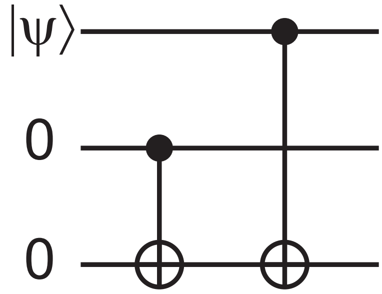
QSIT 2012 - Questions 7

27. April 2012, HIT F 13

1. **Error-correction with superconducting qubits**
In the simplest error-correction schemes the qubit state $|\psi\rangle = a|0\rangle + b|1\rangle$ is encoded into three qubit state $a|000\rangle + b|111\rangle$ with the help of the following protocol:



After encoding the qubits are subject to environmental noise and can, for example, flip their state with certain probability p.

   (a) Show that by two successive measurements of the operators $Z_1 Z_2 = \sigma_z \otimes \sigma_z \otimes I$ and $Z_2 Z_3 = I \otimes \sigma_z \otimes \sigma_z$ we can uniquely determine if any of the qubits flipped and also what qubit flipped. What happens if two qubits flipped at the same time?

   (b) From the information of the two measurements we can correct the state applying appropriate single-qubit operations and eliminated the effect of bit flip on the state. Typically the measurement destroys the coherence of a quantum state, why it works in this case?

   (c) For what probability p the error correction improves the storage of the state $|\psi\rangle$. Sketch the dynamics of fidelity $F = \langle\psi|\rho|\psi\rangle$ where ρ is the density operator of the real state with (and without) error correction.

(d) Measurement and the following correction of the states is technically hard for superconducting qubits. Explain why? Instead we can decode qubits using the scheme on Fig. 2. Evaluate the state after decoding assuming no bit flips and a bit flip on qubit 1. What operation we should apply to correct for the bit flip error?

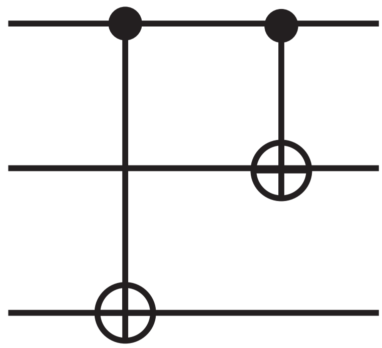


Fig. 2