

I.1 Classical information processing

Freitag, 17. Februar 2012
15:33

⇒ classical bit: possible values 0 1 physical realization $\begin{matrix} \uparrow 5V \\ \downarrow 0V \end{matrix}$
voltage level in a circuit

⇒ manipulation of bits with a physical process:
e.g. magnetization on harddisc, flip flop circuit for RAM

⇒ any logical operation can be decomposed into single & two-bit operations (gate)

examples: identity-gate $a \text{ --- } a$ 'do nothing'

NOT-gate $a \text{ --- } \neg \text{ --- } \text{NOT } a$

AND-gate $\begin{matrix} a \\ b \end{matrix} \text{ --- } \text{AND} \text{ --- } a \text{ AND } b$

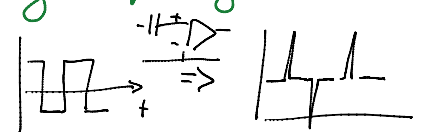
NAND-gate $\begin{matrix} a \\ b \end{matrix} \text{ --- } \text{NAND} \text{ --- } a \text{ NAND } b = \neg(a \text{ AND } b)$

⇒ $\begin{matrix} \uparrow & \uparrow & \uparrow \\ \text{input} & \text{operation} & \text{output} \\ \text{qubit(s)} & & \text{qubit(s)} \end{matrix}$
⇒ OR-gate, XOR gate, NOR gate

Why is digital computing much better than analog computing?

⇒ resilience to errors

⇒ more applications (universality)



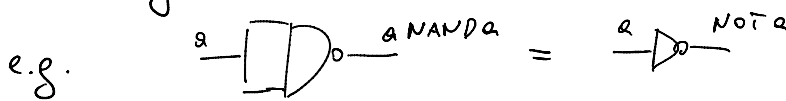
Truth tables

a	b	a AND b
0	0	0
0	1	0
1	0	0
1	1	1



Universal logic gates: NOR, NAND

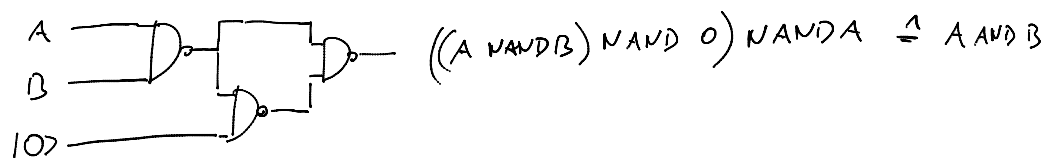
Any other logical gate can be built from using only NOR (or only NAND) gates.



a	a NAND a
0	1
1	0

\Rightarrow universal gates do also exist for quantum computers!

Circuit representation:



- properties:
- * bits can be copied (FANOUT)
 - * additional wrapping bits are allowed (ANCILLAS)
 - * values of bits can be interchanged (CROSSOVER)
 - * number of output bits may be smaller than input bits

Which of these properties are easy/hard to realize in a quantum computer?

- * copying qubits is not possible (no-cloning theorem)
- * ancillas \checkmark
- * crossover \leftrightarrow swap operation \checkmark
- * # outputs = # inputs (reversibility)

\hookrightarrow e.g. AND-gate is not reversible, inputs (a,b) cannot be reconstructed from output

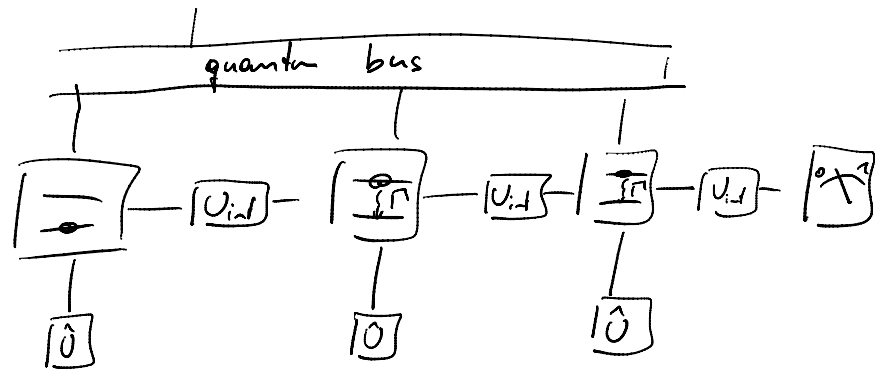
I.2. Generic Quantum Computing Architecture

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features: (Di Vincenzo Criteria)

- ① quantum bits
- ② initialization
- ③ coherence
- ④ universal gates
- ⑤ read out



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- ⑥ conversion stationary/mobile
 - ⑦ transmission of qubits

I. 3. The quantum bit

Montag, 20. Februar 2012
09:04

→ quantum mechanical system with two distinct states



examples: neutron/proton/electron spin,
atomic two levels
⋮

Under which conditions can you consider a quantum mechanical system as an effective two-level system (equivalent to a spin- $\frac{1}{2}$ particle)?
⇒ frequency separation

x) representation of qubit states: vectors in Hilbert space (2-dim, complex)

$$|0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \hat{=} |\downarrow\rangle \hat{=} |H\rangle$$

$| \rangle$ - Dirac notation

$()$ - vector notation

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \hat{=} |\uparrow\rangle \hat{=} |V\rangle$$

→ most general qubit state

$$| \psi \rangle = \alpha |0\rangle + \beta |1\rangle \quad \alpha, \beta \in \mathbb{C}$$

superposition state of $|0\rangle$ & $|1\rangle$

probability to find system in 0 or 1:

$$P_0 = |\alpha|^2$$

$$P_1 = |\beta|^2$$

law of probabilities: $P_0 + P_1 = 1$

⇒ Normalization: $|\alpha|^2 + |\beta|^2 = 1$

What is the main difference to classical bit?

- o) phase matters, that's why decoherence is an issue
- o) parallelism: operations act on both qubit in $|0\rangle$ AND in $|1\rangle$

[extension to many qubits: 2 qubits 00, 01, 10, 11
3 qubits 000, 001, ...
8 possible states]
→ parallel operation on 2^n states!

Bloch Sphere

Montag, 20. Februar 2012
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$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad \text{with } |\alpha|^2 + |\beta|^2 = 1 \Rightarrow 3 \text{ parameters}$$

$$\rightarrow \text{parameterization?} \quad \alpha = \cos \frac{\theta}{2} e^{i\varphi_\alpha}$$
$$\beta = \sin \frac{\theta}{2} e^{i\varphi_\beta}$$

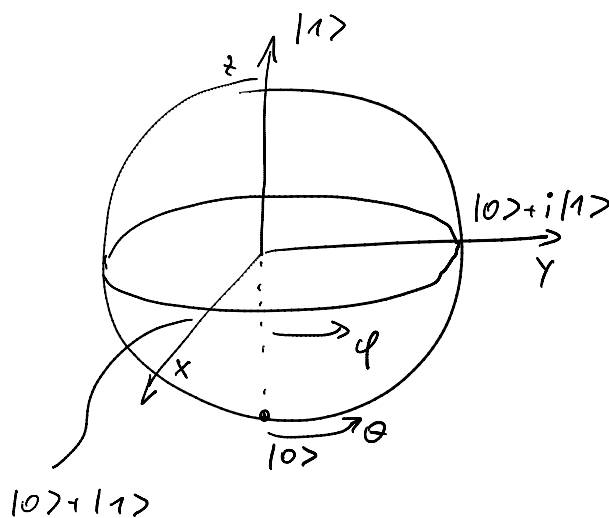
$$|\psi\rangle = \cos \frac{\theta}{2} e^{i\varphi_\alpha} |0\rangle + \sin \frac{\theta}{2} e^{i\varphi_\beta} |1\rangle =$$
$$= e^{i\varphi_\alpha} \left(\cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle \right)$$

Does the global phase factor $e^{i\varphi_\alpha}$ matter?

Not for a single qubit! All measurements are insensitive to a global phase:

$$\langle \psi | \hat{O} | \psi \rangle = \langle \psi | e^{-i\varphi} \hat{O} e^{i\varphi} | \psi \rangle = \langle \psi | \hat{O} | \psi \rangle$$

\Rightarrow 2 parameters: $\theta \in [0, \pi]$... polar angle
 $\varphi \in [0, 2\pi]$... azimuthal angle



equal superposition states lie on the equatorial plane with $\theta = \frac{\pi}{2}$ ($\sin \frac{\pi}{2} = \cos \frac{\pi}{2} = \frac{1}{\sqrt{2}}$)

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle + e^{i\varphi} |1\rangle)$$

Why ' $\cos \frac{\theta}{2}$ ' and not ' $\cos \theta$ ' ?

spin- $\frac{1}{2}$ / 2-level system shows 4π -symmetry, i.e. state returns to itself only for $\theta = 4\pi$, otherwise it obtains additional \ominus sign: (observable in interference experiment)

$$|\psi\rangle_{\theta=0} = -|\psi\rangle_{\theta=2\pi} = |\psi\rangle_{\theta=4\pi}$$

(Representation of group $SU(2)$ instead of $SO(3)$)