QSIT 2012 - Questions 10

18. May 2012, HIT F 13

1. **Bell inequality with photons: classical correlations**
Consider the experimental scheme shown in Fig. 1. The source S sends two photons with some polarizations in different direction. The polarization of each photon can be measured by two polarizes (B and A) aligned with the measurement axes (denoted by a and b labels for the vectors lying in the plane perpendicular to the propagation direction of the photons). The result of each single-short measurement returns $+1$ if a photon has a polarization parallel to the measurement axis and -1 if the polarization is perpendicular. Let's denote the single-short measurement outcomes as $A(a)$ and $B(b)$ where first letter denote the polarizer and the argument denotes the polarization (measurement) axis. What are the possible values for the following combination of single-short outcomes for four different measurements in the classical case:

$$A(a)B(b) - A(a)B(b') + A(a')B(b) + A(a')B(b'), \qquad (1)$$

where a, a' and b, b' are different measurement axes set to each of the polarizers. One can average the results over many experiments to obtain the expectation values for photon polarization denoted as $E(a,b) = \langle A(a)B(b)\rangle$. How would the expectation value

$$S = E(a,b) - E(a,b') + E(a',b) + E(a',b') \qquad (2)$$

differ from the single-short value of (1)

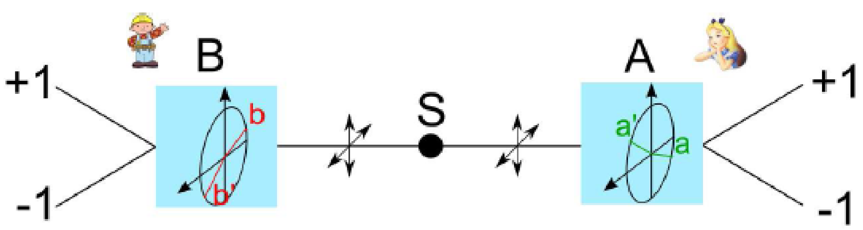

Fig. 1

2. **Bell inequality with photons: quantum correlations**
   In the quantum case the expectation value will be equal to

$$E(a,b) = \langle \phi | (\vec{r}(a)\vec{\sigma}) \otimes (\vec{r}(a)\vec{\sigma}) | \phi \rangle, \tag{3}$$

   where $\vec{r}(a)\vec{\sigma} = \sin(a)\sigma_x + \cos(a)\sigma_z$ gives the measurement operator of a polarizer, and $|\phi\rangle$ is the quantum state describing the photons. If photons are prepared in the maximally entangled state $\phi^+ = (1/\sqrt{2})(|00\rangle + |11\rangle)$ what is the expectation value $E(a,b)$? Pick the angles a, b to maximize (2) (without proof).

3. **Expectation value, photon counts**
   In a real experiment each of expectation values $E(a,b)$ are typically measured with use of fourfold coincidence technique shown in Fig. 2 where each polarizer from Fig. 1 is replaced by a two-channel polarizer. If a photon has vertical (horizontal) polarization (relative to polarizer internal axis which can be adjusted) it propagates through channel $V(H)$ and the corresponding photon detector makes a "count"which can be recorded by the "black box". The "black box"can tells us a number of coincident counts denoted by $C_{ij}(a,b)$ where a and b are directions of the polarizers (measurement axes) and $i,j \in \{H,V\}$ denote two outputs of a two-channel polarizer, respectively. Write down the expectation value $E(a,b)$ with the help of coincidences $C_{ij}(a,b)$. Outline the possible reasons to use the fourfold scheme.

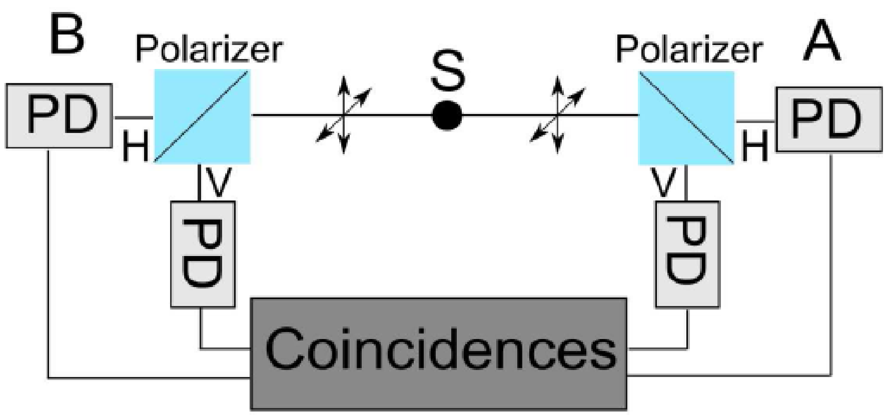

Fig. 2