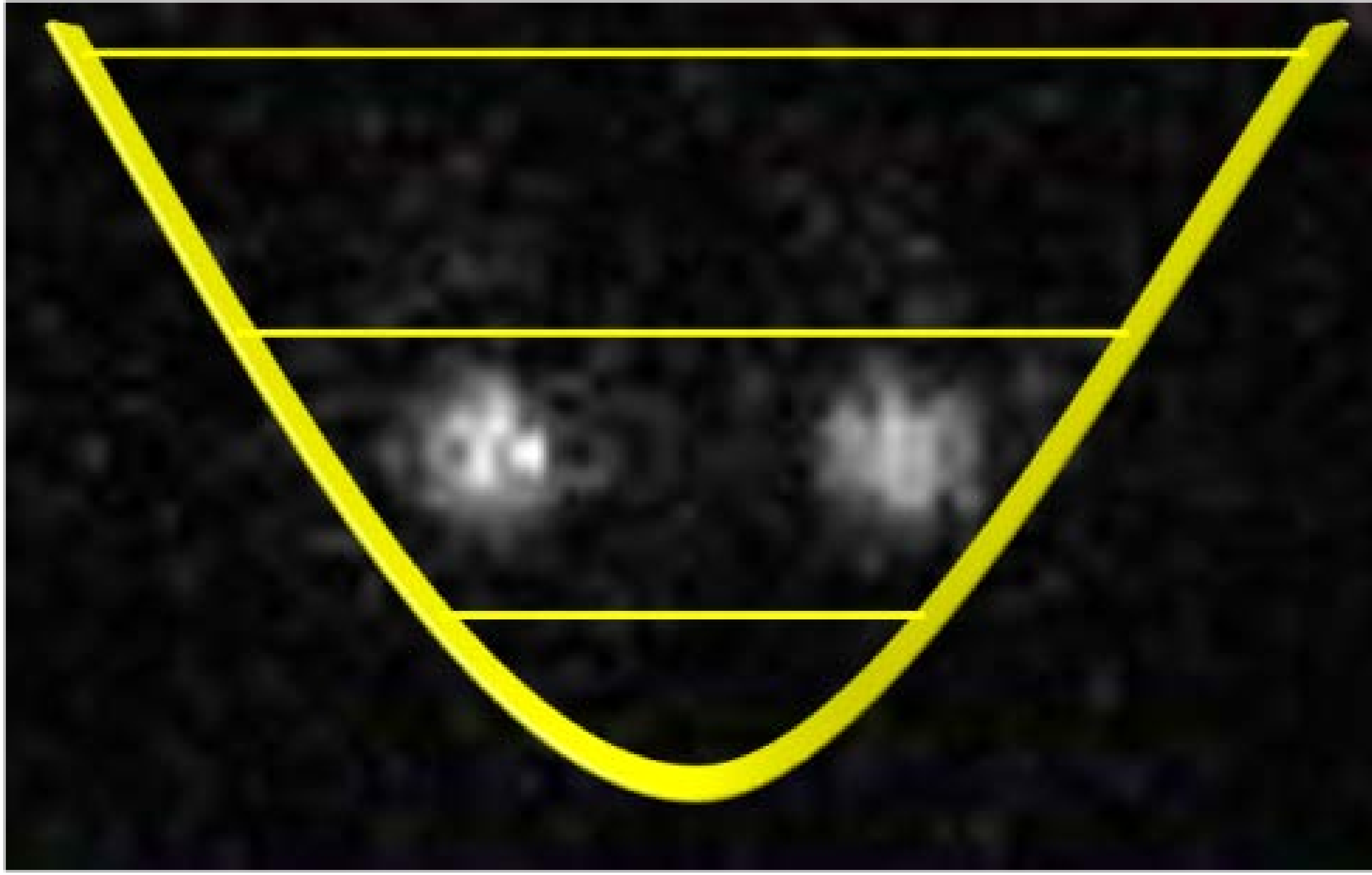
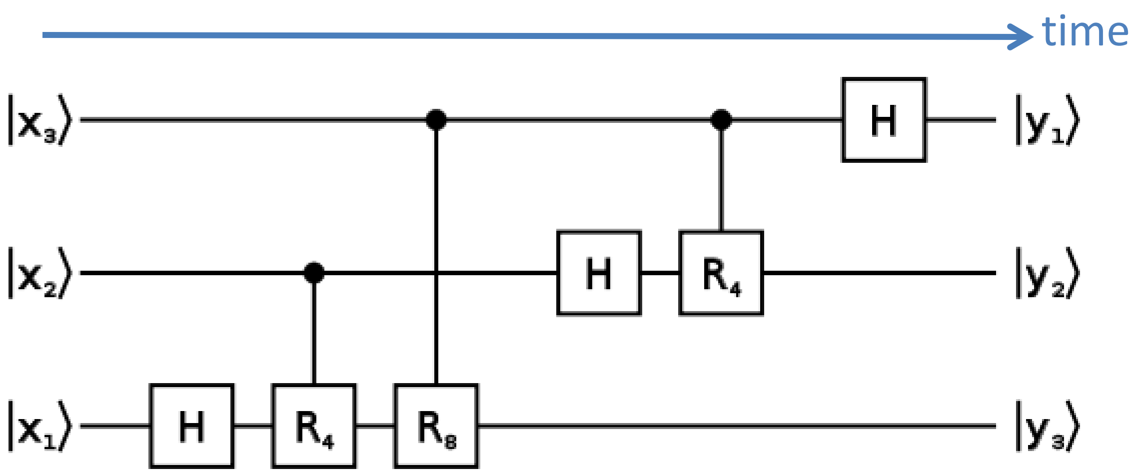


Pre-requisites for quantum computation

Collection of two-state quantum systems - Deutsch, Proc. Roy. Soc. 1985

Initialise qubit to single state



Detect qubit state

Operations which manipulate isolated qubits or pairs of qubits

Large scale device:

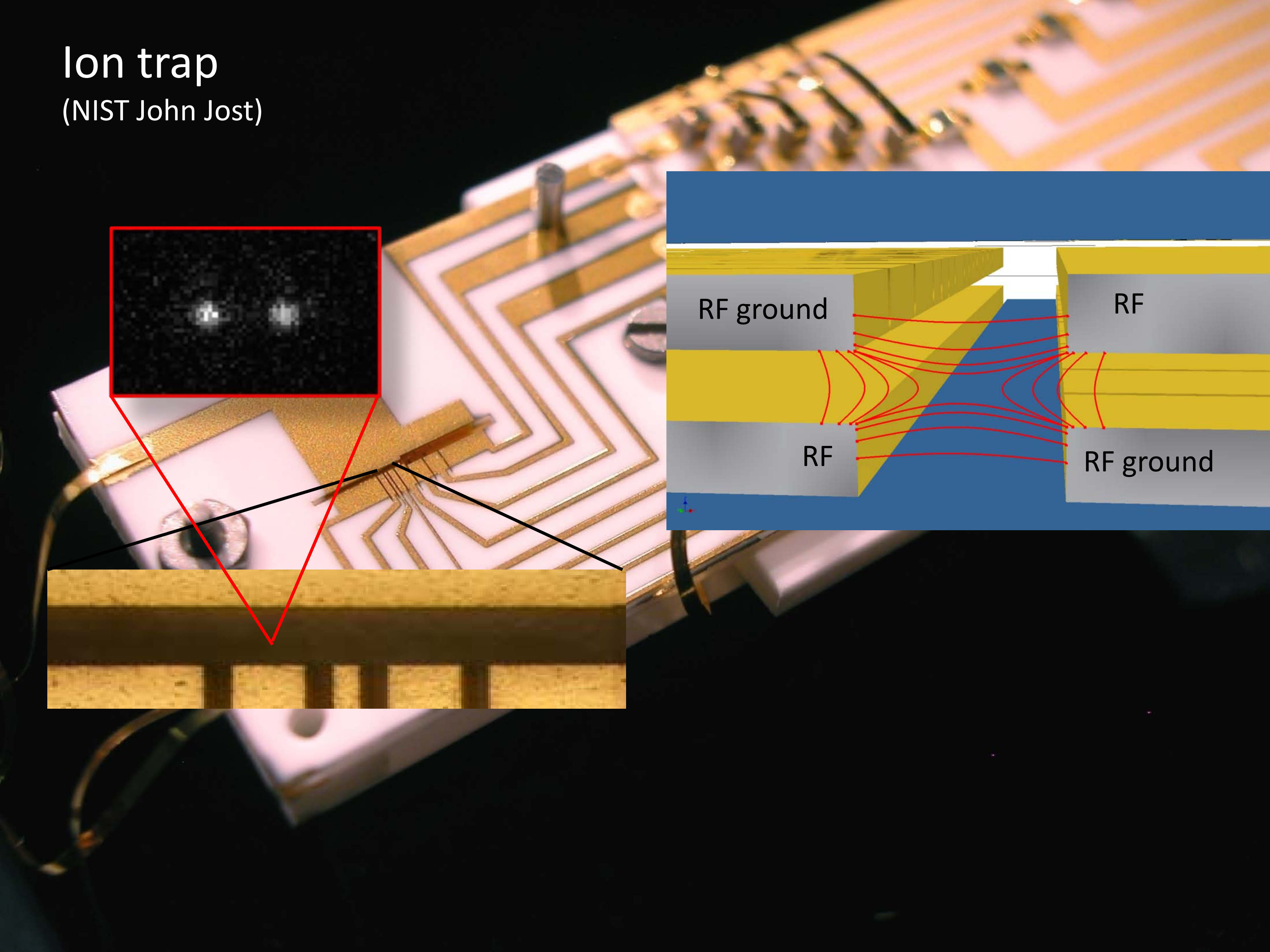
Transport information around processor/distribute entangled states

Perform operations accurately enough to achieve fault-tolerant error-correction

(accuracy ~ 0.9999 required)

Ion trap

(NIST John Jost)



Isolating single charged atoms

Laplace‘s equation
– no chance to trap with static fields

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Paul trap: Use a ponderomotive potential – change potential fast compared to speed of ion

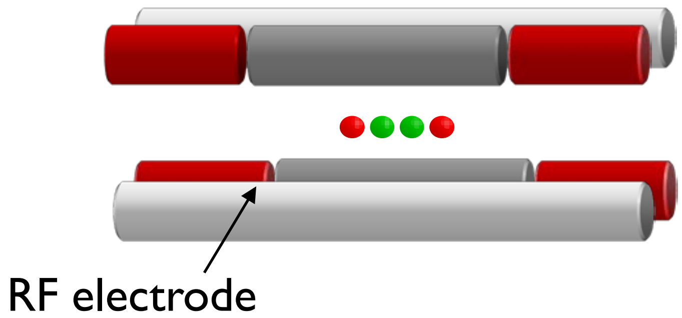
Consider ion in time-varying field

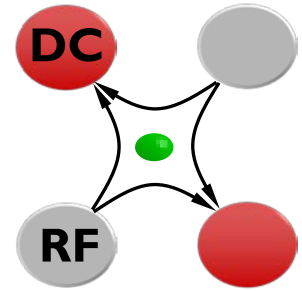
$$M\frac{d^2 x}{dt^2} = qE\cos\Omega t$$

$$\frac{1}{2}M\left(\frac{dx}{dt}\right)^2 = U_{\mathrm{PP}} = \frac{q^2E^2}{2M\Omega^2}\sin^2\Omega t$$

Time average - Effective potential energy which is minimal at minimum *E*

Traps – traditional style

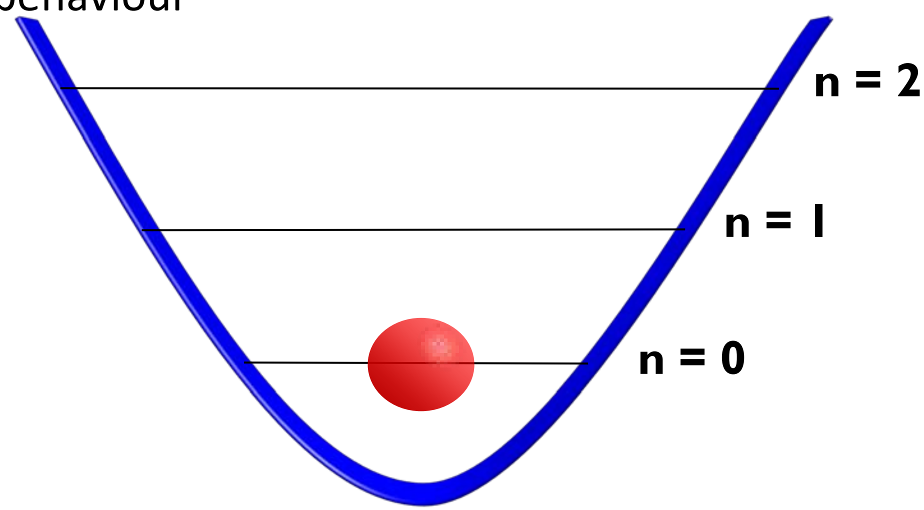




$$V_{RF} = \frac{\alpha V_0}{2R^2}(x^2 - y^2)\cos(\Omega t) \longrightarrow \Phi_{\text{pseudo}} = \left(\frac{\alpha V_0}{R^2}\right)^2 \frac{1}{4m\Omega^2}(x^2 + y^2)$$

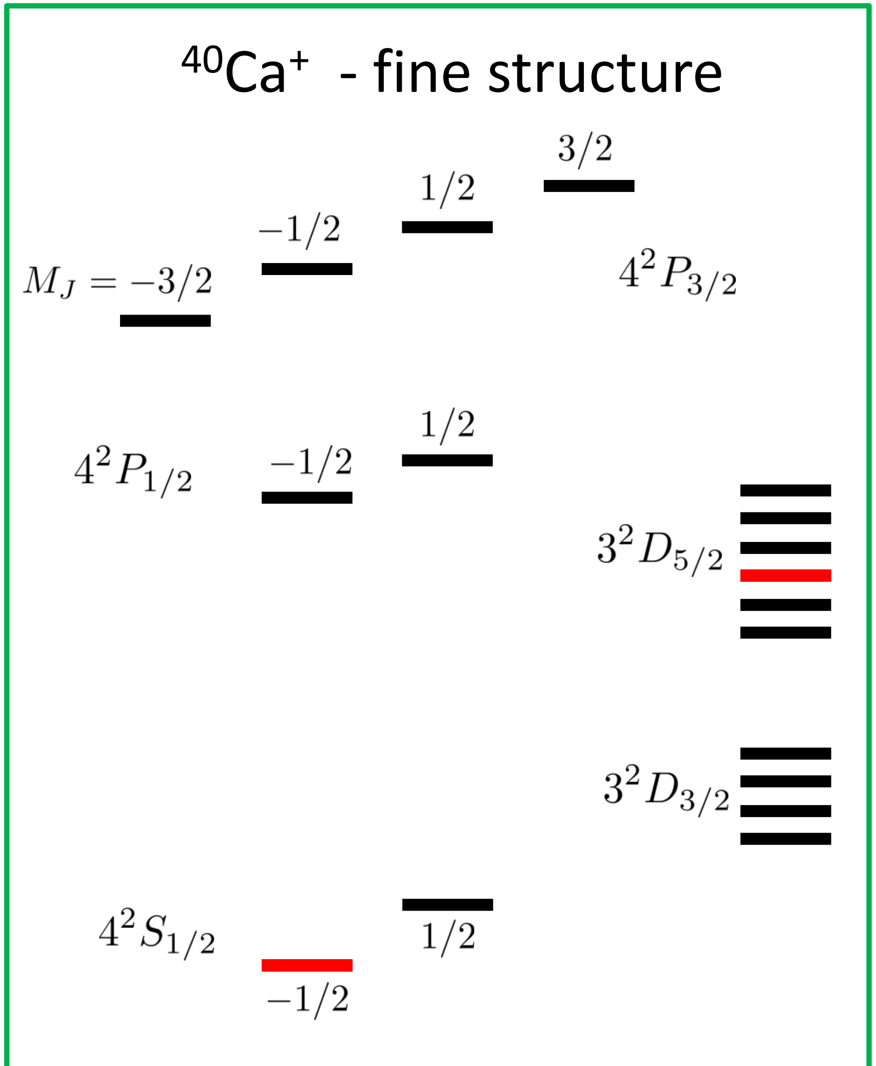
For cold ions, almost ideal harmonic behaviour

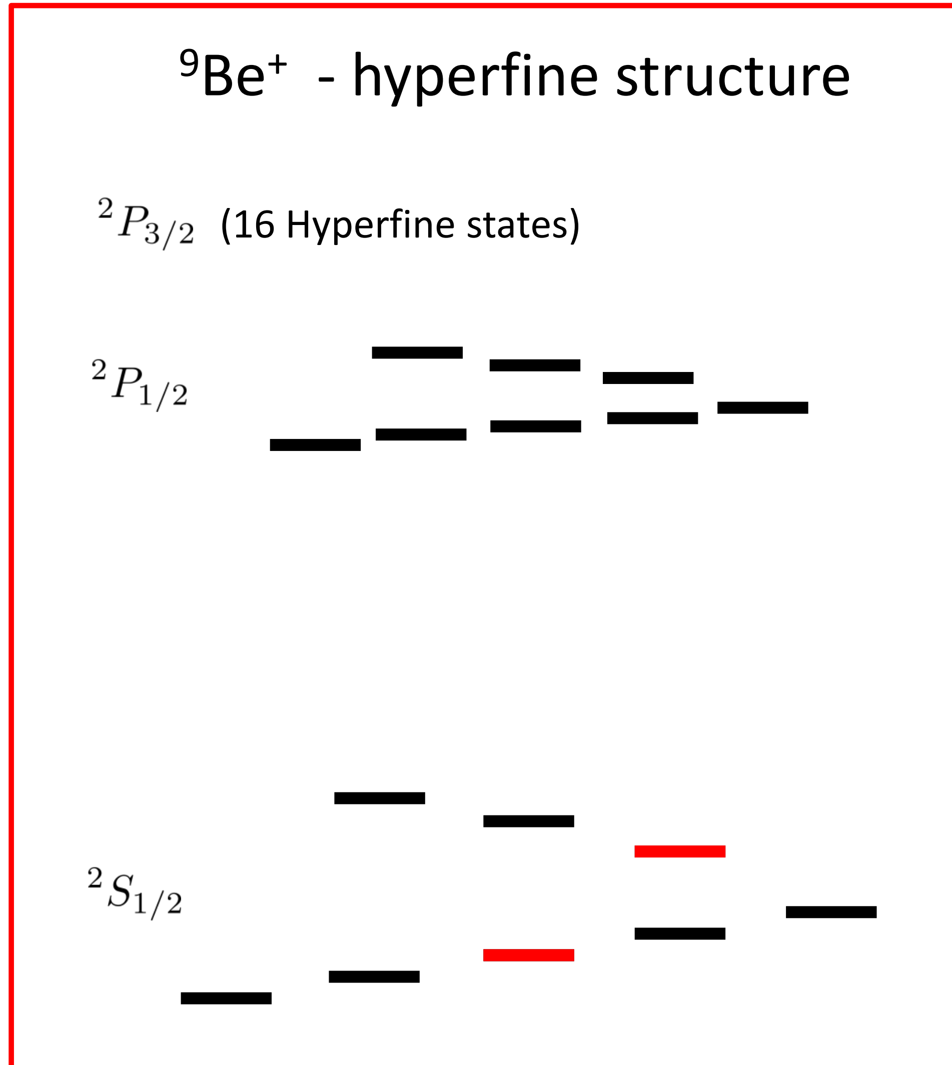
$$\hat{H} = \hbar\omega(\hat{a}^\dagger \hat{a} + 1/2)$$



Multi-level atoms

$$|\psi\rangle = (a|0\rangle + b|1\rangle)$$



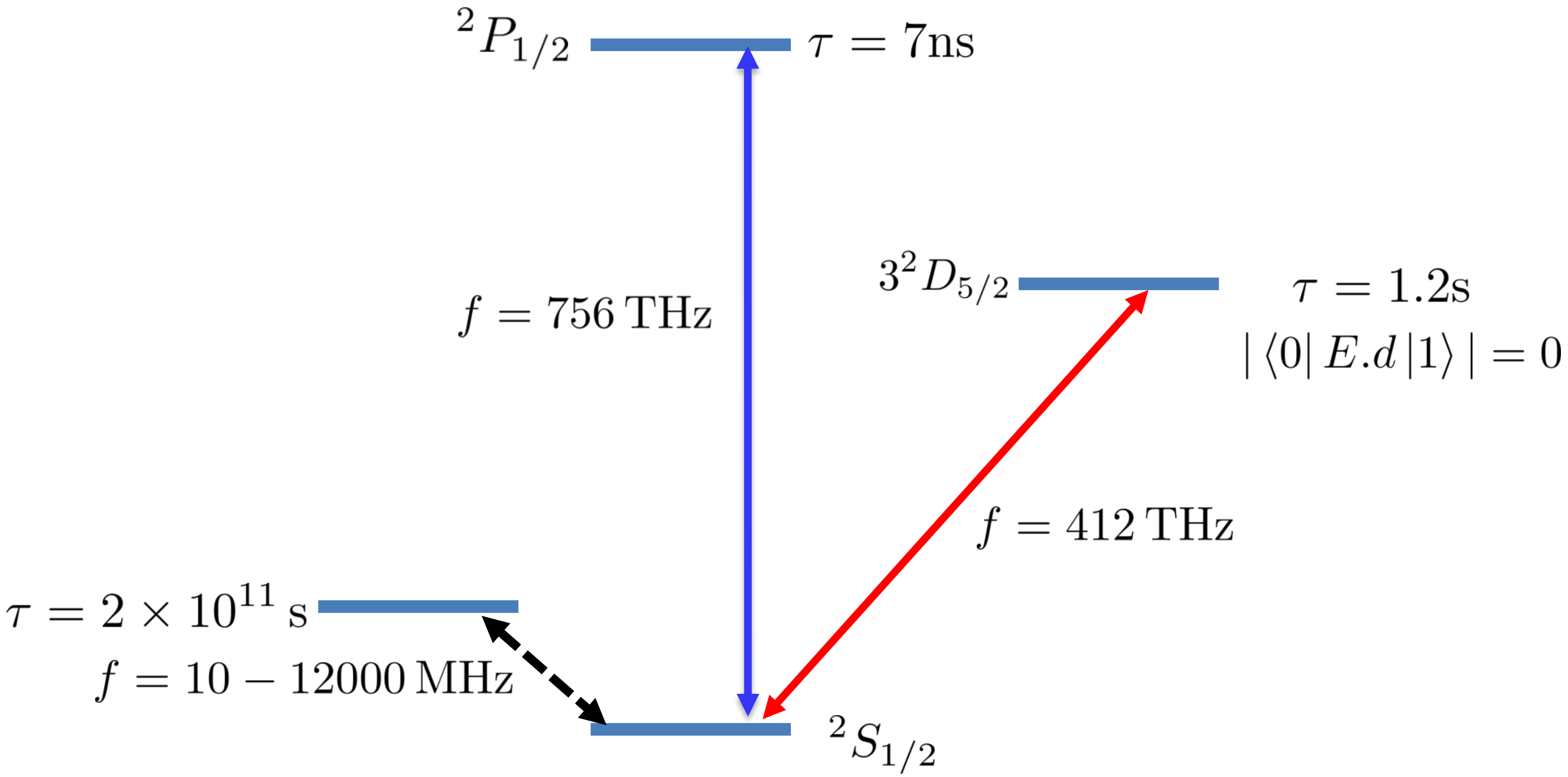


Storing qubits in an atom

$$|\psi\rangle = (a|0\rangle + b|1\rangle)$$

Requirement: long decay time for upper level.

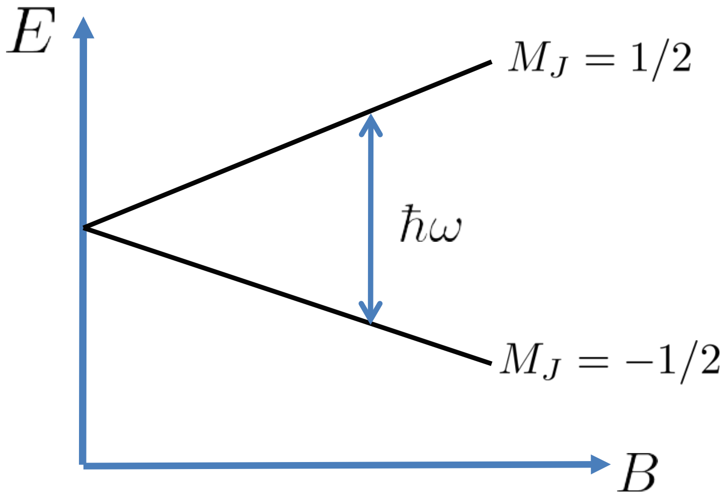
$$\tau \propto \frac{1}{\omega^3}\frac{1}{|\langle 0|\, E.d\, |1\rangle|^2}$$



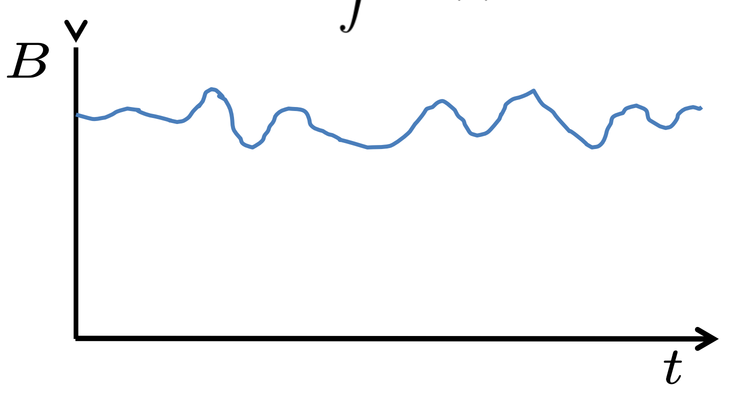
Storing qubits in an atom - phase coherence

$$|\psi\rangle = (a|0\rangle + be^{i\phi}|1\rangle)$$

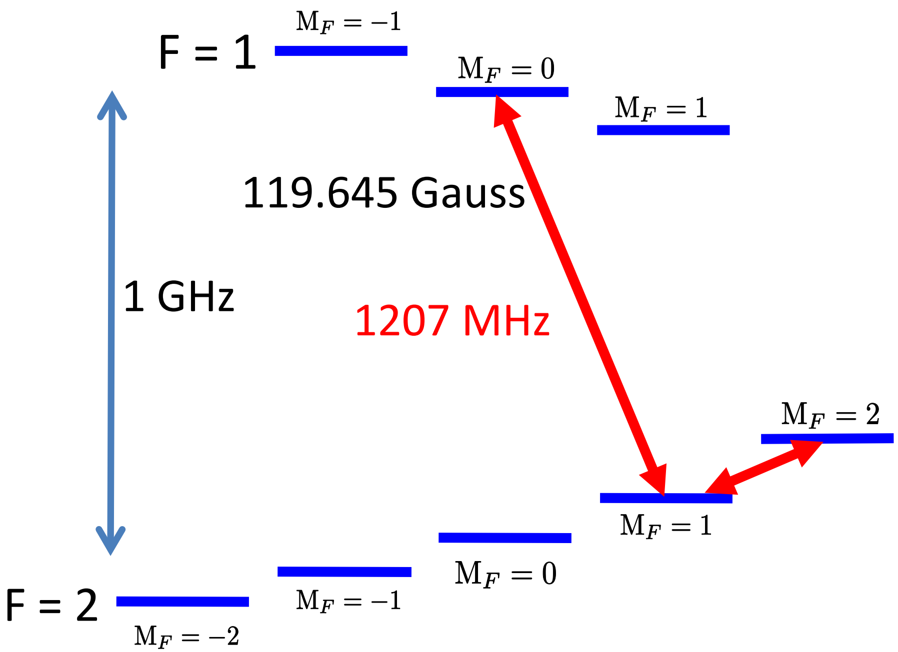
Problem: noise! – mainly from classical fields

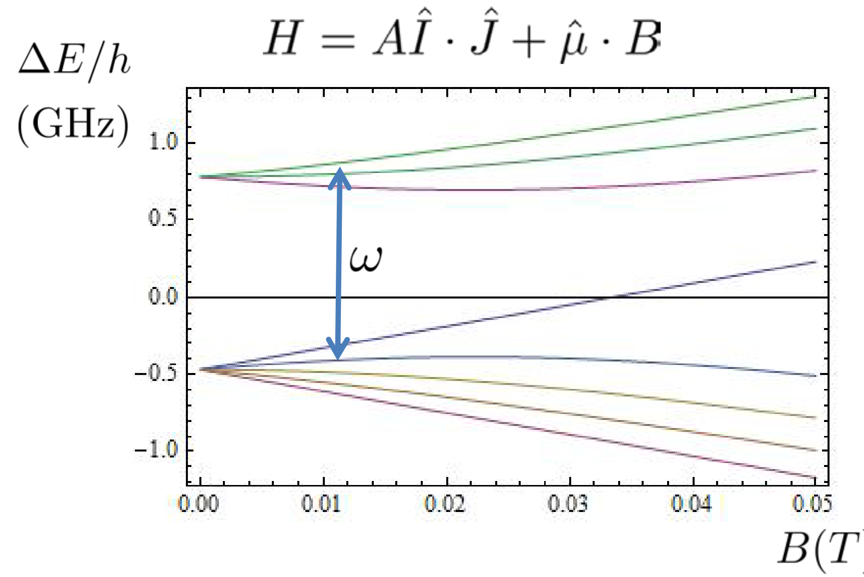


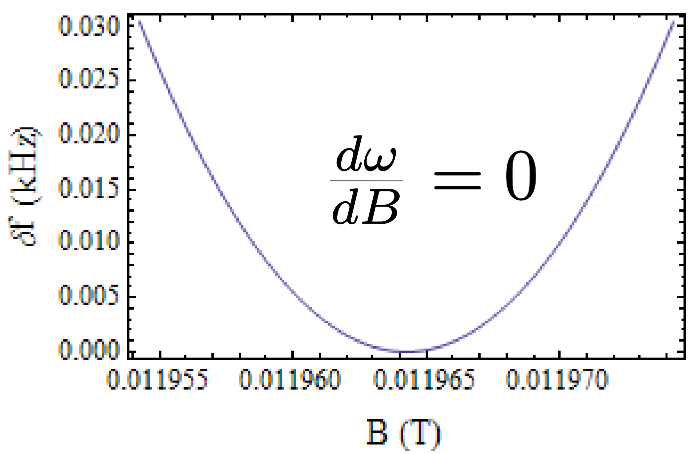
$$\phi \propto \int B(t)dt$$

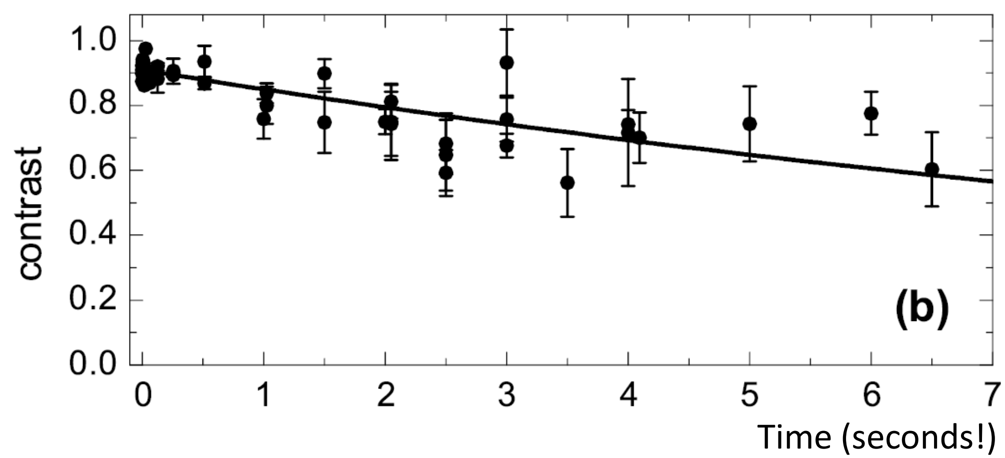


Storing qubits in an atom
Field-independent transitions









Langer et al. PRL 95, 060502 (2005)

Entanglement for protection

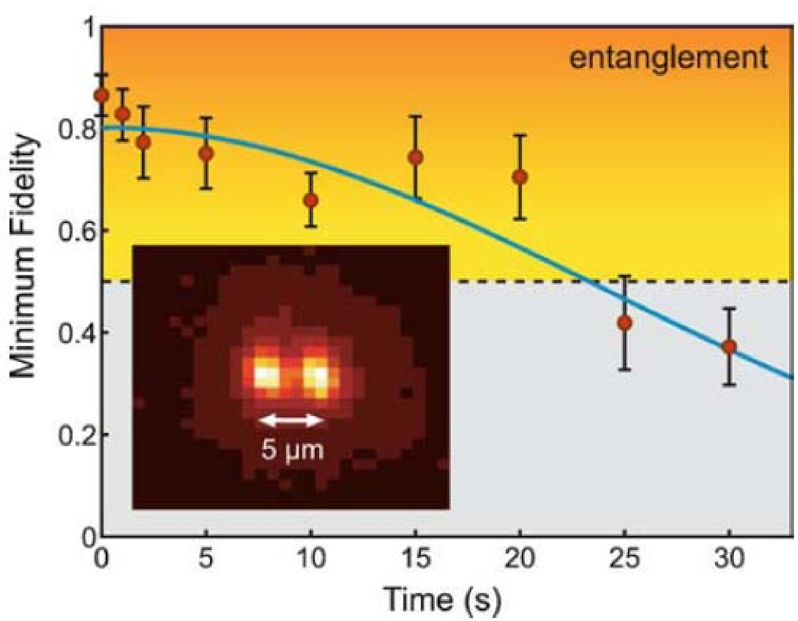
Rejection of common-mode noise

$$|0\rangle + e^{i\omega'(t)t}|1\rangle \qquad\qquad |0\rangle + e^{i\omega(t)t}|1\rangle$$

Now consider entangled state

$$e^{i\omega(t)t}|01\rangle + e^{i\omega'(t)t}|10\rangle = e^{i\omega(t)t}\left(|01\rangle + e^{i(\omega'(t)-\omega(t))t}|10\rangle\right)$$

If noise is common mode, entangled states can have very long coherence times



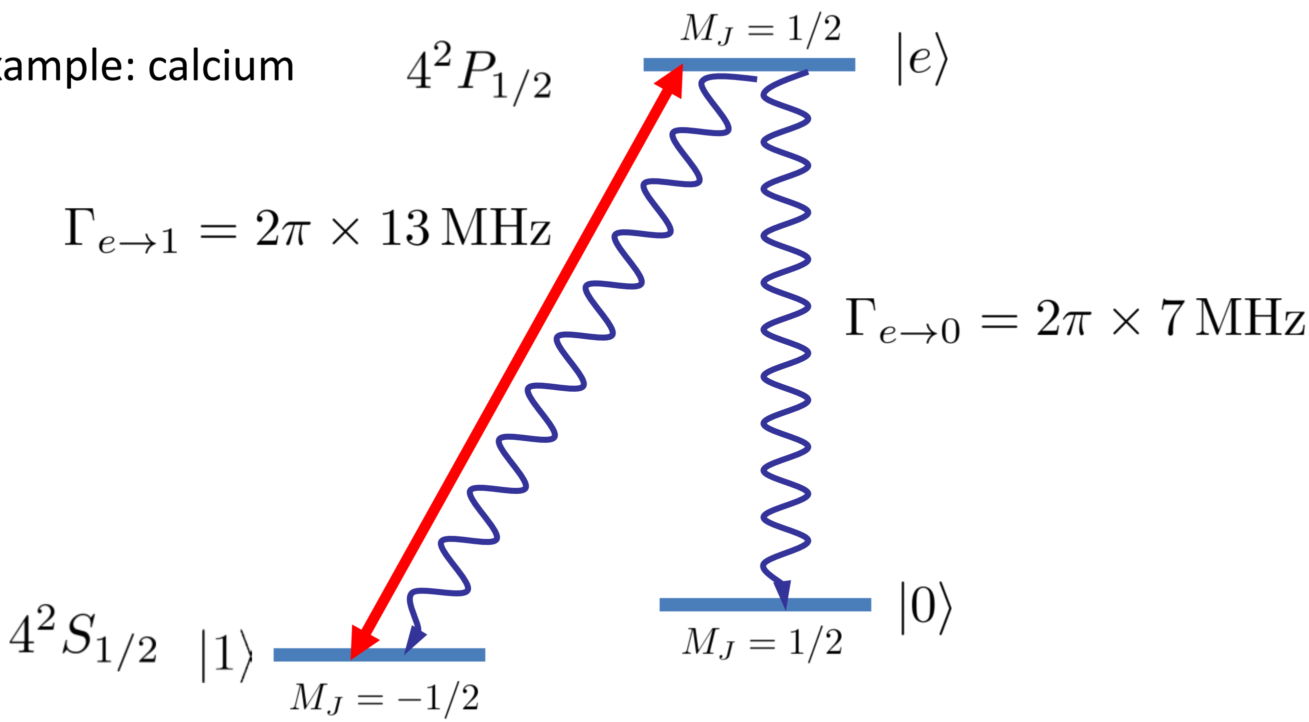
Haffner et al., Appl. Phys. B 81, 151-153 (2005)

Preparing the states of ions

Optical pumping – state initialisation

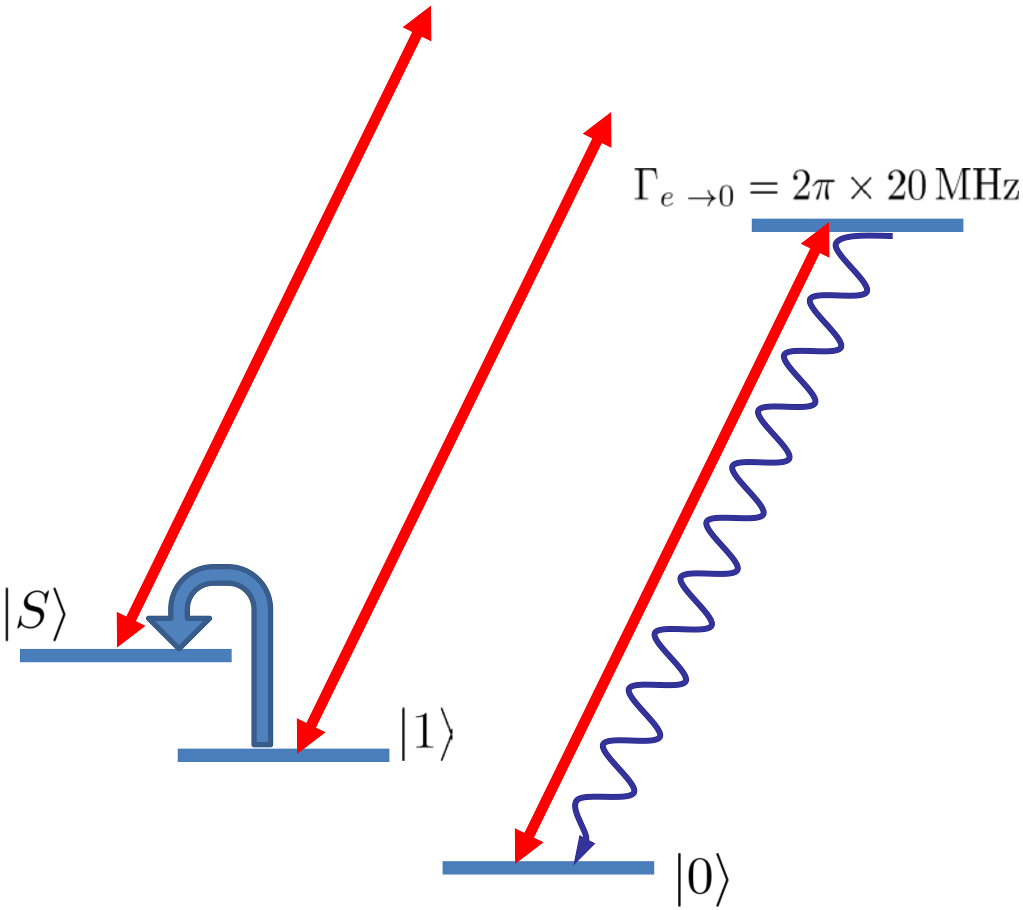
Use a dipole transition for speed

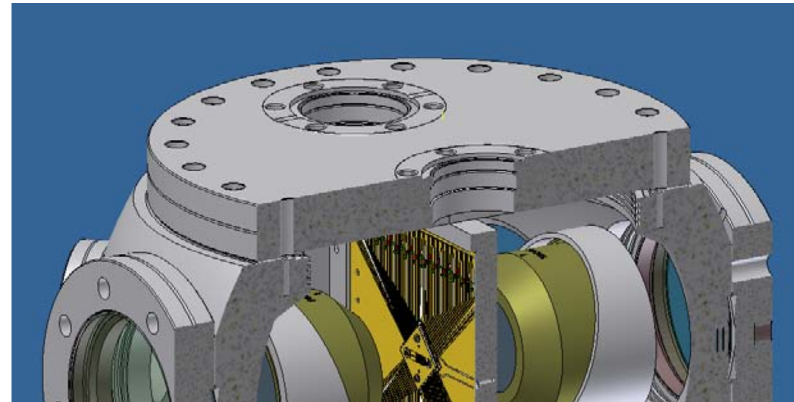
Example: calcium

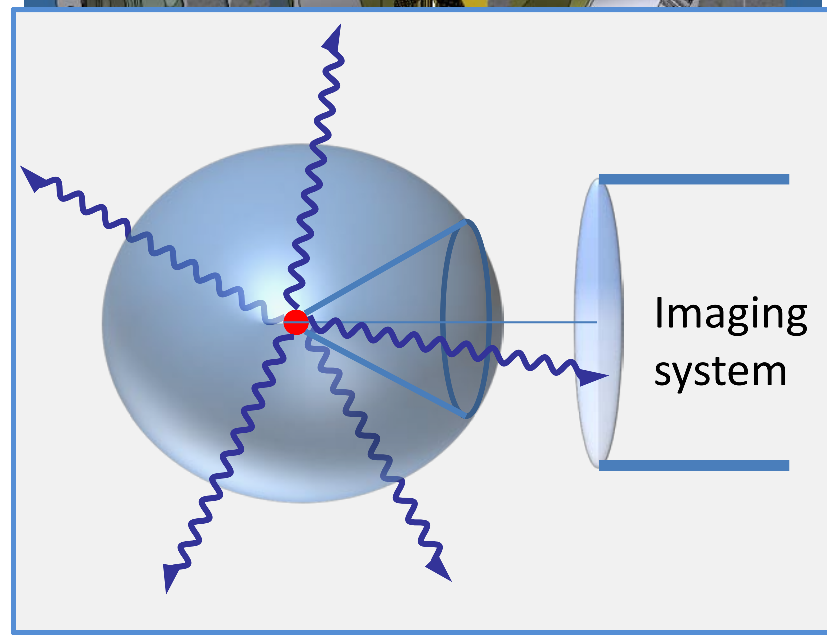


Calcium: scatter around 3 photons to prepare $|0\rangle$ $\tau_{\mathrm{prep}} \sim 20\,\mathrm{ns}$

Reading out the quantum state







Photon scattered every 7 ns
BUT
we only collect a small fraction of these

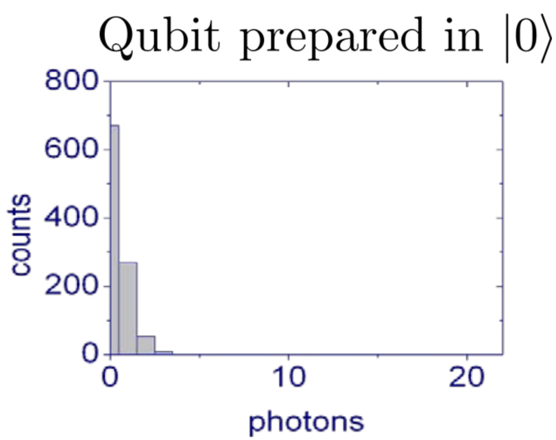
Need to scatter 1000 photons to detect atom $T_{\mathrm{readout}} \sim 100 \to 1000\,\mu s$

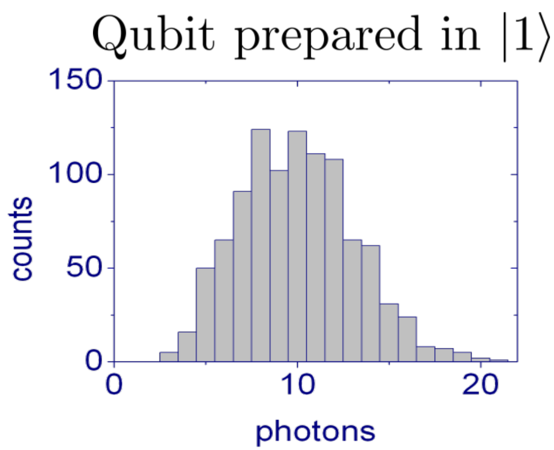
Measurement – experiment sequence

How many photons?

Statistics: repeat the experiment many (1000) times

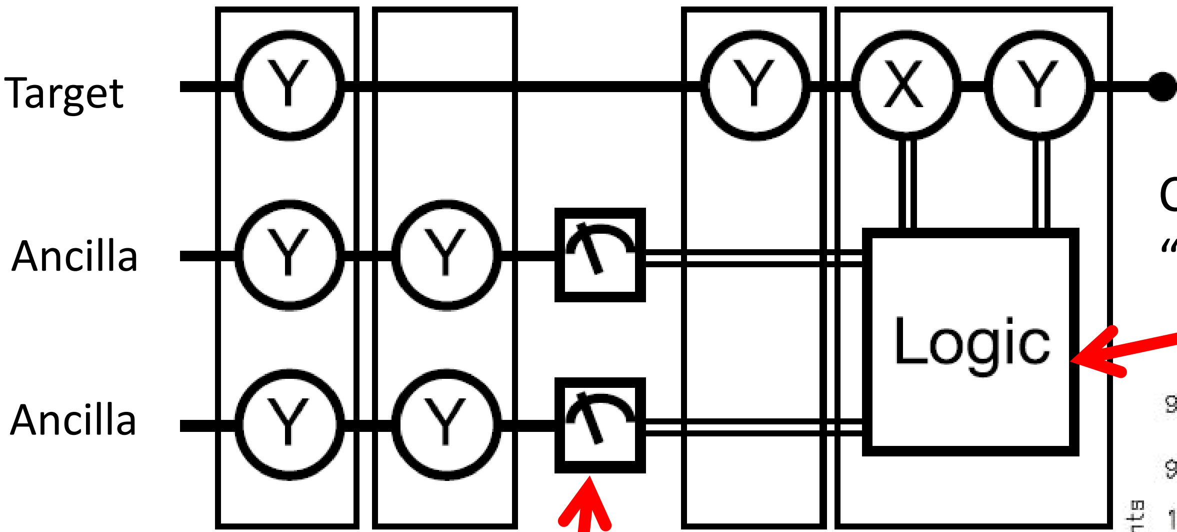
Number of photons = 8, 4, 2, 0, 0, 1, 5, 0, 0, 8 ....





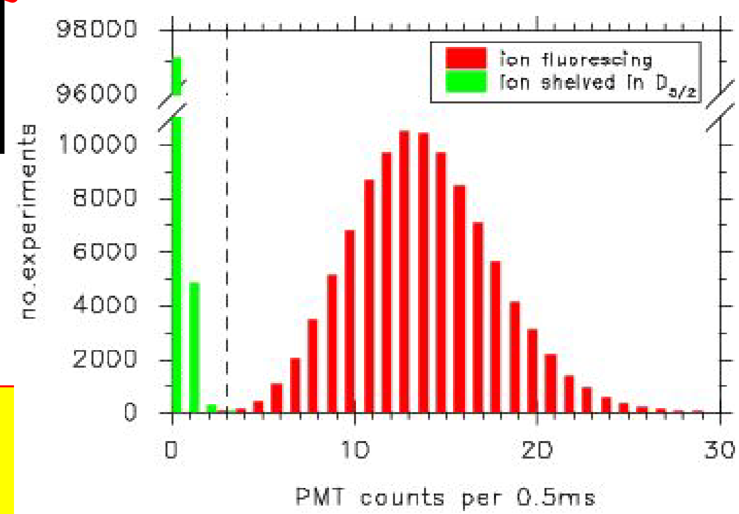
Single shot measurement

"Realization of quantum error-correction", Chiaverini et al., Nature 432, 602, (2004)



Classical processing
"If you get 1, 0, do Y, else do X"

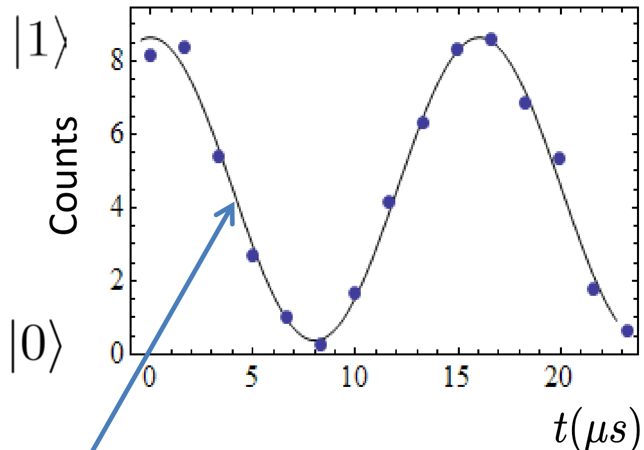
Measurement: "8 counts, this qubit is 1!"



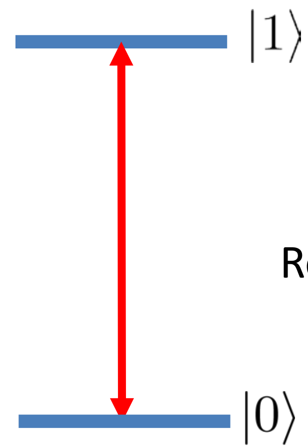
Accuracy of 0.9999 achieved in 150 microseconds
Myerson et al. Phys. Rev. Lett. 100, 200502, (2008)

Manipulating single qubits

$$H = \Omega \left(|0\rangle \langle 1| + |1\rangle \langle 0| \right) \cos\left(\omega t + \phi \right)$$

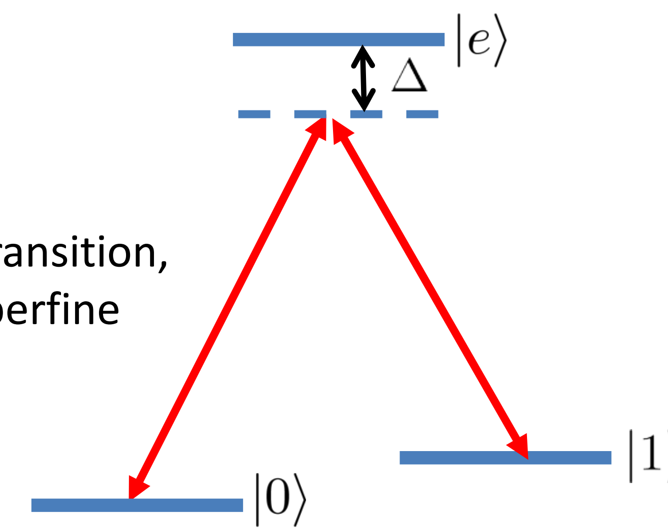


$$|\psi\rangle = (|0\rangle + e^{i\phi}|1\rangle)/\sqrt{2}$$



Laser-driven, quadrupole

Resonant microwaves, hyperfine



Raman transition, hyperfine

Lowest error rate $\sim 2 \times 10^{-5}$ K. R. Brown et al. PRA 2011

Addressing individual qubits

Intensity addressing

Shine laser beam at one ion in string

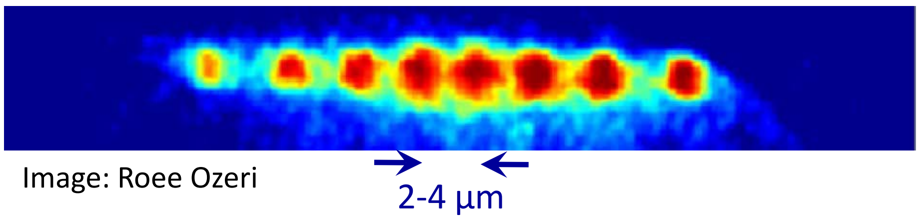
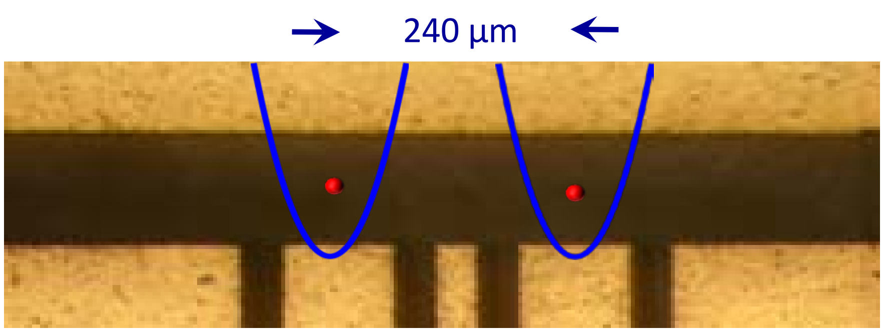
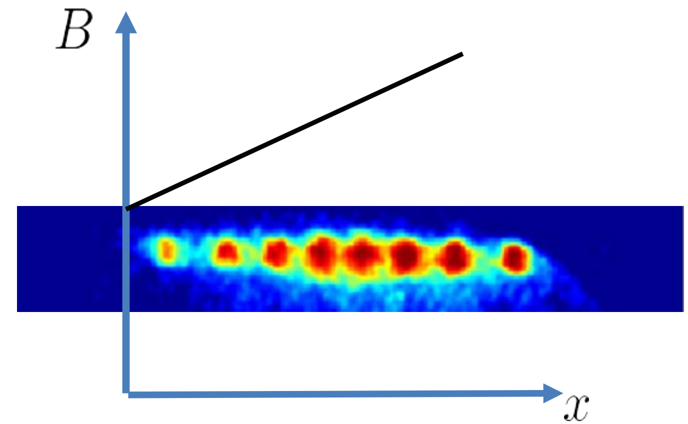


Image: Roee Ozeri

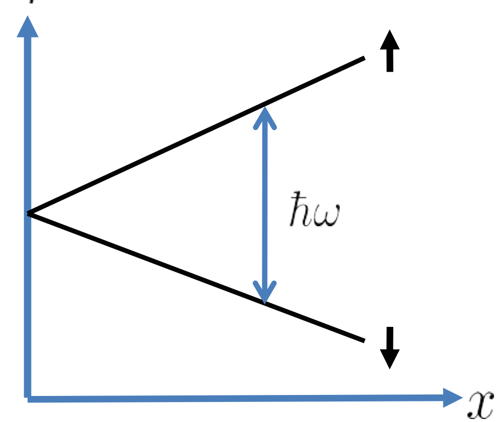
Separate ions by a distance much larger than laser beam size



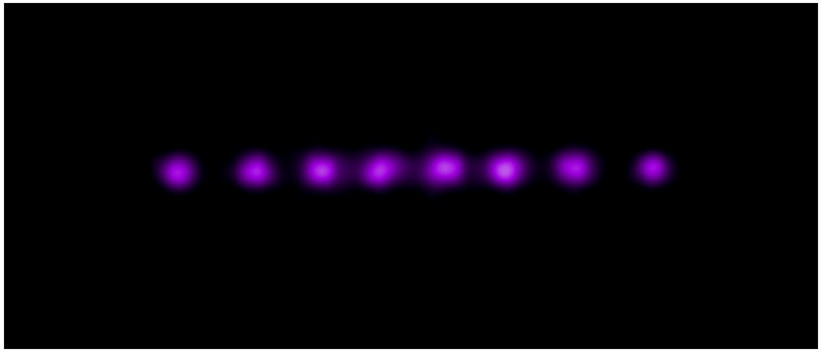
Frequency addressing



$$E = -\mu \cdot B$$

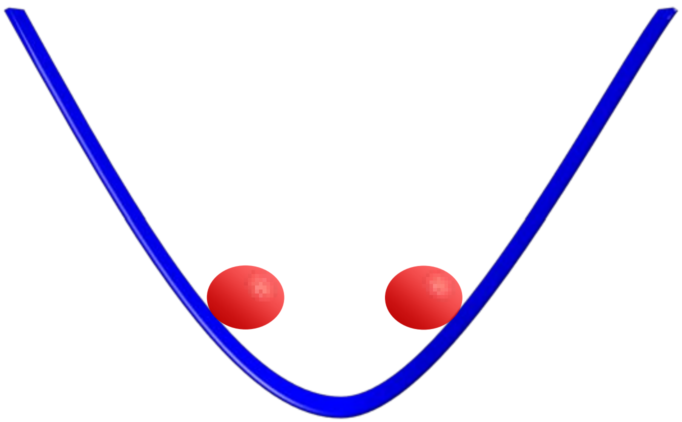


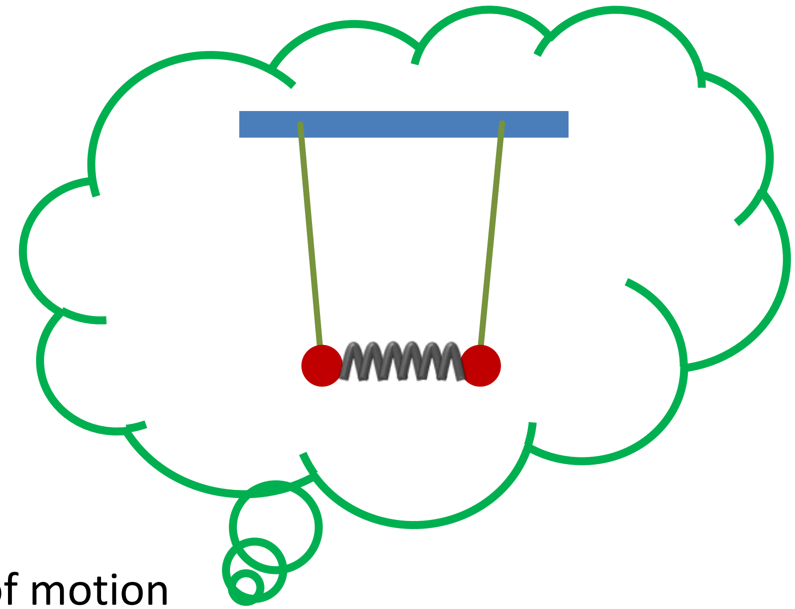
Multiple qubits: interactions



Multiple ions: coupled harmonic oscillators

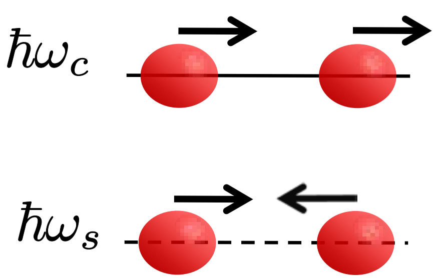
$$V = \frac{k}{2} z_1^2 + \frac{k}{2} z_2^2 + \frac{\alpha}{|z_1 - z_2|}$$





Expand about equilibrium – equation of motion

$$\begin{pmatrix} \ddot{\epsilon}_1 \\ \ddot{\epsilon}_2 \end{pmatrix} = -\omega_z^2 \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} = \begin{pmatrix} k & \alpha \\ \alpha & k \end{pmatrix} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

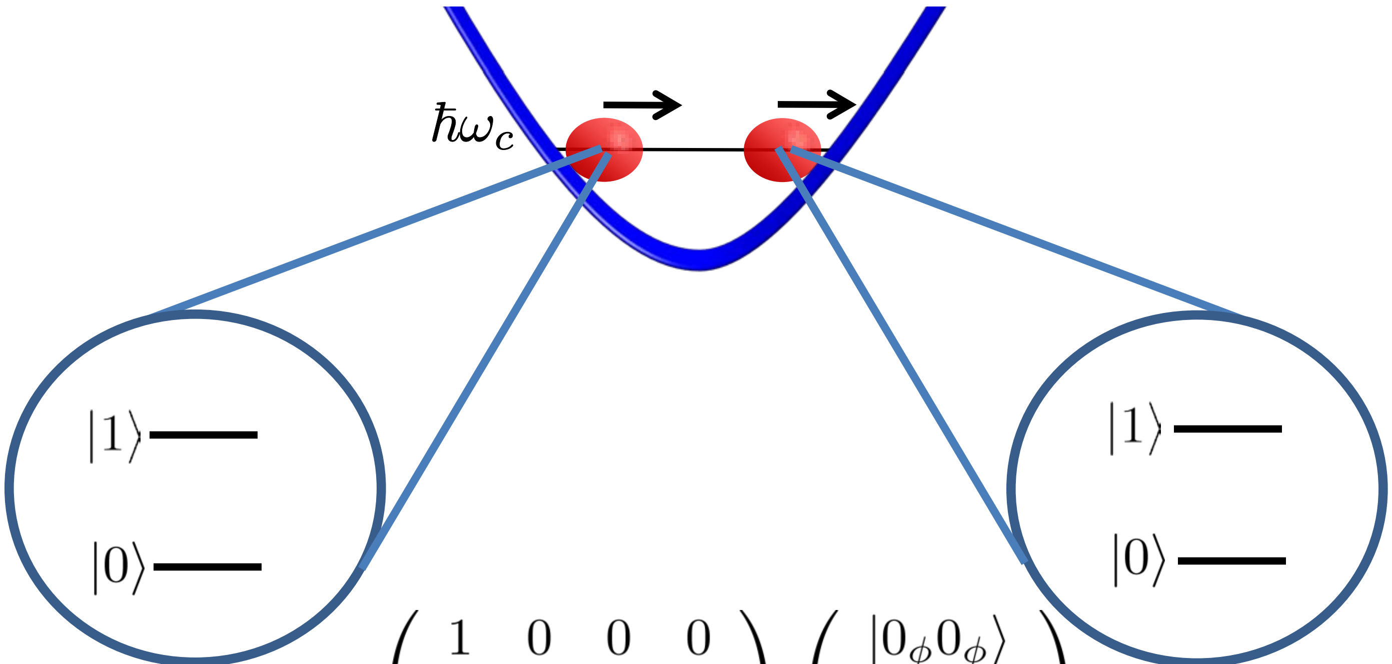


Independent oscillators
- shared motion

The original thought

Cirac and Zoller, PRL (1995)

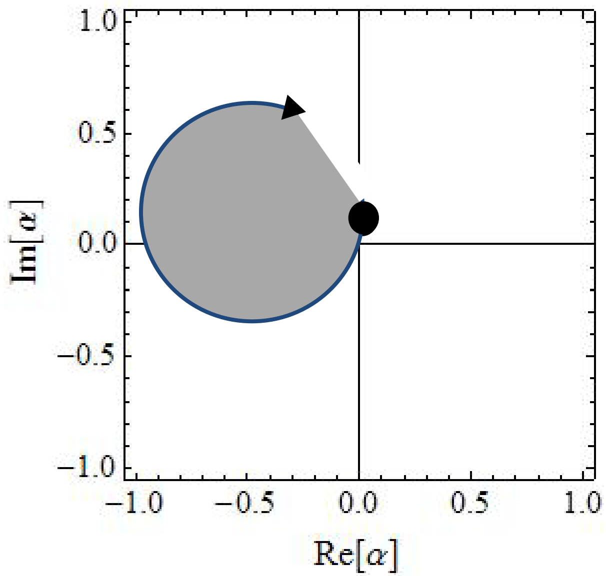
“The collective oscillator is a quantum bus“



$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} |0_\phi 0_\phi\rangle \\ |1_\phi 0_\phi\rangle \\ |0_\phi 1_\phi\rangle \\ |1_\phi 1_\phi\rangle \end{pmatrix}$$

The forced harmonic oscillator

Classical forced oscillator $$\frac{d^2x}{dt^2} = -\omega_z^2 x + \frac{F}{m}\cos(\omega t + \phi)$$



"returns" after $t = \frac{2\pi}{\delta}$

Radius of loop $\propto \frac{F}{\delta}$

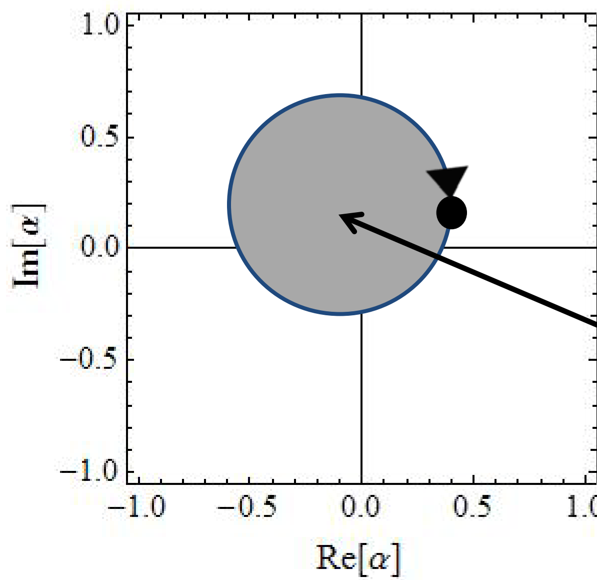
Forced quantum oscillators

$$\begin{aligned} H(t) &= \Omega\cos(\omega t)e^{ikz} \\ &\simeq \Omega\cos(\omega t)\left(1 + ikz_0\left(\hat{a}e^{i\omega_z t} + \hat{a}^\dagger e^{-i\omega_z t}\right)\right) \end{aligned}$$

$$[H(t), H(t')] \neq 0$$

$$U = \exp\left(\frac{i}{\hbar}\int^t H(t')dt' - \frac{1}{2\hbar^2}\int^t\int^{t'}[H(t'), H(t'')]dt'dt'' + \ldots\right)$$

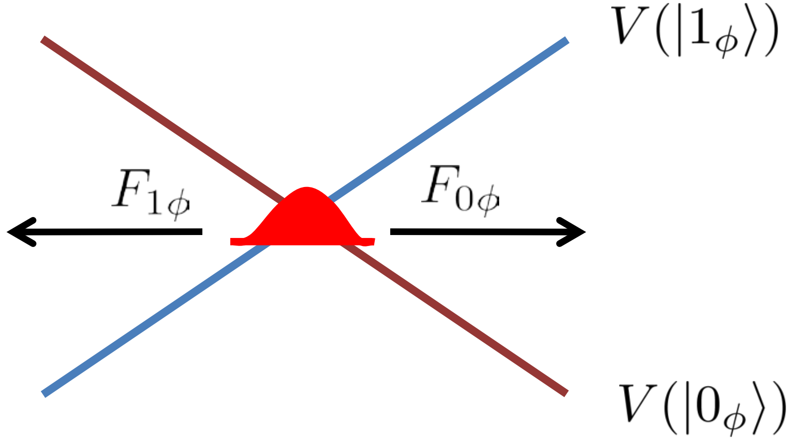
Transient excitation, phase acquired



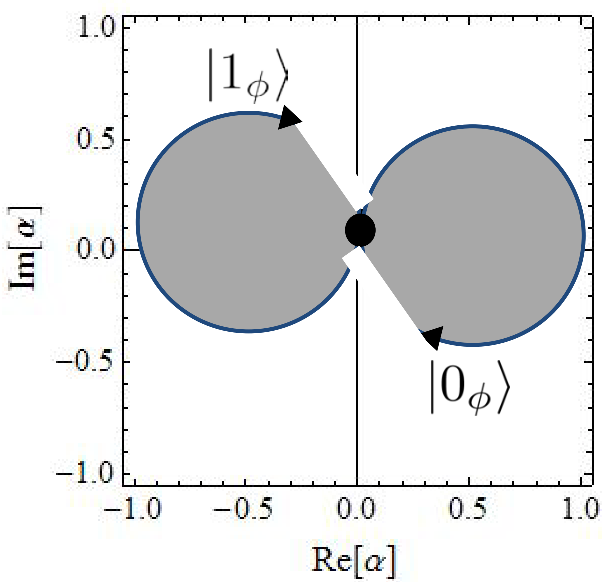
$$|1\rangle\,|\alpha_0\rangle \rightarrow e^{i\Phi}\,|1\rangle\,|\alpha_0\rangle$$

$$\Phi \propto A$$

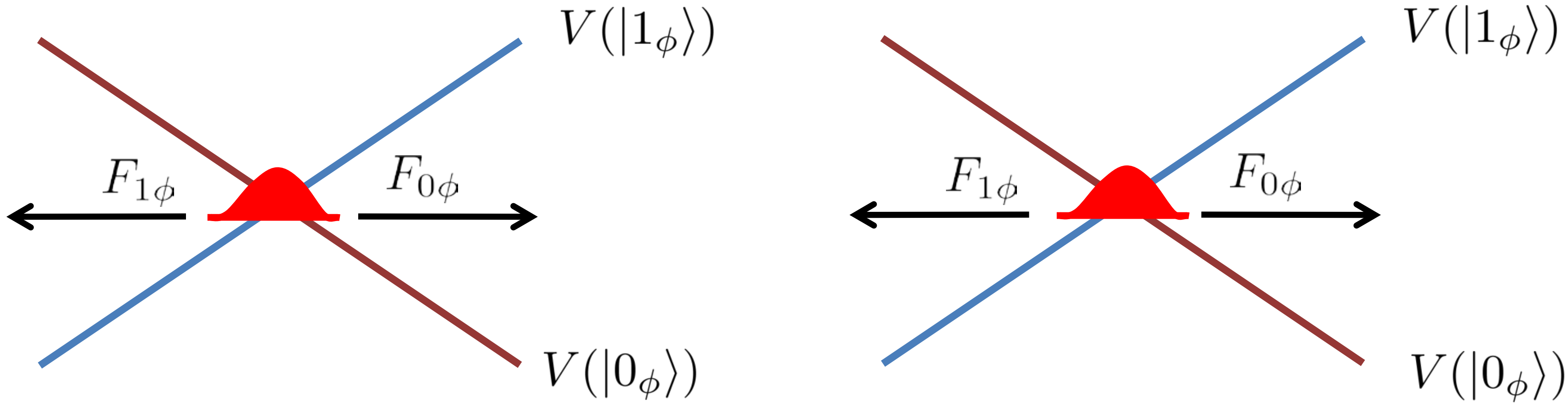
State-dependent excitation



$$|\psi\rangle = |1_\phi\rangle\,|\alpha\rangle + |0_\phi\rangle\,|{-\alpha}\rangle$$

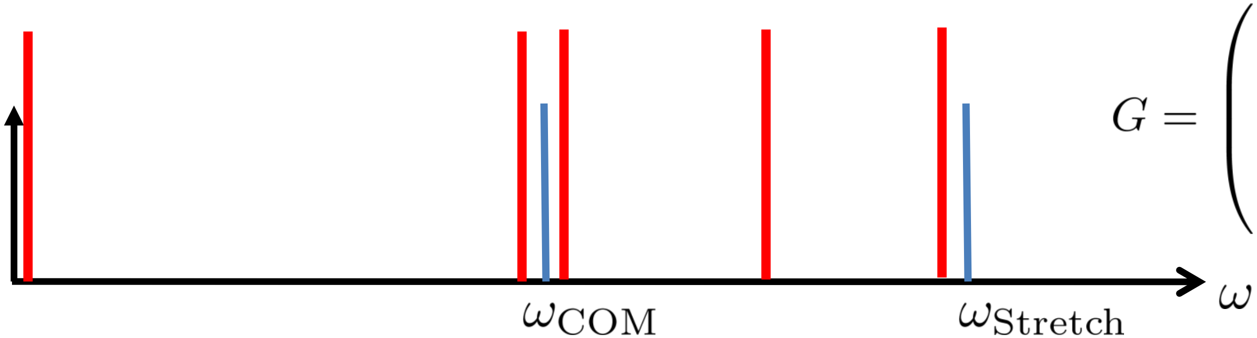


Two-qubit gate, state-dependent excitation



$|0_\phi 1_\phi\rangle, |1_\phi 0_\phi\rangle$ → Force is out of phase; excite Stretch mode

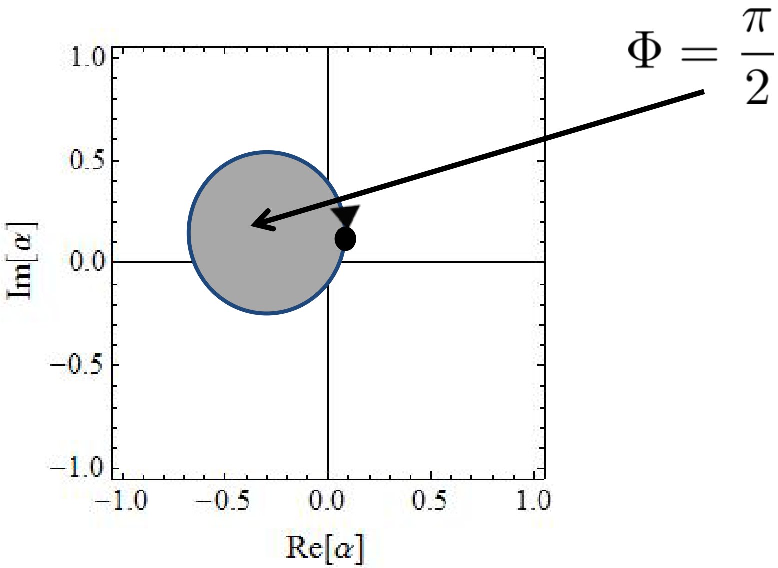
$|1_\phi 1_\phi\rangle, |0_\phi 0_\phi\rangle$ → Force is in-phase; excite COM mode



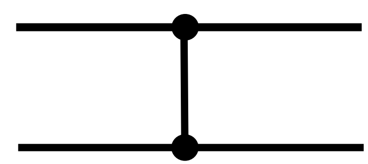
$$G = \begin{pmatrix} e^{i\phi_{00}} & 0 & 0 & 0 \\ 0 & e^{i\phi_{01}} & 0 & 0 \\ 0 & 0 & e^{i\phi_{01}} & 0 \\ 0 & 0 & 0 & e^{i\phi_{01}} \end{pmatrix}$$

Examples: quantum computing

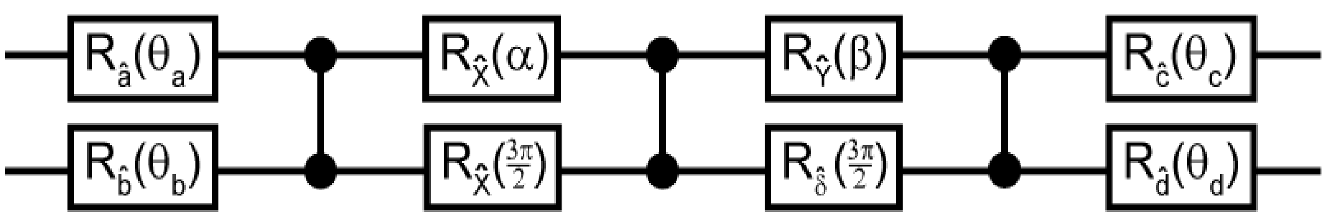
Choose the duration and power: $t_g = 2\pi/\delta \sim 7 \rightarrow 100\mu s$



$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$\hat{U} =$

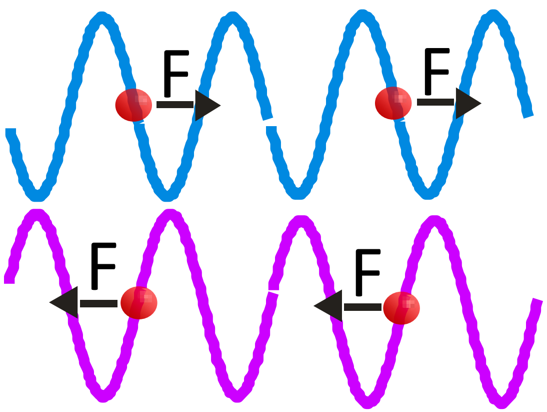


Universal two-qubit ion trap quantum processor: Hanneke et al. Nature Physics 6, 13-16 (2010)

Laser-driven multi-qubit gates

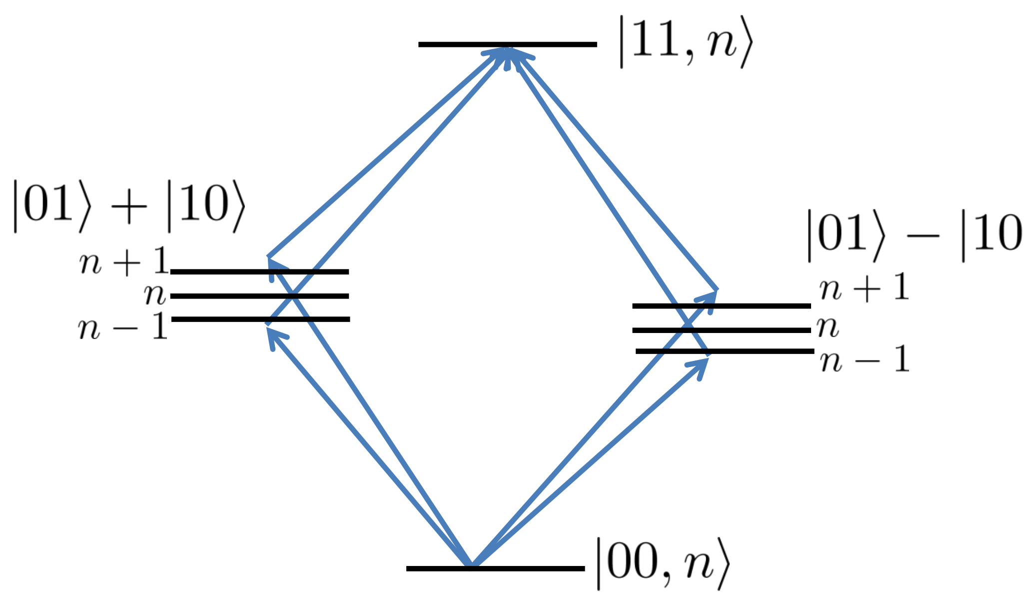
σ_z basis, polarisation standing wave

Leibfried et al. Nature 422, 412-415 (2003)

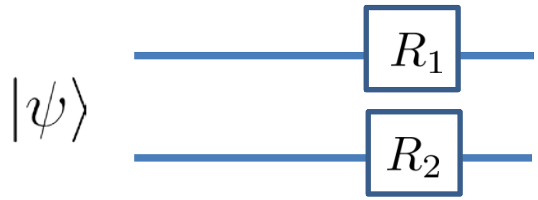


$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

σ_x, σ_y basis, interference effect



State and entanglement characterisation



Detect

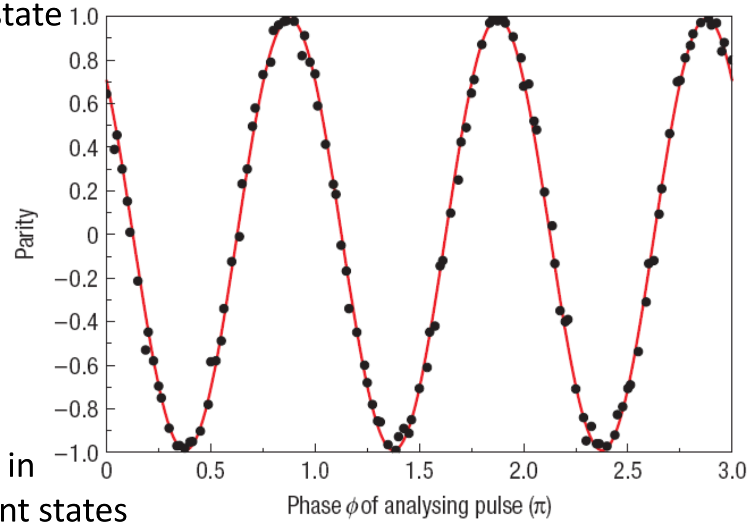
8, 6, 7, 4, 9, 0, 0, 1, 1, 6, 1, 9, 0, 0…
5, 4, 3,11, 4, 1, 0, 0, 1, 8, 0, 8, 1, 0…

Entanglement – correlations…

$$(|11\rangle + |00\rangle)\sqrt{2}$$

$$R_1 = R_2 = R(\theta = \pi/2, \phi)$$

Qubits in the same state



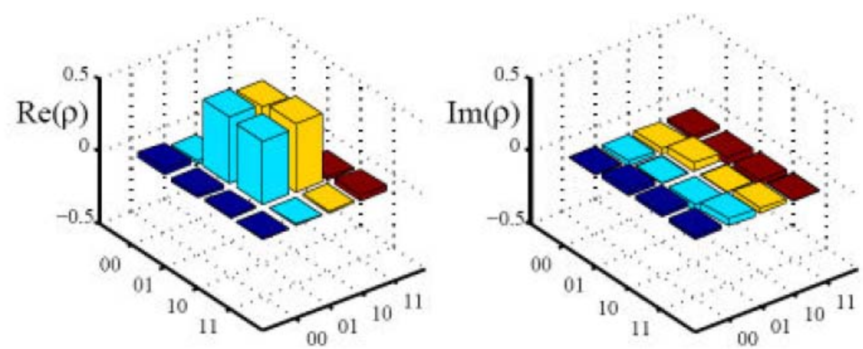
Qubits in different states

F = 0.993 (Innsbruck)

Benhelm et al. Nat. Phys 4, 463(2008)

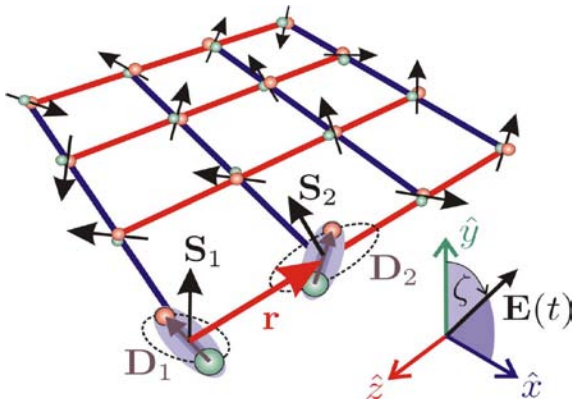
Choose 12 different settings of R_1, R_2

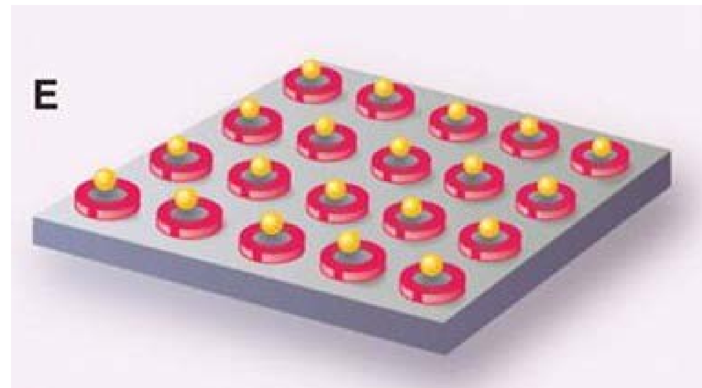
Reconstruct density matrix



Quantum simulation with trapped-ions

Creation of "condensed-matter" Hamiltonians
(Friedenauer et al. Nat. Phys 4, 757-761 (2008)
Kim et al. Nature 465, 7298 (2010))





Go to limit of large motional detuning
(very little entanglement between spin and motion)

$$\Omega \ll \delta$$

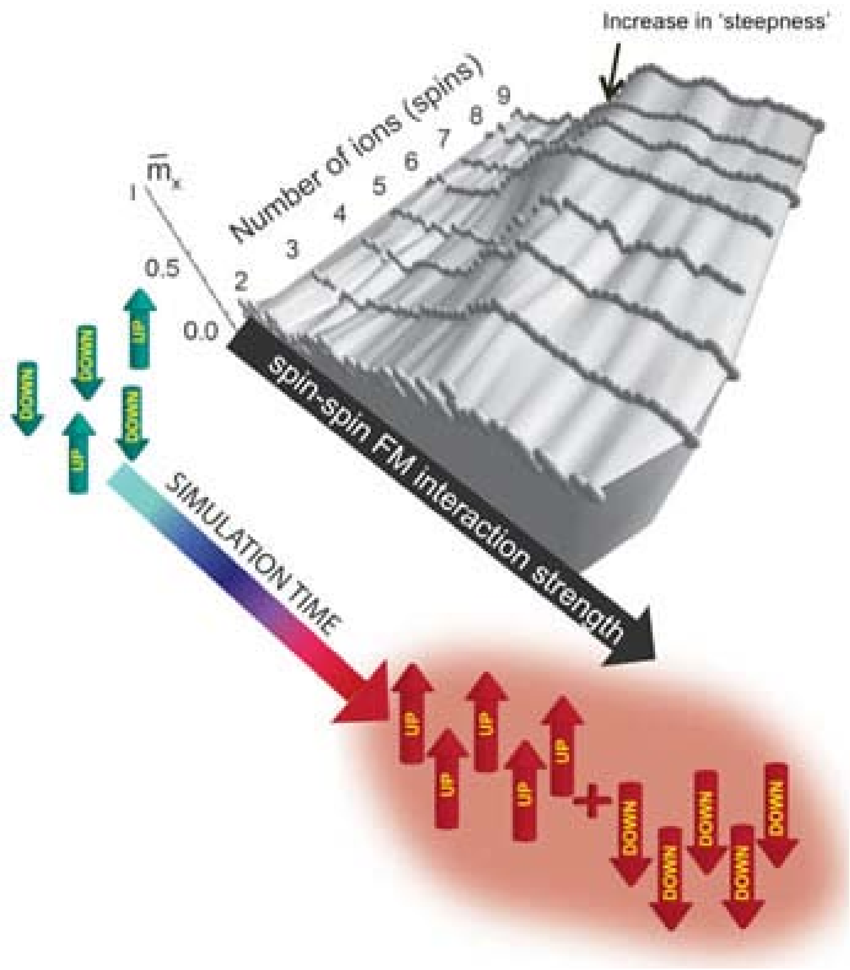
$$\Phi_{10} = \Phi_{01} \simeq i\frac{\Omega^2}{\delta}t$$

$$H_{\mathrm{eff}} \simeq \frac{\Omega^2}{\delta} s_1^x s_2^x \qquad H_{\mathrm{eff}} \simeq \frac{\Omega^2}{\delta} \sum_{i \neq j}^{N} s_i^x s_j^x$$

Analog and digital simulation

Analog: engineer full Hamiltonian

$$H_{\text{eff}} \simeq \underbrace{\frac{\Omega^2}{\delta}\sum_{i\neq j}^{N} s_i^x s_j^x}_{\text{Complicated}} \; \underbrace{+\frac{\Omega^2}{\delta}\sum_{i\neq j}^{N} s_i^y s_j^y}_{\text{Complicated}}$$



Problem: engineering of different terms interfere

Digital

Digital: Trotter approach, apply terms of Hamiltonian sequentially

$$H_2 = \frac{\Omega^2}{\delta} \sum_{i \neq j}^{N} s_i^x s_j^x \qquad H_1 = \frac{\Omega^2}{\delta} \sum_{i \neq j}^{N} s_i^y s_j^y$$

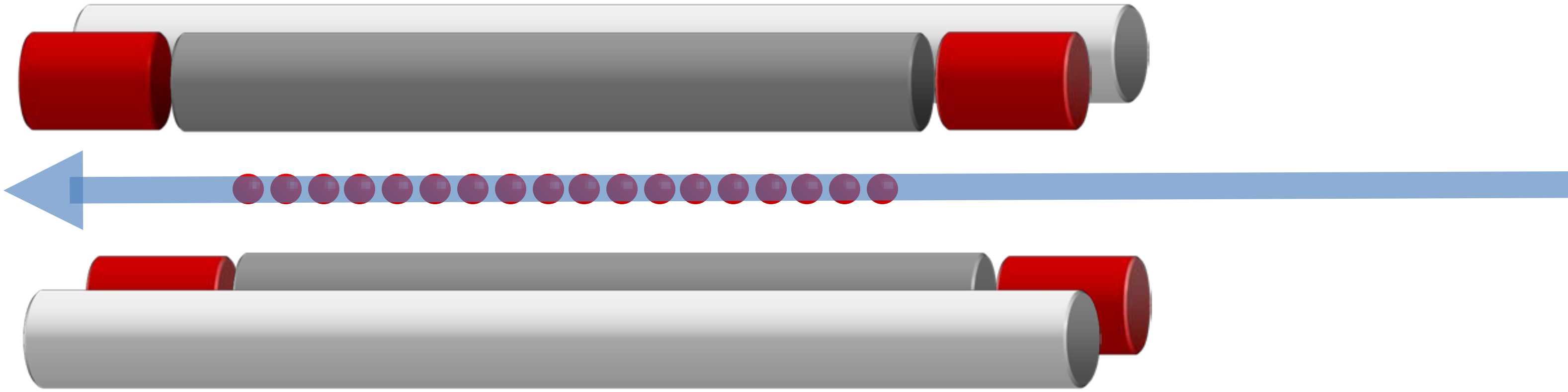
$$U(t) = e^{-iH_1 \delta t} e^{-iH_2 \delta t} e^{-iH_1 \delta t} \ldots$$

Brings error, but can be kept small if time interval short

Difficulty, lots of pulses,
Experiment example: Innsbruck group
http://www.sciencemag.org/content/early/2011/08/31/science.1208001.full.pdf

100 gate pulses, up to 6 ions.

Dealing with large numbers of ions

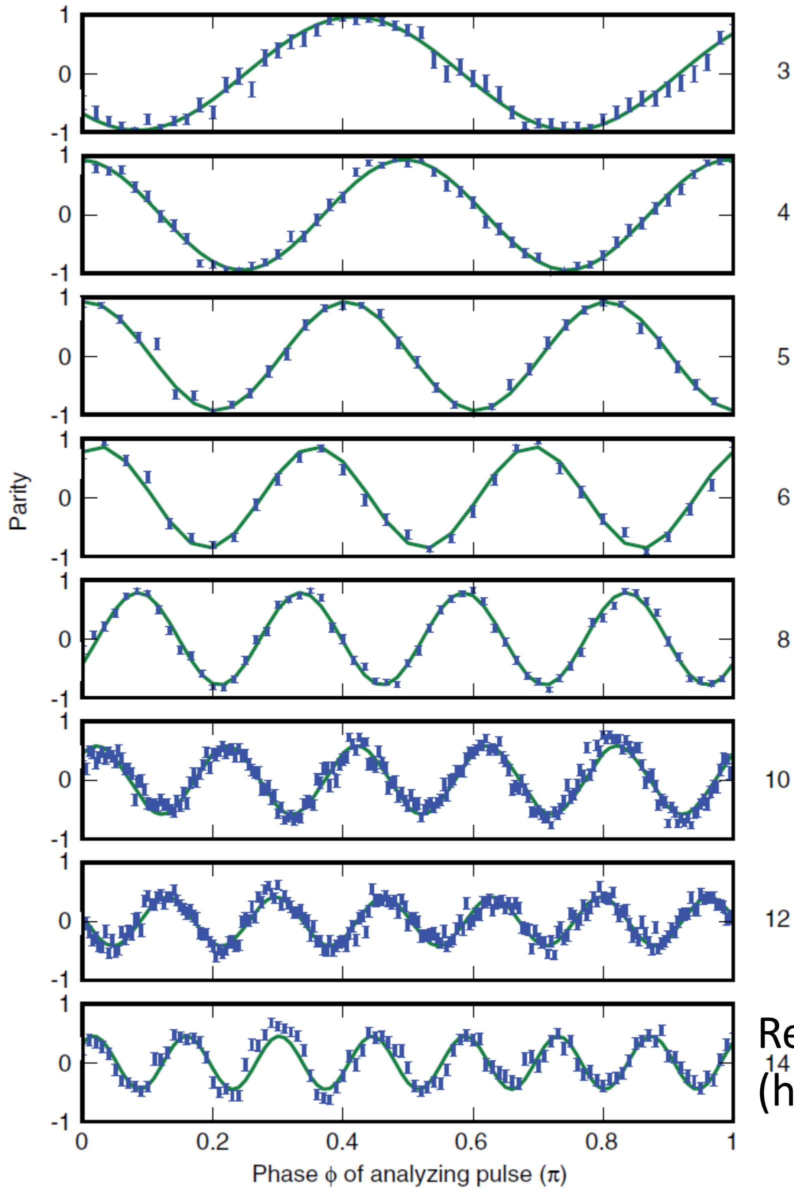


Technical requirement	Limitation
Spectral mode addressing	Mode density increases
Many ions	Heating rates proportional to N
Simultaneous laser addressing	Ions take up space (separation > 2 micron) Laser beams are finite-size

Entanglement of multiple ions

Monz et al., PRL 106, 130506 (2011)

$$\left(|11...1\rangle + |00...0\rangle\right)/\sqrt{2}$$

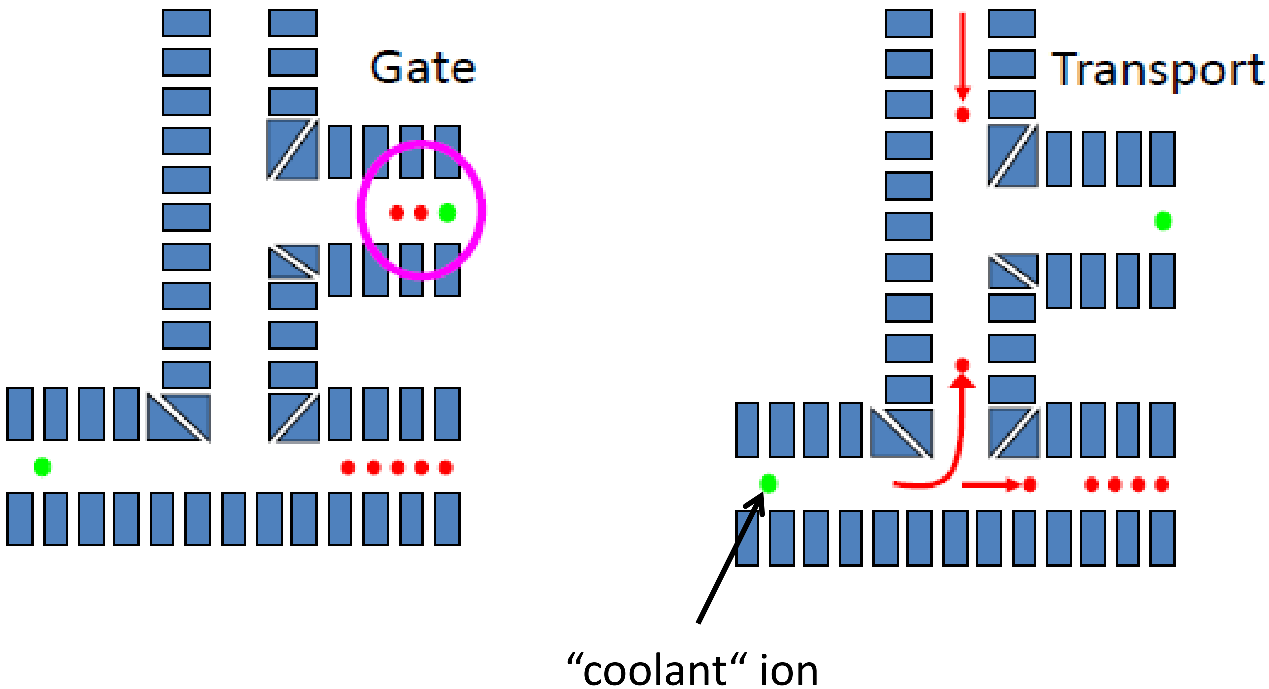


High contrast – 3 ions

Reduced contrast – 14 ions
(hard to fit ions in laser beam!)

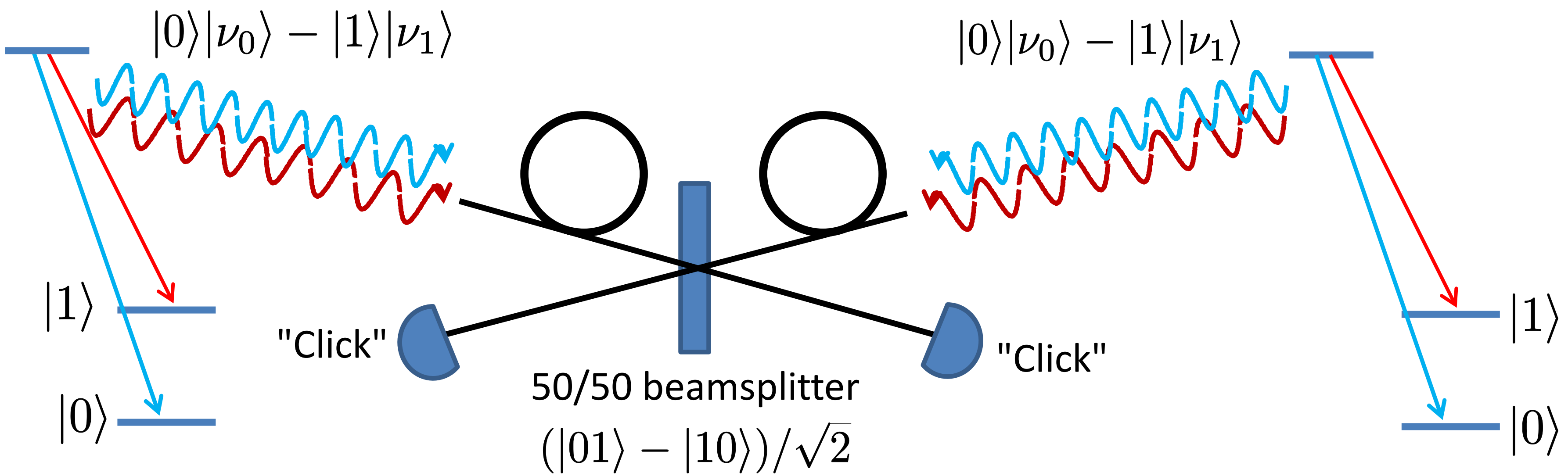
Isolate small numbers of ions

Wineland et al. J. Res. Nat. Inst. St. Tech, (1998)

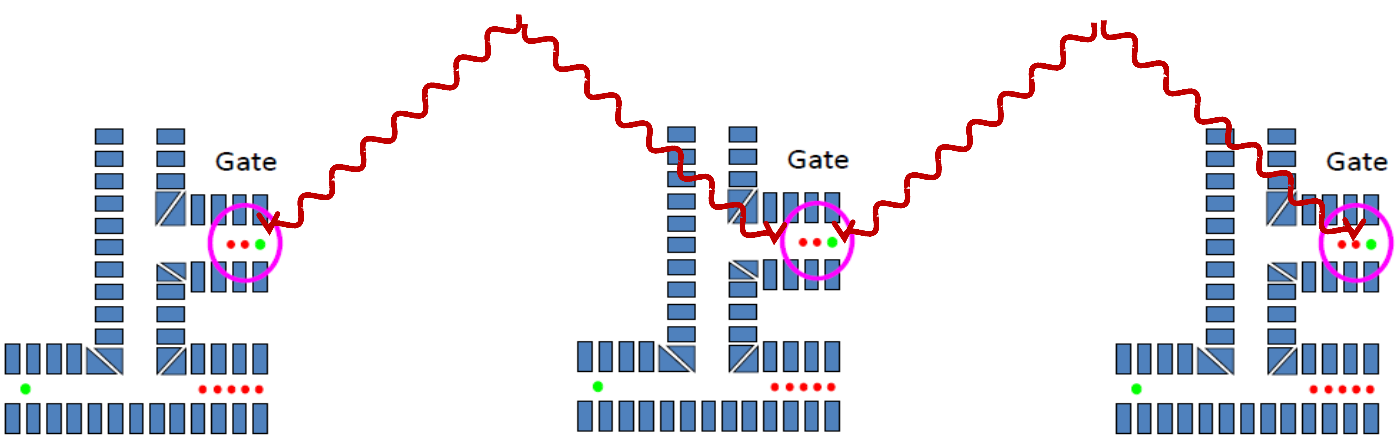


Technological challenge – large numbers of electrodes, many control regions

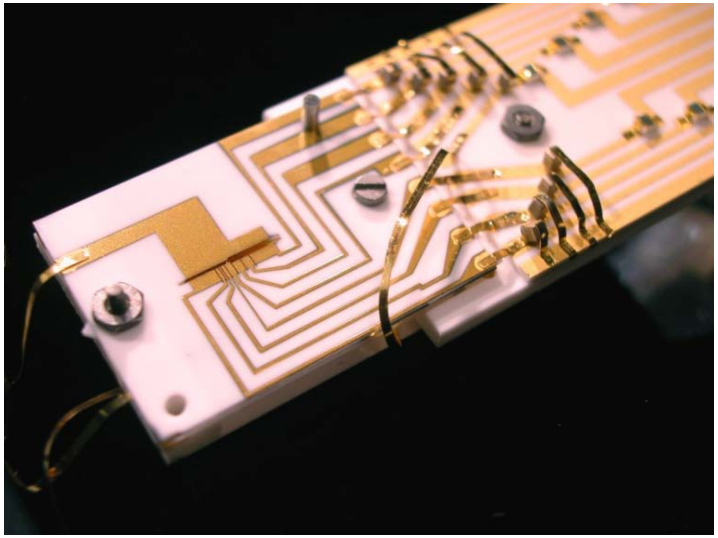
Distributing entanglement: probabilistic

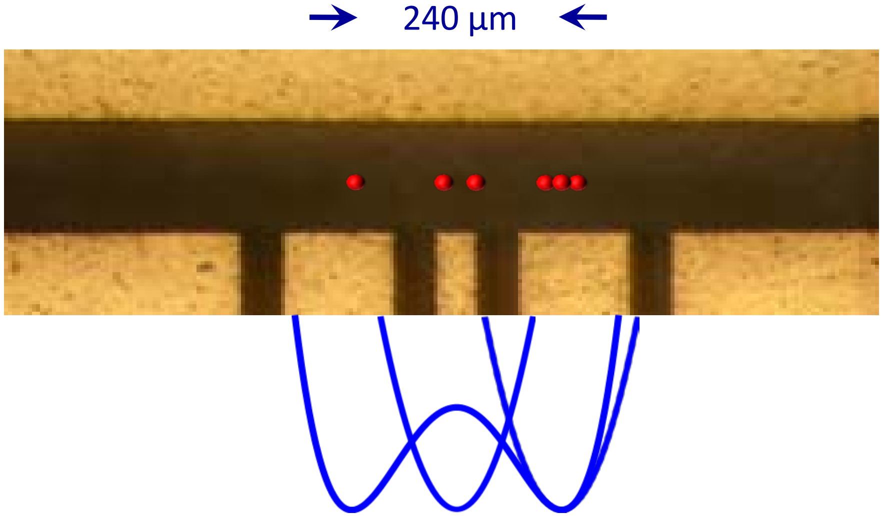


Entangled ions separated by **1m** (Moehring et al. Nature 449, 68 (2008))



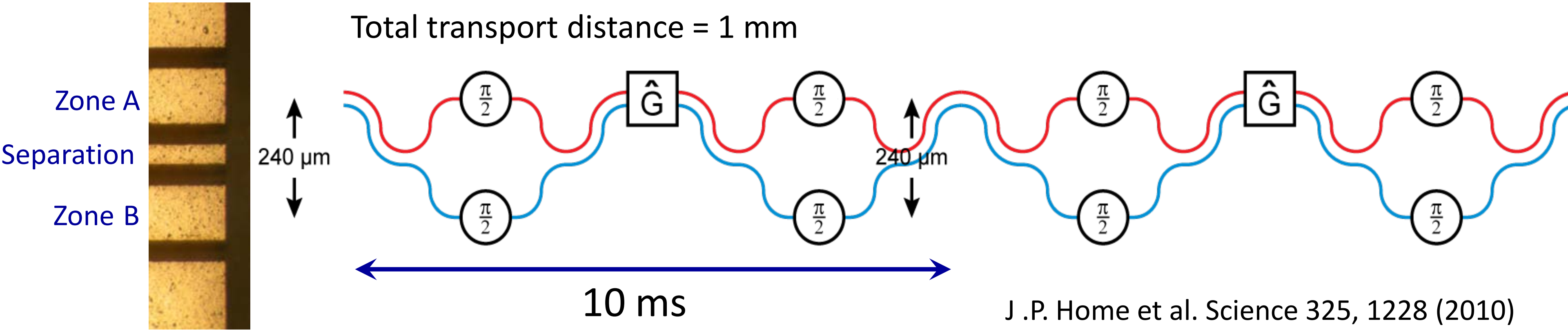
Transport with ions





Move: 20 us, Separate 340 us, 0.5 quanta/separation

Internal quantum states of ions unaffected by transport
Motional states are affected – can be re-initialised

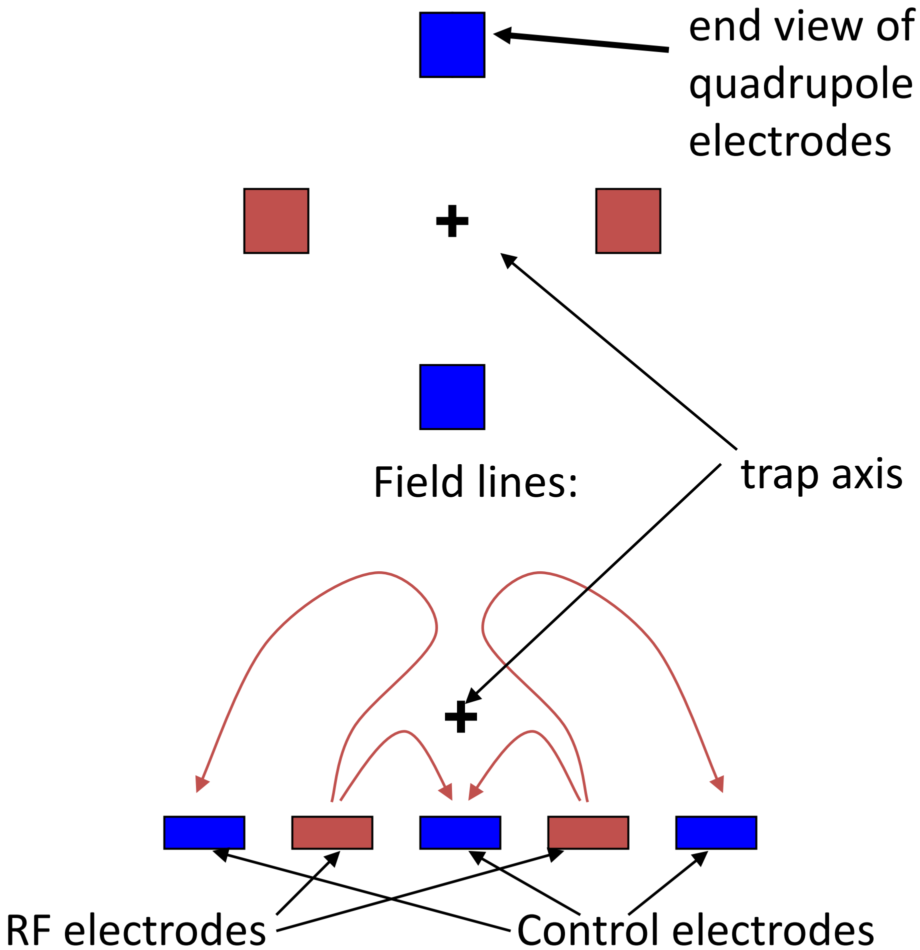


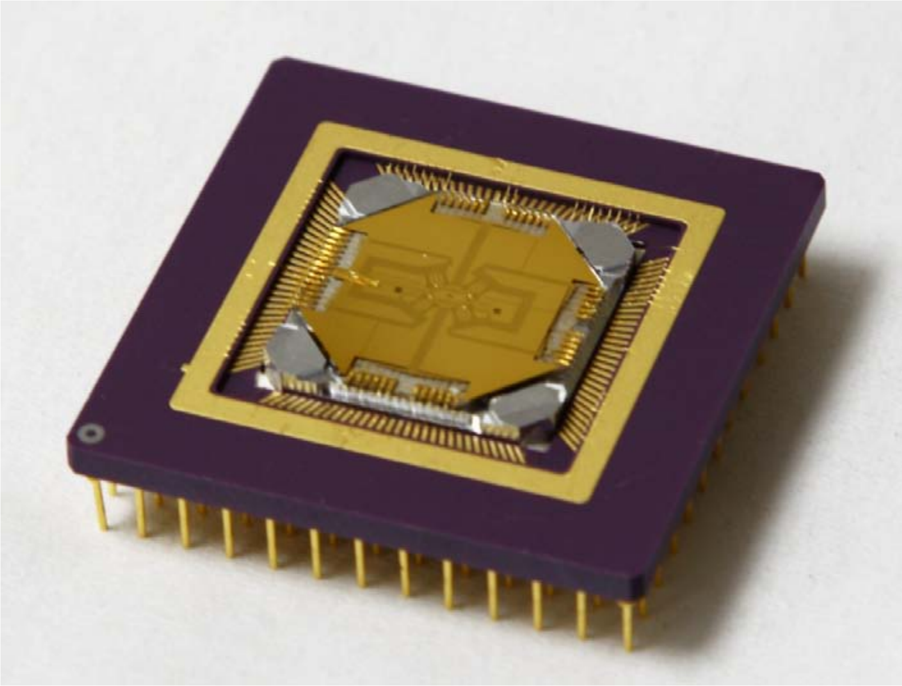
J .P. Home et al. Science 325, 1228 (2010)

Trapping ions on a chip

For microfabrication purposes, desirable to deposit trap structures on a surface

(Chiaverini *et al*., Quant. Inf. & Computation (2005), Seidelin et al. PRL 96, 253003 (2006))

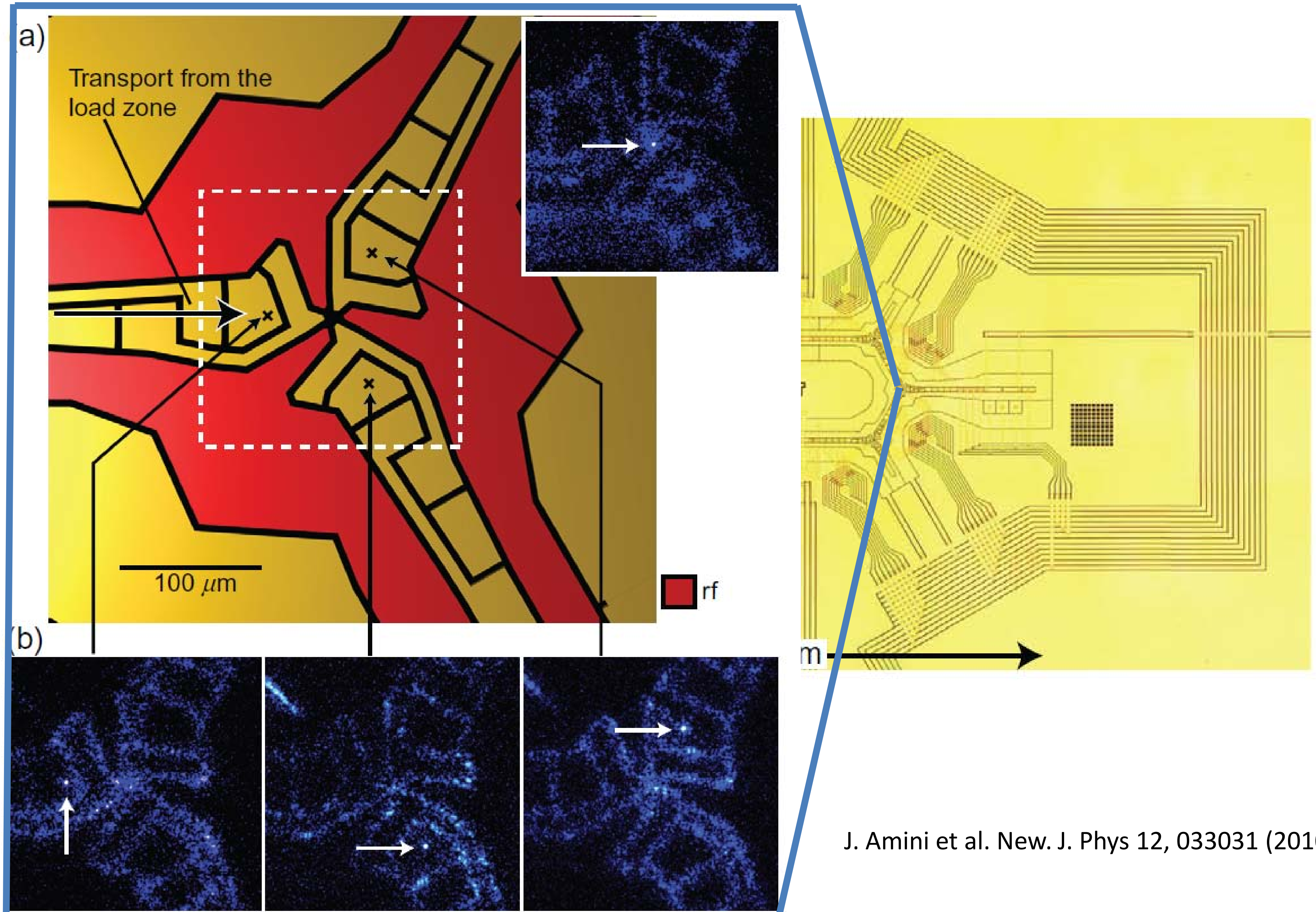




Challenges: shallow trap depth (100 meV)
charging of electrodes

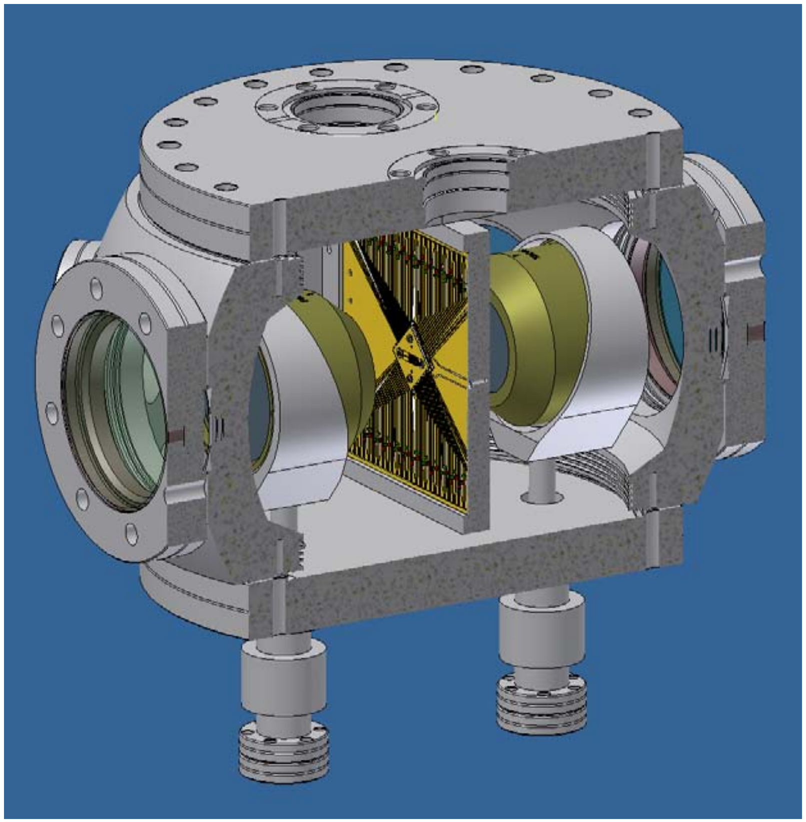
Opportunities: high gradients

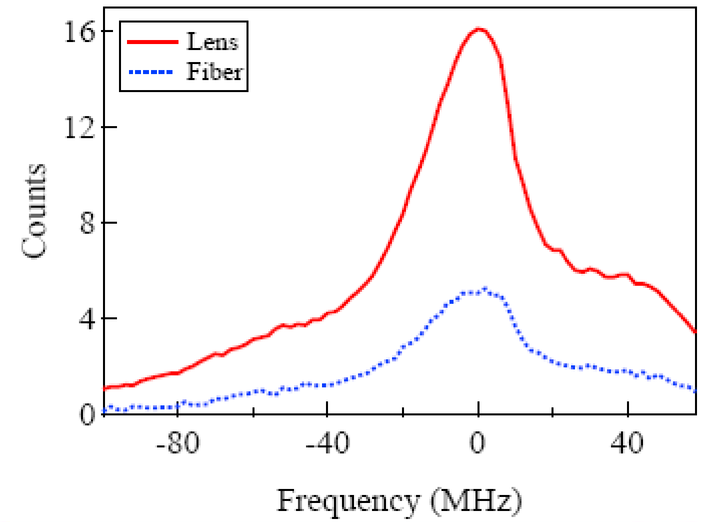
Transporting ions on a (complicated) chip

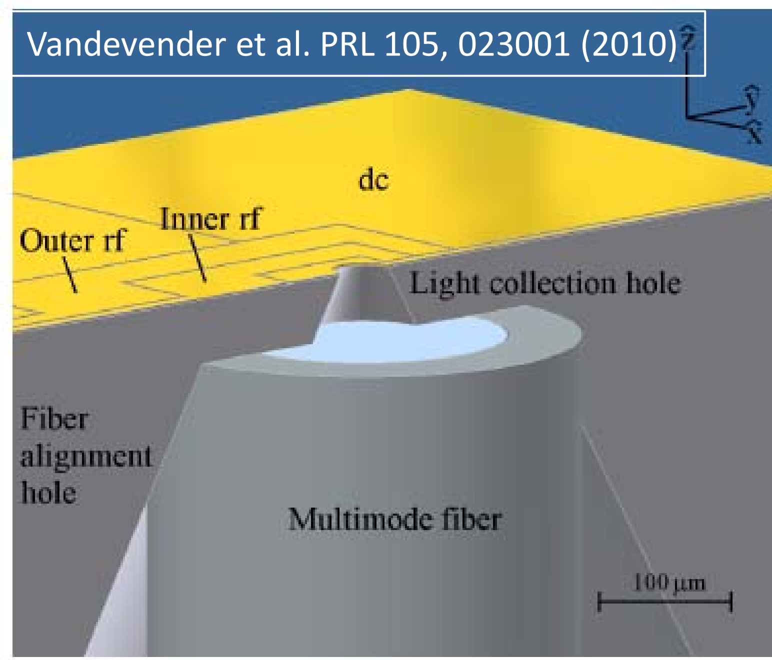


J. Amini et al. New. J. Phys 12, 033031 (2010)

Integrated components 1

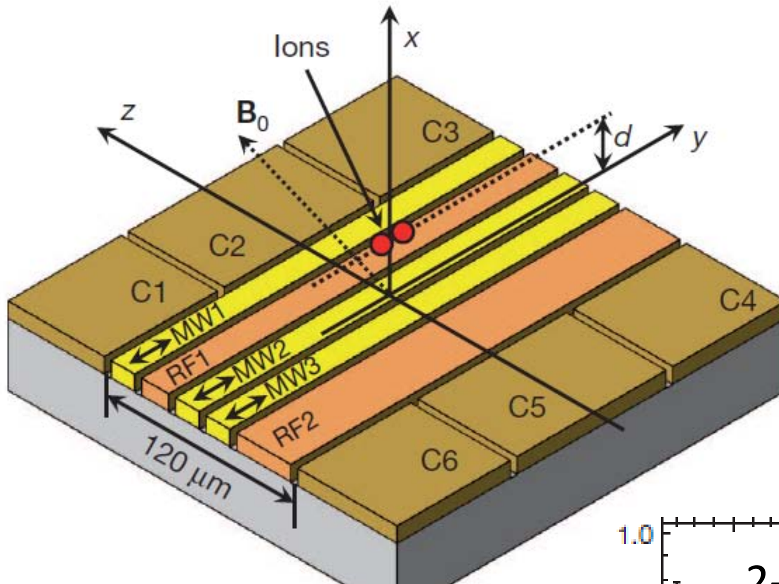






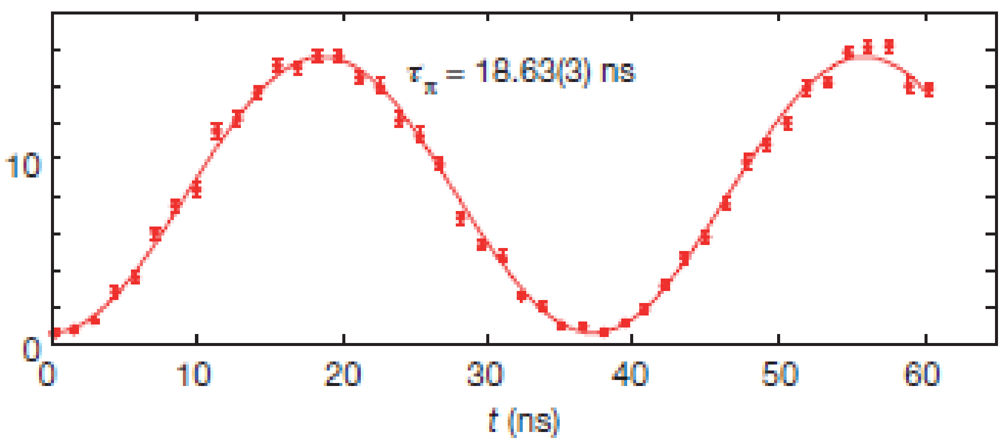
Integrated components

eg. Quantum control using microwaves – removes the need for high-power lasers

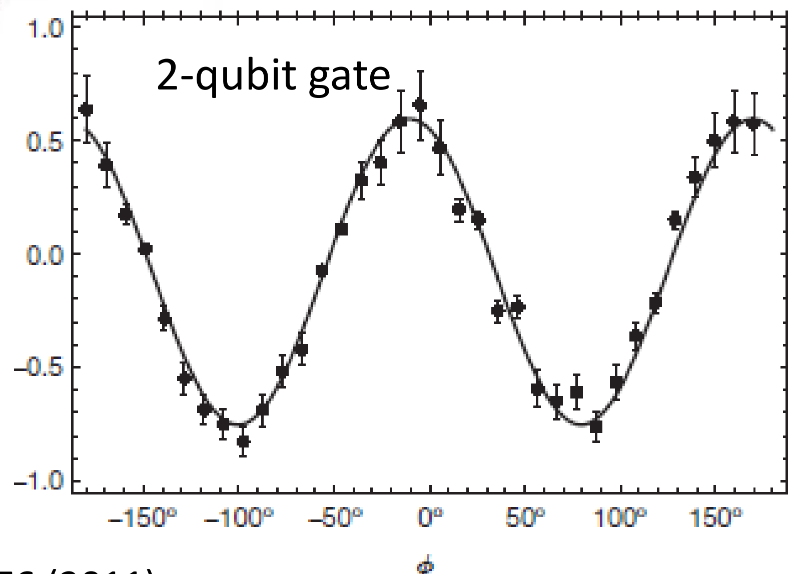


Gradients – produce state-dependent potentials through Zeeman shifts

Single-qubit gate



2-qubit gate

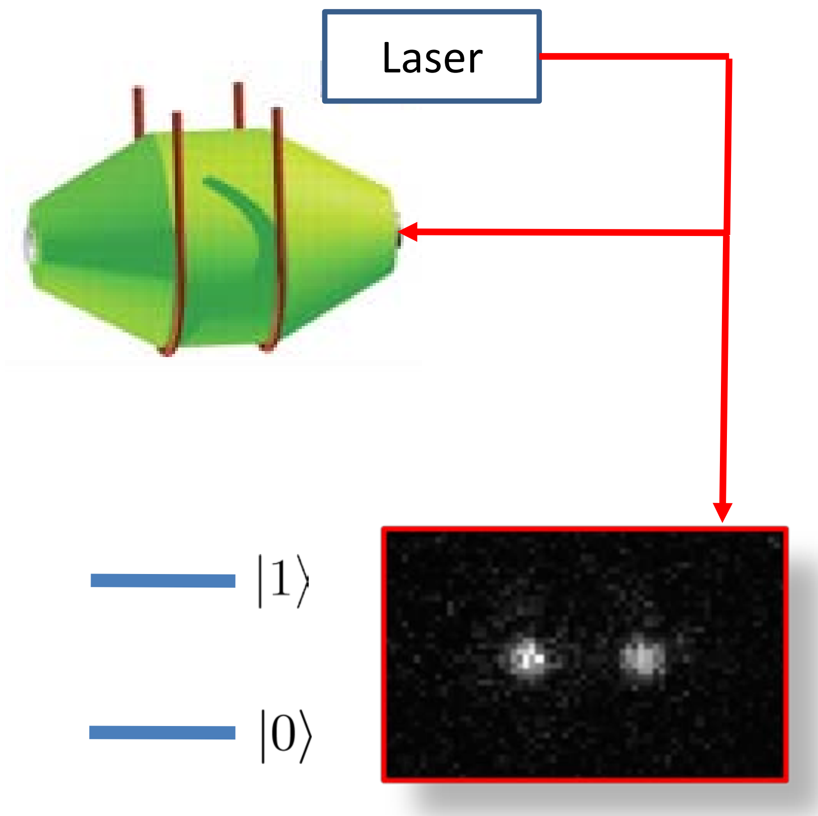


C. Ospelkaus et al. Nature 182, 476 (2011)

Trapped-ions and optical clocks

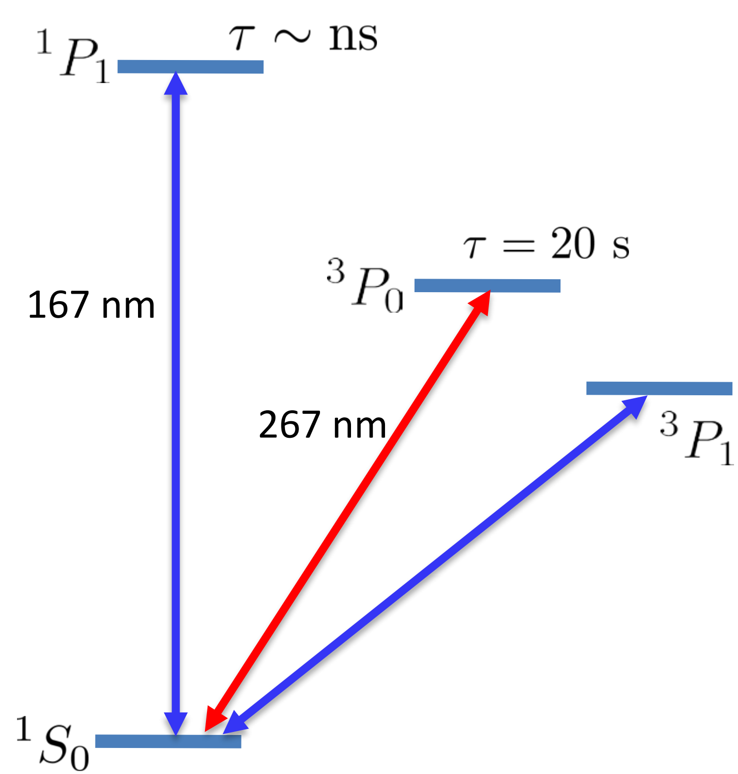
e.g. Rosenband et al., Science 319, 1808 (2008)

Frequency standards



Require very stable ion transition

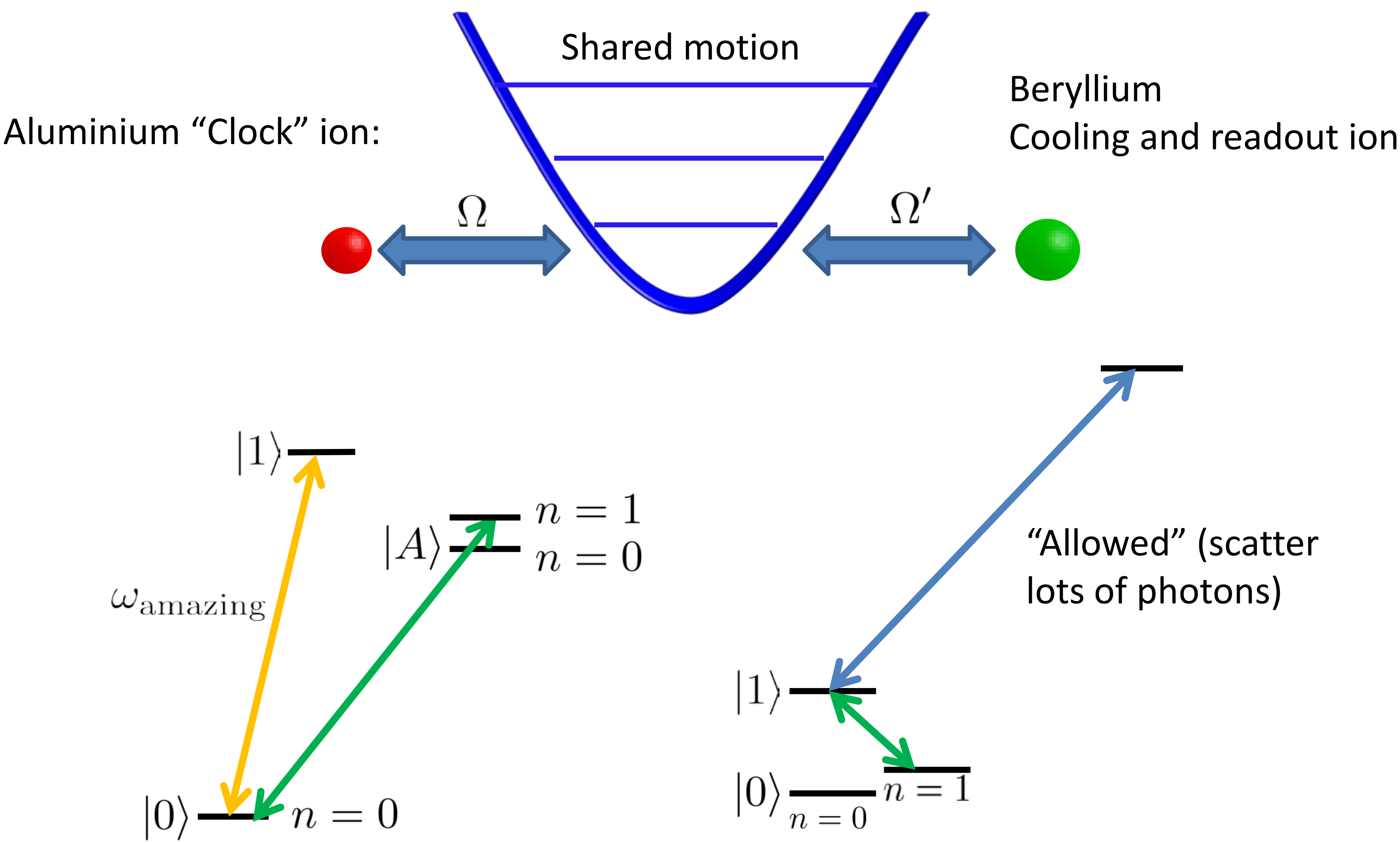
Aluminium ion



Has a very stable transition

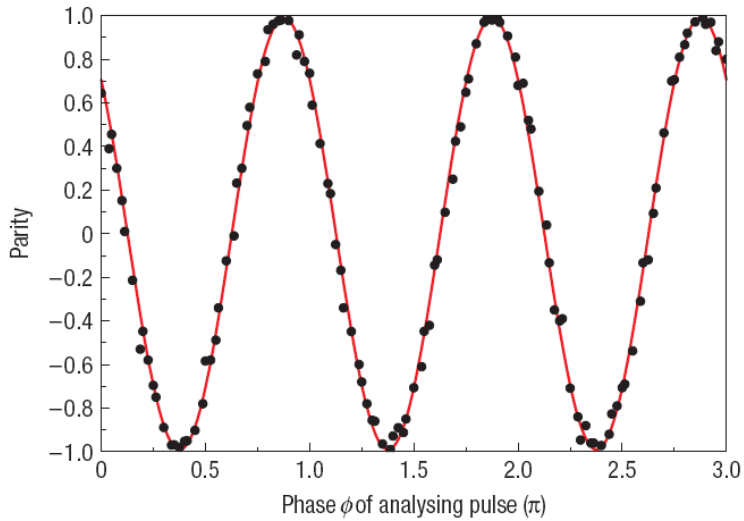
BUT 167 nm is vacuum UV

Atomic clocks – quantum logic readout



Most accurate and precise frequency standards – 8e-18 fractional uncertainty
(Chou et al. PRL *104*, 070802 (2010))

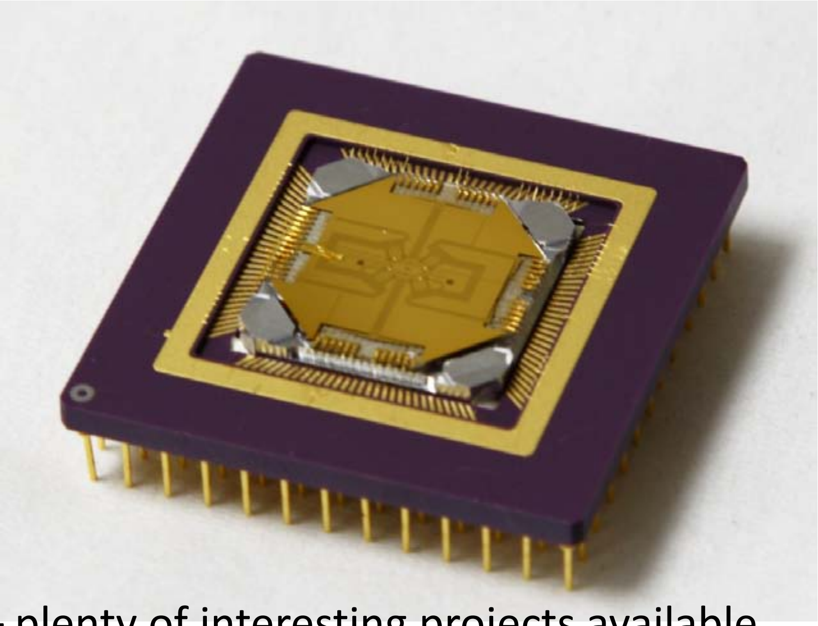
Trapped-ion summary



Have achieved quantum control of up to N ions

Have demonstrated all basic components required to create large scale entangled states

Algorithms & gates include Dense-coding, error-correction, Toffoli, Teleportation, Entanglement purification Entanglment swapping



Working on:

Higher precision

New manipulation methods

Scaling to many ions

ETH group always looking for Masters students – plenty of interesting projects available