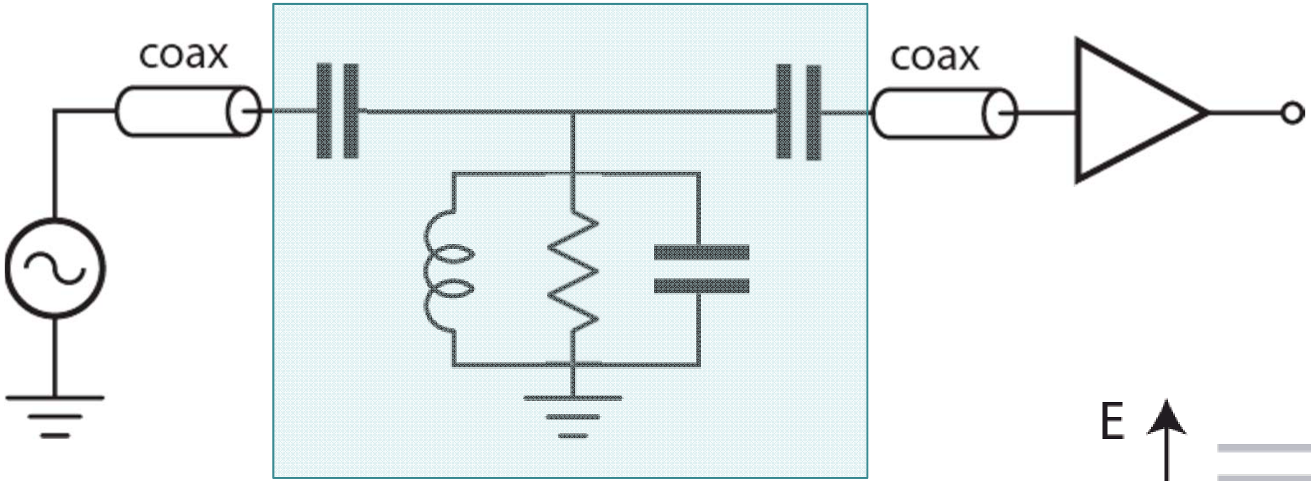
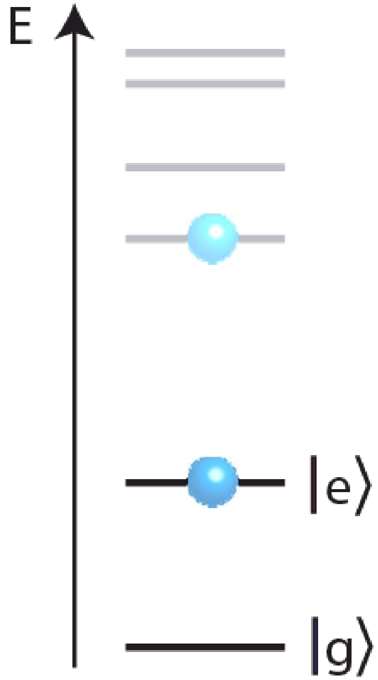
# How to Operate Circuits Quantum Mechanically?




$T \sim 0.01\,\mathrm{K}$



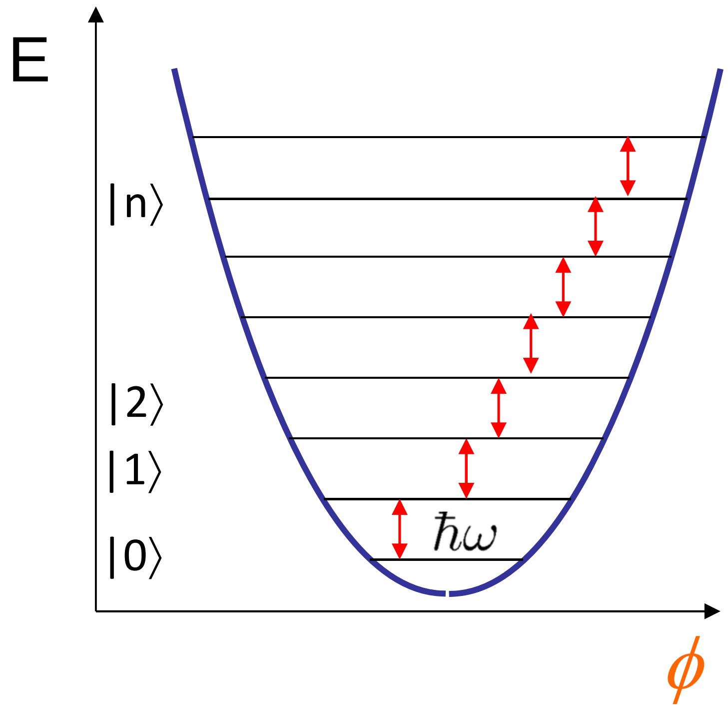

recipe:

- avoid dissipation
- work at low temperatures
- isolate quantum circuit from environment



Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

# Quantum Harmonic Oscillator at Finite Temperature




thermal occupation:

$$\langle n_{\mathrm{th}} \rangle = \frac{1}{\exp\left(h\nu / k_B T\right) - 1}$$

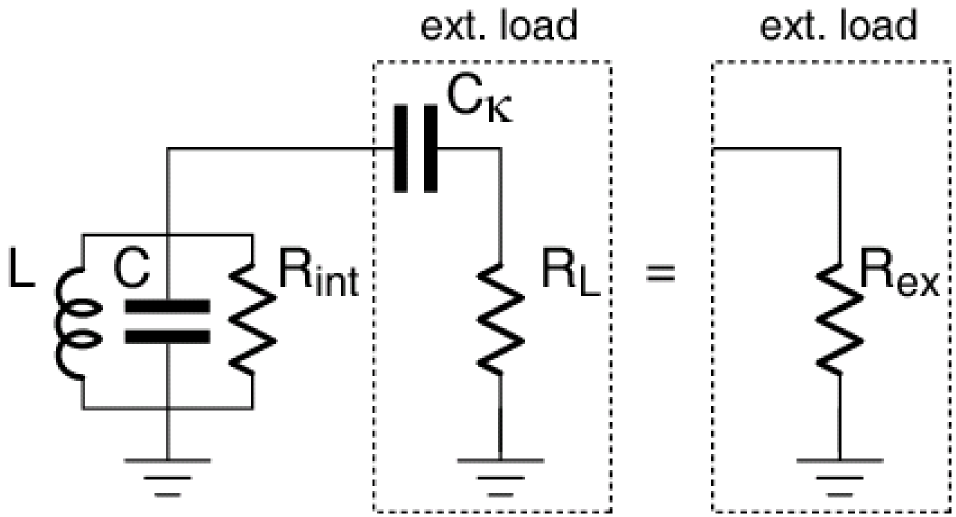
low temperature required:

$$\hbar\omega \gg k_B T$$

10 GHz ~ 500 mK 20 mK

$$\langle n_{\mathrm{th}} \rangle \sim 10^{-11}$$

# Internal and External Dissipation in an LC Oscillator




internal losses: $R_{\rm int}$
conductor, dielectric

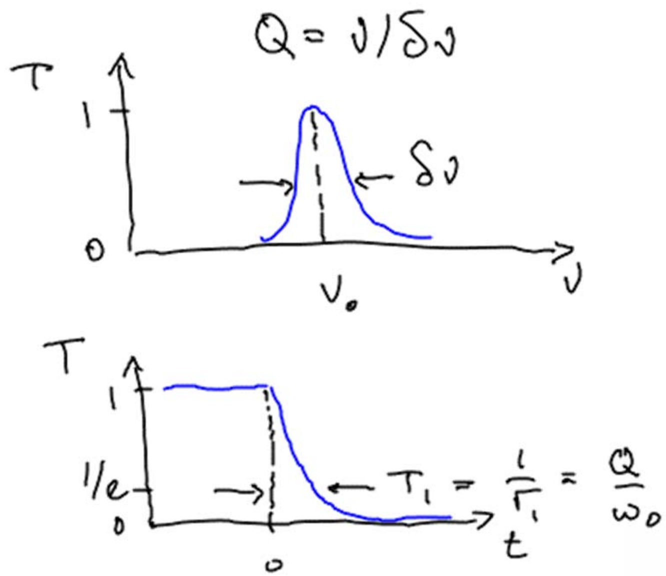
external losses: $R_{\rm ext}$
radiation, coupling

total losses $$\frac{1}{R} = \frac{1}{R_{\rm int}} + \frac{1}{R_{\rm ext}}$$

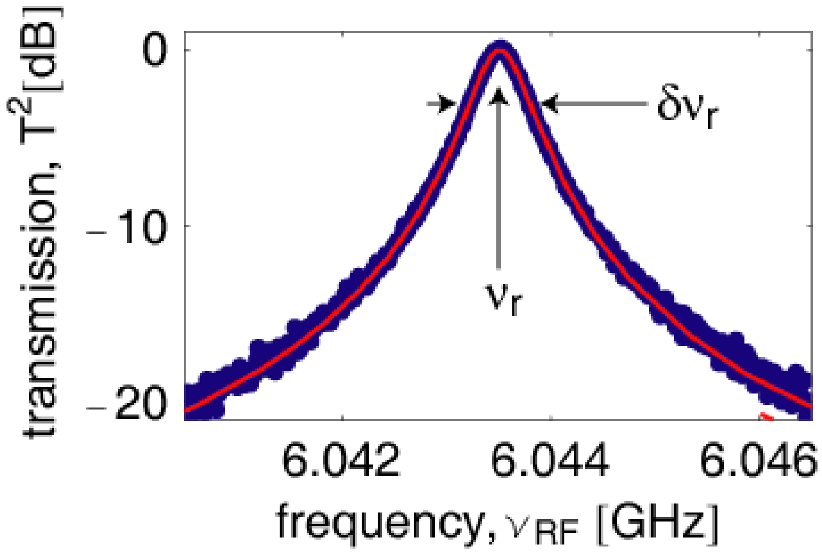
impedance $$Z = \sqrt{\frac{L}{C}}$$

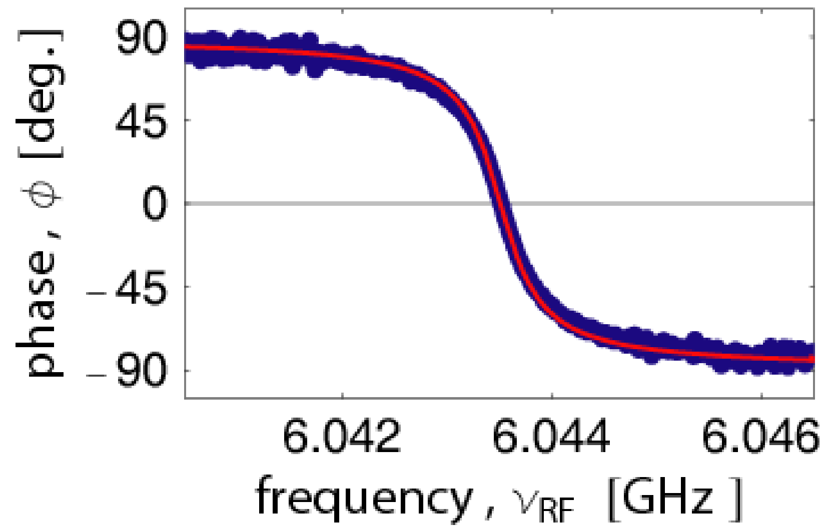
quality factor $$Q = \frac{R}{Z} = \omega_0 RC$$

excited state decay rate $$\Gamma_1 = \frac{\omega_0}{Q} = \frac{1}{RC}$$



# Resonator Quality Factor and Photon Lifetime




resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

photon decay rate:

$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

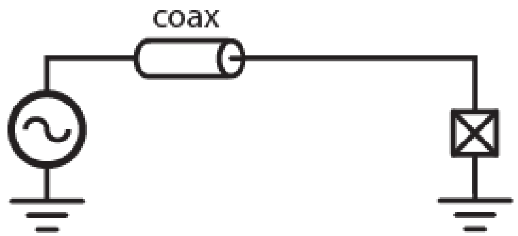
photon lifetime:

$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

# Controlling Coupling to the E.M. Environment

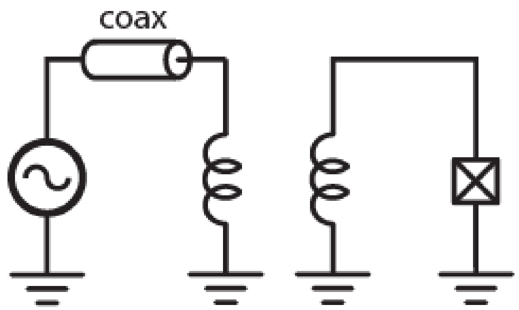
coupling to environment (bias wires):

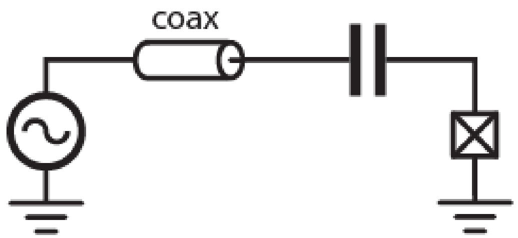
$Z_{\text{line}} \sim 50\,\Omega$



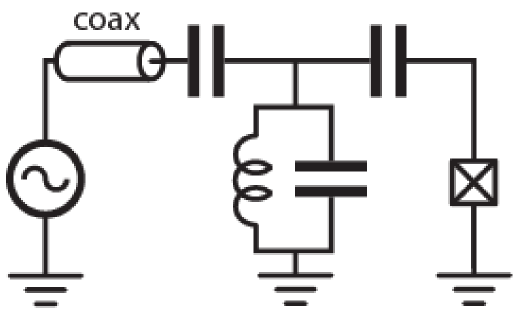

decoherence
from energy relaxation
(spontaneous emission)

decoupling using non-resonant impedance transformers:




using resonant impedance transformers

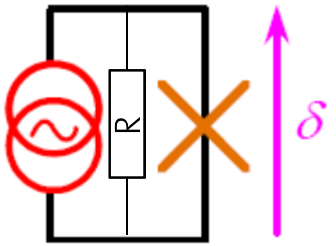



control spontaneous emission
by circuit design

# How to Make Use of the Josephson Junction in Qubits?

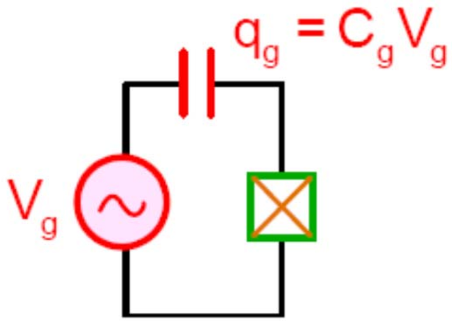
different bias (control) circuits:

phase qubit



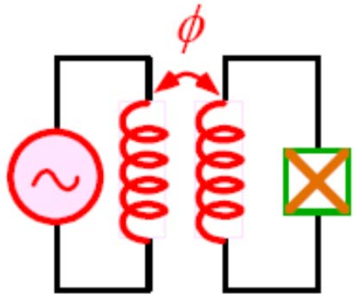

current bias

charge qubit




charge bias

flux qubit

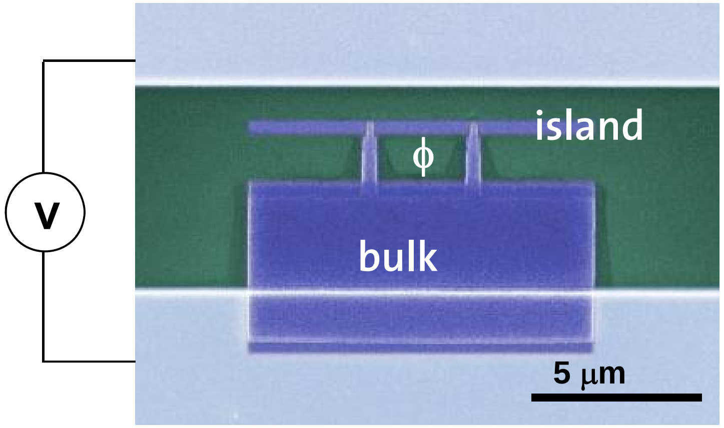


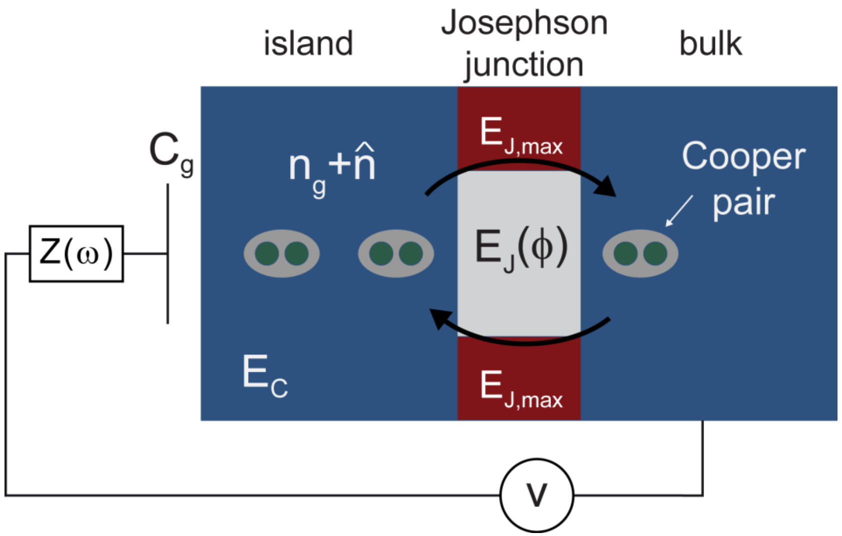

flux bias

# The Cooper Pair Box …

… a charge qubit.

# Cooper Pair Box Qubit



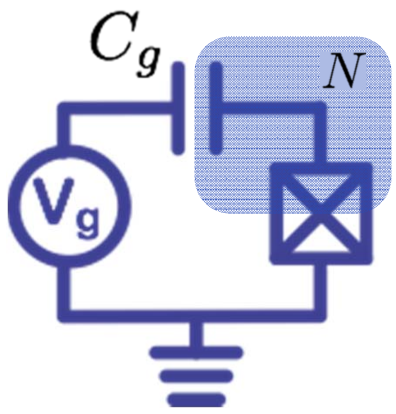




- superconducting island connected via Josephson junctions to grounded reservoir (bulk)
- Cooper pairs can tunnel onto island
- relevant degree of freedom: number of Cooper pairs on island (N)
- polarization charge adjustable via voltage bias
- energy scales: charging energy $E_C$ (energy to add another Cooper pair)
  Josephson energy $E_J$ (coupling energy)



[Bouchiat, Vion, Joyez, Esteve, Devoret, *Physica Scripta* T76, 165 (1998).]

# A Charge Qubit: The Cooper Pair Box

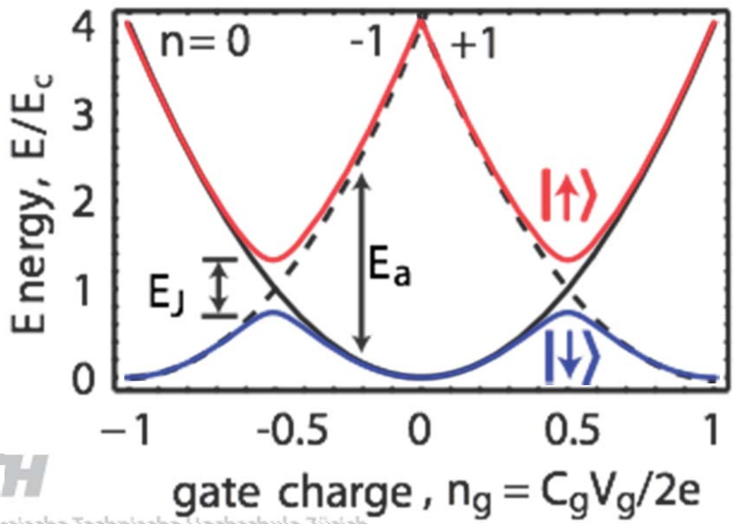



$$H_{el} = E_C N^2$$

$$H = E_C (N - N_g)^2 - E_J \cos\delta$$

$$[\delta, N] = i \quad \Rightarrow \quad e^{\pm i\delta}|n\rangle = |n \pm 1\rangle$$

$$H = \sum_N \left[ E_C (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} (|N\rangle\langle N+1| + |N+1\rangle\langle N|) \right]$$




Charging energy: $E_C = \frac{(2e)^2}{2C_\Sigma}$

Gate charge: $N_g = \frac{C_g V_g}{2e}$

Josephson energy: $E_J = \frac{I_0 \Phi_0}{2\pi}$

Cooper pair box Hamiltonian:

Hamiltonian:

$$\hat{H} = \underbrace{E_c(\hat{N} - N_g)^2}_{\text{electrostatic}} - \underbrace{E_J \cos\hat{\delta}}_{\text{magnetic energy}} \qquad E_J\cos\hat{\delta} = \frac{E_J}{2}\left(e^{i\hat{\delta}} + e^{-i\hat{\delta}}\right)$$

gate charge $N_g = \frac{C_g V_g}{2e}$

charging energy    Josephson coupling Energy

$$E_c = \frac{(2e)^2}{2 C_\Sigma} \qquad E_J = \frac{\phi I_c}{2\pi}$$

Hamiltonian in charge representation:

$$\hat{H} = E_c (N - N_g)^2 \, |N\rangle\langle N| - \frac{E_J}{2} \sum_N \left( |N+1\rangle\langle N| + |N\rangle\langle N+1| \right)$$

easy to diagonalize numerically

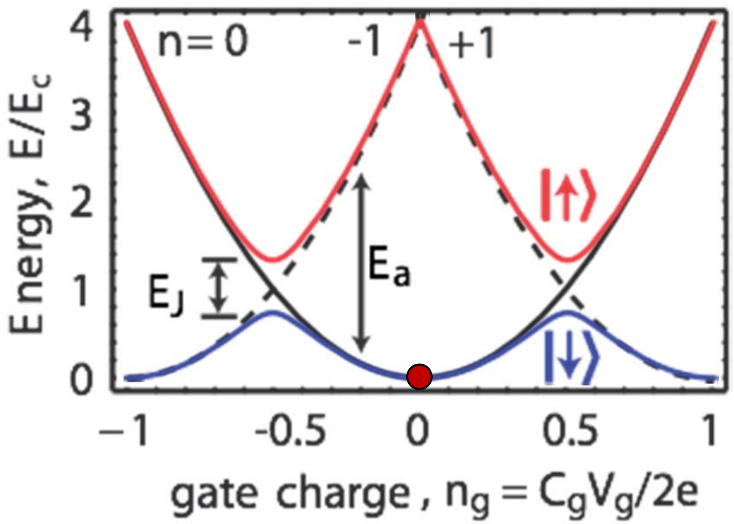
$$\hat{H} = \begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & E_c(-1-N_g)^2 & -E_J/2 & 0 & \cdots \\ \cdots & -E_J/2 & E_c(0-N_g)^2 & -E_J/2 & \cdots \\ \cdots & 0 & -E_J/2 & E_c(1-N_g)^2 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

relation between **phase** and **number** basis:

$$|\delta\rangle = \frac{1}{\sqrt{2\pi}} \sum_N e^{i\hat{N}\delta} |N\rangle \qquad \text{with} \qquad e^{i\hat{\delta}} |N\rangle = |N+1\rangle$$

# Questions:

How does the charge configuration (charge on island) of the system change, when the bias voltage is tuned slowly from $V(0)=0$ to $V(t) = V_f$? In particular, what happens at $n_g=0.5$?




What's the relevant timescale for 'slow'? And what happens, if the voltage is tuned faster?

# Two State Approximation

$$\mathbf{H}_{\mathrm{CPB}} = \mathbf{H}_{\mathrm{el}} + \mathbf{H}_{\mathrm{J}} = E_C(N - N_g)^2 - E_J \cos\delta$$

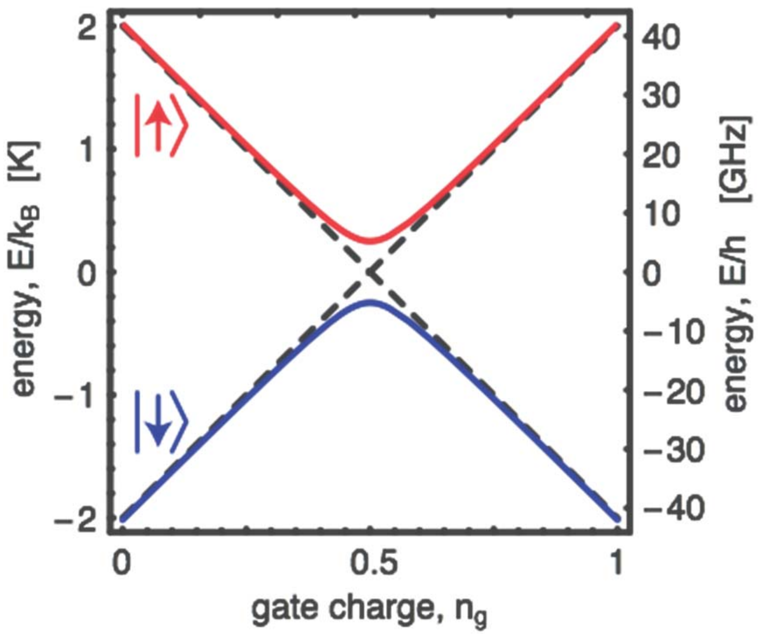
$$\mathbf{H}_{\mathrm{CPB}} = \sum_N \left[ E_C(N - N_g)^2 \left|N\right\rangle \left\langle N\right| - \frac{E_{\mathrm{J}}}{2} \left( \left|N\right\rangle \left\langle N+1\right| + \left|N+1\right\rangle \left\langle N\right| \right) \right]$$

Restricting to a two-charge Hilbert space:

$$N = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1 - \sigma_z}{2}$$

$$\cos\delta = \frac{\sigma_x}{2}$$

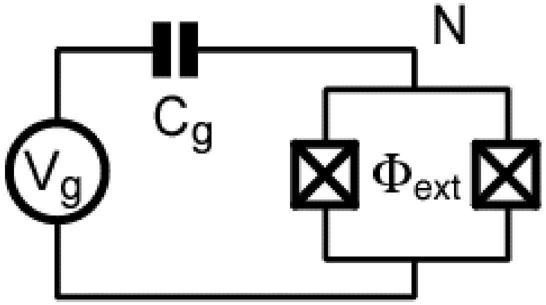
$$\begin{aligned} \mathbf{H}_{\mathrm{CPB}} &= -\frac{E_C}{2}(1 - 2N_g)\sigma_z - \frac{E_J}{2}\sigma_x \\ &= -\frac{1}{2}(E_{\mathrm{el}}\sigma_z + E_J\sigma_x) \end{aligned}$$




Shnirman *et al.*, *Phys. Rev. Lett.* **79**, 2371 (1997)

# Tuning the Josephson Energy

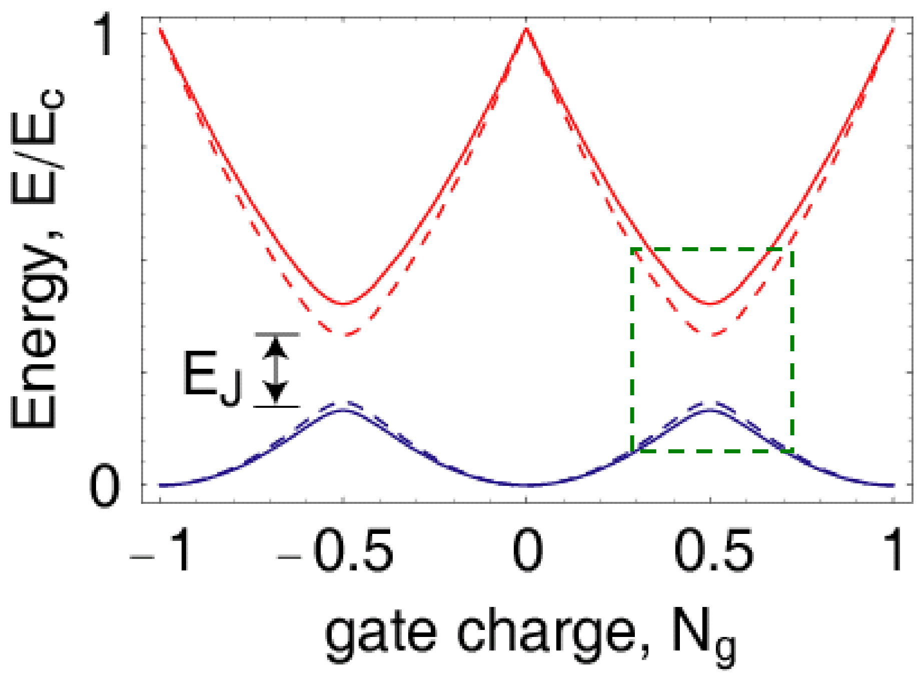
split Cooper pair box in perpendicular field




$$H = E_C\,(N - N_g)^2 - E_{J,\max}\cos\left(\pi\frac{\Phi_{\rm ext}}{\Phi_0}\right)\cos\hat{\delta}$$

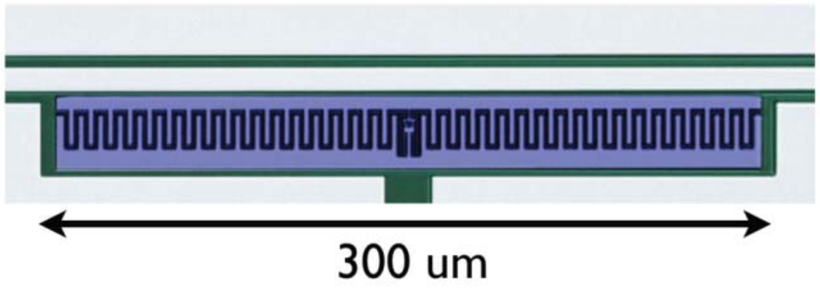
SQUID modulation of Josephson energy

$$E_J = E_{J,\max}\cos\left(\pi\frac{\Phi_{\rm ext}}{\Phi_0}\right)$$

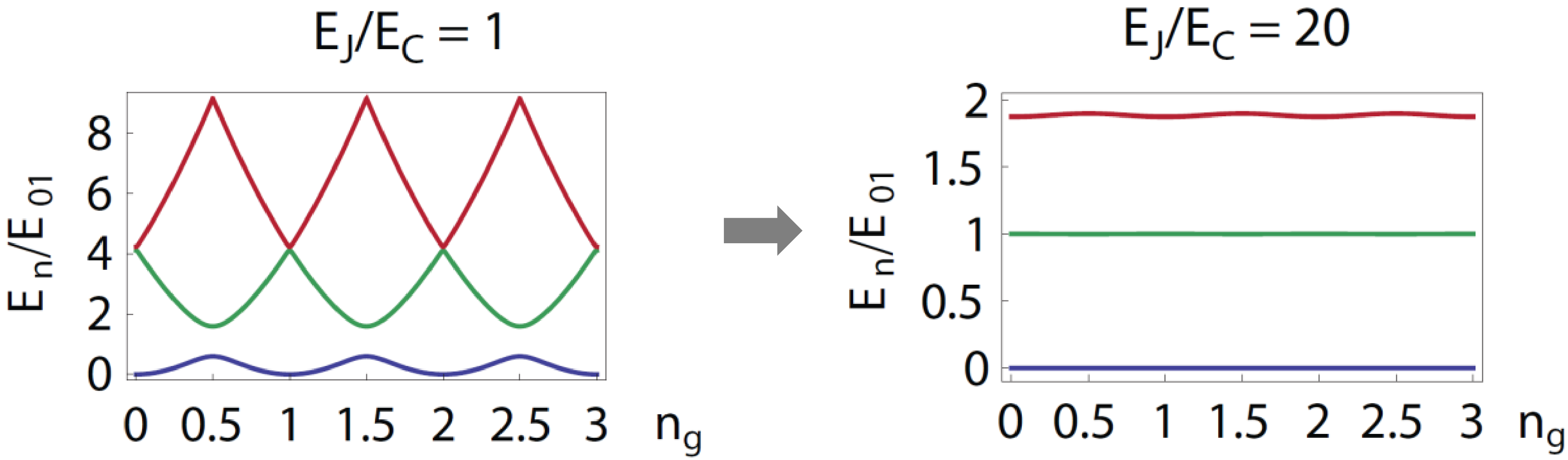



[J. Clarke, *Proc. IEEE* **77**, 1208 (1989)]

# Transmon qubit – a charge noise resilient qubit



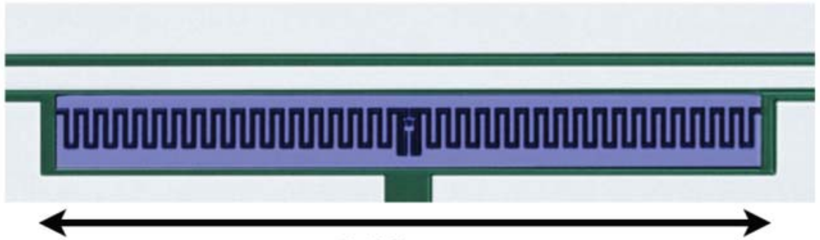

- shunting capacitor reduces $E_c$
- Increased $E_J$/ $E_c$ ratio flattens energy bands
- Less sensitivity to charge noise



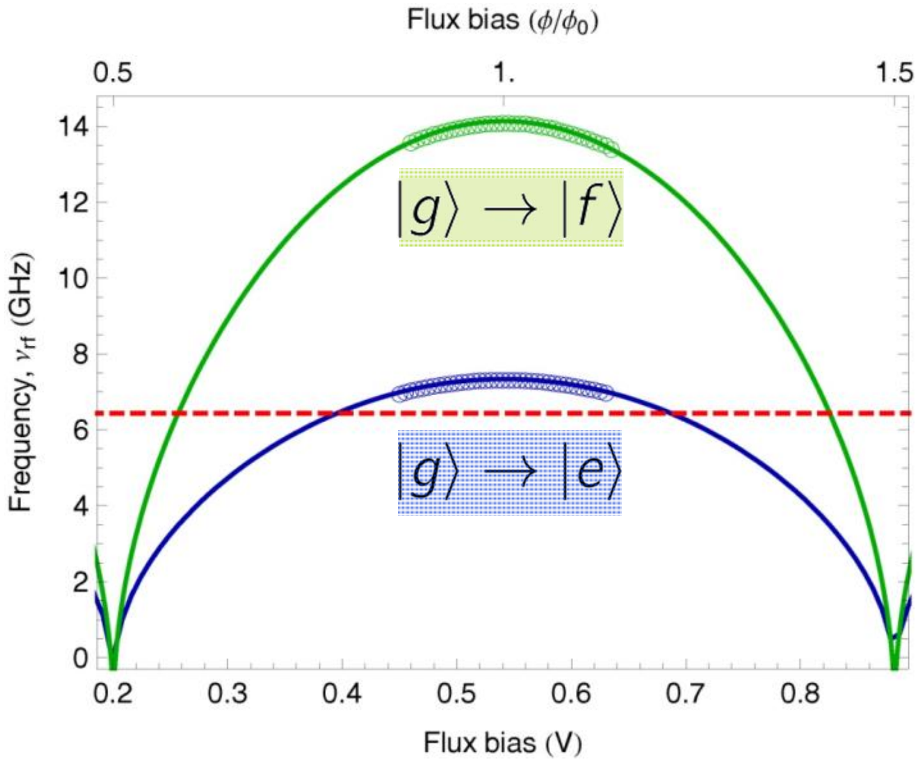

$$H = E_C(\hat{N} - N_g)^2 - E_J \cos\left(\pi \frac{\Phi_{ext}}{\Phi_0}\right) \cos\hat{\delta}$$

[Koch *et al. PRA 76, 042319 (2007);* Schreier *et al. PRB (2008)]*

# Transmon qubit – tuning of transition frequency




transition frequency can be adjusted by external flux bias




$$H = E_C(\hat{N} - N_g)^2 - E_J \cos\left(\pi \frac{\Phi_{ext}}{\Phi_0}\right) \cos\hat{\delta}$$

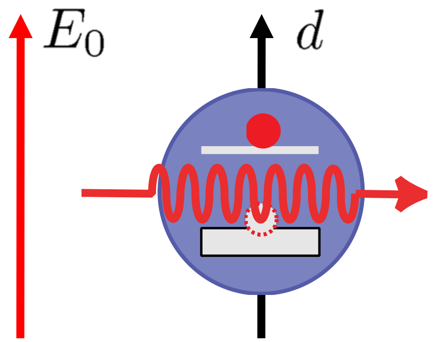


[Koch *et al. PRA 76, 042319 (2007);* Schreier *et al. PRB (2008)]*

# Cavity QED with Electronic Circuits

# Free Atom

weak interaction with single photons:



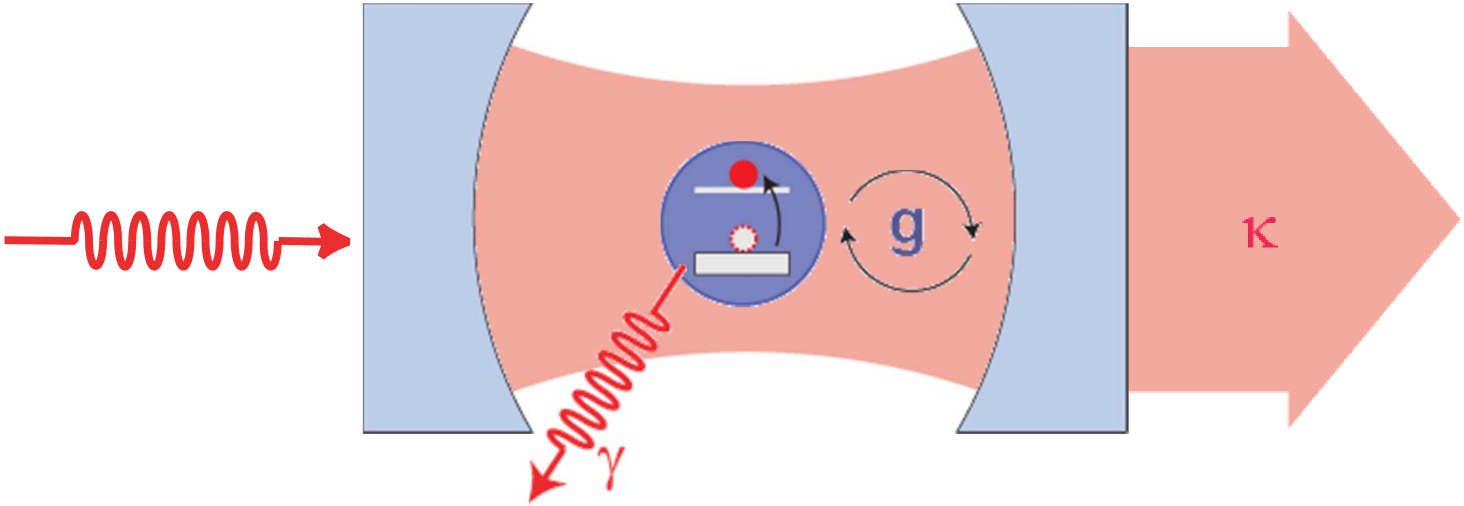

- dipole moment *d* (usually small in atoms ~ $ea_o$)
- single photon fields $E_o$ (small in 3D)
- photon/atom interaction $\hbar g \sim dE_0$ (usually small)



[D. Walls, G. Milburn, Quantum Optics (Springer-Verlag, Berlin, 1994)]

# *Cavity Quantum Electrodynamics*

interaction of atom and photon in a cavity




Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit: $g = dE_0/\hbar > \gamma,\ \kappa$

[D. Walls, G. Milburn, Quantum Optics (Springer-Verlag, Berlin, 1994)]

# Dressed States Energy Level Diagram

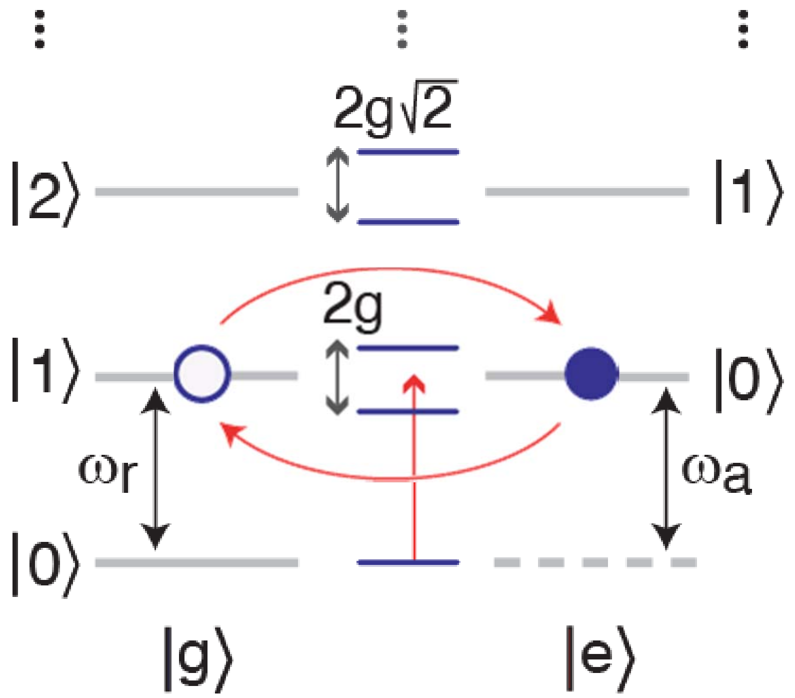
$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

$$g = \frac{dE_0}{\hbar} > \gamma,\, \kappa$$




Jaynes-Cummings Ladder

Atomic cavity quantum electrodynamics reviews:

J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)

S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)