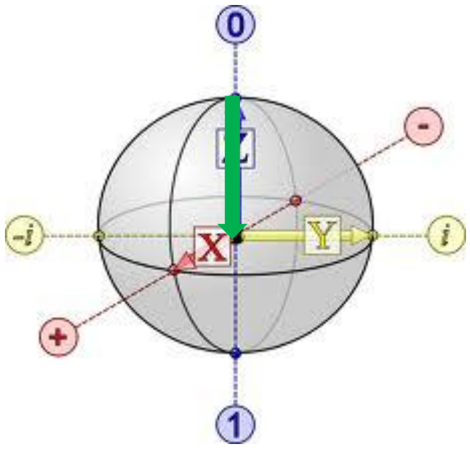


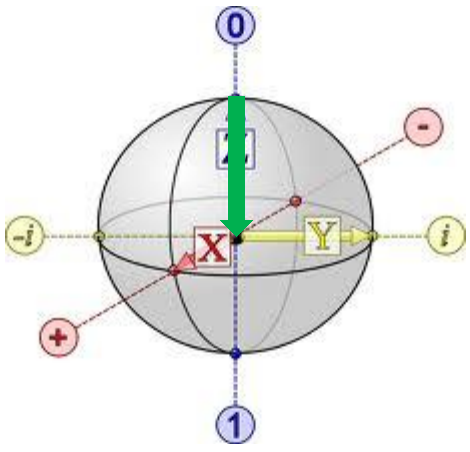


– A universal set of operations

$$O_1(\theta, j) = \exp\left(-i\theta\sigma_z^j\right)$$

DQS with Ions





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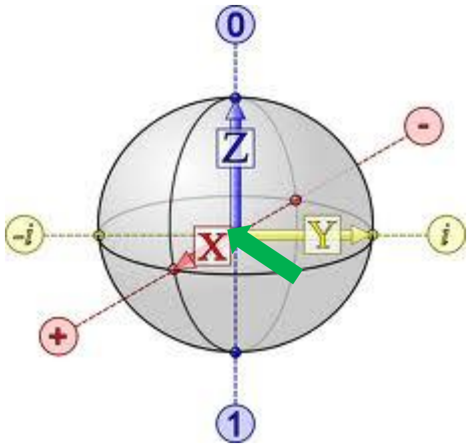
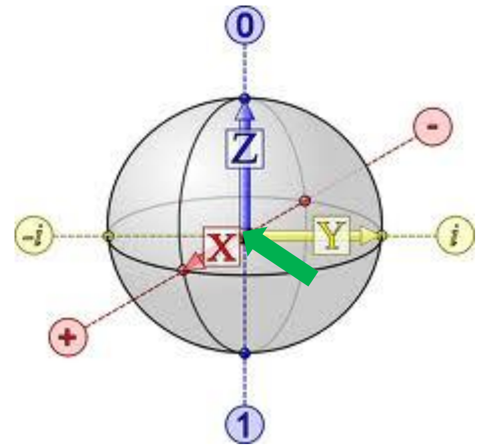
DQS with Ions





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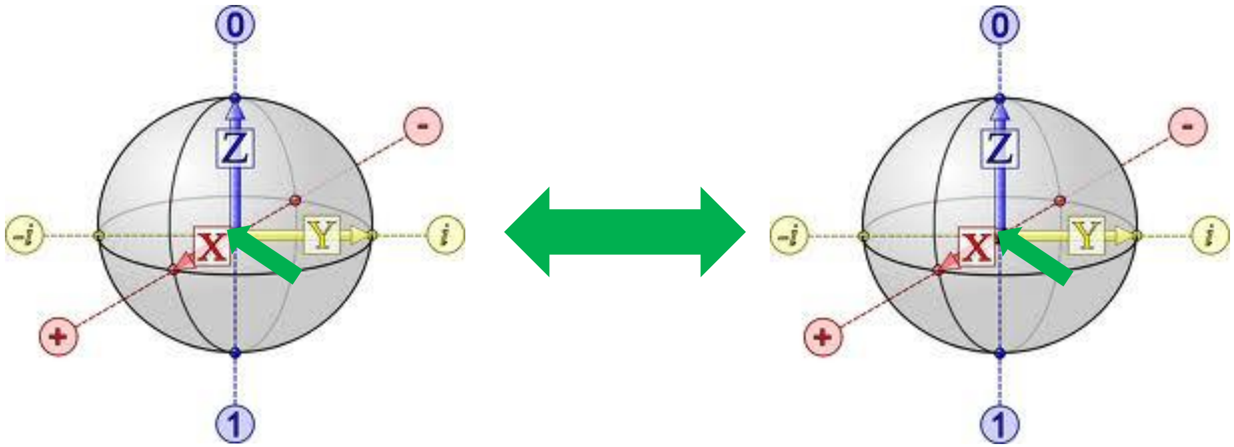

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Ising Model

$$H_X = B(\sigma_z^1 + \sigma_z^2) + J\sigma_x^1\sigma_x^2 = H_1 + H_2$$

Single Qubit Operations

Multi Qubit Operations

- Easy now to implement!

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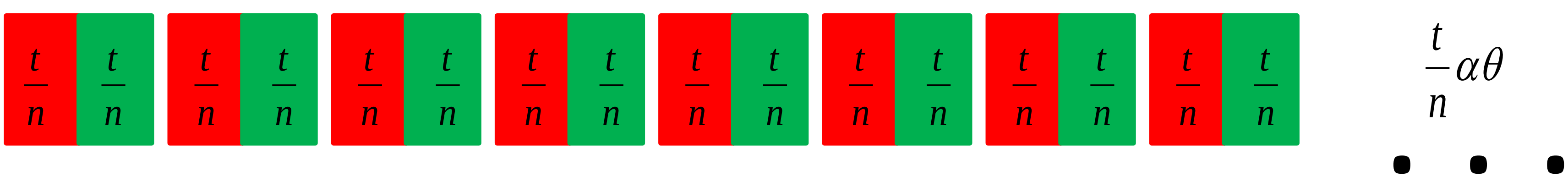

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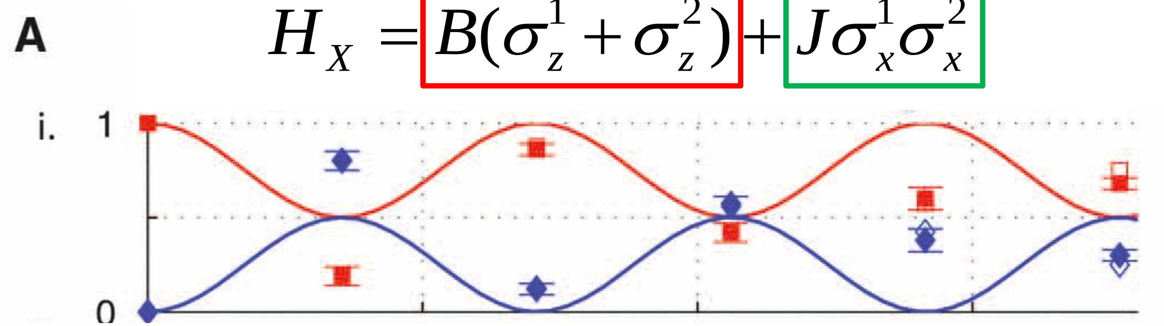
- Measure state at the end

Ising Model-Results

$$H_X = B(\sigma_z^1 + \sigma_z^2) + J\sigma_x^1\sigma_x^2$$

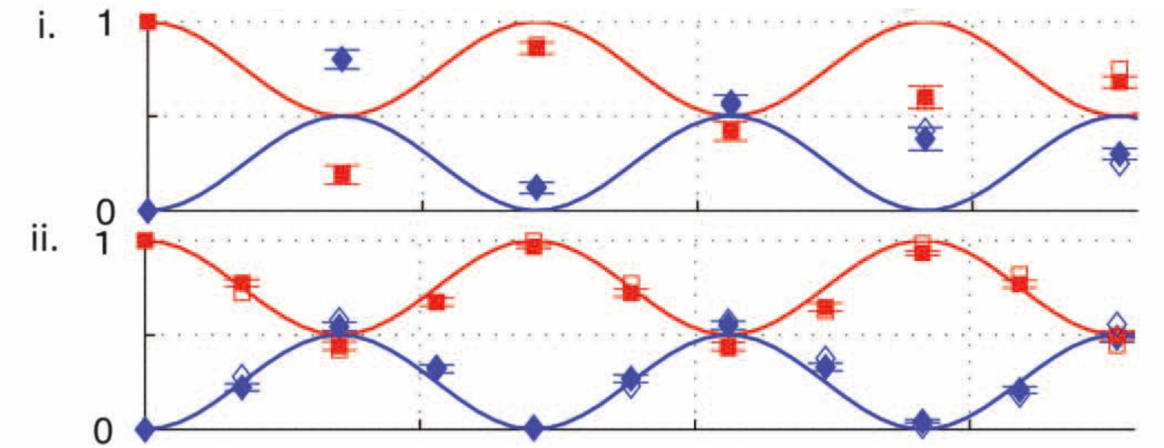


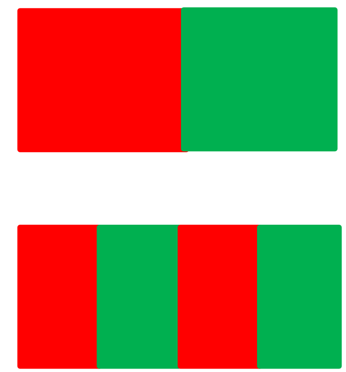
$n = 1$

Ising Model-Results

A

$$H_X = \boxed{B(\sigma_z^1 + \sigma_z^2)} + \boxed{J\sigma_x^1\sigma_x^2}$$



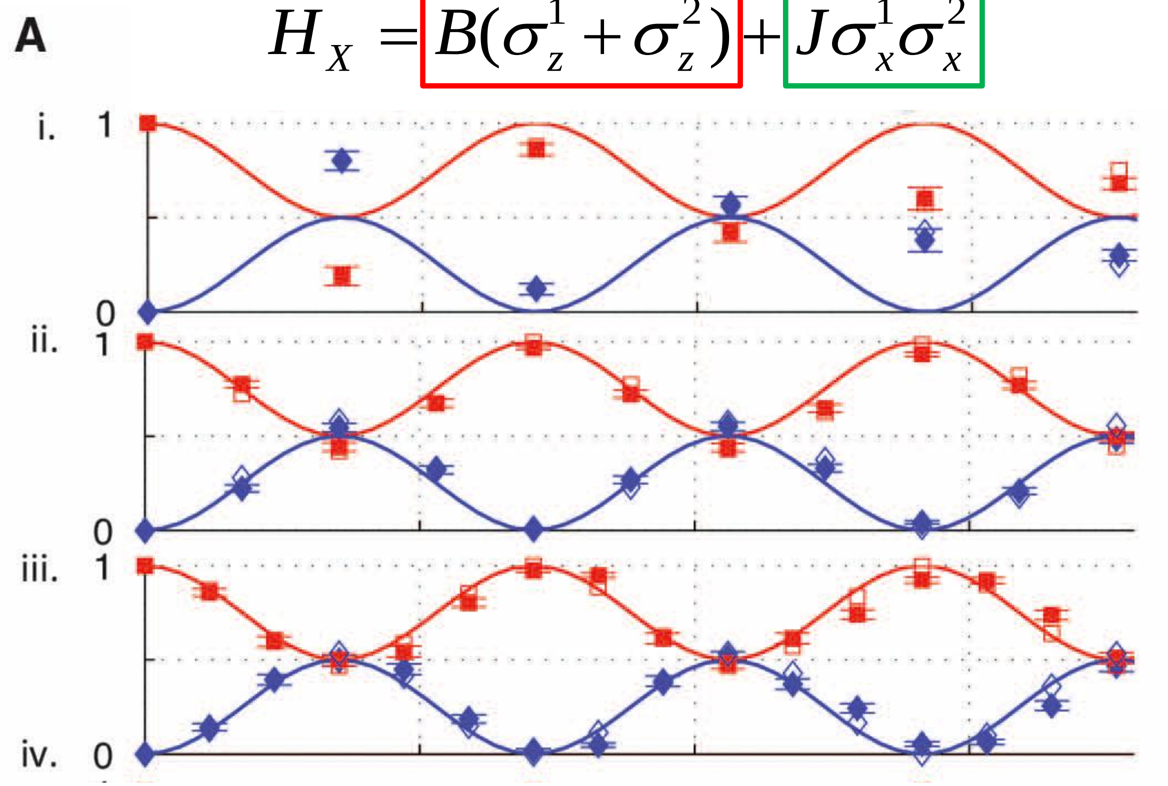


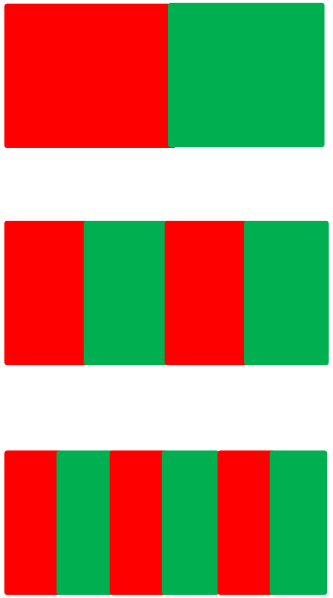
$n = 1$

$n = 2$

Ising Model-Results

$$H_X = B(\sigma_z^1 + \sigma_z^2) + J\sigma_x^1\sigma_x^2$$





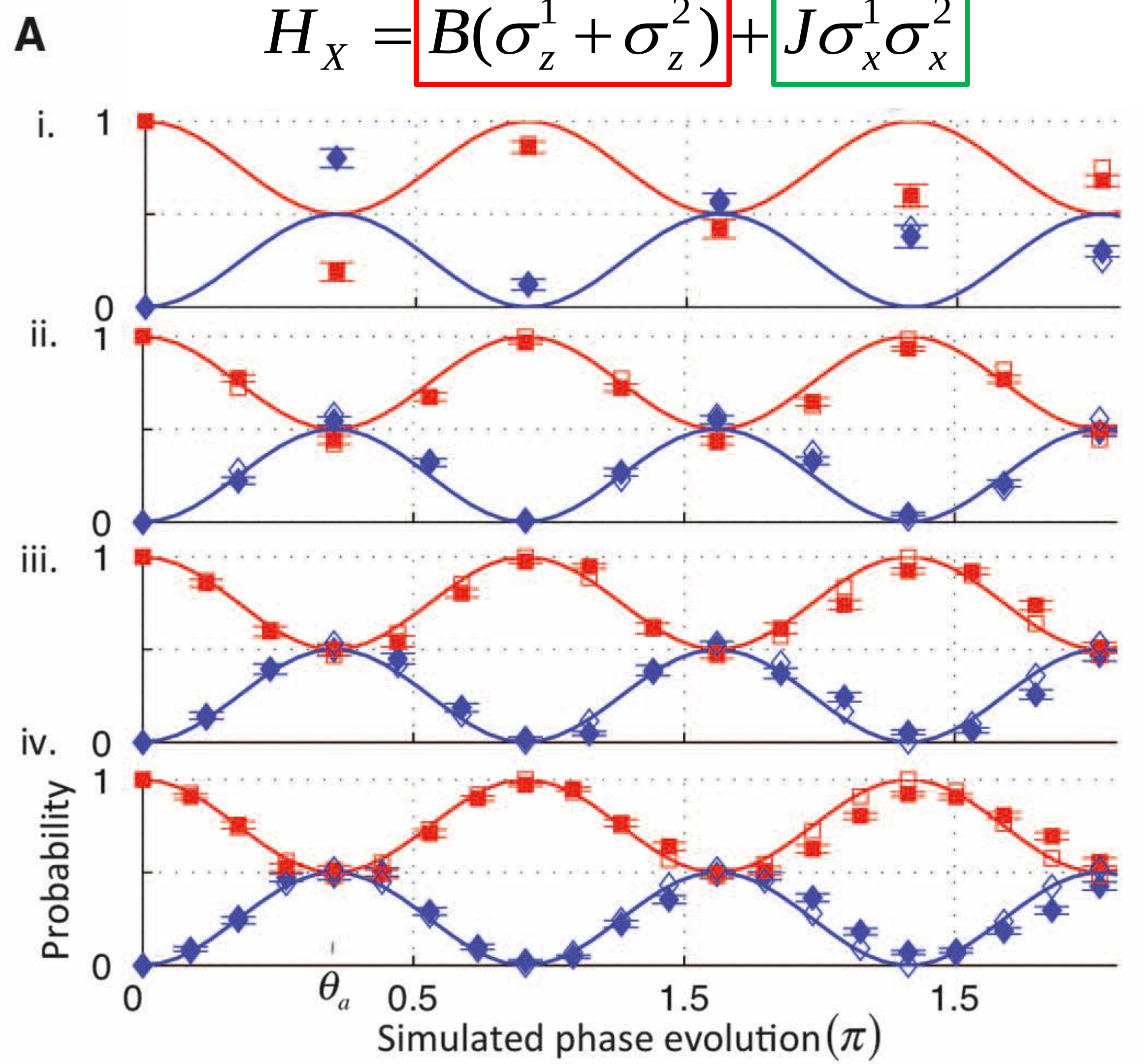
$n = 1$

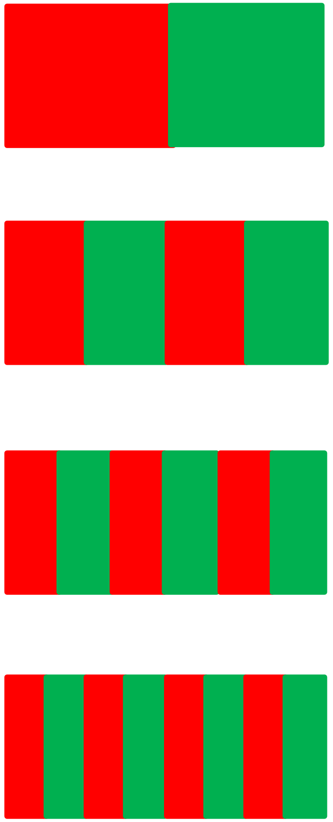
$n = 2$

$n = 3$

Ising Model-Results

$$H_X = \boxed{B(\sigma_z^1 + \sigma_z^2)} + \boxed{J\sigma_x^1\sigma_x^2}$$





$n = 1$

$n = 2$

$n = 3$

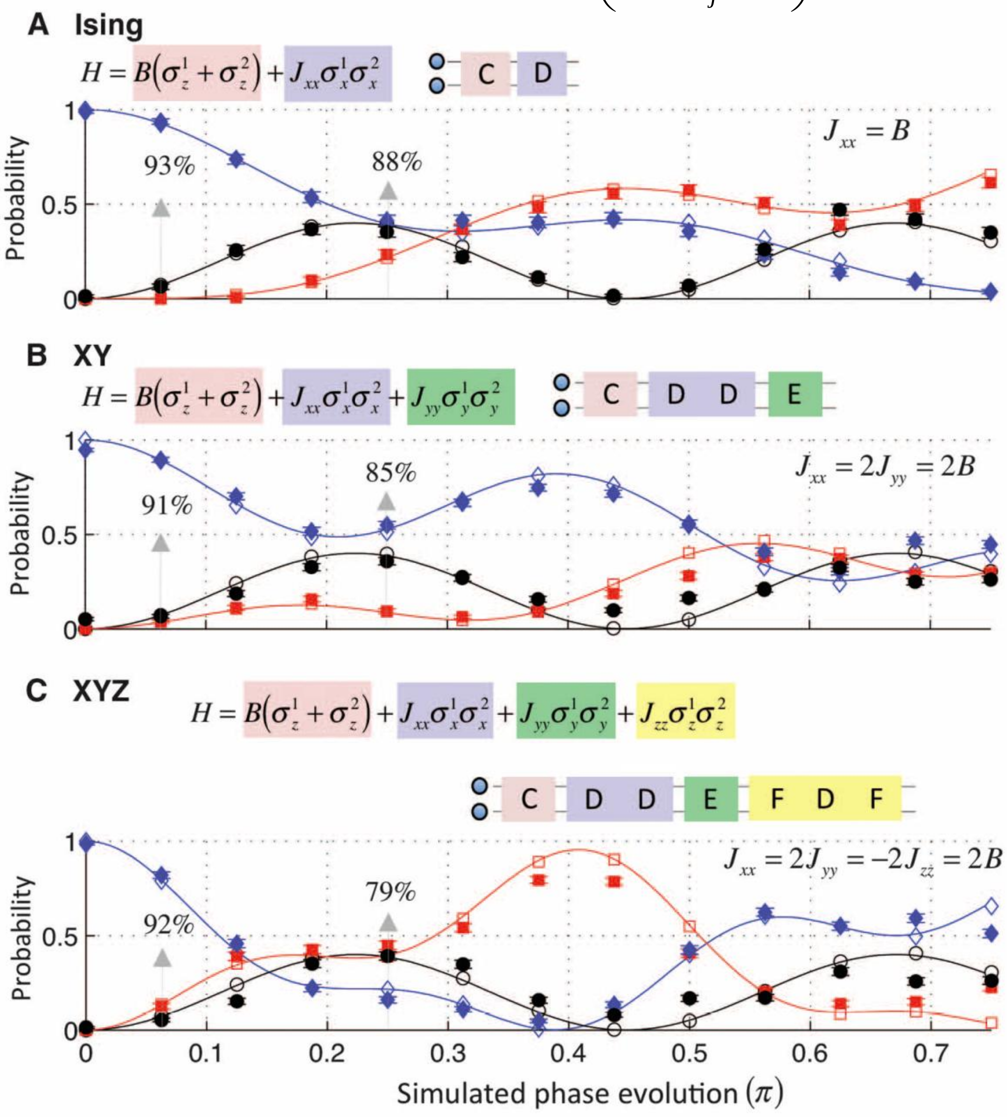
$n = 4$

Content

- Quantum Simulation
  - Analog Quantum Simulation
- Digital Quantum Simulation
  - Ising Model
- Digital Quantum Simulation with Ions
  - More complex system/Results

Complex systems

$$O_1(\theta, j) = \exp\left(-i\theta\sigma_z^j\right) \quad O_2(\theta) = \exp\left(-i\theta\sum_j \sigma_z^j\right) \quad O_3(\theta,\phi) = \exp\left(-i\theta\sum_j \sigma_\phi^j\right) \quad O_4(\theta,\phi) = \exp\left(-i\theta\sum_{i<j} \sigma_\phi^i\sigma_\phi^j\right)$$



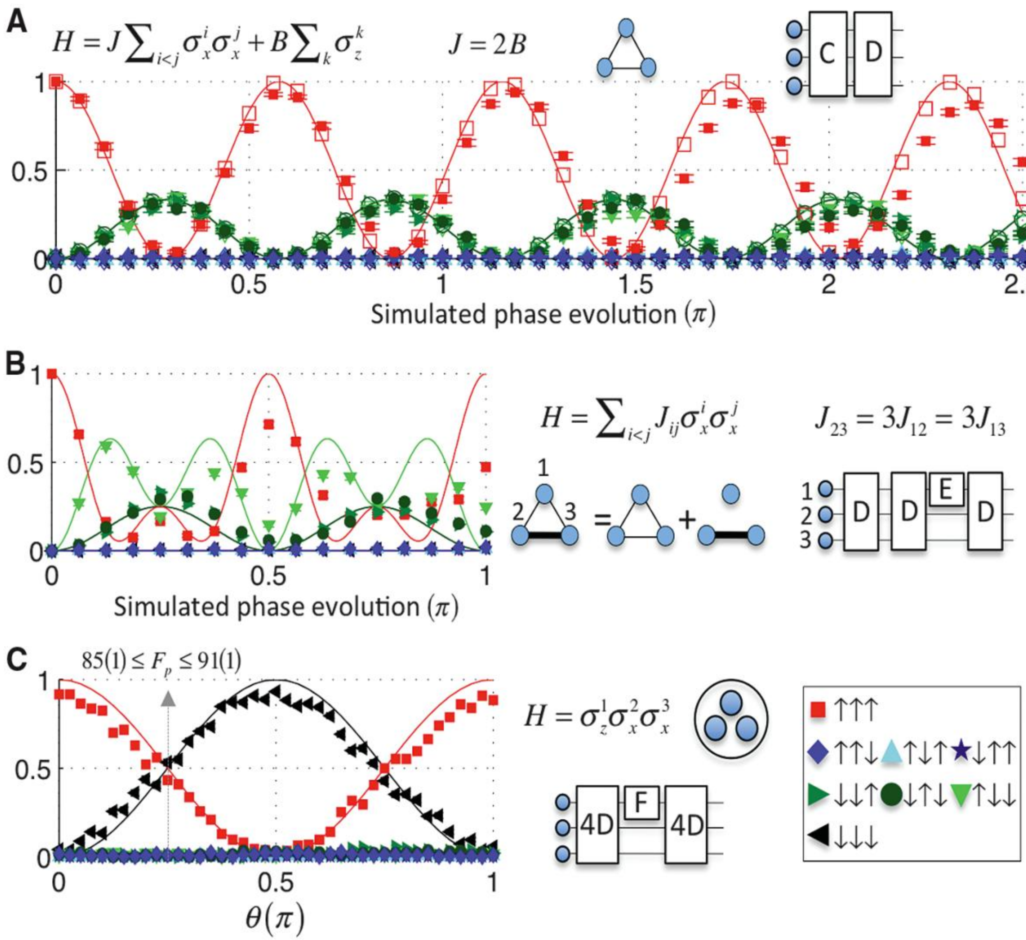
$$O_4(\theta/n,0).O_2(\theta/n)$$

$$O_4(\theta/n,\frac{\pi}{2}).O_4(\theta/n,0).O_2(\theta/n)$$

$$O_3(4\theta/n,0).O_4(\theta/n,0).O_3(4\theta/n,0).$$
$$O_4(\theta/n,\pi).O_4(\theta/n,0).O_2(\theta/n)$$

More complex systems

$$O_1(\theta, j) = \exp\left(-i\theta\sigma_z^j\right) \quad O_2(\theta) = \exp\left(-i\theta\sum_j \sigma_z^j\right) \quad O_3(\theta,\phi) = \exp\left(-i\theta\sum_j \sigma_\phi^j\right) \quad O_4(\theta,\phi) = \exp\left(-i\theta\sum_{i<j} \sigma_\phi^i\sigma_\phi^j\right)$$



$$C = O_2(\pi/32)$$

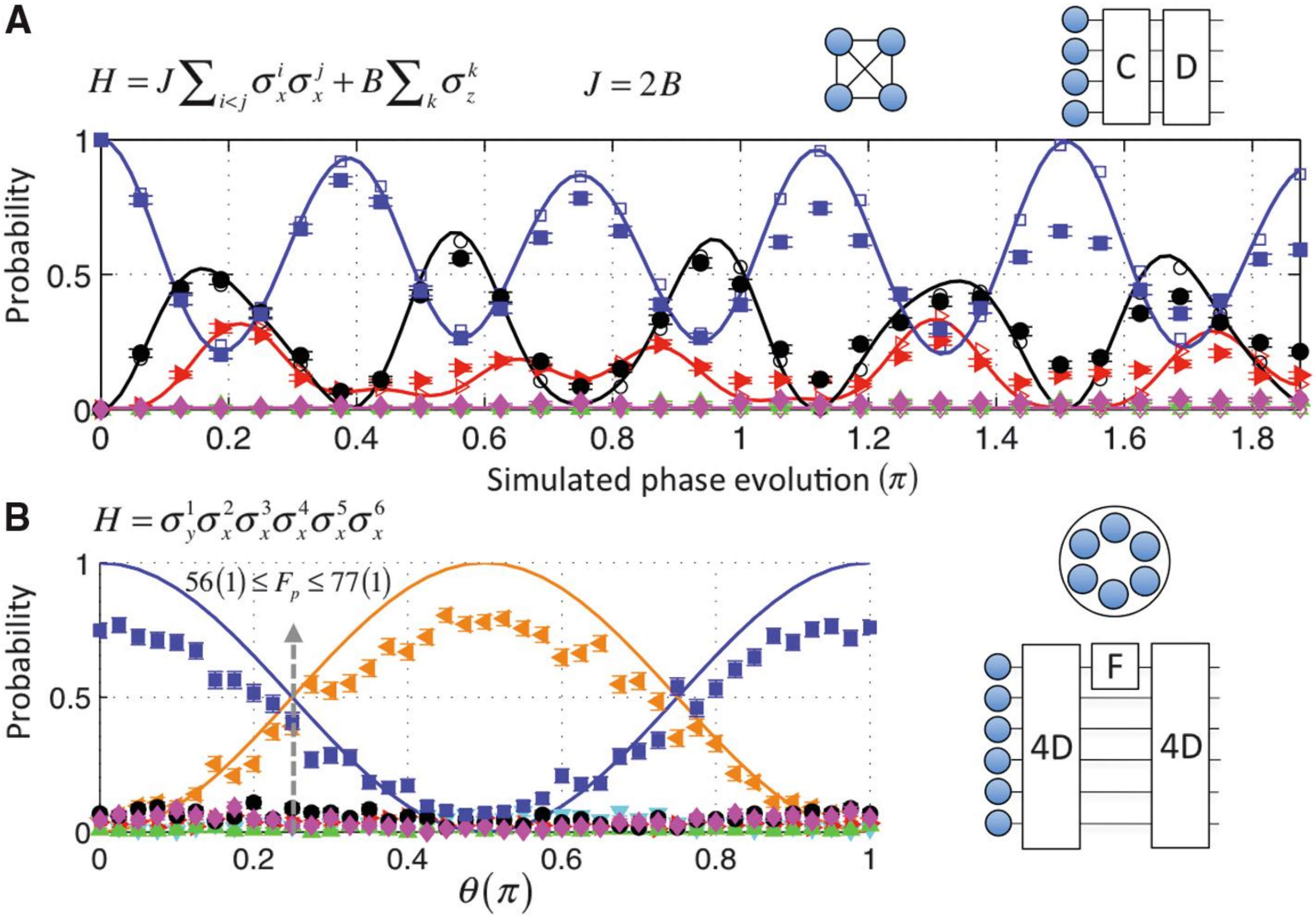
$$D = O_4(\pi/16, 0)$$

$$E = O_1(\pi/2, 1)$$

$$F = O_1(\theta, 1)$$

Even more complex systems

$$O_1(\theta, j) = \exp\left(-i\theta\sigma_z^j\right) \quad O_2(\theta) = \exp\left(-i\theta\sum_j \sigma_z^j\right) \quad O_3(\theta,\phi) = \exp\left(-i\theta\sum_j \sigma_\phi^j\right) \quad O_4(\theta,\phi) = \exp\left(-i\theta\sum_{i<j} \sigma_\phi^i \sigma_\phi^j\right)$$



$$C = O_2(\pi/32)$$

$$D = O_4(\pi/16, 0)$$

$$E = O_1(\pi/2, 1)$$

$$F = O_1(\theta, 1)$$

Summary-Take Home Message

- Analog quantum computing
  - Does well, but only for a single Hamiltonian
  - Needs a system that implements H
- Digital Quantum Computing
  - Possible to implement any local Hamiltonian
  - Needs (only) set of qubits and a universal set of operations
    - The more computational power, the longer and more exact the simulations can be done

Ising Model-Results:time-dependence

$$H_X = B(\sigma_z^1 + \sigma_z^2) + J(t)\sigma_x^1 \sigma_x^2$$