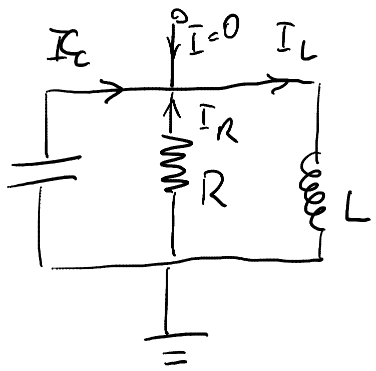
2.5. Dissipation

Mittwoch, 21. März 2012
13:05



$I_C + I_R = I_L$ (Kirchhoff-law)

- current through resistor: $I_R = \frac{V}{R}$
- displacement current:

$$I_C = \dot{Q}_C = C\dot{V}$$

- voltage across inductor $V = -L\dot{I}_L$

$$\Rightarrow -C\dot{V} - \frac{V}{R} + I_L = 0$$

same voltage across elements:

$$\Rightarrow \quad CL\ddot{I}_L + \frac{L}{R}\dot{I}_L - I_L = 0$$

$$\Rightarrow \quad \boxed{\ddot{I}_L + \frac{1}{RC}\dot{I}_L - \frac{1}{LC}I_L = 0}$$

Differential equation for current through inductor

$$\Rightarrow \quad I_L(t) = I_L(0)\, e^{\lambda t} \qquad \lambda_{1,2} = \frac{1}{2LC}\left(-\frac{L}{R} \pm \sqrt{\left(\frac{L}{R}\right)^2 - 4LC}\right)$$

$\left(4LC \gg L/R \; \ldots \text{ underdamped oscillator}\right)$

$$\lambda_{1,2} = -\frac{1}{2RC} \pm i\frac{1}{\sqrt{LC}} = -\frac{\alpha}{2} \pm i\omega$$

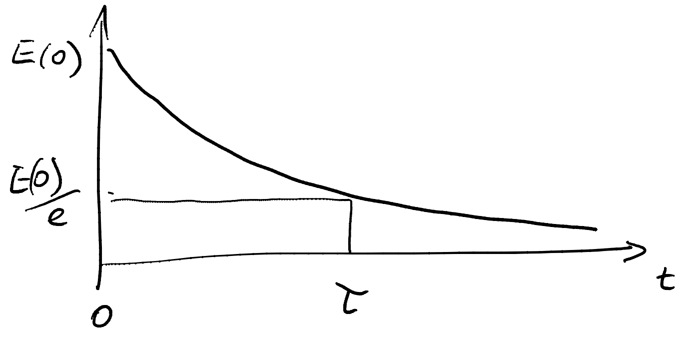
$\alpha = \frac{1}{RC} \equiv \frac{1}{\tau}$ decay constant

$\tau = RC$ decay time

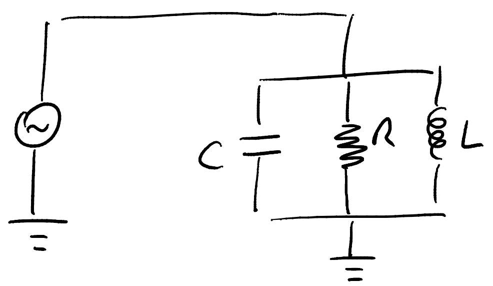
$\omega = 1/\sqrt{LC}$ angular frequency

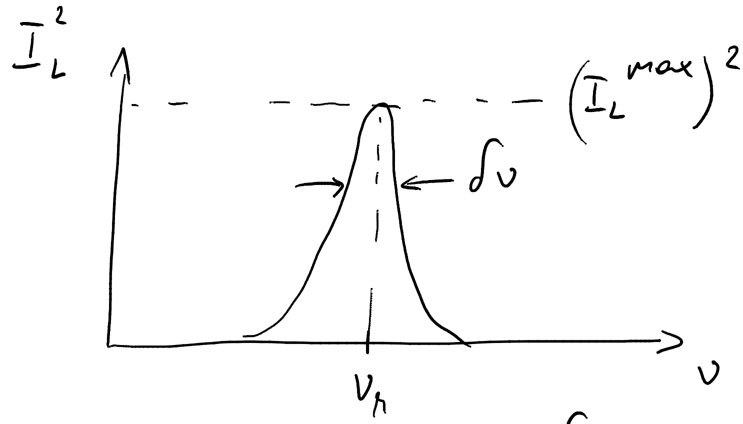
Energy decay rate:

$$E \propto L I_L^2 \propto e^{-\frac{1}{RC}t} = e^{-t/\tau}$$



spectral response: (driven damped oscillator)





Lorentzian line shape: $$I_L^2(\nu) = \left(I_L^{max}\right)^2 \cdot \frac{\frac{\delta\nu}{4}}{(\nu-\nu_r)^2 + \delta\nu^2}$$

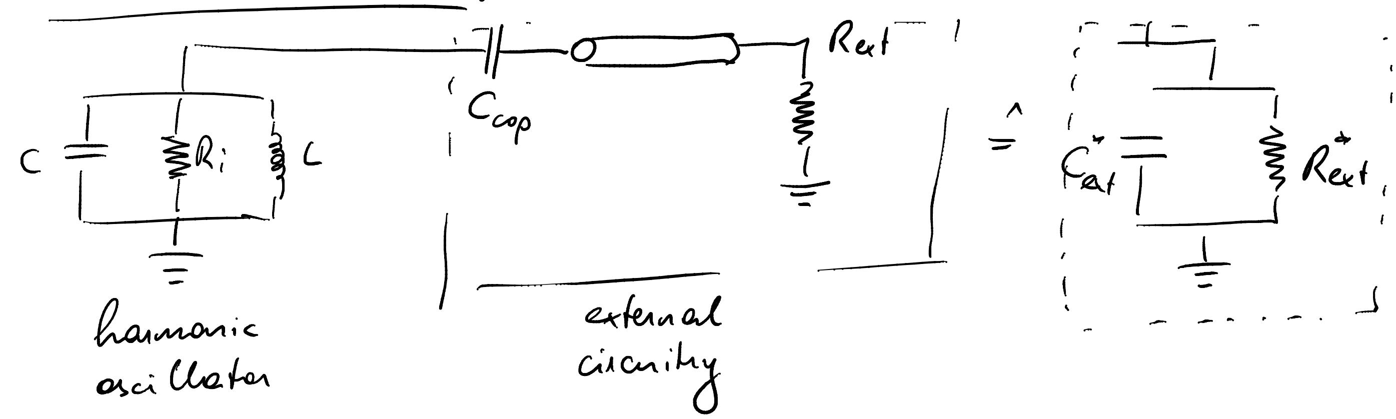
$\delta\nu = \frac{1}{2\pi\tau}$ ... full width at half maximum

Quality factor: $$2\pi \frac{\text{Energy stored}}{\text{Energy lost per cycle}} \hat{=} \frac{\nu_r}{\delta\nu}$$

$$= 2\pi\nu_r \cdot \tau = \underline{\omega_r \cdot R \cdot C}$$

in QM: excited state decay rate. $\Gamma_1 = \frac{1}{\tau} = \frac{1}{RC}$

Internal & external dissipation:



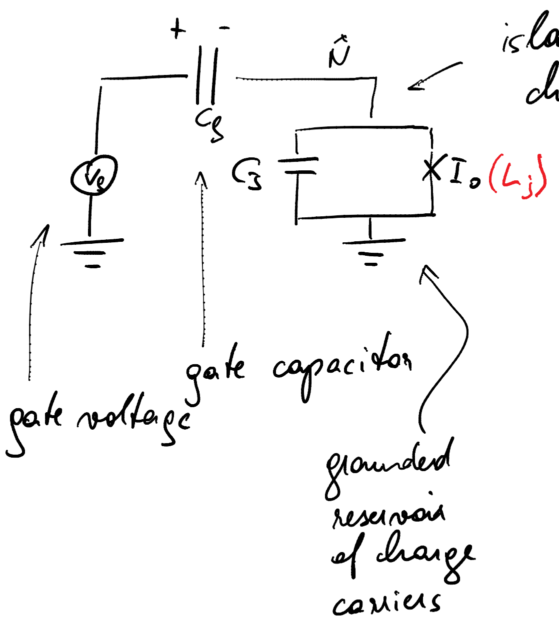
- total effective resistance (parallel capacitance) $\frac{1}{R_{tot}} = \frac{1}{R_{int}} + \frac{1}{R^*_{ext}}$ ⇒ external contribution to energy decay

  — " — capacitance $C_{tot} = C_{int} + C^*_{ext}$

energy decay time $\tau = R_{tot} C_{tot}$

2.6. Cooper Pair Box

Samstag, 22. Oktober 2011
11:31



island on which charges are localized

discrete variable

$$\boxed{\hat{N} = \frac{\hat{Q}}{2e}}$$

number Cooper pairs on island (with respect to charge neutrality)

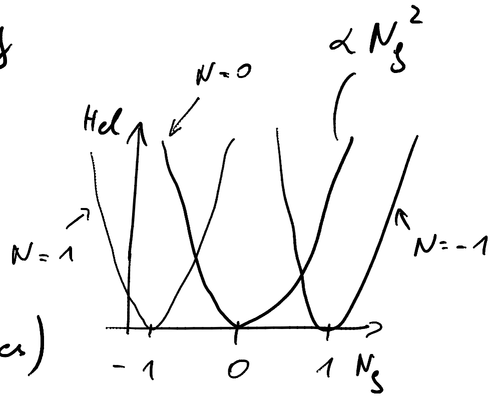
$$\boxed{N_g = \frac{C_g V_g}{2e}}$$

polarization charge on gate capacitor

continuous variable

Hamiltonian: H = electrostatic + magnetic energy

electrostatic energy: $H_{el} = \frac{Q^2}{2C} = \frac{(2e)^2 (N - N_g)^2}{2C_\Sigma}$



with $C_\Sigma = C_J + C_g + \ldots$ (stray capacitances)

and $E_C = \frac{(2e)^2}{2C_\Sigma}$ ... charging energy

magnetic energy: $H_{mag} = -E_J \cos\delta = \frac{\Phi_0 I_0}{2\pi} \underbrace{\cos\delta}_{\approx 1 - \frac{\delta^2}{2}}$

$$\approx -\frac{\Phi_0 I_0}{2\pi}\left(1 - \frac{1}{2}\left(\frac{\Phi}{\Phi_0} 2\pi\right)^2 + \ldots\right)$$

$$\approx \frac{1}{2}\frac{\Phi^2}{L_{J0}}$$

(standard expression for mag. energy)

Hamilton operator:

$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$$

$[\hat{\delta}, \hat{N}] = i$

$\hat{\delta}, \hat{N}$ conjugate variables

$$\cos\hat{\delta} = \frac{1}{2}\left(e^{i\hat{\delta}} + e^{-i\hat{\delta}}\right)$$

$|N\rangle$ .. number eigenstates $\quad [\hat{N}|N\rangle = N|N\rangle]$

$$|\delta\rangle = \frac{1}{\sqrt{2\pi}} \sum_N e^{-iN\delta} |N\rangle \quad \text{(basis trans.)}$$

$|\delta\rangle$ .. phase eigenstates $\quad [\hat{\delta}|\delta\rangle = \delta|\delta\rangle]$

$$|N\rangle = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} d\delta\, e^{+iN\delta} |\delta\rangle$$

properties of $\hat{\delta}$ (phase) and $\hat{N}$ (number) operator:

$$e^{\pm i\hat{\delta}} |N\rangle = |N \pm 1\rangle$$

$$\left[\begin{aligned} e^{\pm i\hat{\delta}} |N\rangle &= \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} d\delta\, e^{iN\delta} e^{\pm i\delta} |\delta\rangle = \\ &= \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} d\delta\, e^{i(N\pm1)\delta} |\delta\rangle \\ &= |N \pm 1\rangle \end{aligned} \right]$$

completeness: $\sum_N |N\rangle\langle N| = \mathbb{1}$ $\quad$ orthogonality: $\langle M|N\rangle = \delta_{M,N}$

Hamiltion operator in charge basis:

$$\hat{H} = \underbrace{\sum_N E_c (\hat{N} - N_g)^2 |N\rangle\langle N|}_{\text{energy of charges on island}} - \underbrace{\frac{E_J}{2}\left(|N\rangle\langle N+1| + |N+1\rangle\langle N|\right)}_{\text{energy to add / remove charges from island}}$$

energy eigenstates:

solve time-independent Schrödingerequation to find $|\psi_n\rangle$

$$\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle$$

either in charge basis (see above) or in phase basis $\hat{\delta}$

$$\hat{N} = \frac{Q}{2e} = -i\hbar \frac{1}{2e}\frac{\partial}{\partial \Phi} = -i\frac{\partial}{\partial \delta}$$

$$\hat{H} = E_c\left(-i\frac{\partial}{\partial \delta} - N_g\right)^2 - E_J \cos\delta$$

exact solutions for $H\psi_n(\delta) = E_n \psi_n(\delta)$ are Mathieu function