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Experimental implementation of Grover's algorithm with transmon qubit architecture

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What is Grover's algorithm?

- Quantum search algorithm
- Task: In a search space of dimension N, find those 0<M<N elements displaying some given characteristics (being in some given states).

Classical search (random guess)	Grover’s algorithm
• Guess randomly the solution • Control whether the guess is actually a solution	• Apply an ORACLE, which *marks* the solution • Decode the marked solution, in order to *recognize* it
O(N) steps O(N) bits needed	O($\sqrt{N}$) x n steps O(log(N)) qubits needed

The oracle

- The oracle MARKS the correct solution

$$f(x) = \begin{cases} 0 & x \ is \ not \ solution \\ 1 & x \ is \ solution \end{cases}$$

Dilution operator
(interpreter)

- Solution is more recognizable

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Grover's algorithm

Procedure

- Preparation of the state

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{x}^{N} |x\rangle$$

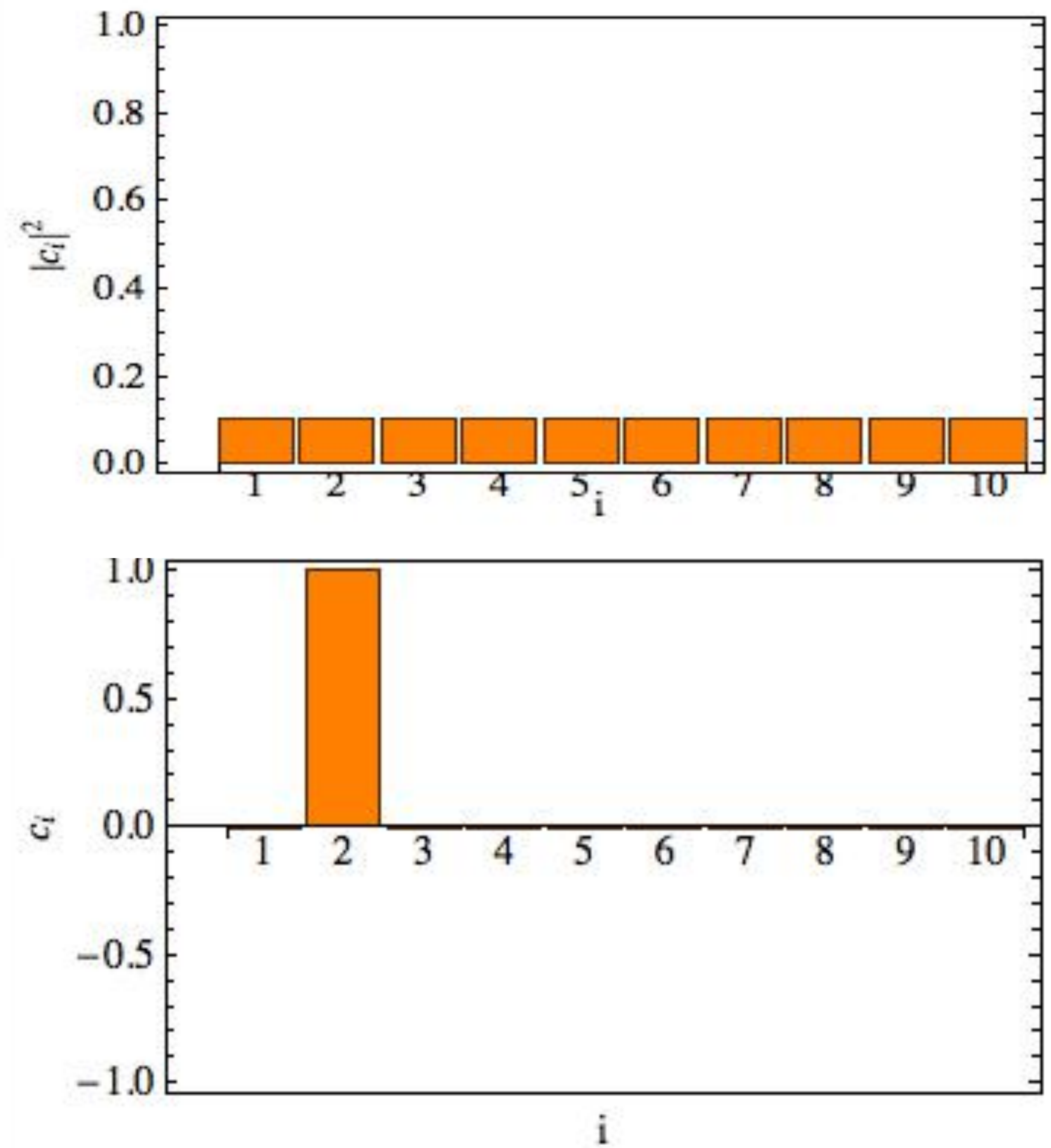
- Oracle application

$$|x\rangle \xrightarrow{O} (-1)^{f(x)} |x\rangle$$

- Dilution of the solution

$$2|\psi\rangle\langle\psi| - I$$

- Readout



Geometric visualization

- Preparation of the state

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_x^N |x\rangle$$

$$|\bar{t}\rangle = \frac{1}{\sqrt{N-M}} \sum_x{}' |x\rangle$$

$$|t\rangle = \frac{1}{\sqrt{M}} \sum_x{}'' |x\rangle \quad G|\psi\rangle$$

$$|\psi\rangle = \sqrt{\frac{N-M}{N}} |\alpha\rangle + \sqrt{\frac{M}{N}} |\beta\rangle$$

Geometric visualization

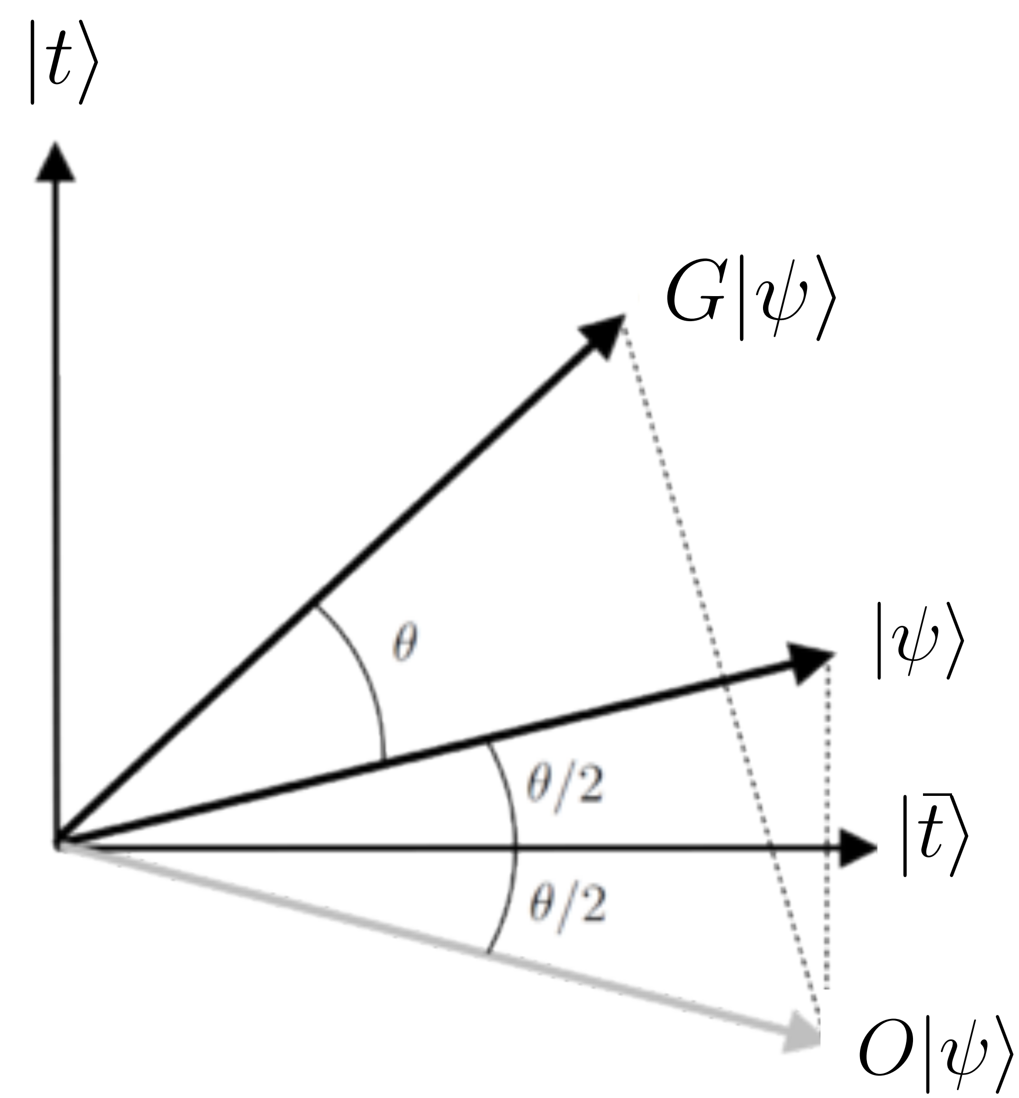
$$|\psi\rangle = \cos(\theta/2)|\bar{t}\rangle + \sin(\theta/2)|t\rangle$$

$\downarrow$ O

$$O|\psi\rangle = \cos(\theta/2)|\bar{t}\rangle - \sin(\theta/2)|t\rangle$$

$\downarrow$ $2|\psi\rangle\langle\psi| - I$

$$G|\psi\rangle = \cos(\frac{3\theta}{2})|\bar{t}\rangle + \sin(\frac{3\theta}{2})|t\rangle$$



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Grover's algorithm

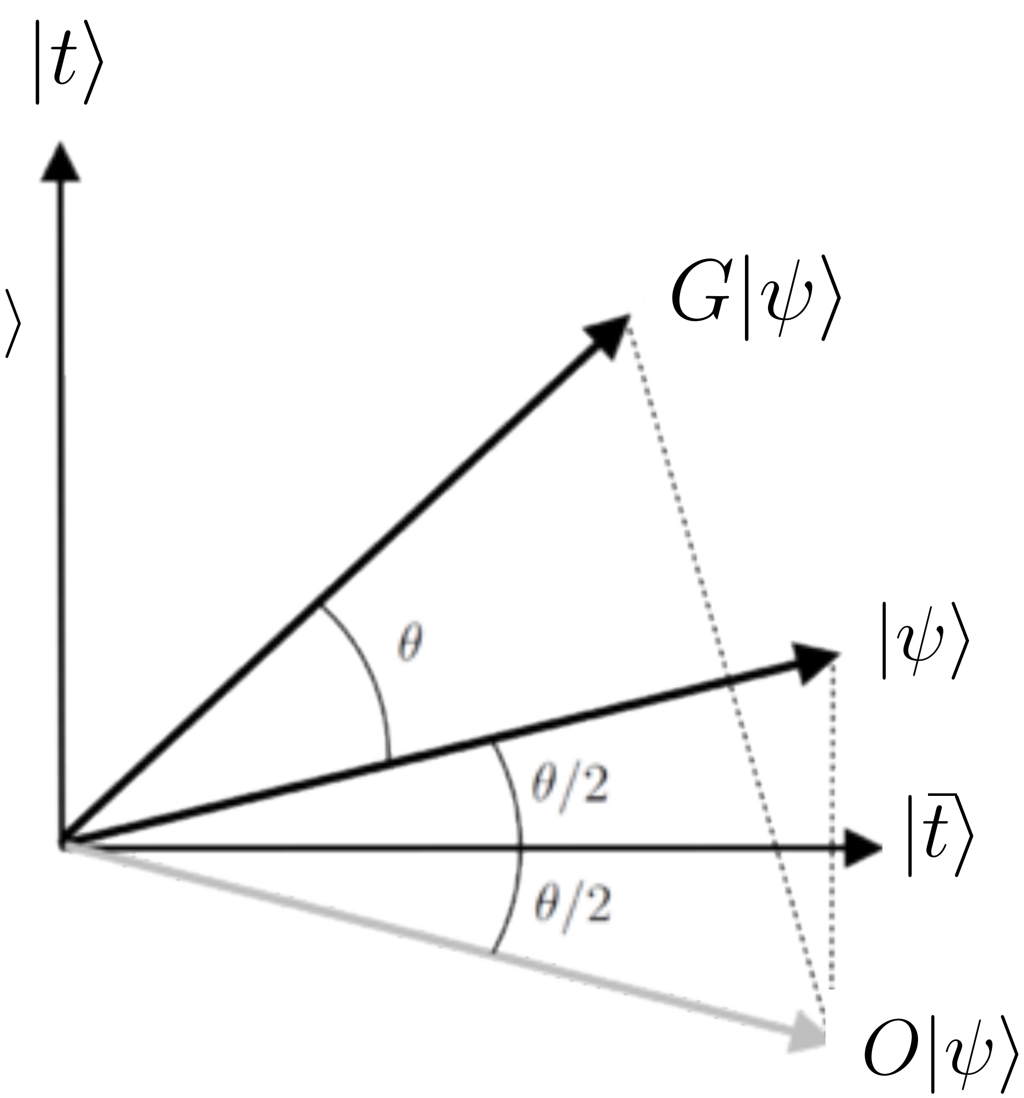
Performance

- Every application of the algorithm is a rotation of θ

$$G^k|\psi\rangle = \cos(\frac{(2k+1)\theta}{2})|\bar{t}\rangle + \sin(\frac{(2k+1)\theta}{2})|t\rangle$$

- The Ideal number of rotations is:

$$R = CI\left[\frac{\arccos\sqrt{M/N}}{\theta}\right]$$



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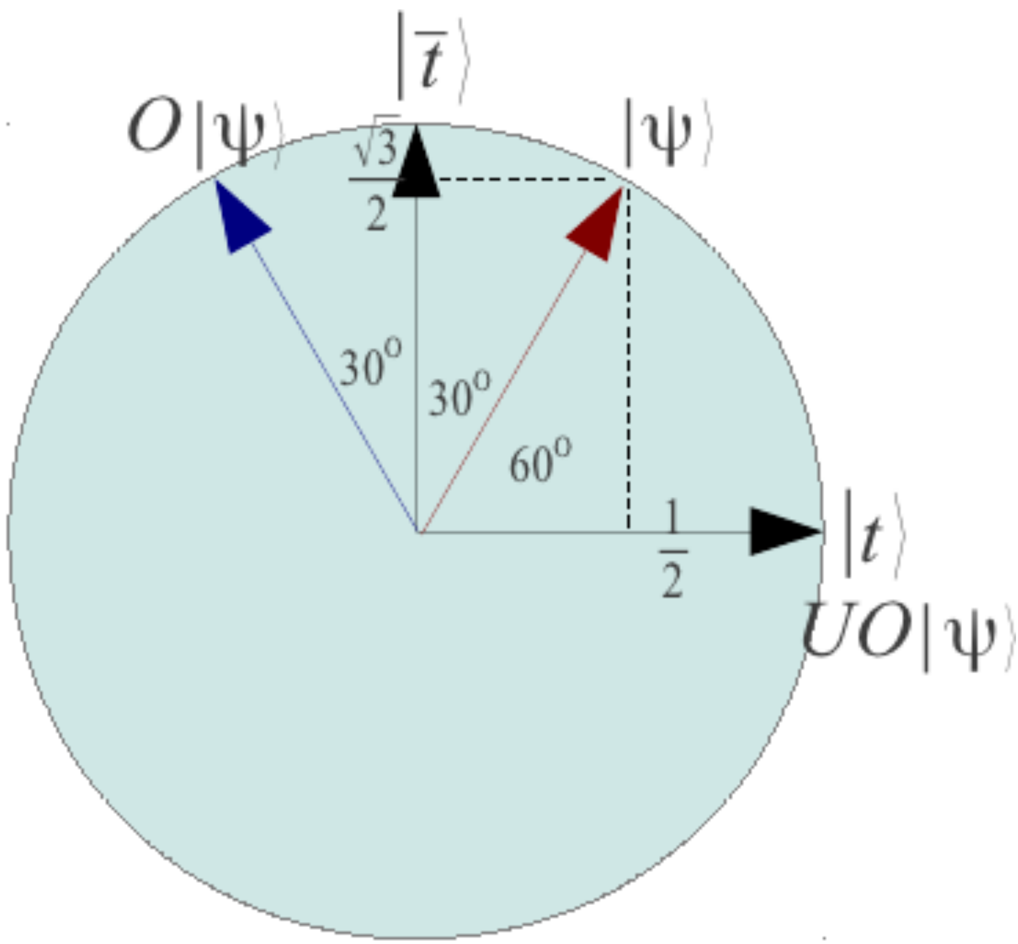
Grover's algorithm
2 qubits



N=4
Oracle marks one state M=1

$$|\psi\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$
$$= \frac{1}{2}|t\rangle + \frac{\sqrt{3}}{2}\frac{1}{\sqrt{3}}\left(|\bar{t}_1\rangle + |\bar{t}_2\rangle + |\bar{t}_3\rangle\right)$$
$$= \frac{1}{2}|t\rangle + \frac{\sqrt{3}}{2}|\bar{t}\rangle$$

After a single run and a projection measurement will get target state with probability 1!

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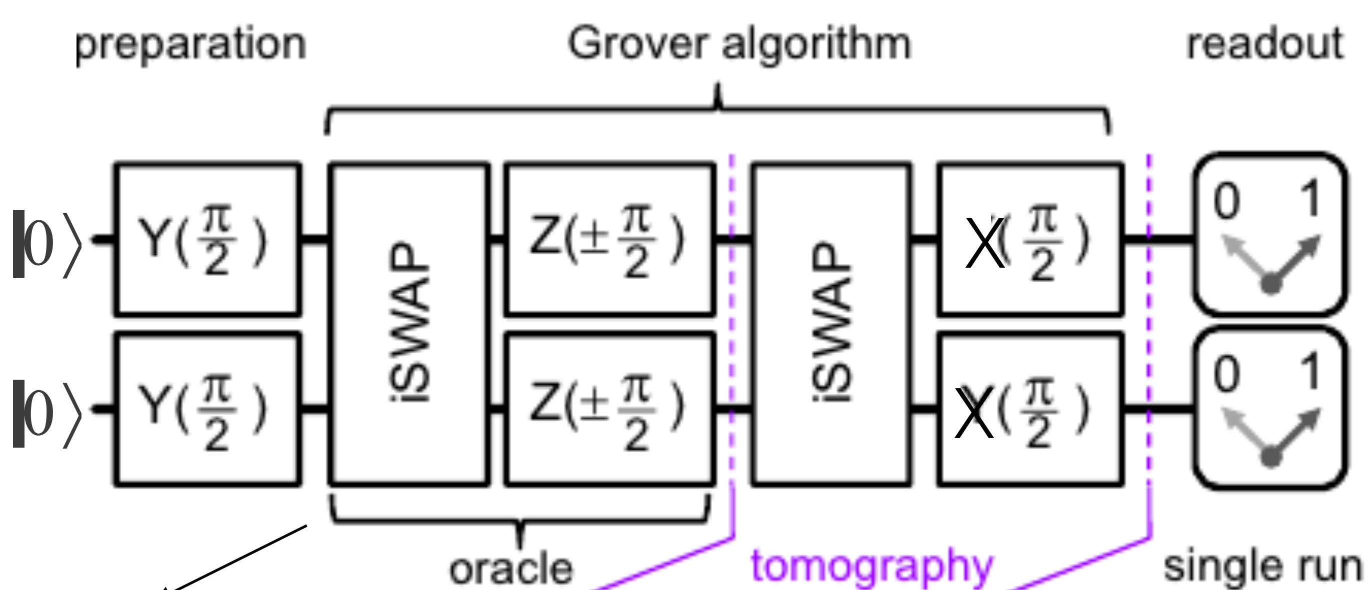
Grover's algorithm

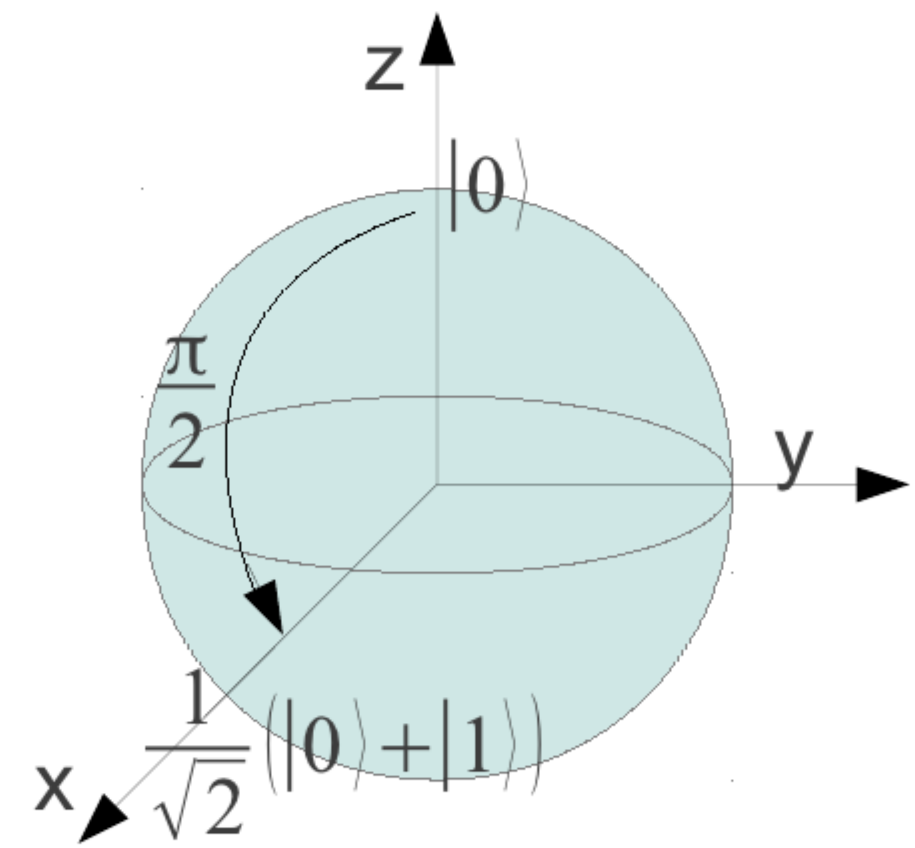
Circuit



$$|\psi\rangle = \frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix} \qquad \frac{1}{2}\begin{pmatrix}1\\-1\\1\\1\end{pmatrix}$$

for t=2 $|01\rangle$

Preparation



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$$

Grover's algorithm

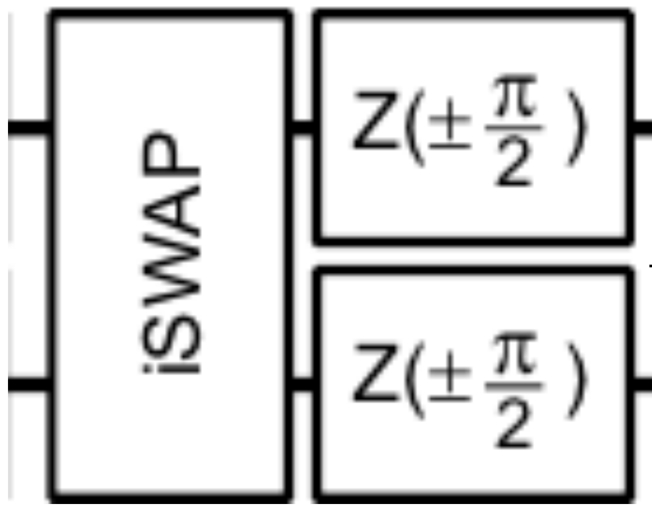
Oracle

Will get 2 cases:

$$e^{\pm i\pi} = \pm i$$

$$e^{\pm i0} = 1$$

$$|\psi\rangle = \frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$$



$$\frac{1}{2}\begin{pmatrix}1\\-1\\1\\1\end{pmatrix}$$

for t=2

$$Z(\theta) = e^{-i\frac{\theta}{2}Z} = \begin{pmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{pmatrix}$$

$$Z(\theta)\otimes Z(\phi) = \begin{pmatrix} e^{i\frac{-\theta-\phi}{2}} & 0 & 0 & 0 \\ 0 & e^{i\frac{-\theta+\phi}{2}} & 0 & 0 \\ 0 & 0 & e^{i\frac{\theta-\phi}{2}} & 0 \\ 0 & 0 & 0 & e^{i\frac{\theta+\phi}{2}} \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix} \xrightarrow{\text{iSWAP}} \frac{1}{2}\begin{pmatrix}1\\-i\\-i\\1\end{pmatrix} \xrightarrow{Z\left(-\frac{\pi}{2}\right)\otimes Z\left(-\frac{\pi}{2}\right)} \frac{1}{2}\begin{pmatrix}i\\-i\\-i\\-i\end{pmatrix}$$

$$\xrightarrow{Z\left(+\frac{\pi}{2}\right)\otimes Z\left(-\frac{\pi}{2}\right)} \frac{1}{2}\begin{pmatrix}1\\-1\\1\\1\end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix} \xrightarrow{\text{iSWAP}} \frac{1}{2}\begin{pmatrix}1\\-i\\-i\\1\end{pmatrix} \xrightarrow{Z\left(-\frac{\pi}{2}\right)\otimes Z\left(+\frac{\pi}{2}\right)} \frac{1}{2}\begin{pmatrix}1\\1\\-1\\1\end{pmatrix}$$

$$\xrightarrow{Z\left(+\frac{\pi}{2}\right)\otimes Z\left(+\frac{\pi}{2}\right)} \frac{1}{2}\begin{pmatrix}-i\\-i\\-i\\i\end{pmatrix}$$

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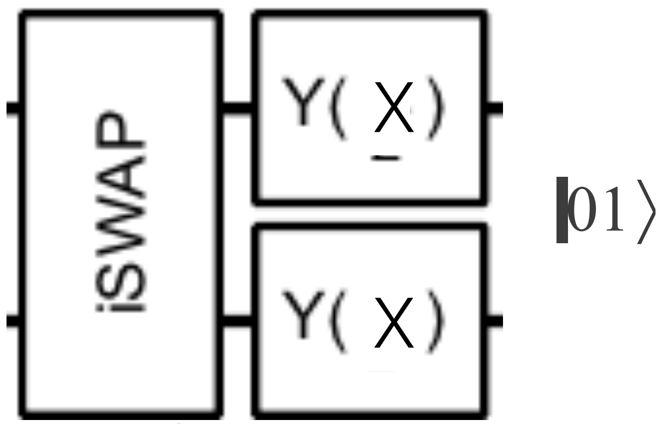
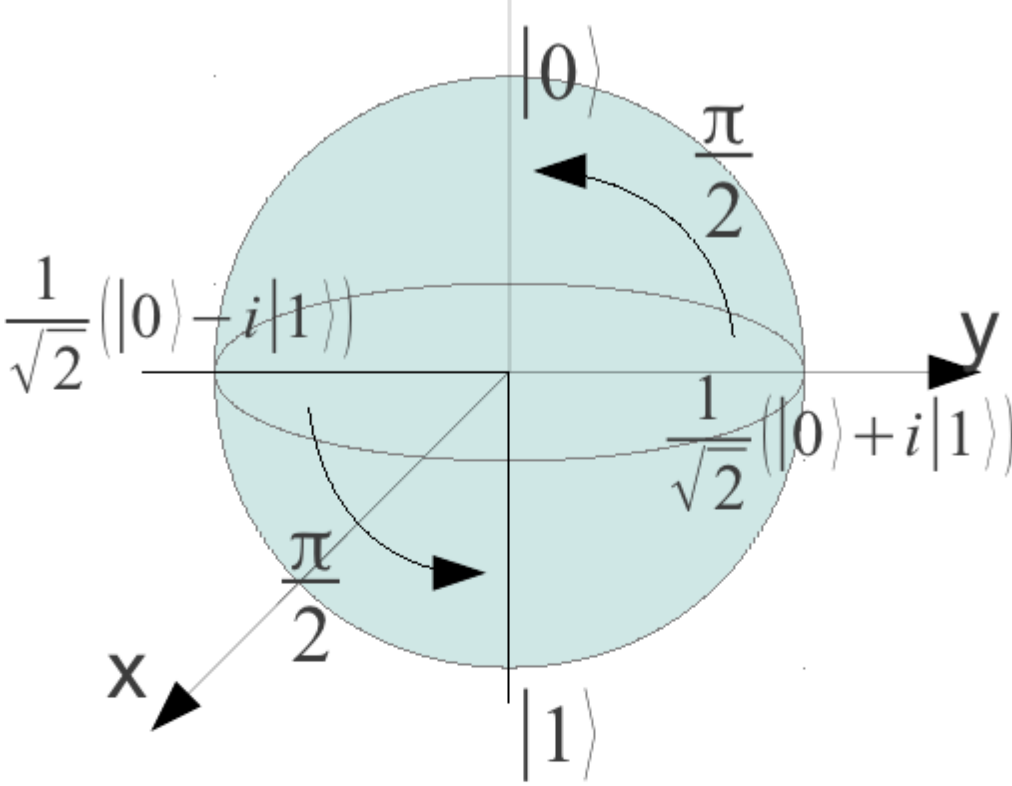
Grover's algorithm

Decoding

$$\frac{1}{2}\begin{pmatrix}1\\-1\\1\\1\end{pmatrix}$$

for t=2





$$\frac{1}{2}\begin{pmatrix}-1\\1\\1\\1\end{pmatrix} \xrightarrow{\text{iSWAP}} \frac{1}{2}\begin{pmatrix}-1\\-i\\-i\\1\end{pmatrix} = -\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

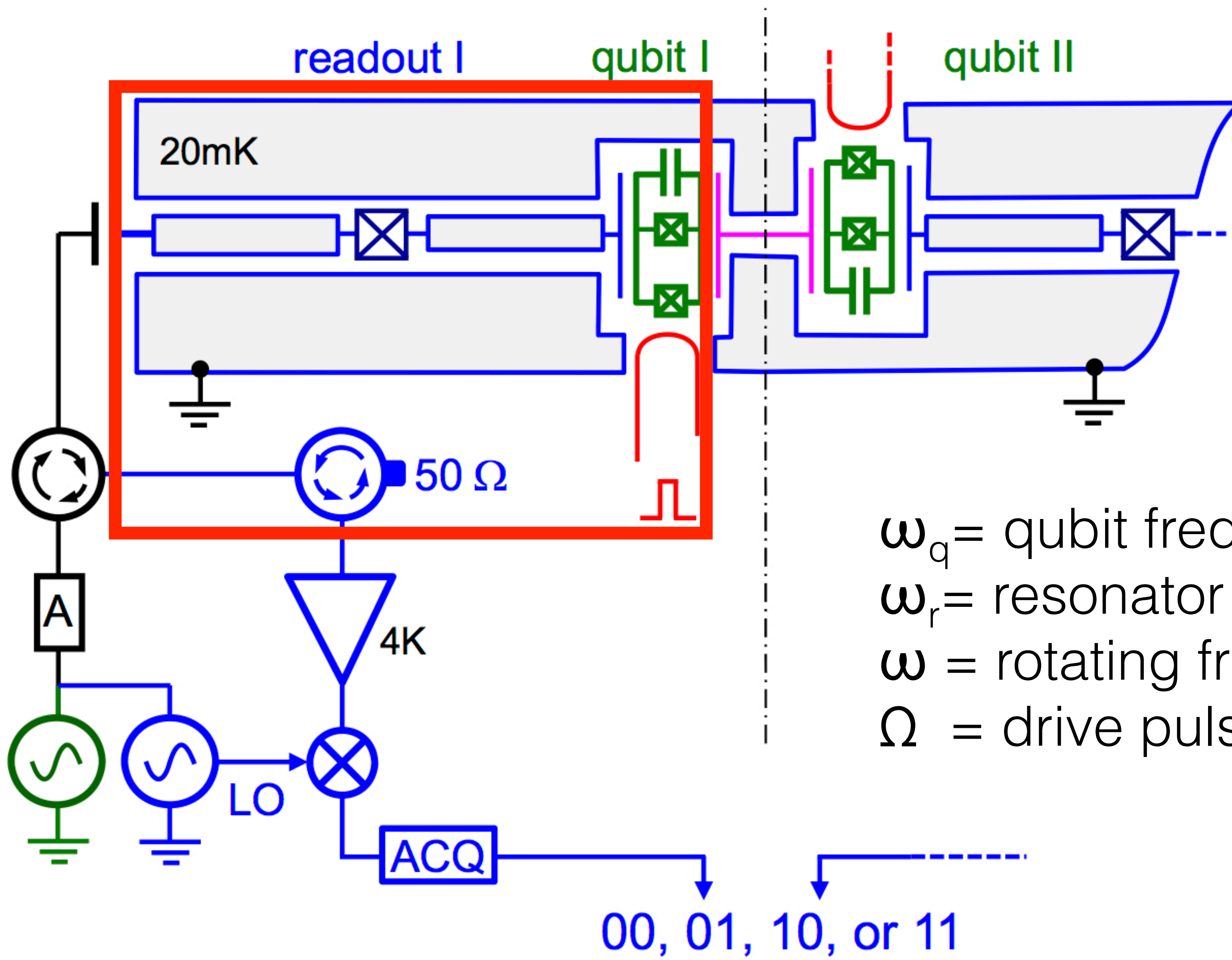
$$= |\circlearrowleft\rangle \otimes |\circlearrowleft\rangle$$

$$\frac{1}{2}\begin{pmatrix}1\\-1\\1\\1\end{pmatrix} \xrightarrow{\text{iSWAP}} \frac{1}{2}\begin{pmatrix}1\\-i\\i\\1\end{pmatrix} = |\circlearrowleft\rangle \otimes |\circlearrowright\rangle$$

$$\frac{1}{2}\begin{pmatrix}1\\1\\-1\\1\end{pmatrix} \xrightarrow{\text{iSWAP}} \frac{1}{2}\begin{pmatrix}1\\i\\-i\\1\end{pmatrix} = |\circlearrowright\rangle \otimes |\circlearrowleft\rangle$$

$$\frac{1}{2}\begin{pmatrix}1\\1\\1\\-1\end{pmatrix} \xrightarrow{\text{iSWAP}} \frac{1}{2}\begin{pmatrix}1\\-i\\-i\\-1\end{pmatrix} = |\circlearrowright\rangle \otimes |\circlearrowright\rangle$$

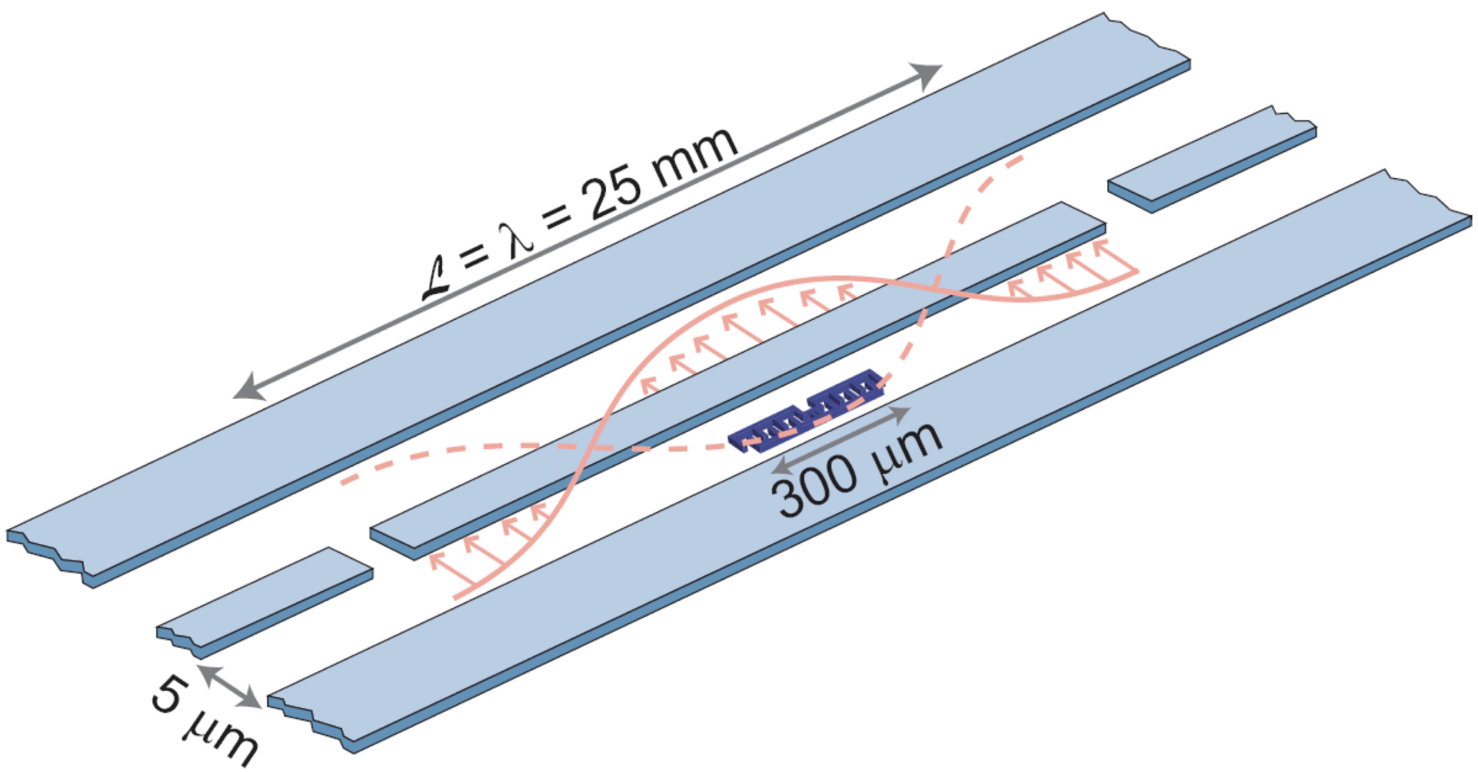
Experimental setup



ω_q = qubit frequency
ω_r = resonator frequency
ω = rotating frame frequency
Ω = drive pulse amplitude

Single qubit manipulation

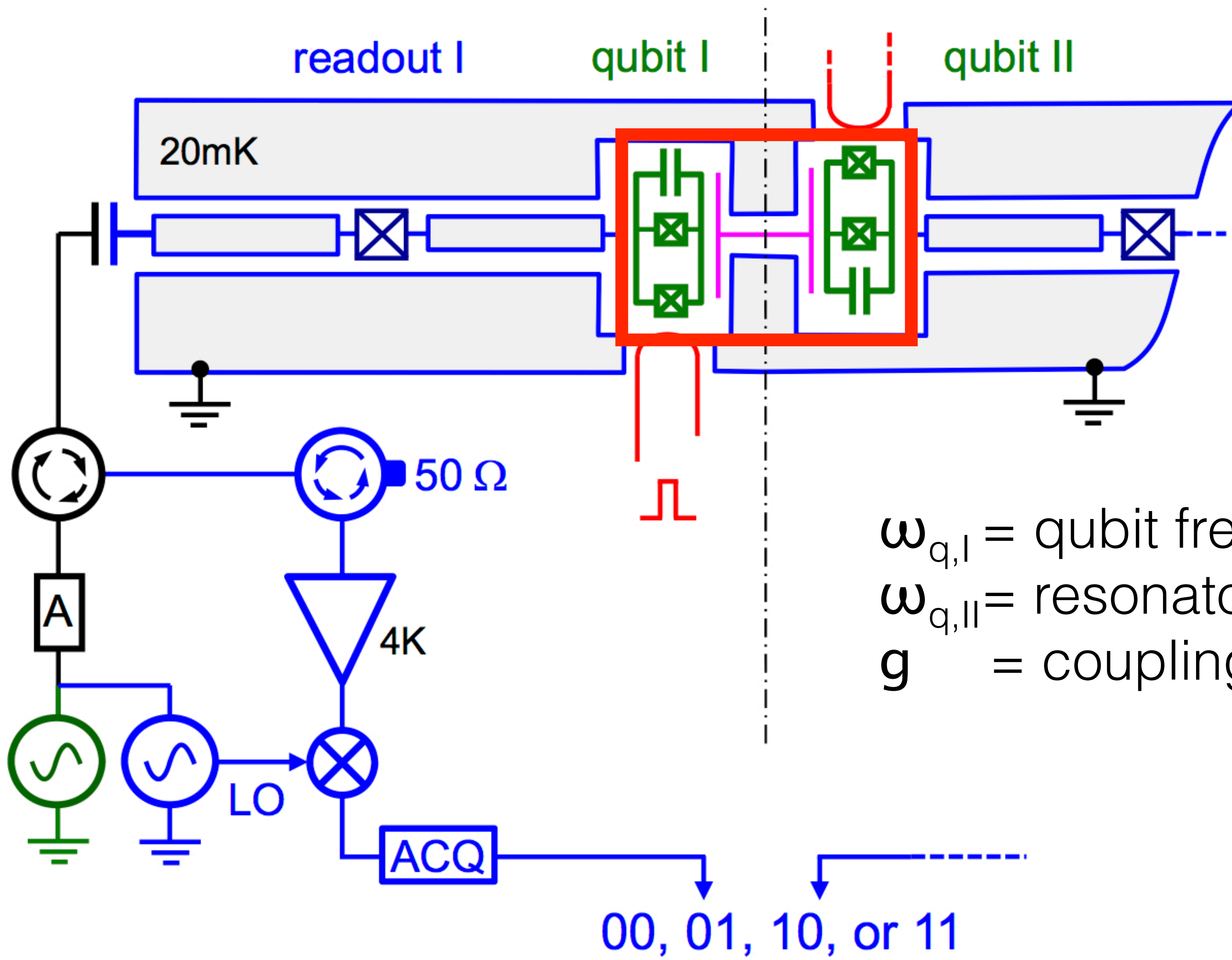
- Qubit frequency control via flux bias
- Rotations around z axis: detuning Δ
- Rotations around x and y axes: resonant pulses Ω



Δ

$$H_{rot} = \frac{\omega_q - \omega}{2}\sigma_z + \frac{\Omega}{2}(\cos\phi\sigma_x + \sin\phi\sigma_y) \equiv \frac{\delta}{2}\sigma_z + \frac{\Omega_x}{2}\sigma_x + \frac{\Omega_y}{2}\sigma_y.$$

Experimental setup



$\omega_{q,I}$ = qubit frequency
$\omega_{q,II}$ = resonator frequency
g = coupling strength

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Qubit capacitive coupling

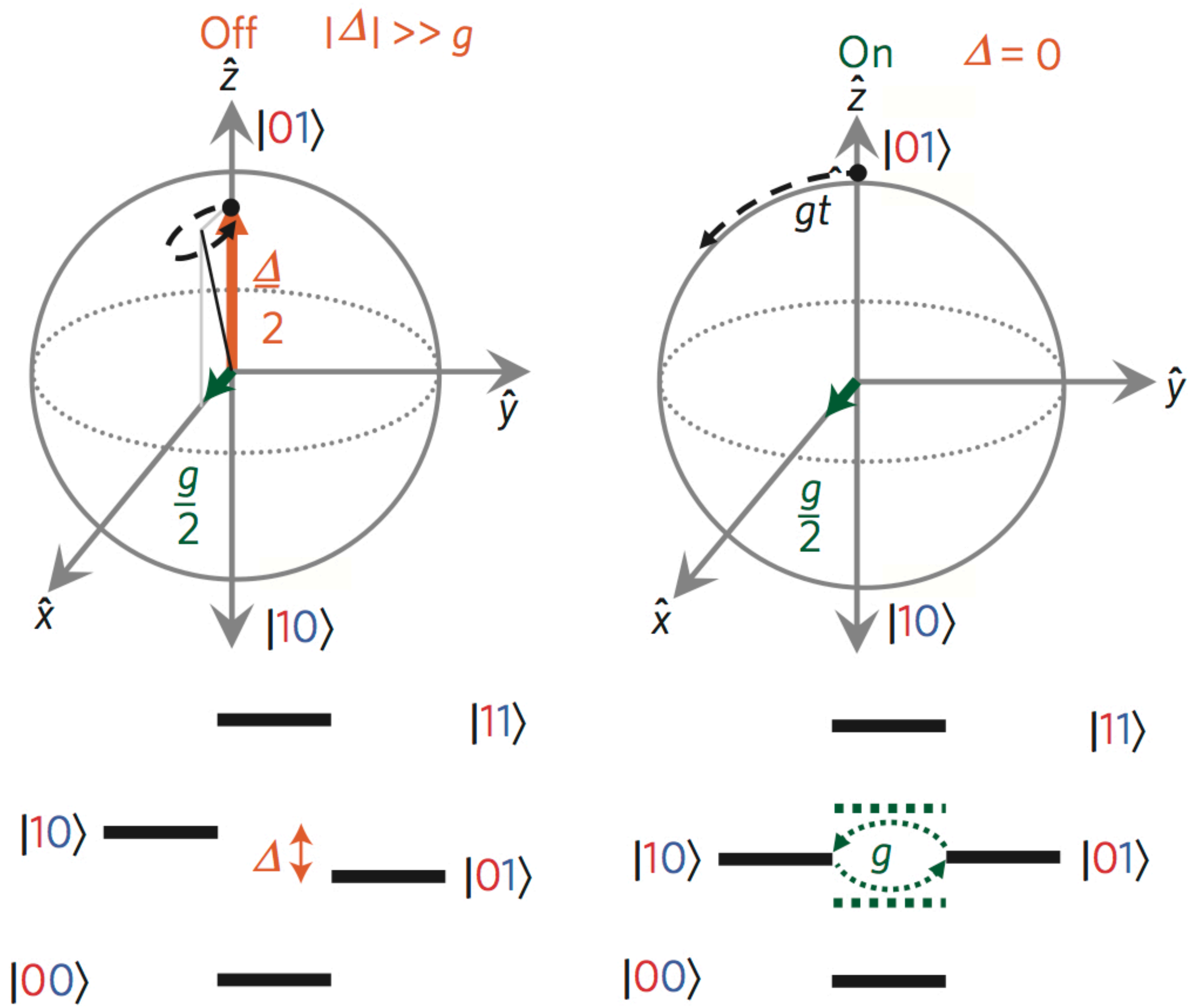
In the rotating frame

$$\omega = \omega_{q,II}$$

the coupling Hamiltonian is:

$$H_{tot} = h(\omega_{q,I}\sigma_z^I + \omega_{q,II}\sigma_z^{II} + H_{int})$$

$$H_{int} = g(|10\rangle\langle 01| + |01\rangle\langle 10|)$$



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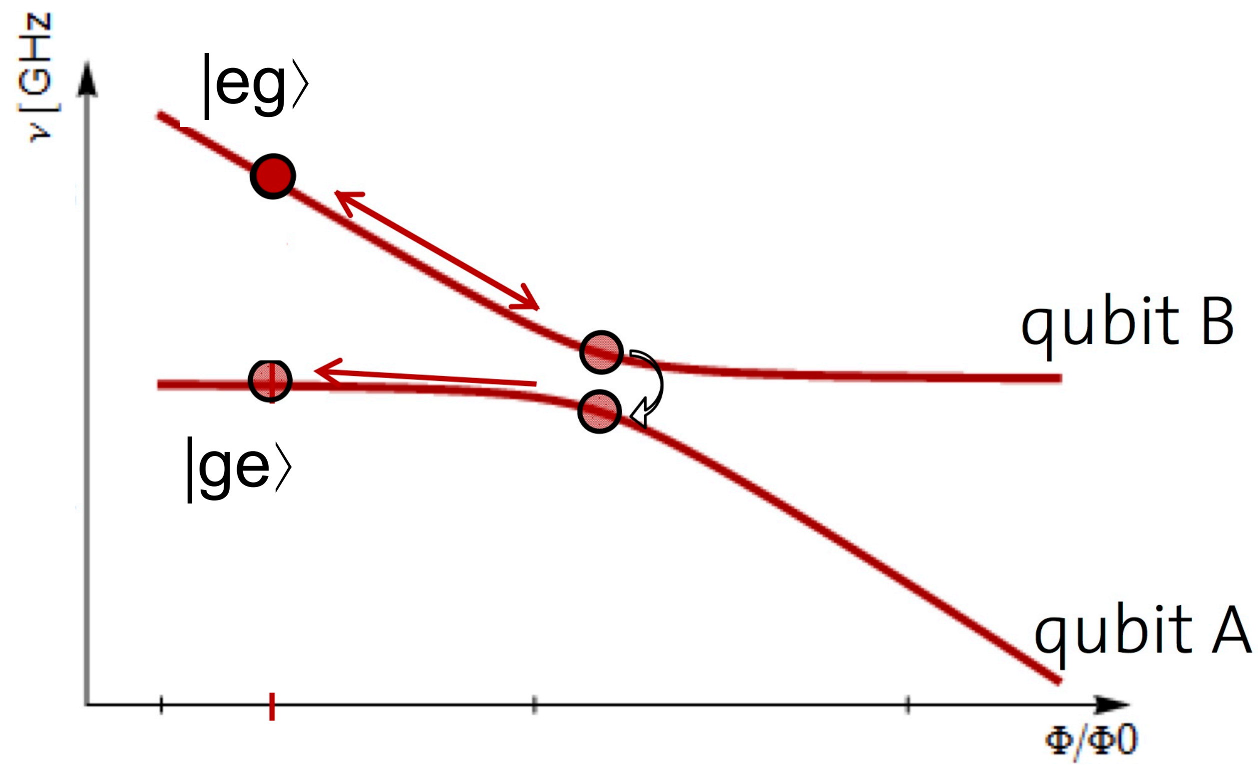
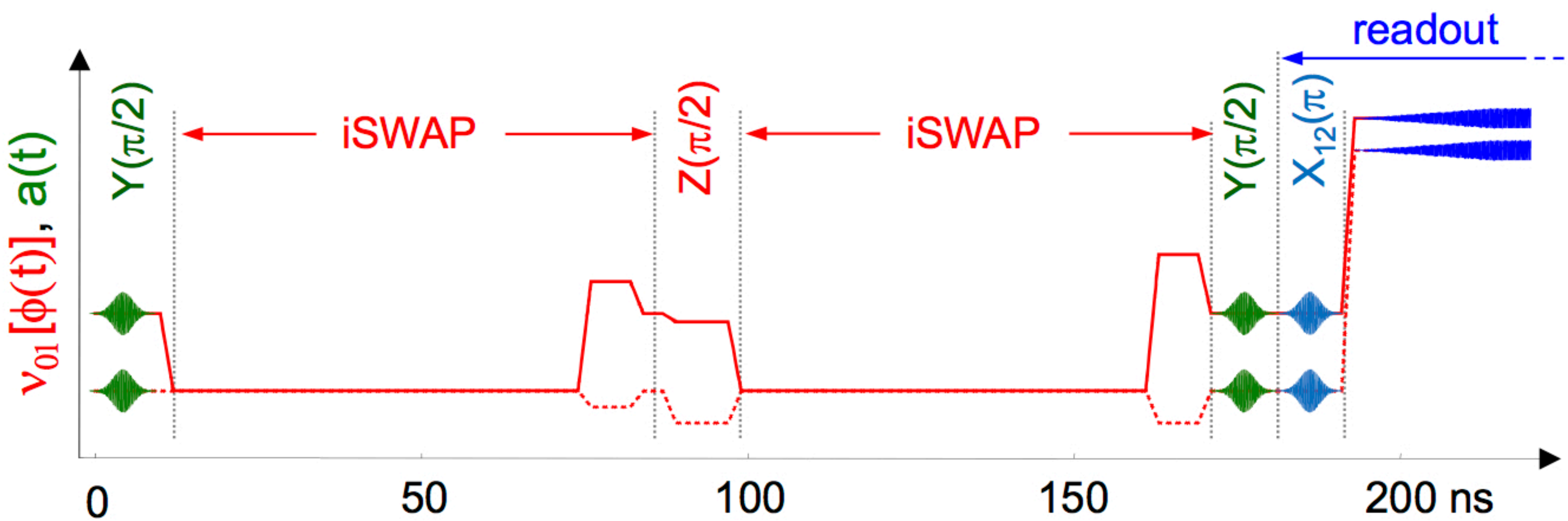
iSWAP gate

- Controlled interaction between $\langle 10|$ and $\langle 01|$
- By letting the two states interact for $t = \pi/g$ we obtain an iSWAP gate!

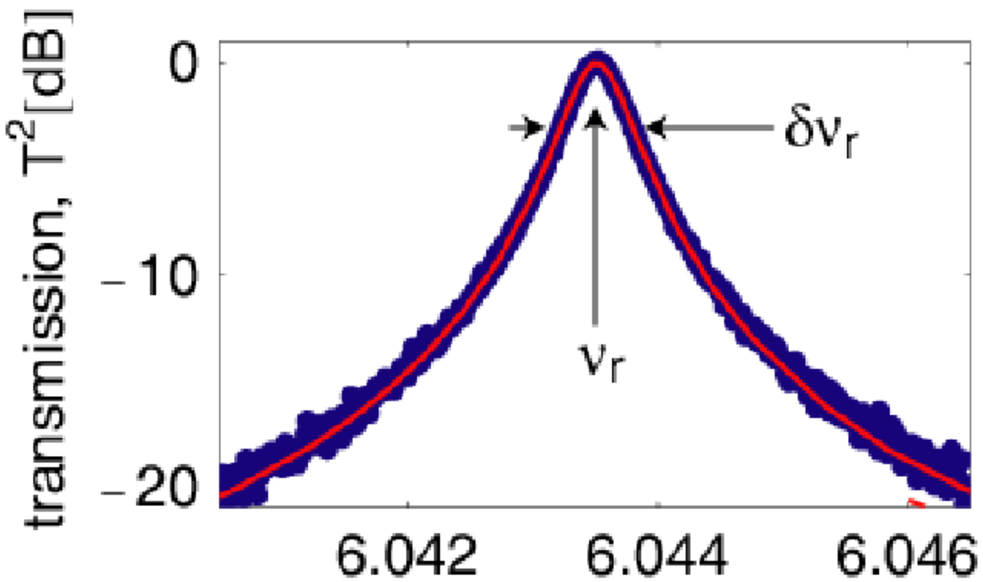


$$U_{\text{int}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(gt/2) & -i\sin(gt/2) & 0 \\ 0 & -i\sin(gt/2) & \cos(gt/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Pulse sequence



Linear transmission line and single-shot measurement

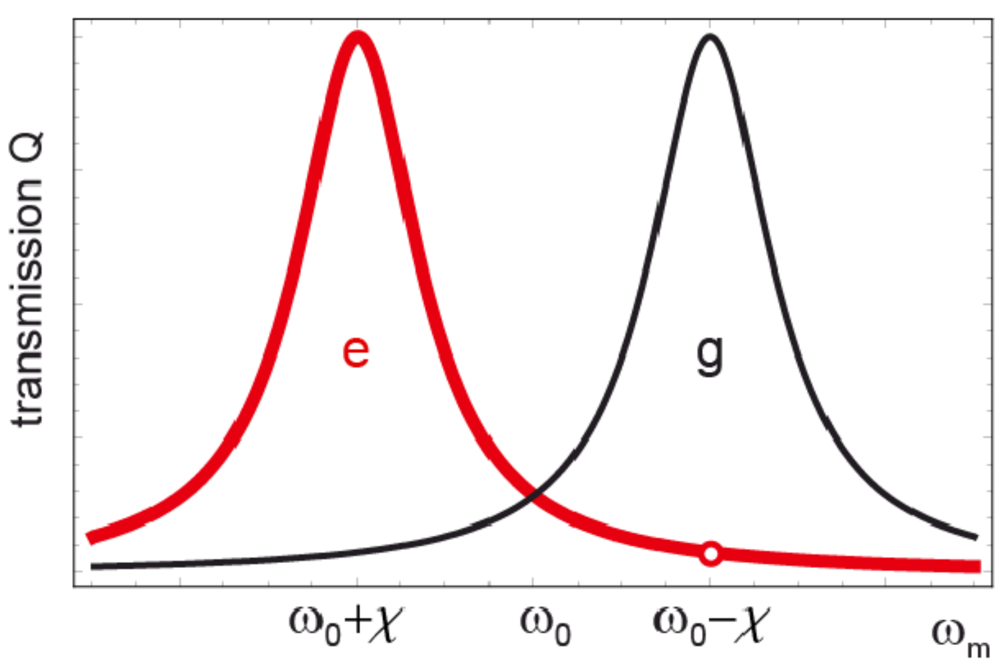


Quality factor of empty linear transmission line $Q = \frac{\nu_r}{\Delta \nu_r}$

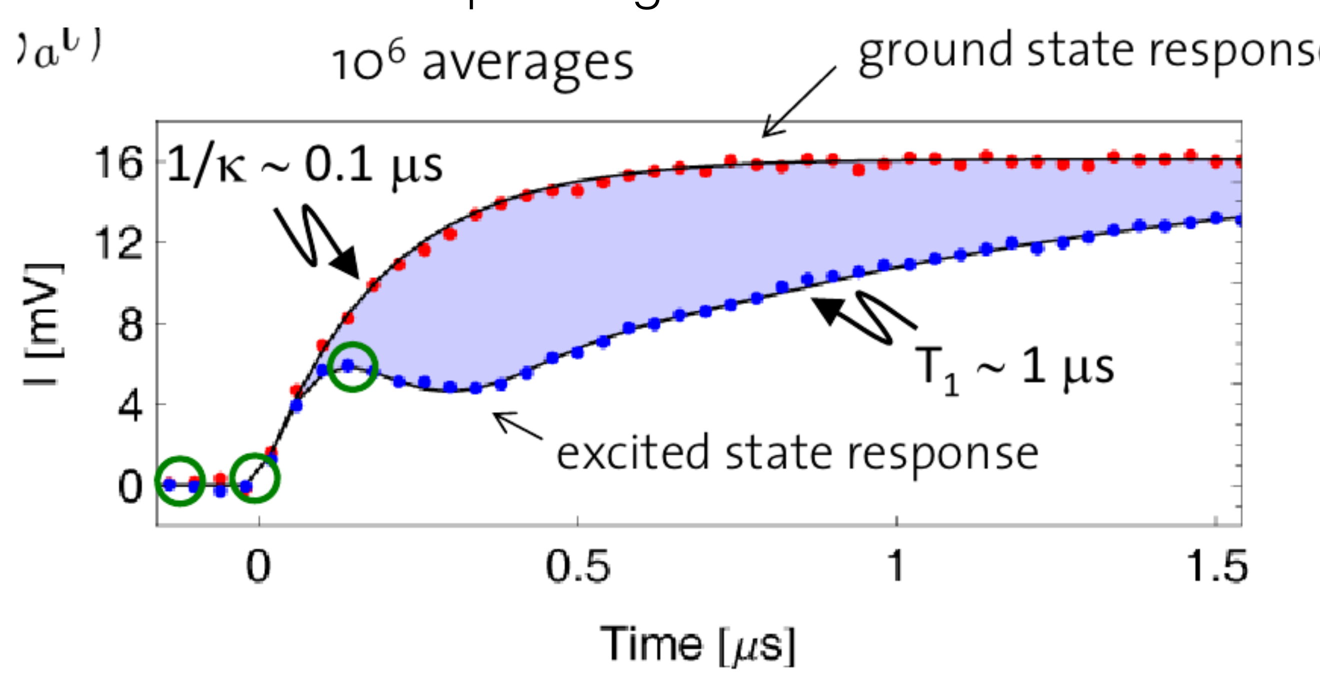
With resonant frequency ν_r

Presence of a transmon in $|g\rangle$ state shifts the resonant frequency of the transmission line $\omega_0 = 2\pi\nu_r \rightarrow \omega_0 - \chi$

If microwave at $\omega_0 - \chi$ full transmission for $|g\rangle$ state, partial for $|e\rangle$ state



Current corresponding to transmitted EM



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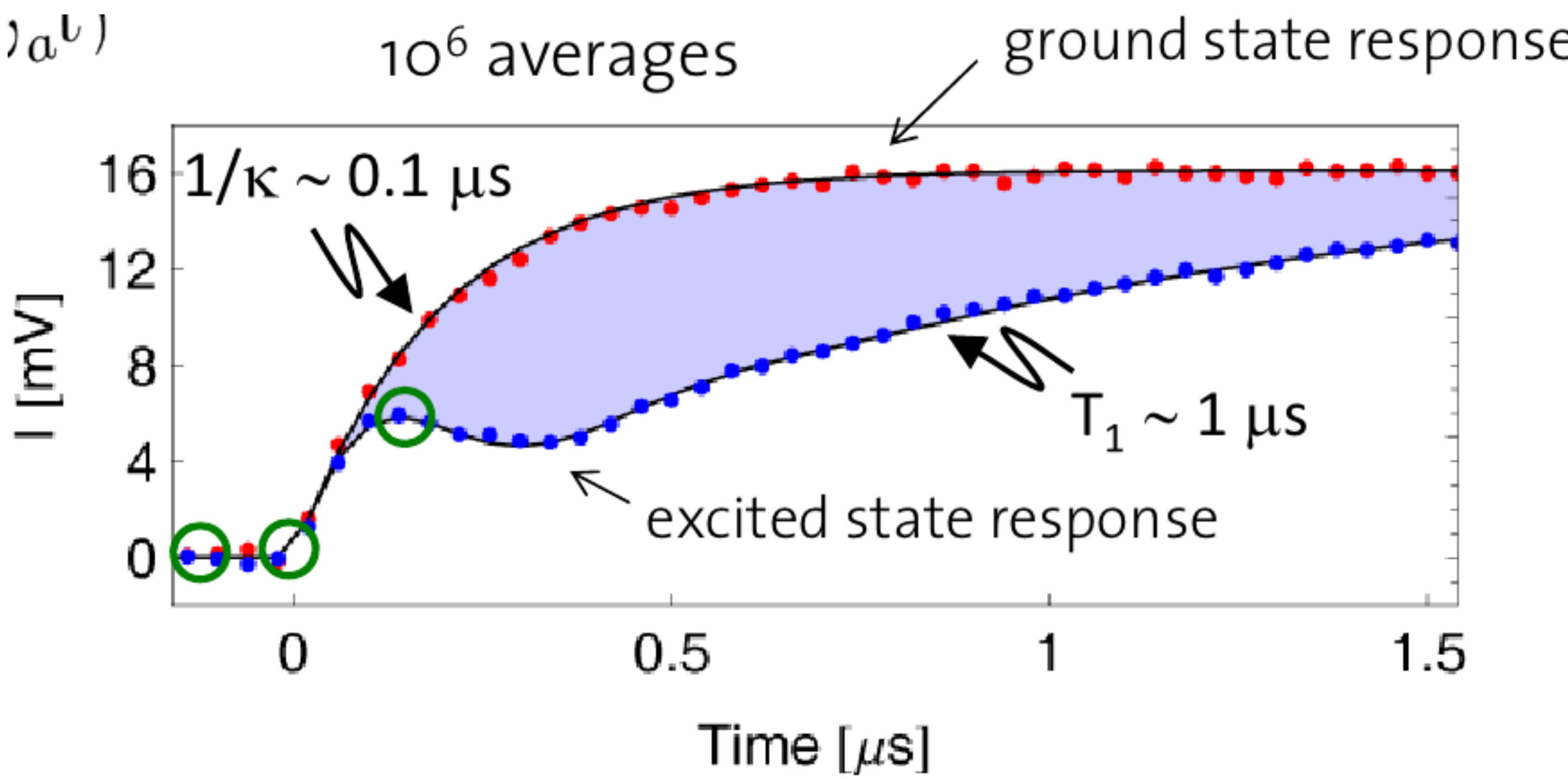
Linear transmission line and single-shot measurement



Single shot

- If there was no noise, would get either blue or red curve
- Real curve so noisy that cannot tell whether $|g\rangle$ or $|e\rangle$

Cannot do single-shot readout

We need an amplifier which increases the area between $|g\rangle$ and $|e\rangle$ curves, but does not amplify the noise

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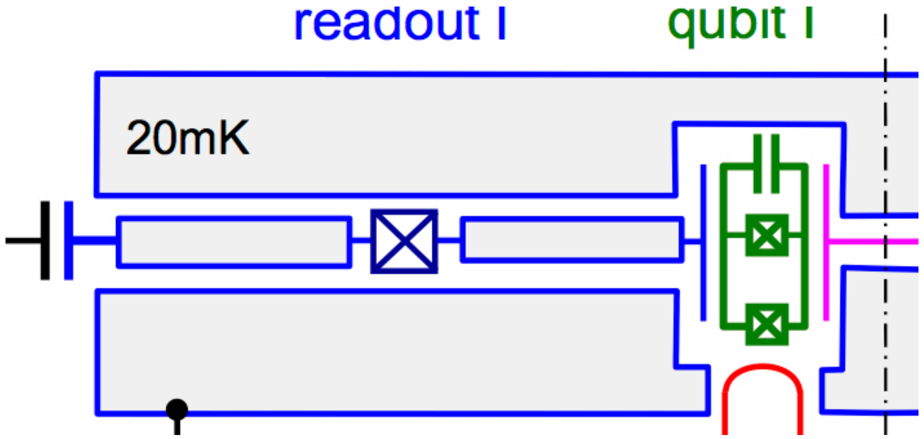
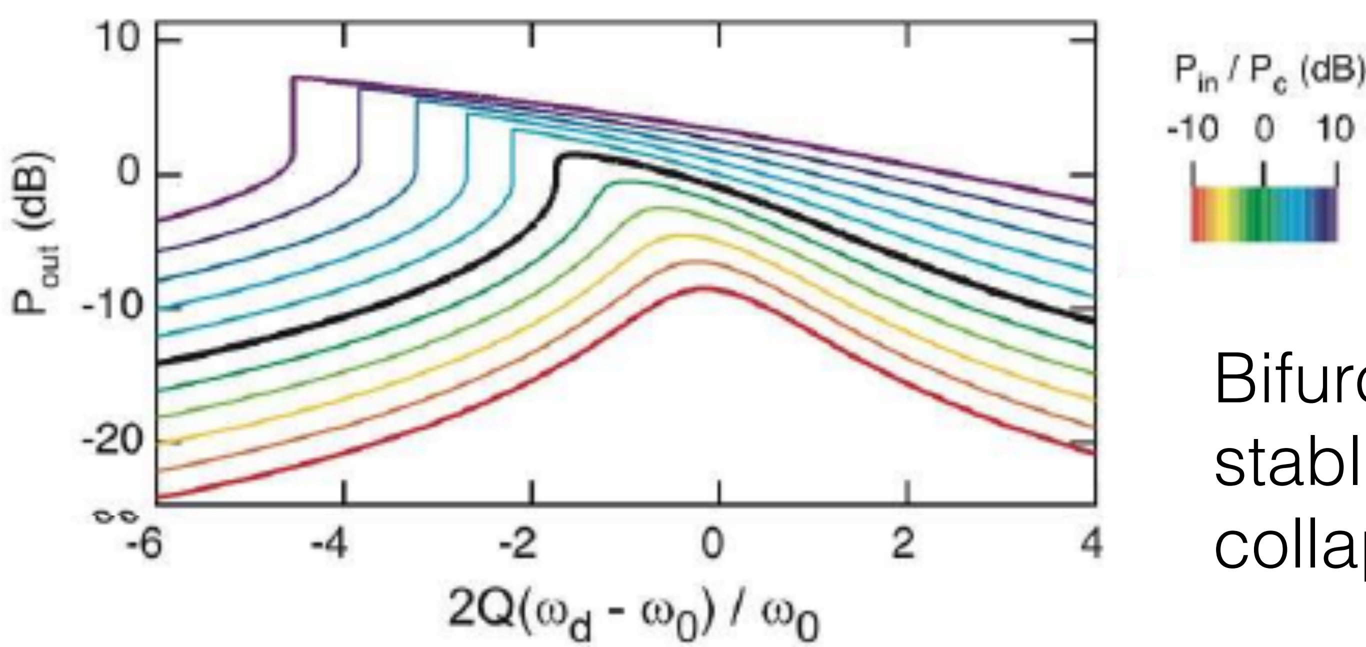
Josephson Bifurcation Amplifier (JBA)



Nonlinear transmission line due to Josephson junction
Resonant frequency ω_0



At $P_{in} = P_C$ max. slope diverges

Bifurcation: at the correct (P_{in}, ω_d) two stable solutions, can map the collapsed state of the qubit to them

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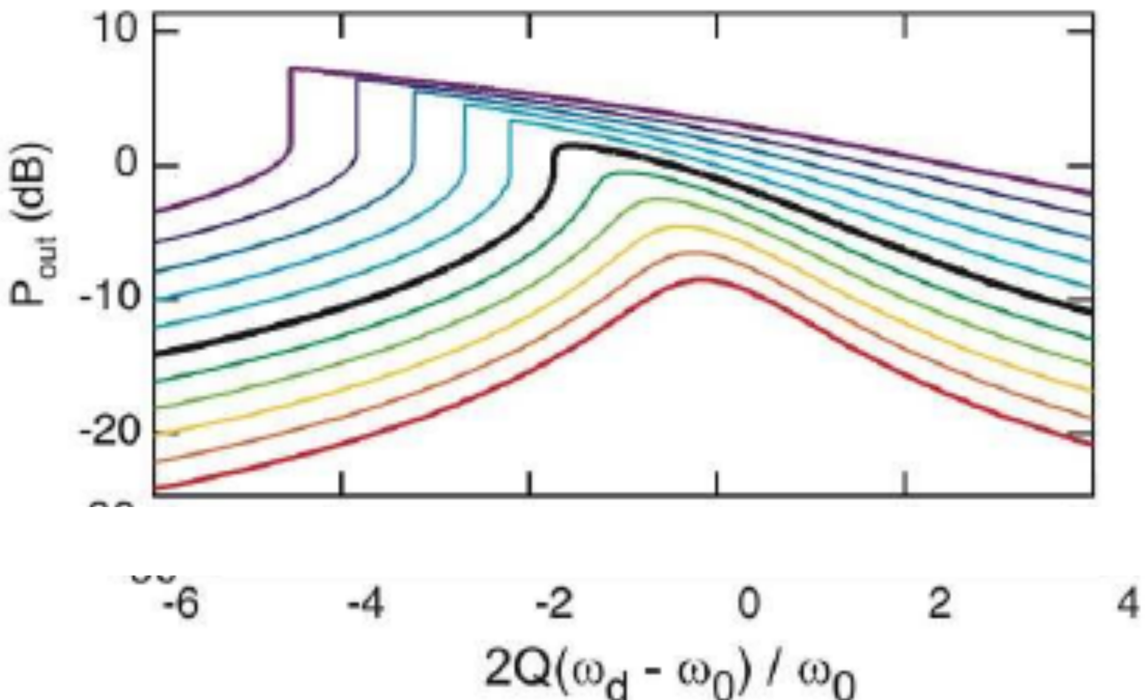
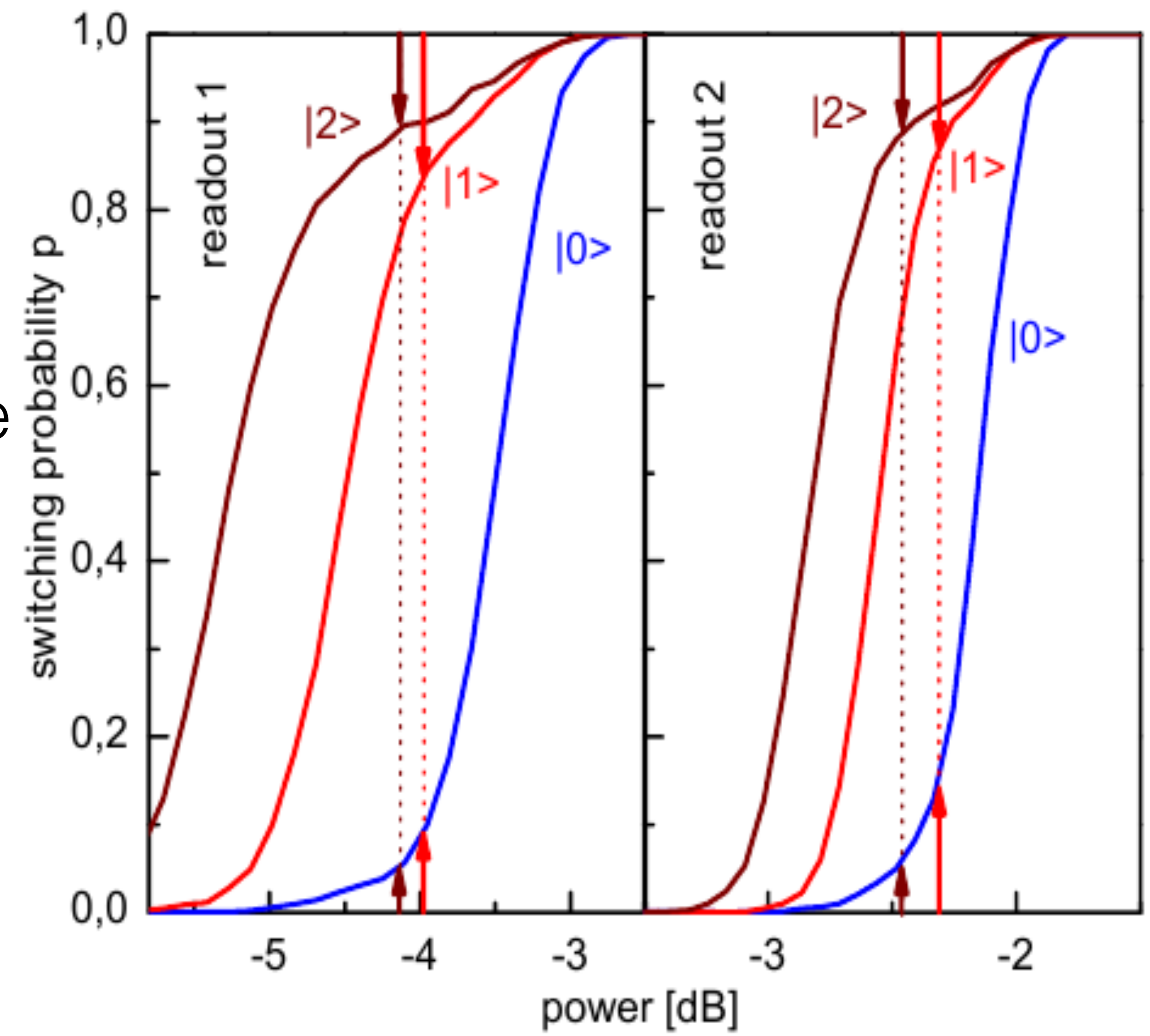


Switching probability p: probability that the JBA changes to the higher-amplitude solution

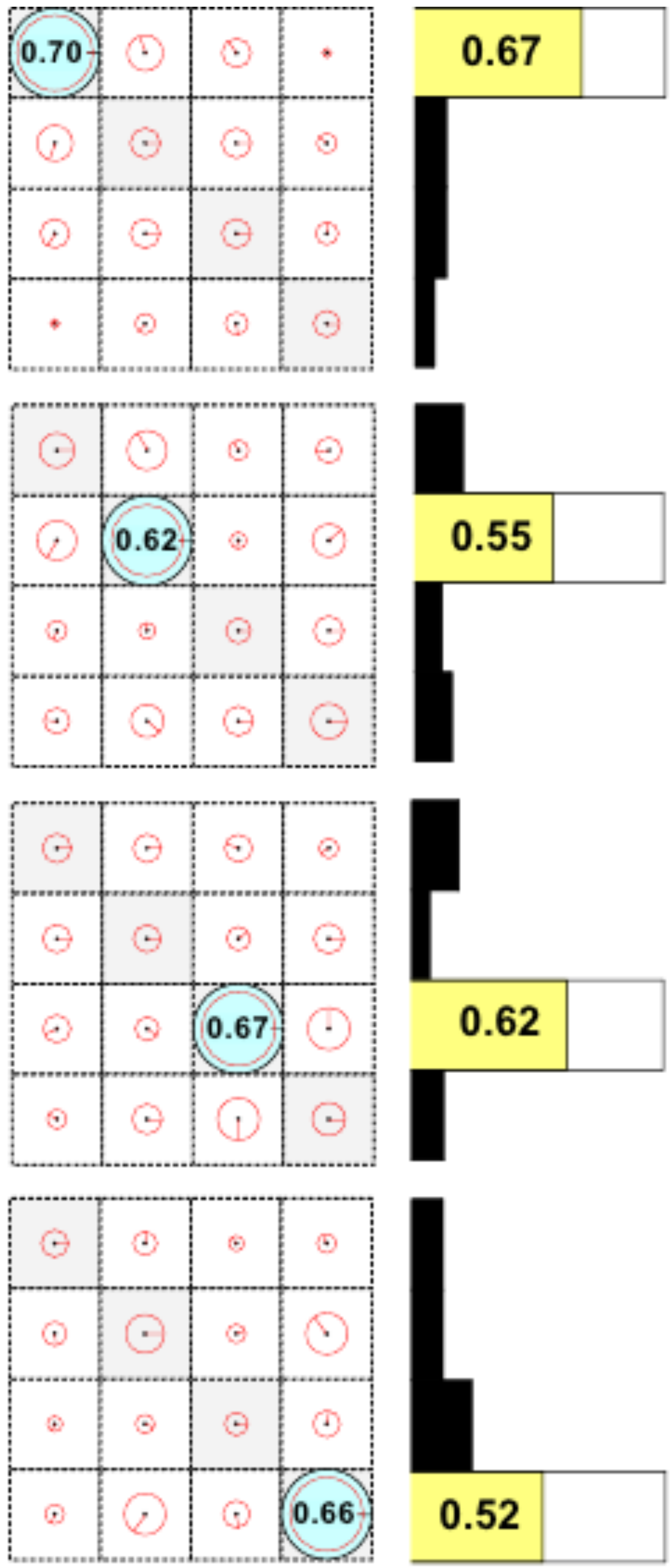
- Excite $|1\rangle \rightarrow |2\rangle$
- Choose power corresponding to the biggest difference of switching probabilities

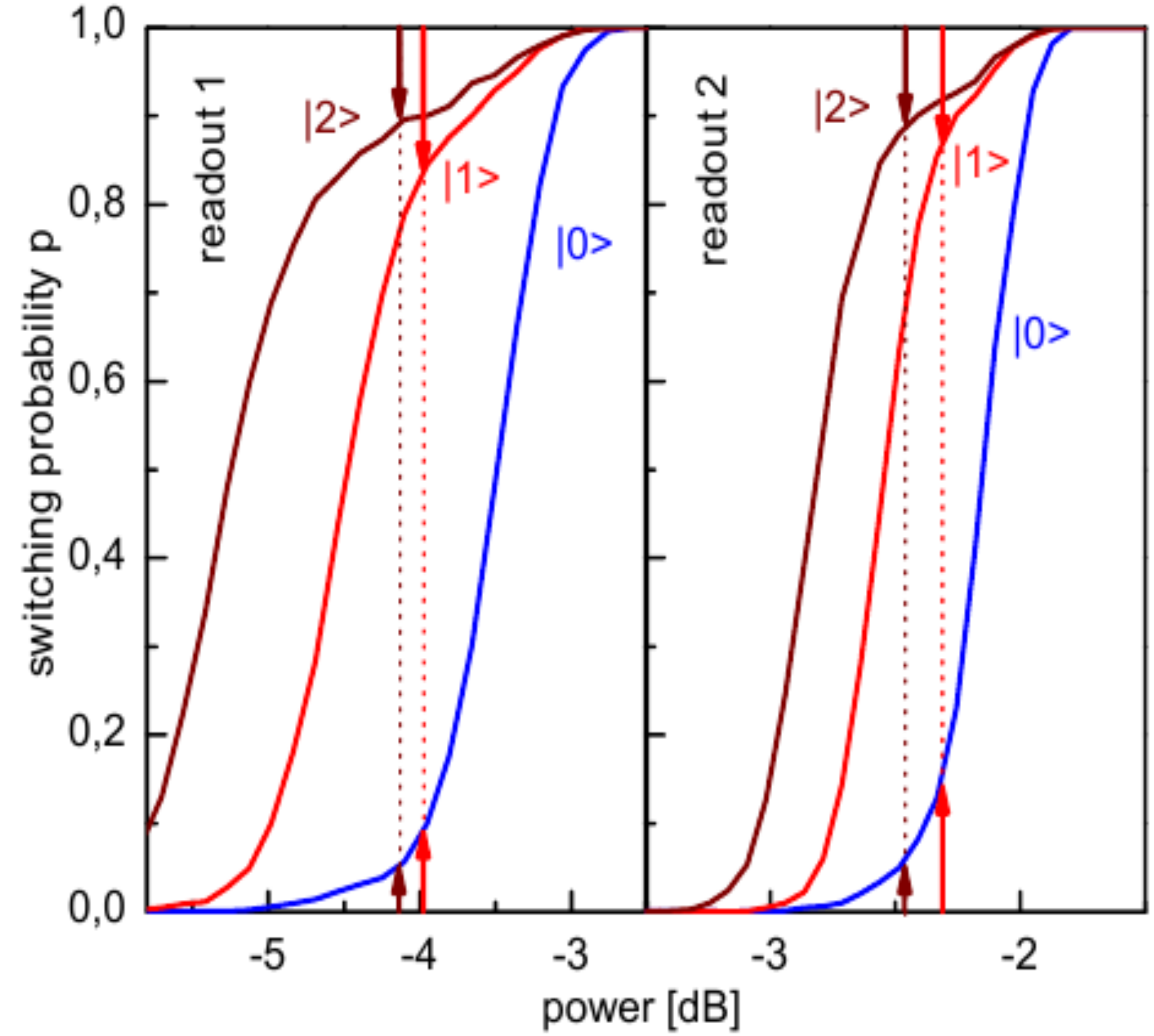


Errors

- Nonzero probability of incorrect mapping
- Crosstalk

Errors





- Nonzero probability of incorrect mapping
- Crosstalk

Conclusions

- Gate operations of Grover algorithm successfully implemented with capacitively coupled transmon qubits
- Arrive at the target state with probability 0.62 – 0.77 (tomography)
- Single-shot readout with JBA (no quantum speed-up without it)
- Measure the target state in single shot with prob 0.52 – 0.67 (higher than 0.25 classically)

Sources

- Dewes, A; Lauro, R; Ong, FR; et al., "*Demonstrating quantum speed-up in a superconducting two-qubit processor*", arXiv:1109.6735 (2011)

- Bialczak, RC; Ansmann, M; Hofheinz, M; et al., "*Quantum process tomography of a universal entangling gate implemented with Josephson phase qubits*", Nature Physics 6, 409 (2007)

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Thank you for your attention

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