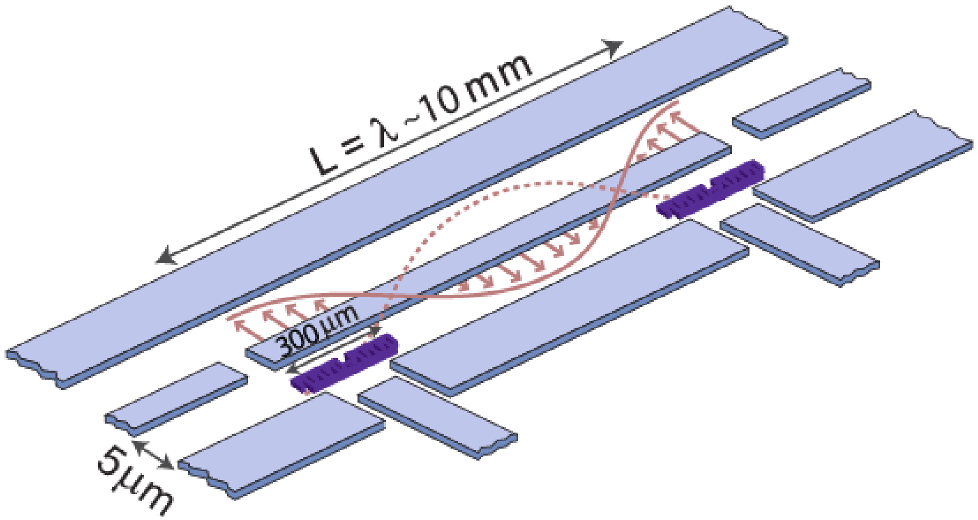
Coupling Superconducting Qubits and Generation of Entanglement

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Entangling two distant qubits

transmission line resonator can be used as a **'quantum bus'**

to create **entangled** states

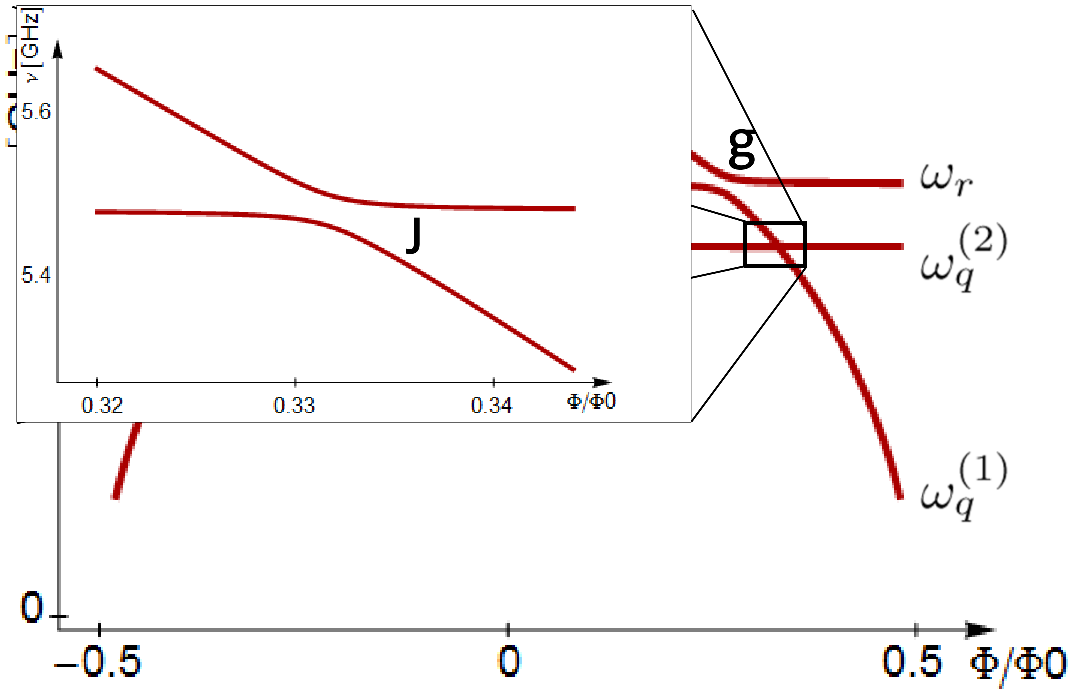


Dispersive two-qubit J-coupling – Energy levels

qubit 1: transition frequency: $\omega_q \approx \sqrt{8E_C E_J} = \sqrt{8E_C E_{J,max}\,|\cos(\pi\Phi/\Phi_0)|}$

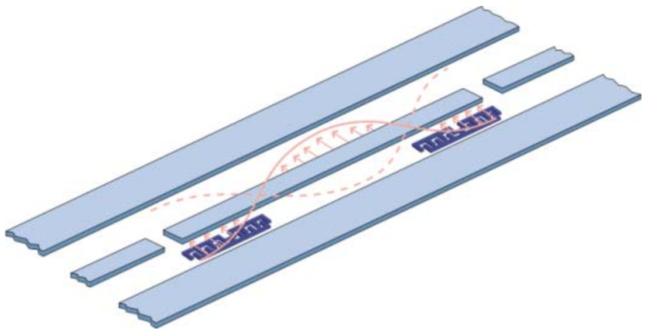
qubit 2: constant frequency (5.5 GHz)

resonator:
- direct coupling (g ~ 130 MHz)
- mediated J-coupling (J ~ 20 MHz)



[Majer *et al.*, Nature 449 (2007)]

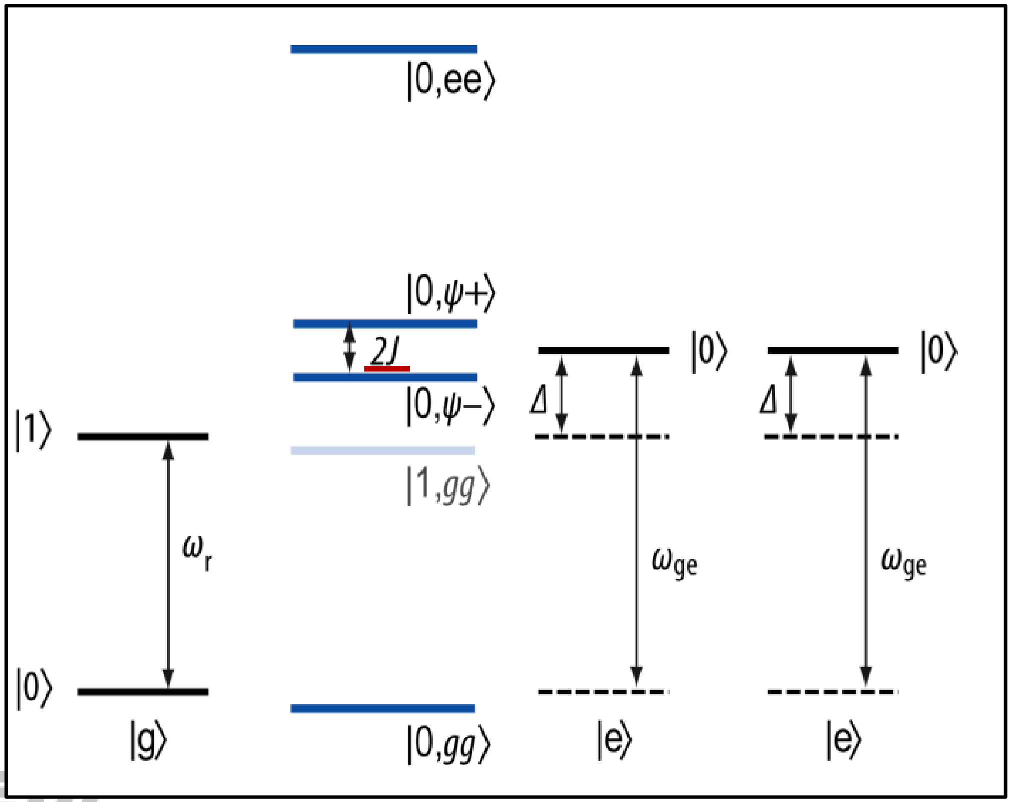
Dispersive regime – 2 qubits



$$H = H_0 + J\left(\sigma_+^{(1)}\sigma_-^{(2)} + \sigma_-^{(2)}\sigma_+^{(1)}\right)$$

transverse exchange (J-) coupling
mediated by virtual photons

$$H_0 = \hbar(\omega_r + \sum_{j=1,2}\chi_j\sigma_{zj})a^\dagger a + \frac{\hbar}{2}\sum_{j=1,2}(\omega_{aj}+\chi_j)\sigma_{zj}$$



coupling strength determined by qubit-cavity coupling g_j and detuning $\Delta = \omega_a - \omega_r$:

$$J = \frac{g_1 g_2}{\Delta}$$

qubit eigenstates (Bell states):

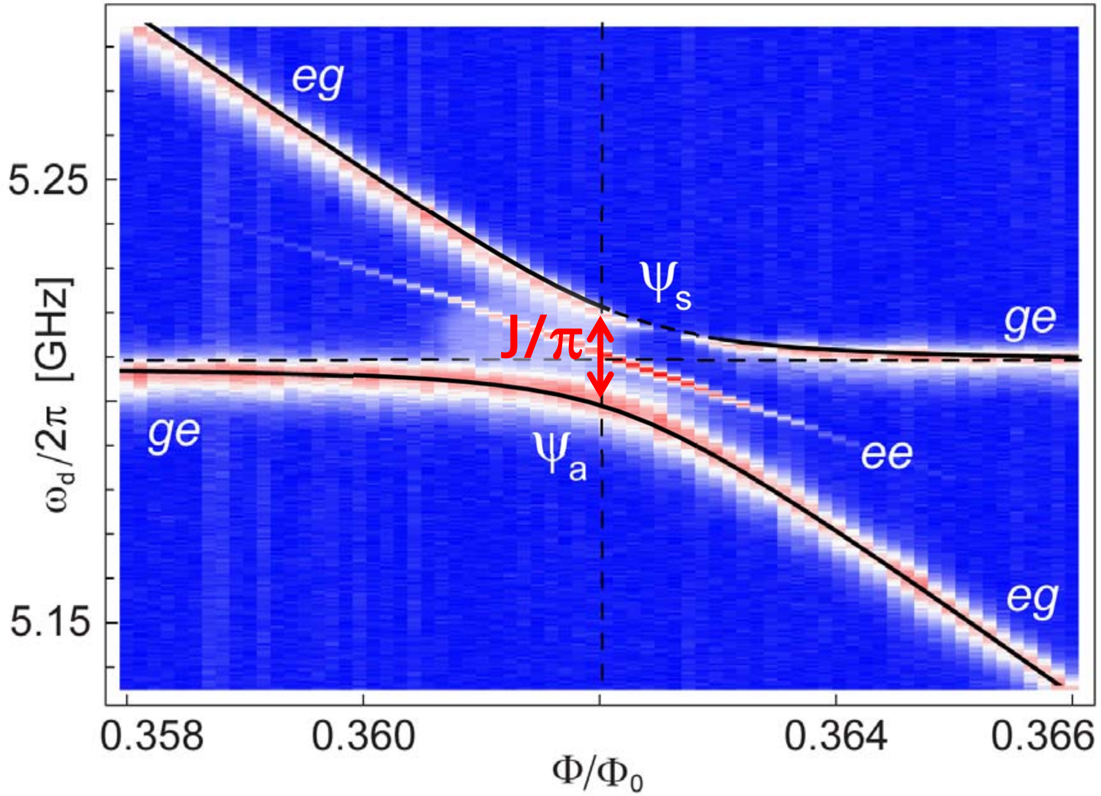
$|\psi+\rangle = (|eg\rangle + |ge\rangle)/\sqrt{2}$

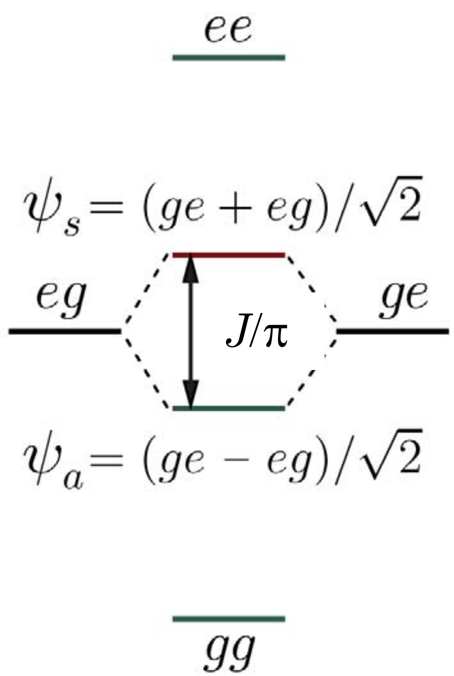
$|\psi-\rangle = (|eg\rangle - |ge\rangle)/\sqrt{2}$

Avoided level crossing

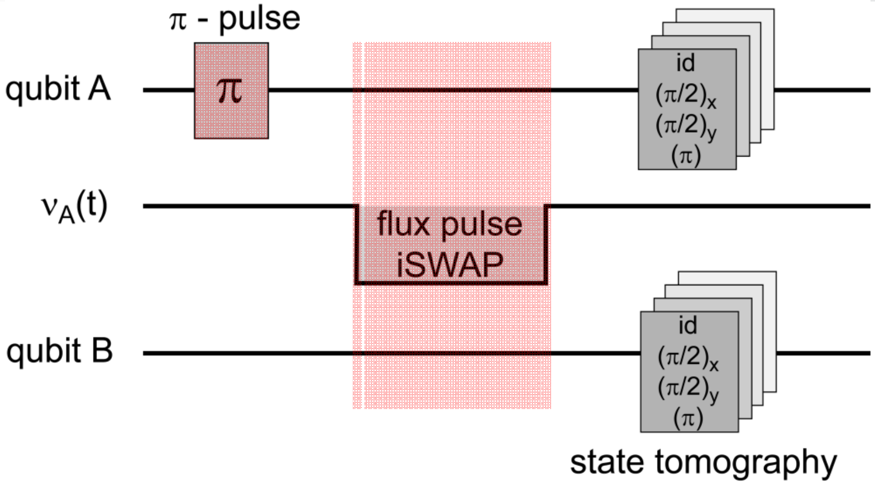
qubit A swept across resonance with fixed qubit B

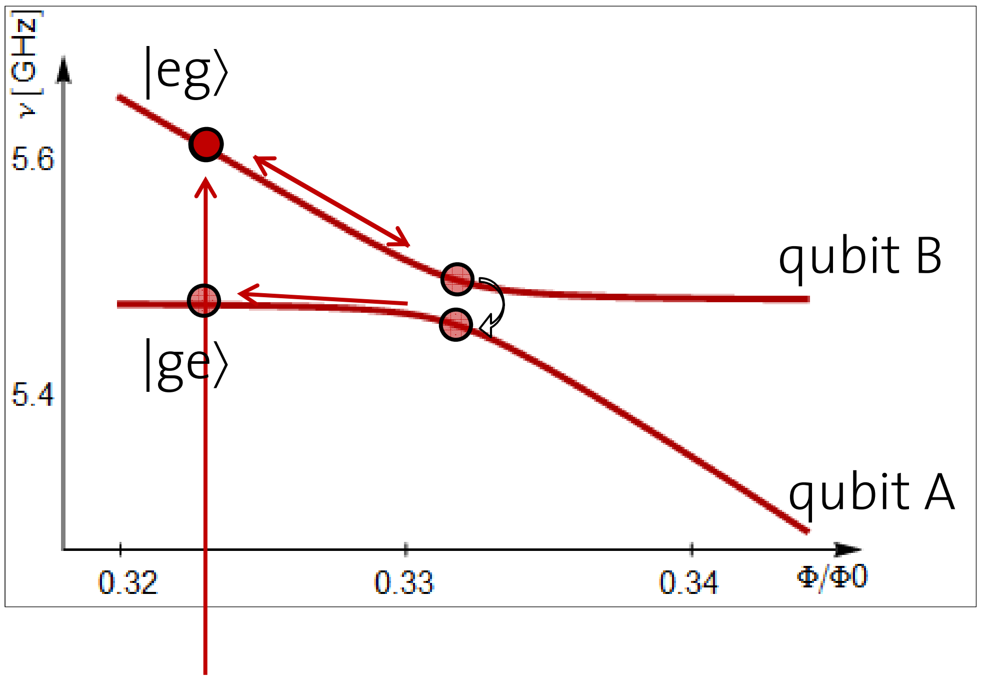
cavity mediated coupling leads to an avoided crossing

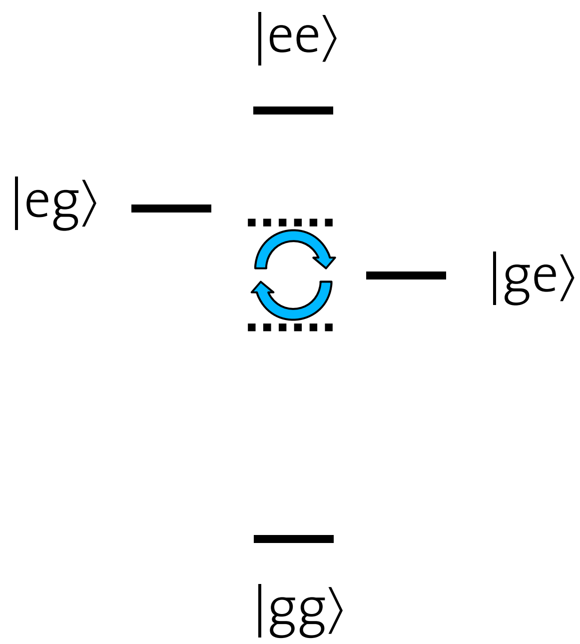




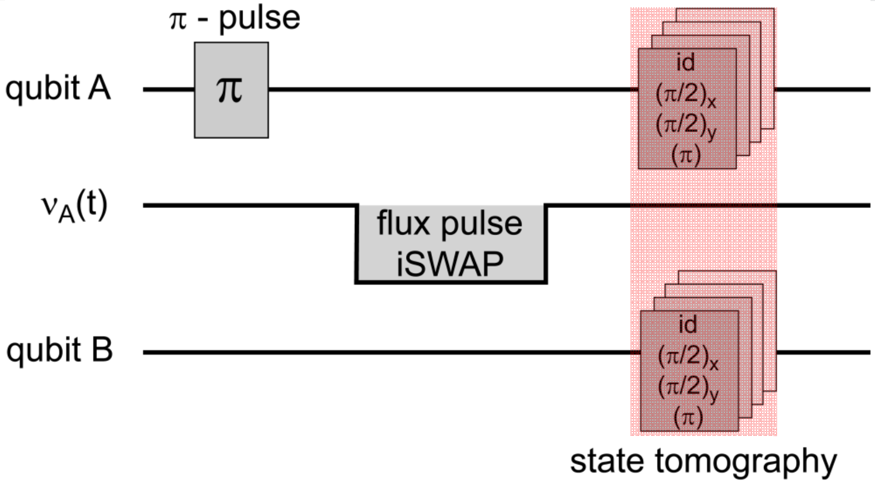
2-qubit gate: √iSWAP gate using ge ↔ eg transitions







2-qubit gate: iSWAP gate using ge $\leftrightarrow$ eg transitions



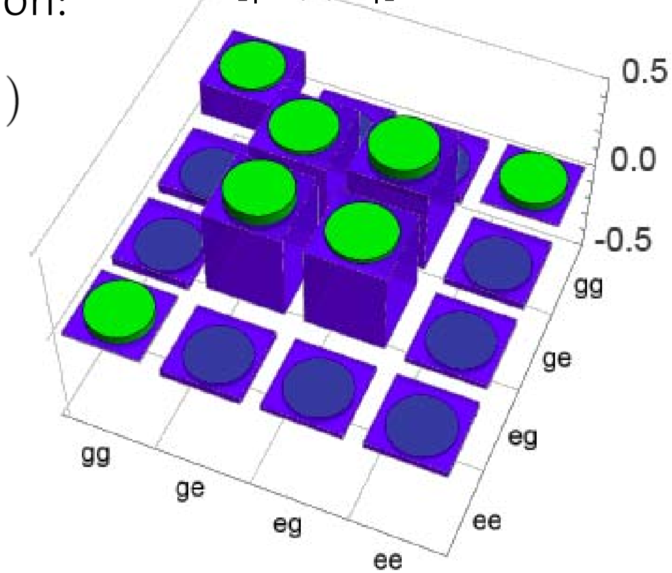
$$|gg\rangle \xrightarrow{\pi_A} |eg\rangle$$

$$\xrightarrow{\sqrt{iSWAP}} \frac{1}{\sqrt{2}}\left(|eg\rangle - i|ge\rangle\right)$$

+ local phase transformation:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|eg\rangle + |ge\rangle\right)$$

Re[|Ψ⟩⟨Ψ|]



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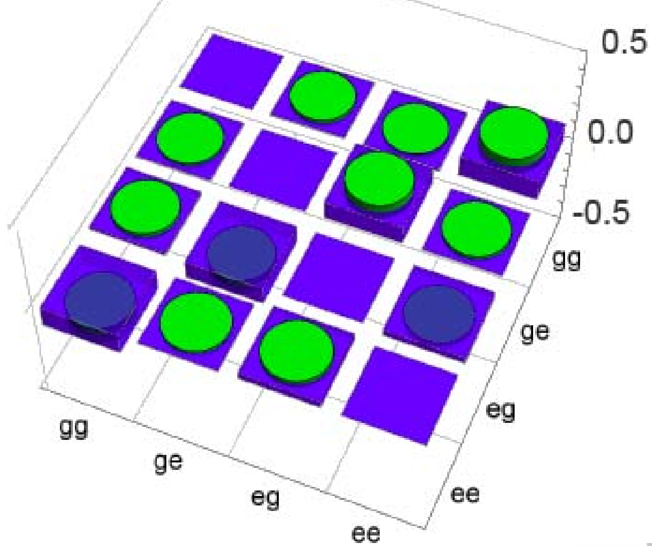


Characterisation of two-qubit state:

Is it sufficient to measure single qubit observables

$$\sigma_x \otimes 1,\ \sigma_y \otimes 1,\ \sigma_z \otimes 1$$

and

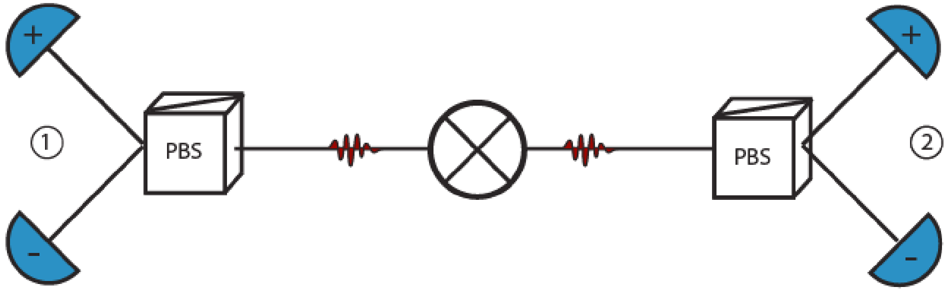
$$1 \otimes \sigma_x,\ 1 \otimes \sigma_y,\ 1 \otimes \sigma_z$$

to fully reconstruct any arbitrary **two**-qubit state?

1. Yes – it is sufficient.
2. No – more observables need to be measured.
3. Maybe.

Correlation measurement

[Photons: Weihs *et al.*, PRL **81** (1998); supercond. qubits: Steffen *et al.*, Science **313** (2006).



Correlation measurement with individual readout

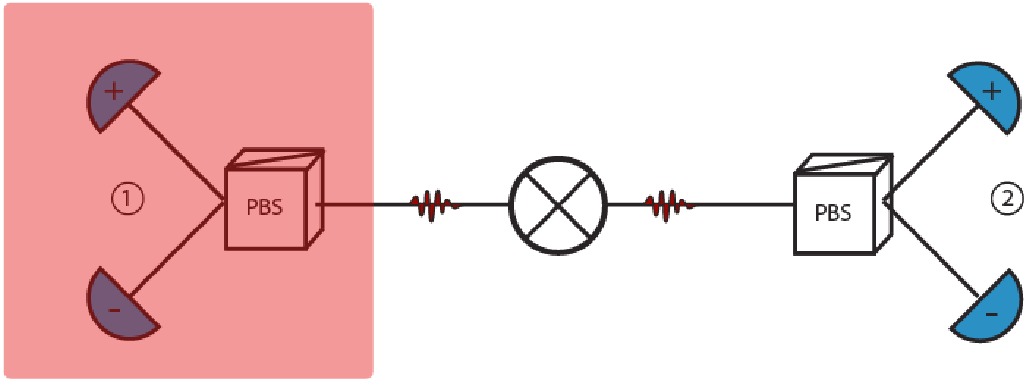


table of single shot values (±1):

k	$\sigma_z^k \ 1$		
1	+1		
2	-1		
...	...		
K	-1		
$\langle \ldots \rangle = \frac{1}{K} \sum_k$	$\langle \sigma_z \otimes 1 \rangle = -1/3$		

Correlation measurement with individual readout

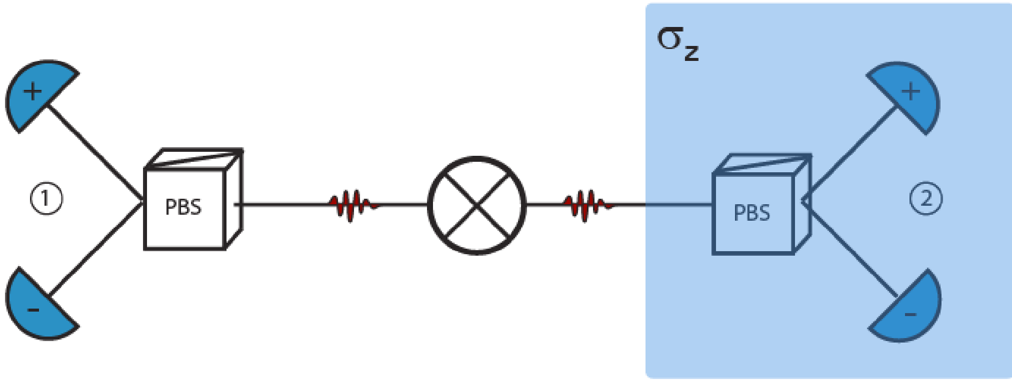


table of single shot values (±1):

k	$\sigma_z^k \quad 1$	$1 \quad \sigma_z^k$	
1	+1	+1	
2	-1	-1	
…	…	…	
K	-1	+1	
$\langle \ldots \rangle = \frac{1}{K}\sum_k$	$\langle \sigma_z \otimes 1 \rangle = -1/3$	$\langle 1 \otimes \sigma_z \rangle = 1/3$	

Correlation measurement with individual readout

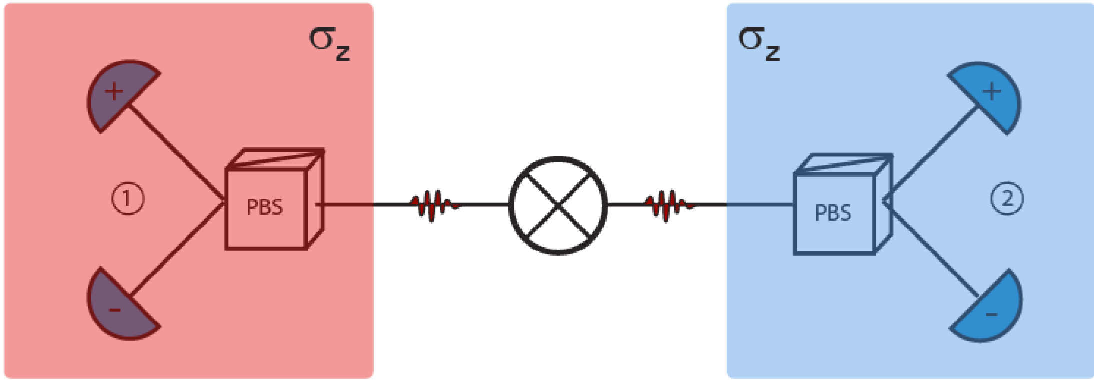
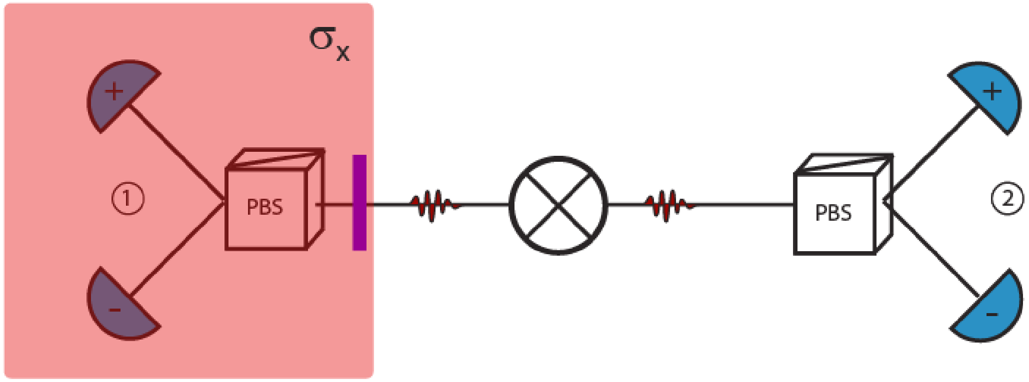


table of single shot values (±1):

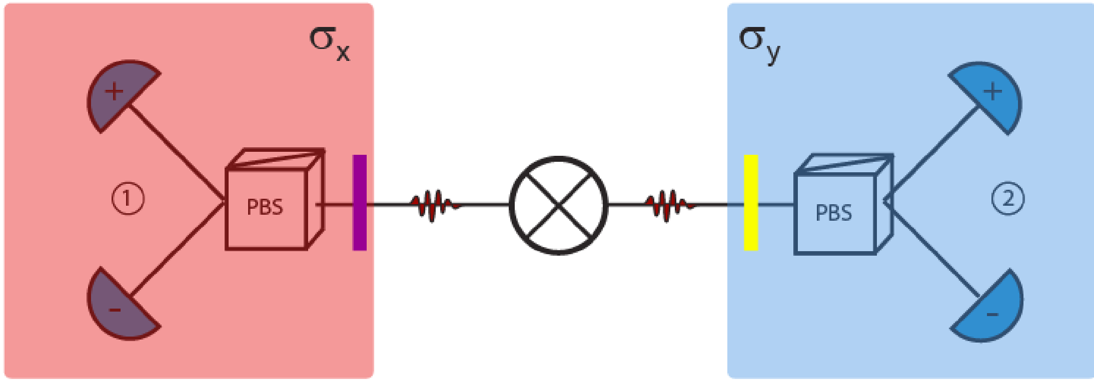
k	$\sigma_z^k \; 1$	$1 \; \sigma_z^k$	$\sigma_z^k \; \sigma_z^k$
1	+1	+1	(+1).(+1) = +1
2	-1	-1	(-1).(-1) = +1
...	...	...	...
K	-1	+1	(-1).(+1)=-1
$\langle \ldots \rangle = \frac{1}{K} \sum_k$	$\langle \sigma_z \otimes 1 \rangle = -1/3$	$\langle 1 \otimes \sigma_z \rangle = 1/3$	$\langle \sigma_z \otimes \sigma_z \rangle = 1/3$

Correlation measurement with individual readout



rotation of qubit: $\langle \sigma_x \; 1 \rangle$, $\langle 1 \; \sigma_z \rangle$ and $\langle \sigma_x \; \sigma_z \rangle$ are measured

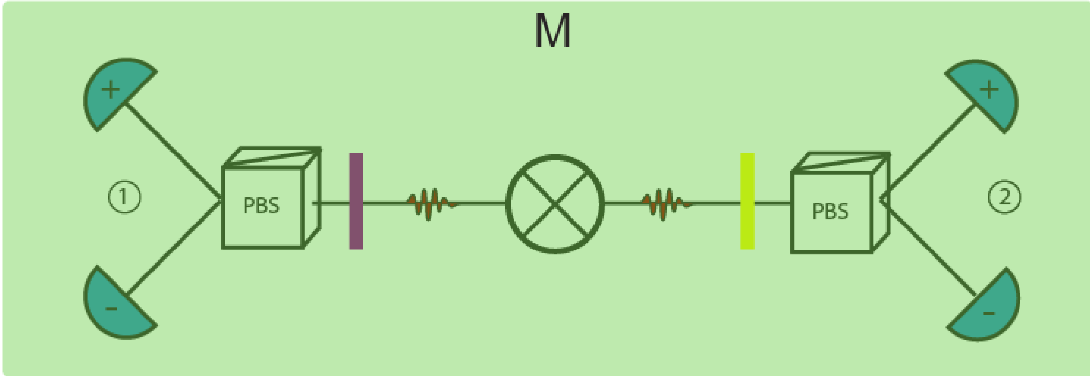
Correlation measurement with individual readout



or $\langle \sigma_x \, 1 \rangle$, $\langle 1 \, \sigma_y \rangle$ and $\langle \sigma_x \, \sigma_y \rangle$, a.s.o.

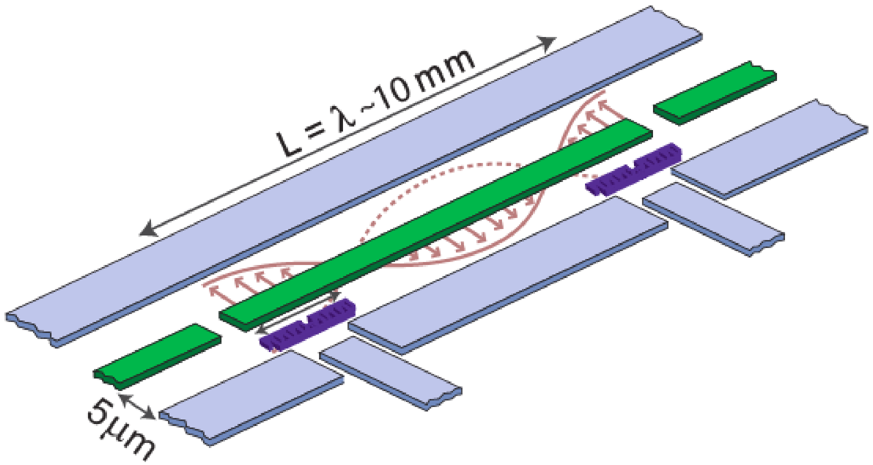
-> all combinations of $\{\sigma_x, \sigma_y, \sigma_z\}$ give full information about the state

Correlation measurement with joint readout



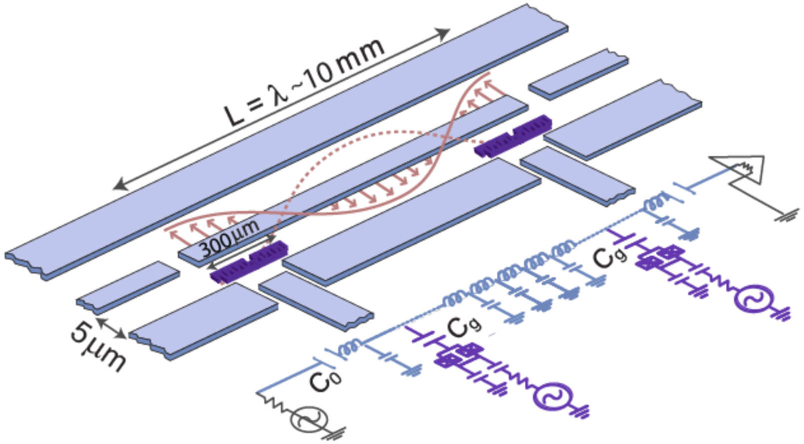
Circuit QED-Setup:

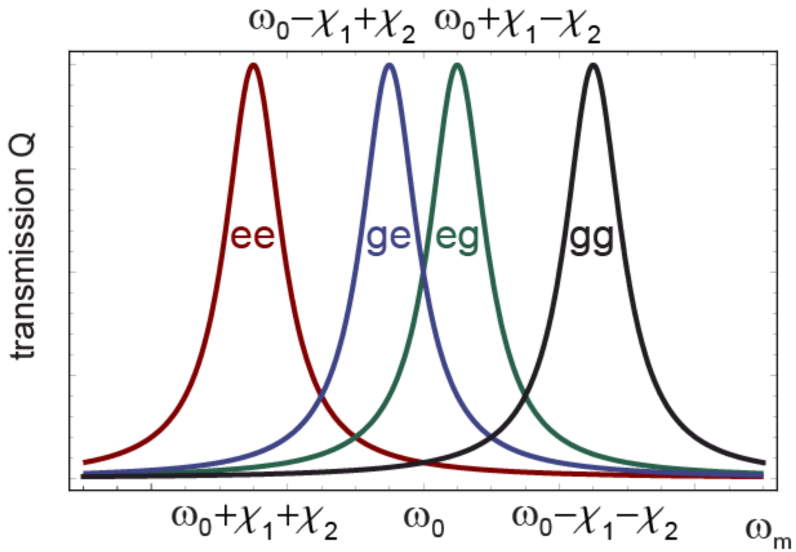
- single qubit operations
- only single detection device



Two qubit setup

joint averaged read-out of two-qubit state:





[Majer *et al.*, Nature **449** (2007)]

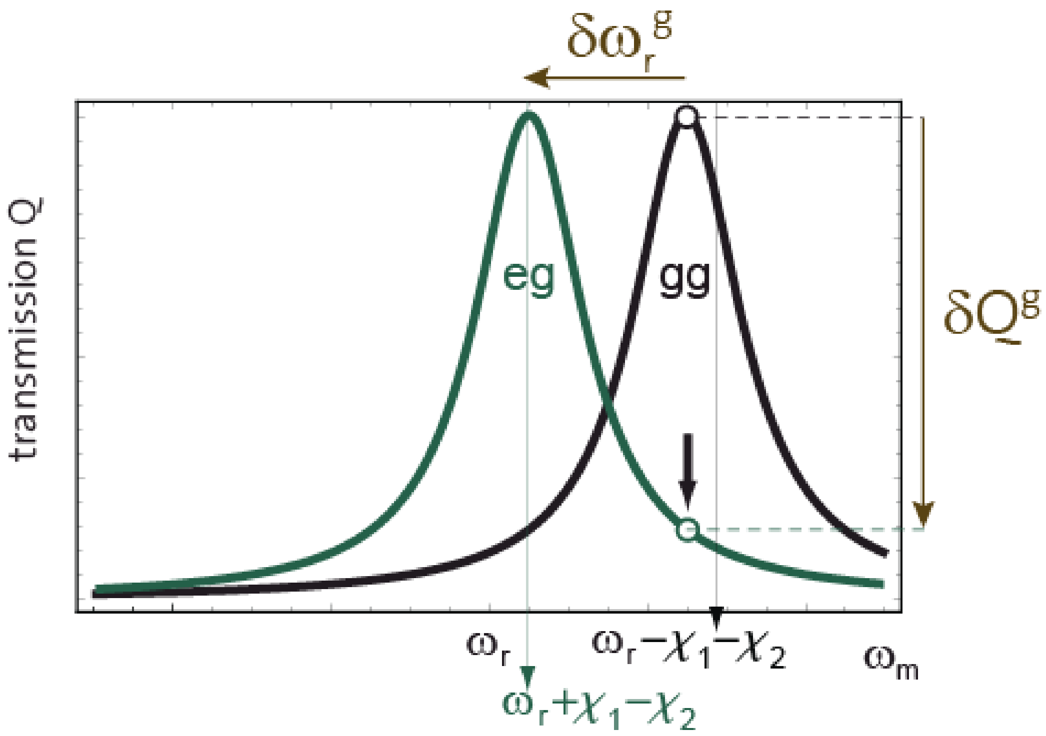
dispersive two-qubit Hamiltonian:

$$(\sigma_{z1} = 1 \otimes \sigma_z; \sigma_{z2} = \sigma_z \otimes 1)$$

$$H_0 = \hbar(\omega_r + \underbrace{\chi_1\hat{\sigma}_{z1} + \chi_2\hat{\sigma}_{z2}}_{\delta\hat{\omega}_r})\hat{a}^\dagger\hat{a} + \frac{\hbar}{2}\sum_{j=1,2}(\omega_{aj} + \chi_j)\hat{\sigma}_{zj}$$

Homodyne measurement

Amplitude difference (δQ) depends on state of second qubit:



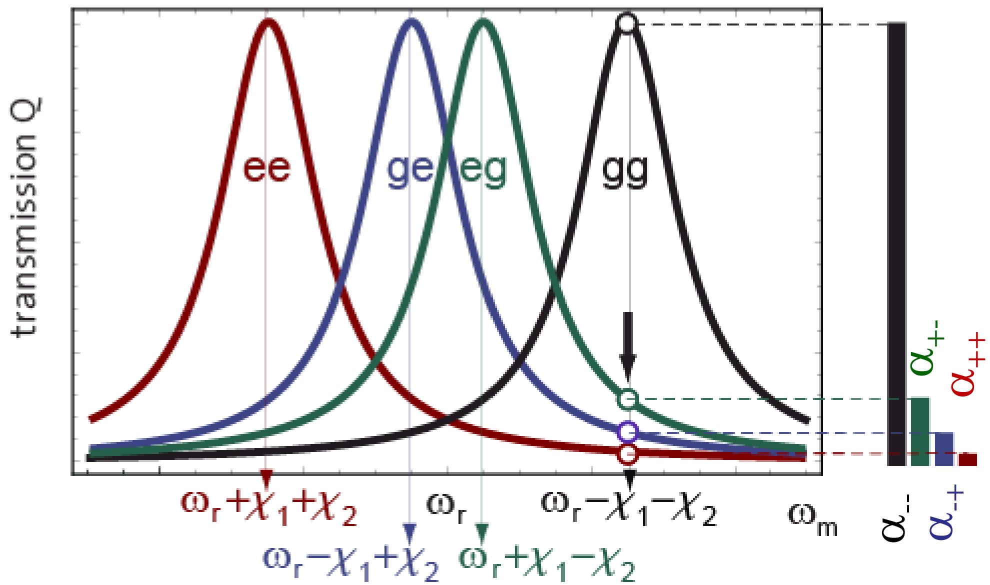
➡ qubit-qubit correlations can be determined from transmission measurement

Measurement operator

Homodyne voltage measurement: $Q = \langle \hat{M}_Q \rangle = \mathrm{Tr}[\rho \hat{M}_Q]$

steady state amplitude:

$$\hat{M}_Q = \frac{\kappa}{(\Delta_{rm} + \hat{\delta\omega})_r^2 + (\kappa/2)^2}$$



$$\hat{M}_Q = \beta_{00}\,\mathrm{id} \otimes \mathrm{id} + \beta_{10}\,\hat{\sigma}_{z1} \otimes \mathrm{id} + \beta_{01}\,\mathrm{id} \otimes \hat{\sigma}_{z2} + \beta_{11}\,\hat{\sigma}_{z1} \otimes \hat{\sigma}_{z2}$$
$$= \alpha_{++}|ee\rangle\langle ee| + \alpha_{+-}|eg\rangle\langle eg| + \alpha_{-+}|ge\rangle\langle ge| + \alpha_{--}|gg\rangle\langle gg|$$

Measurement operator

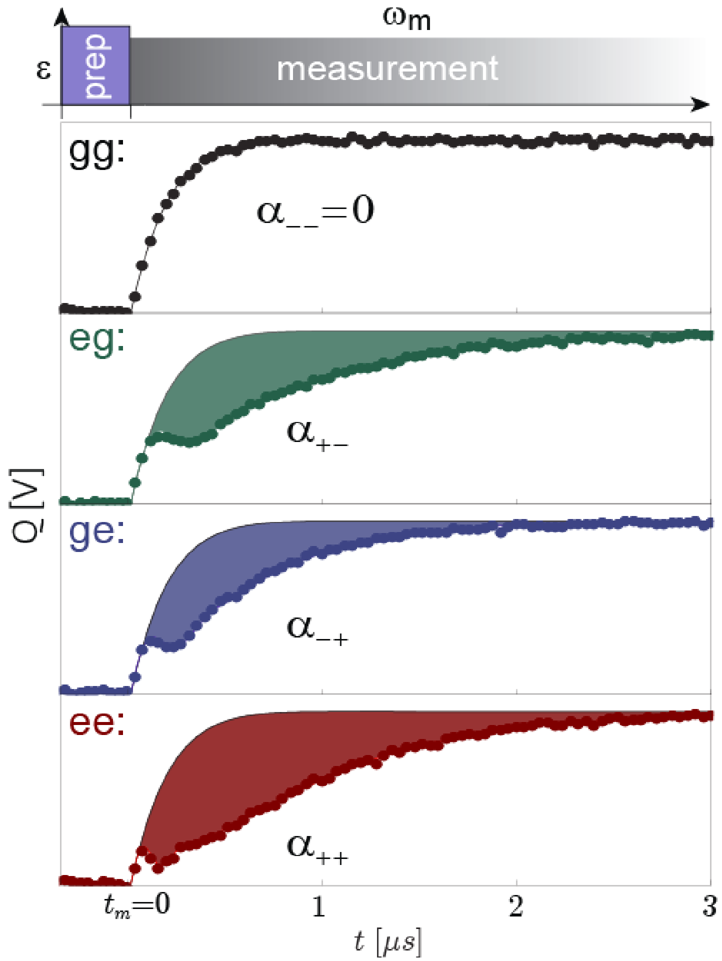
$$\hat{M}_Q = \beta_{00}\,\mathrm{id}\otimes\mathrm{id} + \beta_{10}\,\hat{\sigma}_{z1}\otimes\mathrm{id} + \beta_{01}\,\mathrm{id}\otimes\hat{\sigma}_{z2} + \beta_{11}\,\hat{\sigma}_{z1}\otimes\hat{\sigma}_{z2}$$
$$= \alpha_{++}|ee\rangle\langle ee| + \alpha_{+-}|eg\rangle\langle eg| + \alpha_{-+}|ge\rangle\langle ge| + \alpha_{--}|gg\rangle\langle gg|$$

Integrated Q-quadrature signal determines measurement operator:

$$\alpha_{\pm\pm} \equiv \frac{1}{N}\int_{t_m}^{T} (Q_{\pm\pm}(t) - Q_{--}(t))dt$$

Non-vanishing $\sigma_z \sigma_z$ term for qubit-qubit correlations, e.g.:

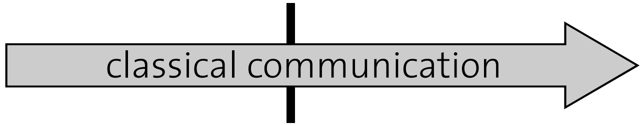
$$\hat{M}_Q = 0.8\mathrm{id} + 0.4\,\sigma_z\otimes\mathrm{id}$$
$$-0.3\mathrm{id}\otimes\sigma_z - 0.1\sigma_z\otimes\sigma_z$$



Quantum Teleportation

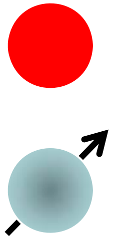
Alice

No local interaction!



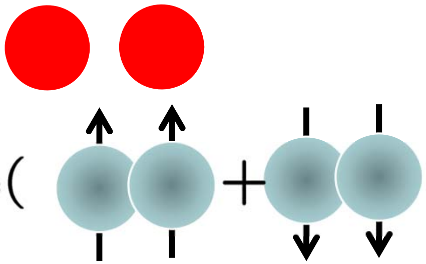
Bob

Qubit A:

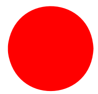


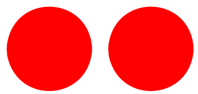
$|\psi\rangle =$

Qubit B, C:



$|\text{Bell}\rangle = \frac{1}{\sqrt{2}}(\quad + \quad)$

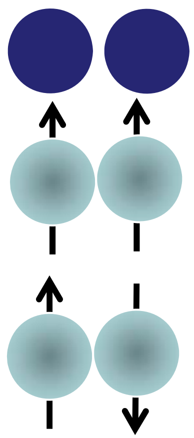




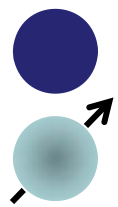
Bell state measurement:

If Bell state 1:

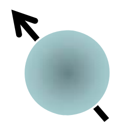
If Bell state 2:



$|\psi\rangle =$



$|\tilde{\psi}\rangle = e^{i\sigma_x/2}|\psi\rangle =$



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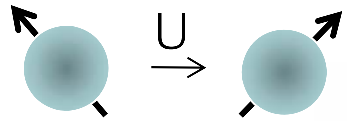
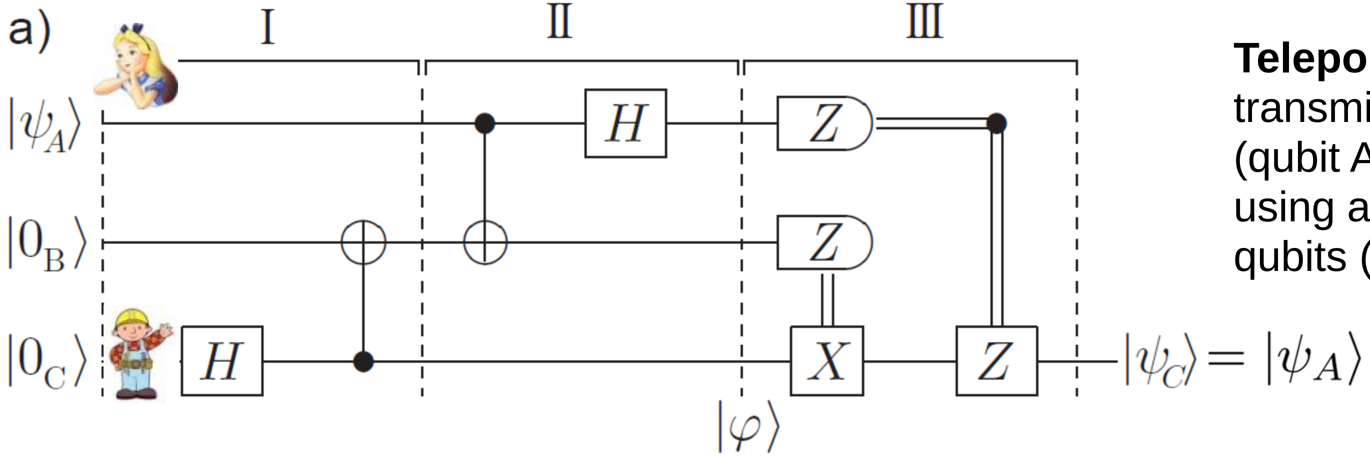


Teleportation Circuit



Teleportation:
transmission of quantum bit (qubit A) from Alice to Bob using a pair of entangled qubits (qubits B+C)

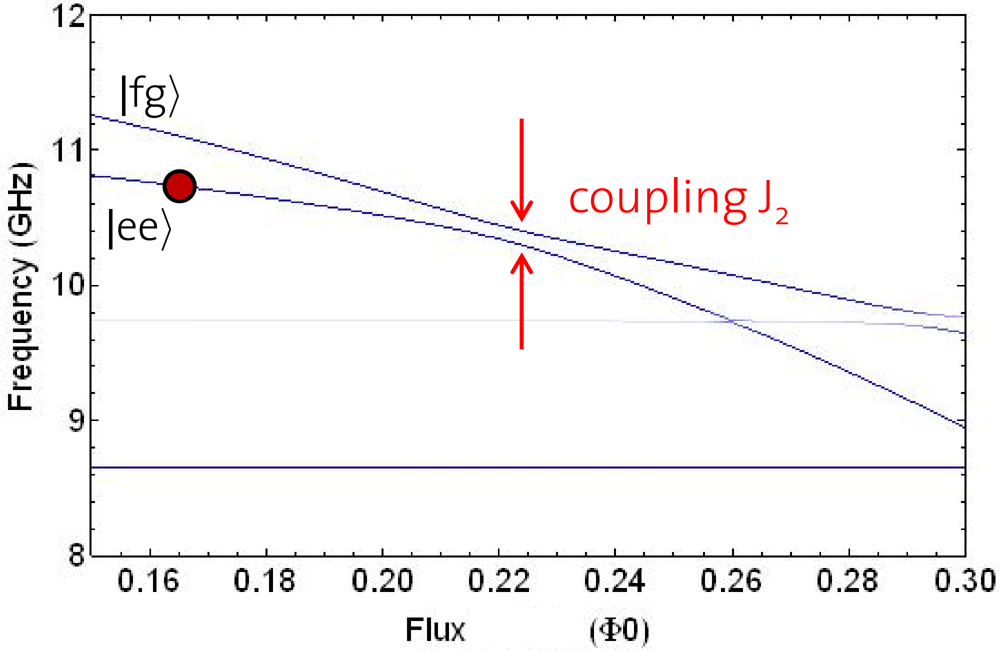
Preparation of Bell state

Measurement + classical communication

Bell Measurement

2-qubit gate: C-Phase gate using ee ↔ fg transitions

- ee-level interacts with fg-level
- coupling strength J_2~ 40-80 MHz (g ~ 300 MHz)
- fast, non-adiabatic tuning of ee/fg levels into resonance
- 2π - rotation after $t=\pi/J_2$
- $|ee\rangle$ -state picks up phase $e^{i2\pi/2}= -1$



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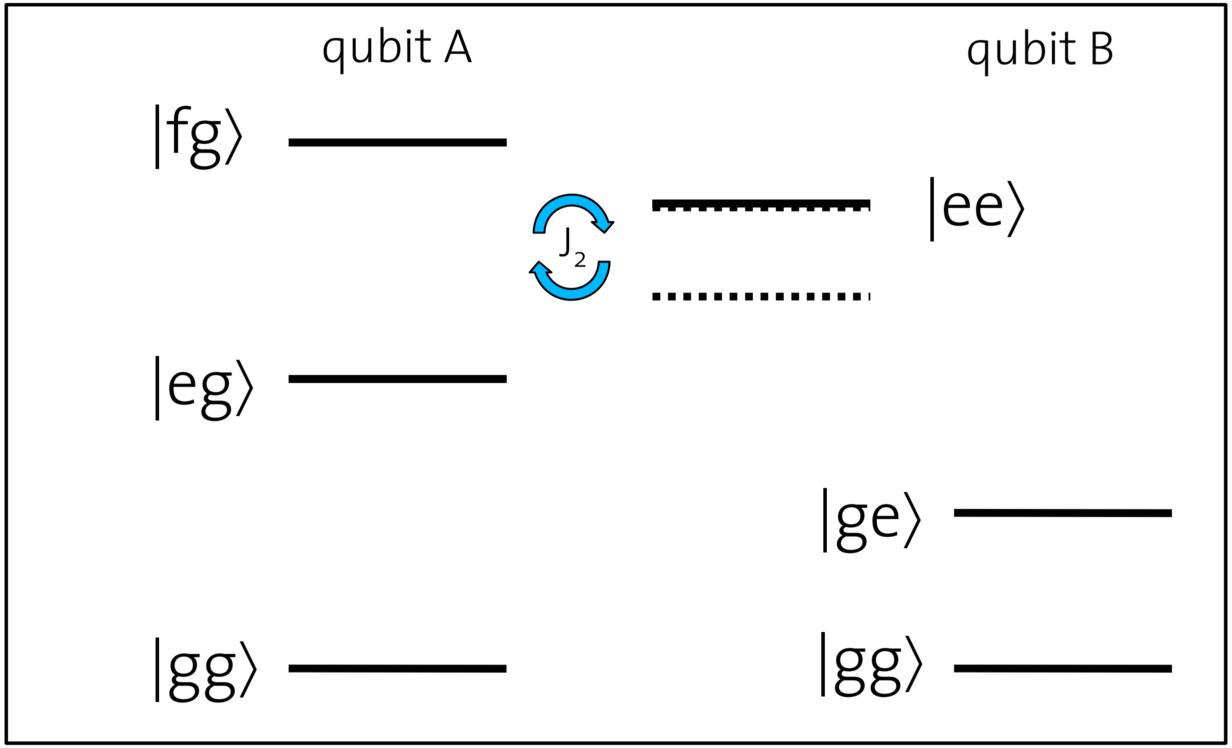


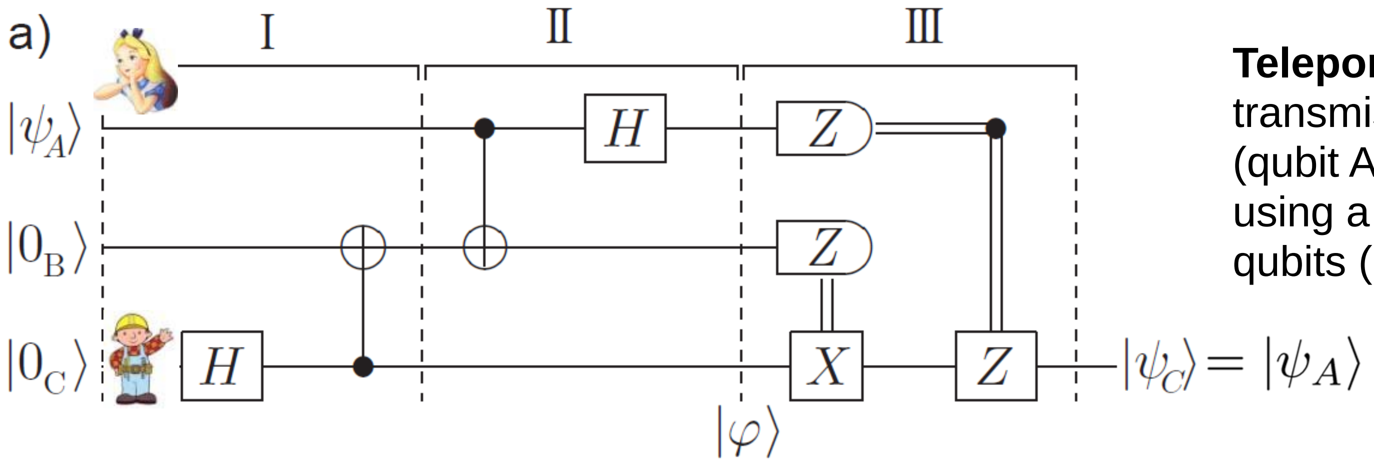
gate operation:

C-Phase = universal 2-qubit gate

$$|ee\rangle \longrightarrow -|ee\rangle$$
$$|ge\rangle \longrightarrow |ge\rangle$$
$$|eg\rangle \longrightarrow |eg\rangle$$
$$|gg\rangle \longrightarrow |gg\rangle$$

$$U_{CPhase} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Teleportation Circuit



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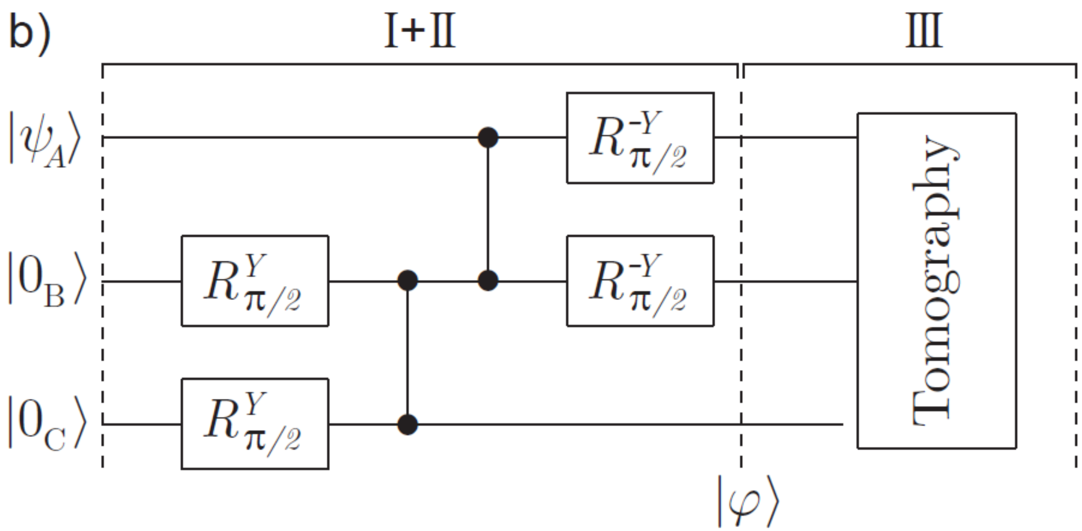
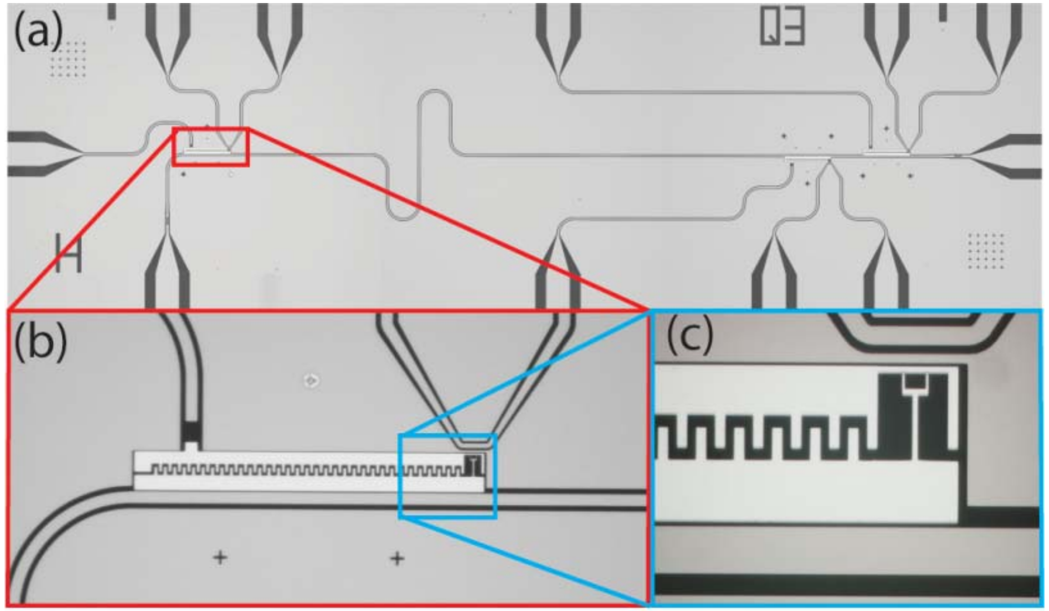


implemented three qubit tomography at step III

ETH Quantum processor platform with 3-Qubits

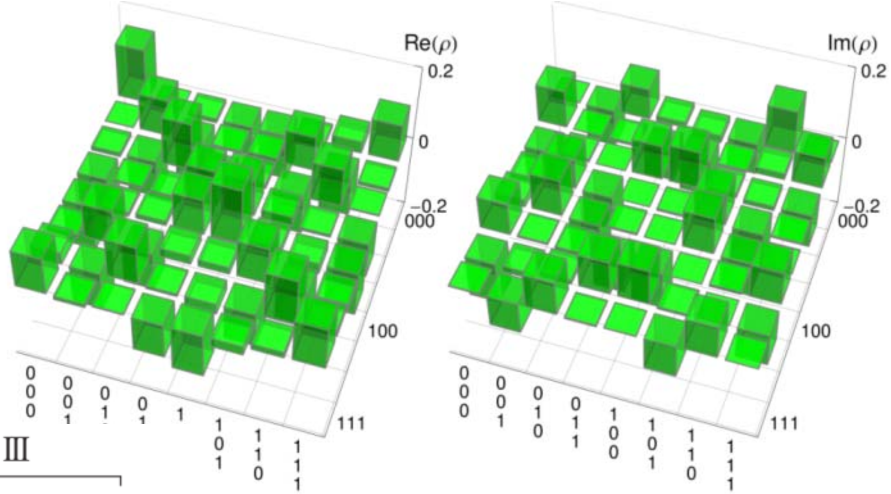


- Full individual coherent qubit control via local charge and flux lines
- Large coupling strength to resonator $g \sim 300 - 350$ MHz
- Transmon coherences times: $T_1 \sim 0.8 - 1.2\ \mu s$, $T_2 \sim 0.4 - 0.7 \mu s$.

[Baur, Fedorov, Steffen, SF, da Silva, Wallraff. arXiv:1107.4774]

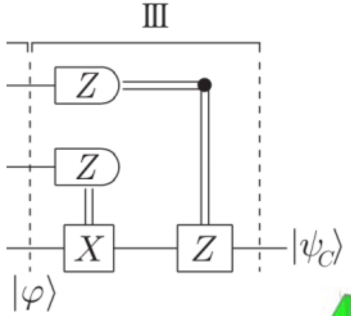
State tomography of the entangled 3-qubit state

Example: State to be teleported on qubit A is $|\Psi\rangle = \frac{1}{\sqrt{2}}(|g\rangle + i|e\rangle)$

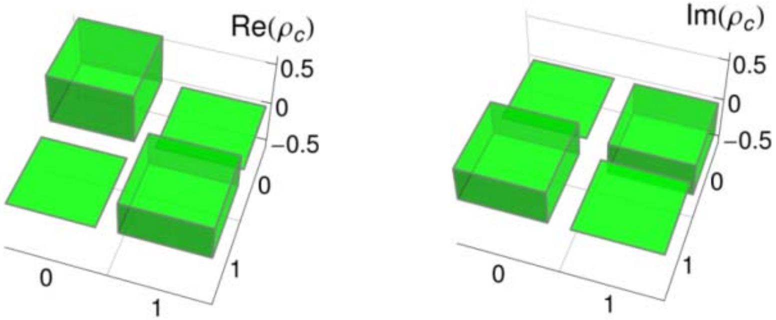


$$|\varphi\rangle = \{|g_A g_B\rangle \otimes |\Psi\rangle_C + |g_A e_B\rangle \otimes \sigma_x|\Psi\rangle_C + |e_A g_B\rangle \otimes \sigma_z|\Psi\rangle_C + |e_A e_B\rangle \otimes (-\sigma_z\sigma_x)|\Psi\rangle_C\}$$

$$\rho = |\varphi\rangle\langle\varphi|$$



Simulating measurement of qubit A and B with projection on $|g_A g_B\rangle$:



$$\rho_C = \langle g_A g_B|\rho|g_A g_B\rangle = |\Psi\rangle\langle\Psi|$$

fidelity 88%

[Baur, Fedorov, Steffen, Filipp, da Silva, Wallraff. PRL 108, 040502 (2012)]

DiVincenzo Criteria fulfilled for Superconducting Qubits

for Implementing a Quantum Computer in the standard (circuit approach) to quantum information processing (QIP):

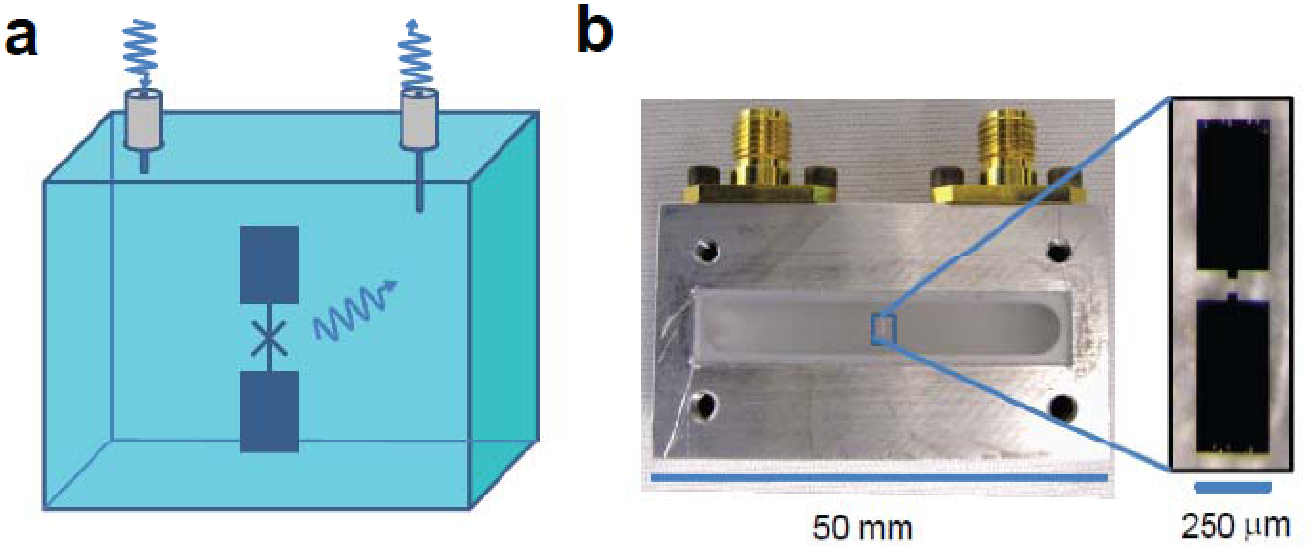
#1. A scalable physical system with well-characterized qubits. ✓
#2. The ability to initialize the state of the qubits. ✓
#3. Long (relative) decoherence times, much longer than the gate-operation time. ✓
#4. A universal set of quantum gates. ✓
#5. A qubit-specific measurement capability. ✓

plus two criteria requiring the possibility to transmit information:

#6. The ability to interconvert stationary and mobile (or flying) qubits. ✓
#7. The ability to faithfully transmit flying qubits between specified locations. ✓

Recent trends I – transmon in 3D cavity

Transmon in a three-dimensional cavity: lifetimes (T1) ~ 100μs



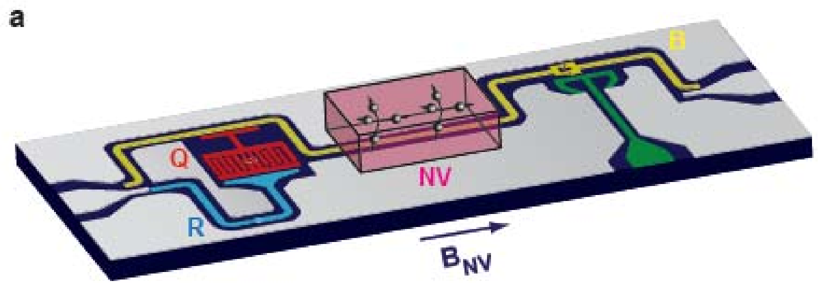
T_1=60μs
T_2=15μs

H. Paik et al., PRL (2012)

Questions: Tunabiliy? Scalability?

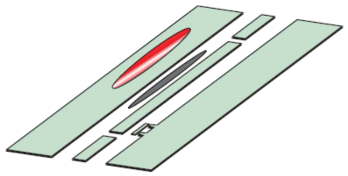
Recent trends II – Hybrid Circuit QED

- Spin ensembles (NV centers) – superconducting circuits
  [cavity-NV: Kubo et al., PRL 105, 140502 (2010); Schuster et al., PRL 205, 140501 (2010)]

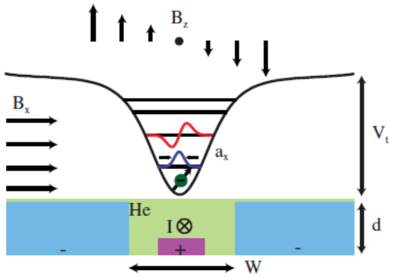


[Kubo, et al. PRL 107, 220501 (2011)]

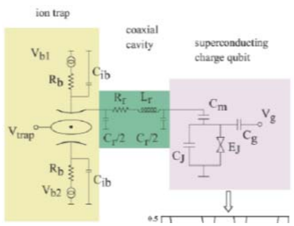
- Atomic ensembles (BEC)
  [Verdu, PRL 103, 043603 (2009)]



- Electrons on Helium
  [Schuster et al., PRL 105, 040503 (2010)]



- Charged particles (Ions)
  [Tian et al., PRL 92, 247902 (2004)]



- Rydberg atoms
  [Hogan et al., PRL 108, 063004 (20012)]