QSIT 2012 - Questions 11

25. May 2012, HIT F 13

1. **Spin dipolar interaction in liquid**
   Consider two spins coupled vias dipole interaction

$$H_D = \frac{\gamma_1 \gamma_2 \hbar}{4r^3} \left(\vec{\sigma}_1 \vec{\sigma}_2 - 3(\vec{\sigma}_1 \cdot \hat{n})(\vec{\sigma}_2 \cdot \hat{n}) \right), \tag{1}$$

   where r distance between spins, $\hat{n}$ is a unit vector connecting two spins, γ_n, $\vec{\sigma}_n$ $n \in \{1, 2\}$ are gyromagnetic ratios and spin operators for each spin, respectively. In liquid all spins are oriented randomly thus the interaction for the whole ensemble of spins is averaged out. Show that spherical average of H_D over $\hat{n}$ is zero.

2. **Elements for quantum computing with NMR**
   Another spin coupling mechanism is called J coupling (or through bond interaction):

$$H_D = \frac{J\hbar}{4} \left(\vec{\sigma}_1 \cdot \vec{\sigma}_2 \right), \tag{2}$$

   where J is the strength interaction. W what happens if the difference for spin frequencies is much more than J?

   Consider two coupled spins

$$H_0 = \frac{\hbar\omega_1}{2}\sigma_z^1 + \frac{\hbar\omega_2}{2}\sigma_z^2 + \frac{J\hbar}{4}\sigma_z^1\sigma_z^2, \tag{3}$$

   where ω_1 and ω_2 spins Larmor frequencies. Driving spins by the transversal magnetic field can be described by

$$H_R = \sum_{j=1,2} (A_i \sigma_x^i \cos(\omega t + \phi)). \tag{4}$$

   Here A_i describes the coupling of the magnetic field to each spin. Outline strategies for creating single- and two-qubit operation with help of unitary evolutions governed by (3) and (4).