QSIT 2012 - Questions 9

11. May 2012, HIT F 13

1. **Trotter Formula**
   Consider quantum evolution $U = \exp[-(i/\hbar)Ht]$ where the Hamiltonian is a sum of two non-commuting Hamiltonians: $H = A + B$, $[A, B] \neq 0$. One can simulate the evolution U by a Trotter formula:

   $$e^{i(A+B)t} = (e^{iAt/n}e^{iBt/n})^n + O(t^2/n),$$

   or by a more precise Trotter-Suzuki approximation:

   $$e^{i(A+B)t} = (e^{iBt/2n}e^{iAt/n}e^{iBt/2n})^n + O(t^3/n^2).$$

   Prove these approximations. What convenience these expansions give for quantum information processing?

2. **Universal set for trapped-ion quantum computing**
   By using laser fields one can induce the following set of Hamiltonians (interactions) for a string of trapped ions:

   (a) By using a single frequency laser beam resonant with the qubit transitions one can induce spins flips on all ions simultaneously described by Hamiltonian $S_x = \sum_k \sigma_x^k$.

   (b) By using a tightly focused off-resonant laser beam interacting with only k-th ion one can induce single-qubit phase shifts described by Hamiltonian $\sigma_z^{(k)}$.

   (c) A bichromatic laser field can be used to for all ions at once described by S_x^2.

   What operations are achieved by the sequences:

   (a) $U_z^{(k)}(-\pi)U_X(-\theta/2)U_z^k(\pi)U_X(\theta/2)$, where $U_z^{(k)}(\phi) = \exp(-i(\phi/2)\sigma_z^{(k)})$ and $U_X(-\phi) = \exp(-i(\phi/2)S_X)$.

(b) $U_z^{(k)}(-\pi)U_{X^2}(\theta/2)U_z^k(\pi)U_{X^2}(\theta/2)$, where $U_{X^2}(\phi) = \exp(-i(\phi/2)S_X^2)$.

Show that the set of Hamiltonians $\{S_x^2, S_x, \sigma_z^{(1)}, \sigma_z^{(1)}, ..., \sigma_z^{(N)}\}$ is universal.