Coupling Superconducting Qubits and Generation of Entanglement
Entangling two distant qubits

transmission line resonator can be used as a ‘quantum bus’
to create entangled states
Dispersive two-qubit J-coupling – Energy levels

qubit 1: transition frequency: $\omega_q \approx \sqrt{8E_C E_J} = \sqrt{8E_C E_{J,\text{max}}} \cos(\pi \Phi / \Phi_0)$

qubit 2: constant frequency (5.5 GHz)

resonator:  
- direct coupling ($g \sim 130$ MHz)
- mediated J-coupling ($J \sim 20$ MHz)

[Major et al., Nature 449 (2007)]
Dispersive regime – 2 qubits

\[ H = H_0 + J \left( \sigma_+^{(1)} \sigma_-^{(2)} + \sigma_-^{(2)} \sigma_+^{(1)} \right) \]

transverse exchange (J-) coupling mediated by virtual photons

\[ H_0 = \hbar (\omega_r + \sum_{j=1,2} \chi_j \sigma_{zj}) a^\dagger a + \]
\[ + \frac{\hbar}{2} \sum_{j=1,2} (\omega_{aj} + \chi_j) \sigma_{zj} \]

coupling strength determined by qubit-cavity coupling \( g_j \) and detuning \( \Delta = \omega_a - \omega_r \):

\[ J = \frac{g_1 g_2}{\Delta} \]

qubit eigenstates (Bell states):

\[ |\psi+\rangle = (|eg\rangle + |ge\rangle)/\sqrt{2} \]
\[ |\psi-\rangle = (|eg\rangle - |ge\rangle)/\sqrt{2} \]
Avoided level crossing

qubit A swept across resonance with fixed qubit B
cavity mediated coupling leads to an avoided crossing
2-qubit gate: $\sqrt{iSWAP}$ gate using $ge \leftrightarrow eg$ transitions
2-qubit gate: iSWAP gate using ge ↔ eg transitions

\[ |gg\rangle \overset{\pi_A}{\rightarrow} |eg\rangle \]
\[ \sqrt{iSWAP} \rightarrow \frac{1}{\sqrt{2}} (|eg\rangle - i|ge\rangle) \]

+ local phase transformation:
\[ |\Psi\rangle = \frac{1}{\sqrt{2}} (|eg\rangle + |ge\rangle) \]
Characterisation of two-qubit state:

Is it sufficient to measure single qubit observables

\[ \sigma_x \otimes 1, \sigma_y \otimes 1, \sigma_z \otimes 1 \]

and

\[ 1 \otimes \sigma_x, 1 \otimes \sigma_y, 1 \otimes \sigma_z \]

to fully reconstruct any arbitrary two-qubit state?

1. Yes – it is sufficient.
2. No – more observables need to be measured.
3. Maybe.
Correlation measurement

Correlation measurement with individual readout

Table of single shot values (±1):

<table>
<thead>
<tr>
<th>k</th>
<th>$\sigma_z^k$</th>
<th>1</th>
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<tr>
<td>1</td>
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$\langle \ldots \rangle = \frac{1}{K} \sum_k \langle \sigma_z \otimes 1 \rangle = -1/3$
Correlation measurement with individual readout

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$\langle \ldots \rangle = \frac{1}{K} \sum_k \langle \sigma_z \otimes 1 \rangle = -1/3$  \hspace{1cm} $\langle 1 \otimes \sigma_z \rangle = 1/3$
Correlation measurement with individual readout

Table of single shot values ($\pm 1$):

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$\langle \ldots \rangle = \frac{1}{K} \sum_k \sigma_z \otimes 1 = -1/3$

$\langle 1 \otimes \sigma_z \rangle = 1/3$

$\langle \sigma_z \otimes \sigma_z \rangle = 1/3$
Correlation measurement with individual readout

rotation of qubit: $\langle \sigma_x \ 1 \rangle$, $\langle 1 \ \sigma_z \rangle$ and $\langle \sigma_x \ \sigma_z \rangle$ are measured
Correlation measurement with individual readout

\[ \langle \sigma_x \sigma_y \rangle, \text{a.s.o.} \]

\[ \text{or} \langle \sigma_x \rangle, \langle 1 \sigma_y \rangle \text{ and } \langle \sigma_x \sigma_y \rangle, \text{a.s.o.} \]

\[ \rightarrow \text{all combinations of } \{\sigma_x, \sigma_y, \sigma_z\} \text{ give full information about the state} \]
Correlation measurement with joint readout

Circuit QED-Setup:

- single qubit operations
- only single detection device
Two qubit setup

joint averaged read-out of two-qubit state:

dispersive two-qubit Hamiltonian:

$$H_0 = \hbar (\omega_r + \chi_1 \hat{\sigma}_z + \chi_2 \hat{\sigma}_{z2}) \hat{a}^\dagger \hat{a} + \frac{\hbar}{2} \sum_{j=1,2} (\omega_{a,j} + \chi_j) \hat{\sigma}_{zj}$$

Homodyne measurement

Amplitude difference ($\delta Q$) depends on state of second qubit:

qubit-qubit correlations can be determined from transmission measurement
Measurement operator

Homodyne voltage measurement: \[ Q = \langle \hat{M}_Q \rangle = \text{Tr}[\rho \hat{M}_Q] \]

steady state amplitude:

\[
\hat{M}_Q = \frac{\kappa}{(\Delta_{rm} + \hat{\delta}\omega)^2 + (\kappa/2)^2}
\]

\[
\hat{M}_Q = \beta_{00} \text{id} \otimes \text{id} + \beta_{10} \hat{\sigma}_z \otimes \text{id} + \beta_{01} \text{id} \otimes \hat{\sigma}_z + \beta_{11} \hat{\sigma}_z \otimes \hat{\sigma}_z
\]

\[
= \alpha_{++} |ee\rangle\langle ee| + \alpha_{+-} |eg\rangle\langle eg| + \alpha_{-+} |ge\rangle\langle ge| + \alpha_{--} |gg\rangle\langle gg|)
\]
Measurement operator

\[ \hat{M}_Q = \beta_{00} \text{id} \otimes \text{id} + \beta_{10} \hat{\sigma}_z \otimes \text{id} + \beta_{01} \text{id} \otimes \hat{\sigma}_z + \beta_{11} \hat{\sigma}_z \otimes \hat{\sigma}_z \]

\[ = \alpha_{++}|ee\rangle\langle ee| + \alpha_{+-}|eg\rangle\langle eg| + \alpha_{-+}|ge\rangle\langle ge| + \alpha_{--}|gg\rangle\langle gg| \]

Integrated Q-quadrature signal determines measurement operator:

\[ \alpha_{\pm \pm} = \frac{1}{N} \int_{t_m}^{T} (Q_{\pm \pm}(t) - Q_{--}(t)) dt \]

Non-vanishing \( \sigma_z \sigma_z \) term for qubit-qubit correlations, e.g.:

\[ \hat{M}_Q = 0.8\text{id} + 0.4 \sigma_z \otimes \text{id} - 0.3\text{id} \otimes \sigma_z - 0.1 \sigma_z \otimes \sigma_z \]
Quantum Teleportation

Alice

No local interaction!

Bob

classical communication

Qubit A:

\[ |\psi\rangle = \begin{pmatrix} \uparrow \cr \downarrow \end{pmatrix} \]

Qubit B, C:

\[ |\text{Bell}\rangle = \frac{1}{\sqrt{2}} \left( \begin{pmatrix} \uparrow \cr \uparrow \end{pmatrix} + \begin{pmatrix} \downarrow \cr \downarrow \end{pmatrix} \right) \]

Bell state measurement:

If Bell state 1:

\[ |\tilde{\psi}\rangle = e^{i\sigma_x/2} |\psi\rangle = \begin{pmatrix} \uparrow \cr \downarrow \end{pmatrix} \]

If Bell state 2:

\[ |\tilde{\psi}\rangle = e^{i\sigma_y/2} |\psi\rangle = \begin{pmatrix} \uparrow \cr \downarrow \end{pmatrix} \]

\[ \Rightarrow \begin{pmatrix} \uparrow \cr \downarrow \end{pmatrix} \]

...
Teleportation: transmission of quantum bit (qubit A) from Alice to Bob using a pair of entangled qubits (qubits B+C)

Preparation of Bell state
Bell Measurement
Measurement + classical communication
2-qubit gate: C-Phase gate using ee ↔ fg transitions

- ee-level interacts with fg-level
- coupling strength $J_2 \sim 40-80$ MHz ($g \sim 300$ MHz)
- fast, non-adiabatic tuning of ee/fg levels into resonance
- $2\pi$ - rotation after $t = \pi / J_2$
- $|ee\rangle$ - state picks up phase $e^{i2\pi / 2} = -1$
2-qubit gate: C-Phase gate using ee <-> fg transitions

gate operation: 

\[ \begin{align*} 
|ee\rangle & \rightarrow -|ee\rangle \\
|ge\rangle & \rightarrow |ge\rangle \\
|eg\rangle & \rightarrow |eg\rangle \\
|gg\rangle & \rightarrow |gg\rangle \\
\end{align*} \]

**C-Phase** = universal 2-qubit gate

\[ U_{CPhase} = \begin{pmatrix} 
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 \\
\end{pmatrix} \]
Teleportation: transmission of quantum bit (qubit A) from Alice to Bob using a pair of entangled qubits (qubits B+C)

Preparation of Bell state
Bell Measurement

implemented three qubit tomography at step III
ETH Quantum processor platform with 3-Qubits

- Full individual coherent qubit control via local charge and flux lines
- Large coupling strength to resonator $g \sim 300 - 350 \text{ MHz}$
- Transmon coherences times: $T_1 \sim 0.8 - 1.2 \mu s$, $T_2 \sim 0.4 - 0.7 \mu s$.

State tomography of the entangled 3-qubit state

Example: State to be teleported on qubit A is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|g\rangle + i|e\rangle)$$

$$|\varphi\rangle = \{|g_ag_B\rangle \otimes |\Psi\rangle_C$$

$$+ |g_a e_B\rangle \otimes \sigma_x |\Psi\rangle_C$$

$$+ |e_A g_B\rangle \otimes \sigma_z |\Psi\rangle_C$$

$$+ |e_a e_B\rangle \otimes (-\sigma_z \sigma_x) |\Psi\rangle_C\}$$

$$\rho = |\varphi\rangle \langle \varphi|$$

Simulating measurement of qubit A and B with projection on $|g_ag_B\rangle$:

$$\rho_C = \langle g_A g_B | \rho | g_A g_B \rangle$$

$$= |\Psi\rangle \langle \Psi|$$

fidelity 88%

DiVincenzo Criteria fulfilled for Superconducting Qubits for Implementing a Quantum Computer in the standard (circuit approach) to quantum information processing (QIP):

1. A scalable physical system with well-characterized qubits. ✓
2. The ability to initialize the state of the qubits. ✓
3. Long (relative) decoherence times, much longer than the gate-operation time. ✓
5. A qubit-specific measurement capability. ✓

plus two criteria requiring the possibility to transmit information:

6. The ability to interconvert stationary and mobile (or flying) qubits. ✓
7. The ability to faithfully transmit flying qubits between specified locations. ✓
Recent trends I – transmon in 3D cavity

Transmon in a three-dimensional cavity: lifetimes ($T_1$) ~ 100 $\mu$s

Questions: Tunability? Scalability?
Recent trends II – Hybrid Circuit QED

- Spin ensembles (NV centers) – superconducting circuits
  [cavity-NV: Kubo et al., PRL 105, 140502 (2010); Schuster et al., PRL 205, 140501 (2010)]

- Atomic ensembles (BEC)
  [Verdu, PRL 103, 043603 (2009)]

- Charged particles (Ions)
  [Tian et al., PRL 92, 247902 (2004)]

- Electrons on Helium
  [Schuster et al., PRL 105, 040503 (2010)]

- Rydberg atoms
  [Hogan et al., PRL 108, 063004 (2012)]