Multiparticle Entanglement

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(our names are entangled?)

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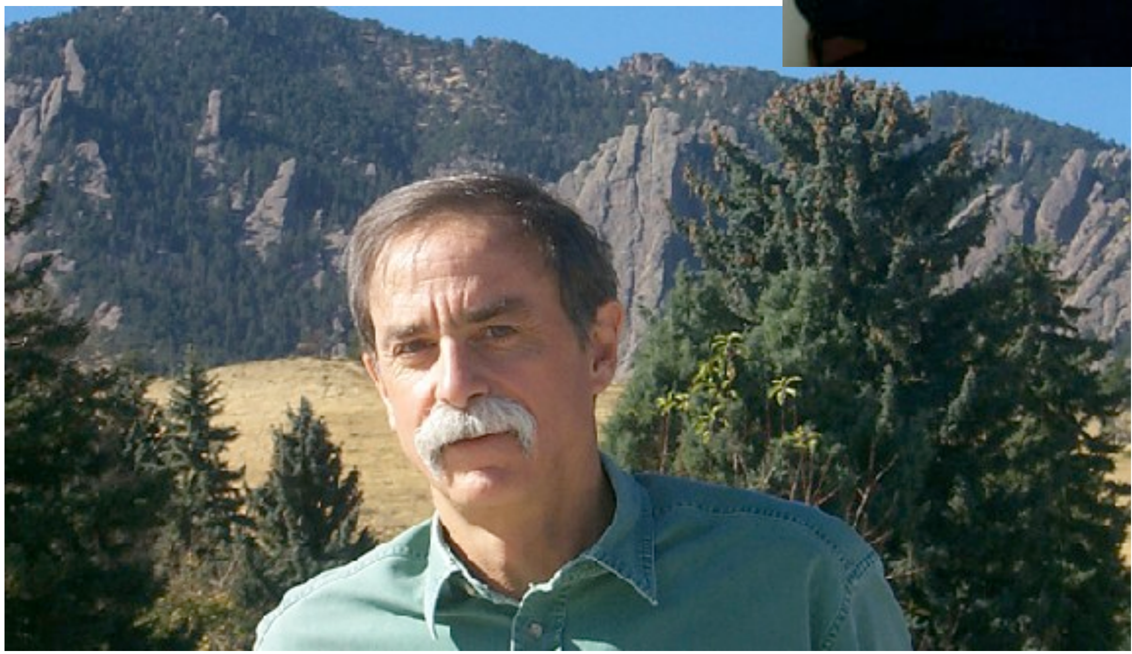
Entangled states of trapped atomic ions

Rainer Blatt[1,2] & David Wineland[3]

Der Würdigungspreis
im Bereich
Naturwissenschaft ging
an den Innsbrucker
Physiker Rainer **Blatt**



David **Wineland**, a member
of the NIST Ion Storage
Group

PRL **106,** 130506 (2011) PHYSICAL REVIEW LETTERS week ending 1 APRIL 2011

14-Qubit Entanglement: Creation and Coherence

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We report the creation of Greenberger-Horne-Zeilinger states with up to 14 qubits. By investigating the coherence of up to 8 ions over time, we observe a decay proportional to the square of the number of qubits. The observed decay agrees with a theoretical model which assumes a system affected by correlated, Gaussian phase noise. This model holds for the majority of current experimental systems developed towards quantum computation and quantum metrology.

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LETTERS

Creation of a six-atom 'Schrödinger cat' state

D. Leibfried[1], E. Knill[1], S. Seidelin[1], J. Britton[1], R. B. Blakestad[1], J. Chiaverini[1]†, D. B. Hume[1], W. M. Itano[1], J. D. Jost[1], C. Langer[1], R. Ozeri[1], R. Reichle[1] & D. J. Wineland[1]

Among the classes of highly entangled states of multiple quantum systems, the so-called 'Schrödinger cat' states are particularly useful. Cat states are equal superpositions of two maximally different quantum states. They are a fundamental resource in fault-tolerant quantum computing[1–3] and quantum communication, where they can enable protocols such as open-destination teleportation[4] and secret sharing[5]. They play a role in fundamental tests of quantum mechanics[6] and enable improved signal-to-noise

$\tilde{S}_z|\uparrow\rangle = \frac{1}{2}|\uparrow\rangle$ and $\tilde{S}_z|\downarrow\rangle = -\frac{1}{2}|\downarrow\rangle$ (for simplicity we set $\hbar = 1$). We define $|\uparrow, N\rangle \equiv |\uparrow\rangle_1|\uparrow\rangle_2 \ldots |\uparrow\rangle_N$ and $|\downarrow, N\rangle \equiv |\downarrow\rangle_1|\downarrow\rangle_2 \ldots |\downarrow\rangle_N$. In this notation, prototypical cat states of N qubits can be written as:

$$|N\,\mathrm{Cat}\rangle = \frac{1}{\sqrt{2}}(|\uparrow, N\rangle + e^{i\vartheta}|\downarrow, N\rangle) \qquad (1)$$

To generate such states we initially prepare the ions in state $|\downarrow, N\rangle$ and then apply the following unitary operation to transform the

Cat states

- Cat states: „equal superpositions of two maximally different states".
- For more than two qubits (subsystems):

GHZ-states (Greenberger-Horne-Zeilinger

$$|\Psi\rangle = 1/\sqrt{2}\ (|0...0\rangle + |1...1\rangle)$$

here: $|N\ Cat\rangle = 1/\sqrt{2}\ (|\uparrow ... \uparrow\rangle + e^{i\theta}|\downarrow ... \downarrow\rangle)$

Ion traps

String of ions

in a linear Paul trap

6 Qubits

$^{9}Be^{+}$ ions

14 Qubits

$^{40}Ca^{+}$ ions

Ingredients

- atomic ions
  - confined in electromagnetic traps
  - manipulated with laser beams -> pumping
- Centre-of-mass (COM) frequencies?
  - Axial COM f. between $\omega_{COM}/2\pi = 2.6\text{MHz}$ and 3.4MHz
  - Radial COM f. ~ 8 MHz
- Operations U_N using two-photon stimulated Raman transitions
  - Laser Pulses with a certain freq., duration, intensity & phase; same for all qubits.

Preparation of entangled state

- Start with $|\downarrow, N\rangle$
- Apply unitary operation:

$$U_N=(\exp[i\pi/2\ J_x]\exp[i\xi\pi/2\ J_z])(\exp[i\pi/2\ J_z^2])(\exp[i\pi/2\ J_x])$$

with $\xi=1$ if N is odd and $\xi=0$ otherwise

  - Goal: $|N\ Cat\rangle=1/\sqrt{2}\ (|\uparrow\ldots\uparrow\rangle+e^{i\theta}|\downarrow\ldots\downarrow\rangle)$
- Measure Entanglement!

We need a measure

- Fidelity: $F = |\langle \Psi_N | N\ Cat \rangle|^2$
- Useful:

$$F = ½(P_{\uparrow N}+P_{\downarrow N})+|C_{\downarrow N;\uparrow N}|$$

- That is not all there is:

  for N>2 there is no single measure that quantifies entanglement!

- Only comparable if $|\Phi\rangle$ -- via LOCC --> $|\Psi\rangle$

Measure (cont.)

- Witness Operator: $W = 1 - 2|N\ Cat\rangle\langle N\ Cat|$

- $\langle W \rangle = 1 - 2 * \text{Fidelity}$
- If $\langle W \rangle < 0$ significantly $\Rightarrow$ entanglement

- then states can be purified by LOCC

Measure (cont.)

- ‚depolarization' method: Using LOCC, density matrix gets transformed, then:

  N-Particle entanglement if:

  $2\,|C_{\downarrow N;\uparrow N}| > \max_j(P_j + P_{j'})$

- The most important information resides in the magnitude of coherence $C_{\downarrow N;\uparrow N}$

Results

From amplitude of parity-oscillations:

$$|C_{\downarrow 4;\uparrow 4}| \geq 0.349(2)$$ ✓

$$|C_{\downarrow 5;\uparrow 5}| \geq 0.264(2)$$ ✓

$$|C_{\downarrow 6;\uparrow 6}| \geq 0.210(2)$$ ✗

from poissonian fits:

$$F_{4Cat} \geq 0.76(1)$$ ✓

$$F_{5Cat} \geq 0.60(2)$$ ✓

$$F_{6Cat} \geq 0.509(4)$$

$$\langle W_4 \rangle \leq 0.51(2)$$ ✓

$$\langle W_5 \rangle \leq 0.20(2)$$ ✓

$$\langle W_6 \rangle \leq 0.018(8)$$ ~

Results

$\langle W_6 \rangle \leq 0.018(8)$ ~

Use 'depolarization' method:

$$2\,|C_{\downarrow N;\uparrow N}| > \max_j(P_j + P_{j'})$$

knowing: $\max_j(P_j + P_{j'}) \leq 2 \max(P_j)$

For the $|6\ Cat\rangle$ state:

$$|C_{\downarrow 6;\uparrow 6}| \geq 0.210(2)$$
$$\geq \max_j(P_{\downarrow j}\,|_{\,j \in \{1,2,3,4,5\}}) = 0.119(9)$$

✓

Take Home Message

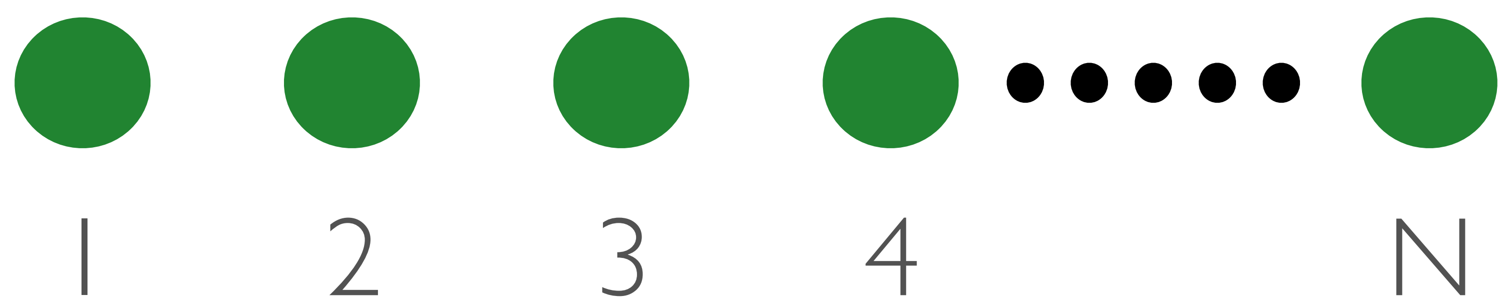
- Entanglement of N particles:
  (theoretical) problem to quantify if $N>2$

DECOHERENCE IN MULTIPARTICLE SYSTEMS

Andres Vargas
Andreas Wanner

EXPERIMENT DESCRIPTION

N $^{40}Ca^+$ Ions



$$D_{5/2}(m=-1/2) \equiv |0\rangle$$

$$S_{1/2}(m=-1/2) \equiv |1\rangle$$

$$|\widetilde{0}\rangle = |0\ldots0\rangle$$

$$|\widetilde{1}\rangle = |1\ldots1\rangle$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(|0\ldots0\rangle + |1\ldots1\rangle)$$

EXPERIMENT DESCRIPTION

$|1 \ldots 1\rangle$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(|0 \ldots 0\rangle + |1 \ldots 1\rangle)$$

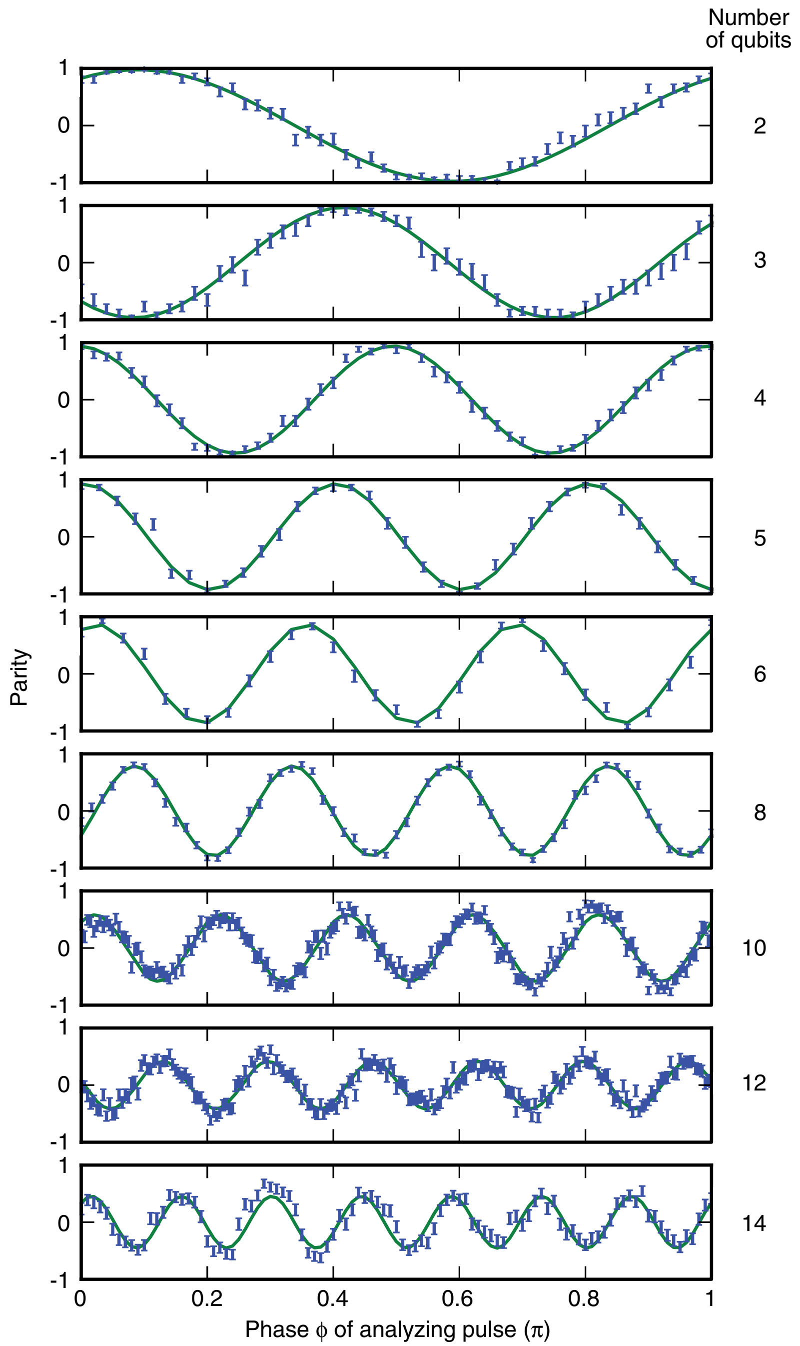
Initialize → **Entangle** → **Characterize**

optical pumping

Scattering light at 397 nm

$S_{1/2} \leftrightarrow P_{1/2}$

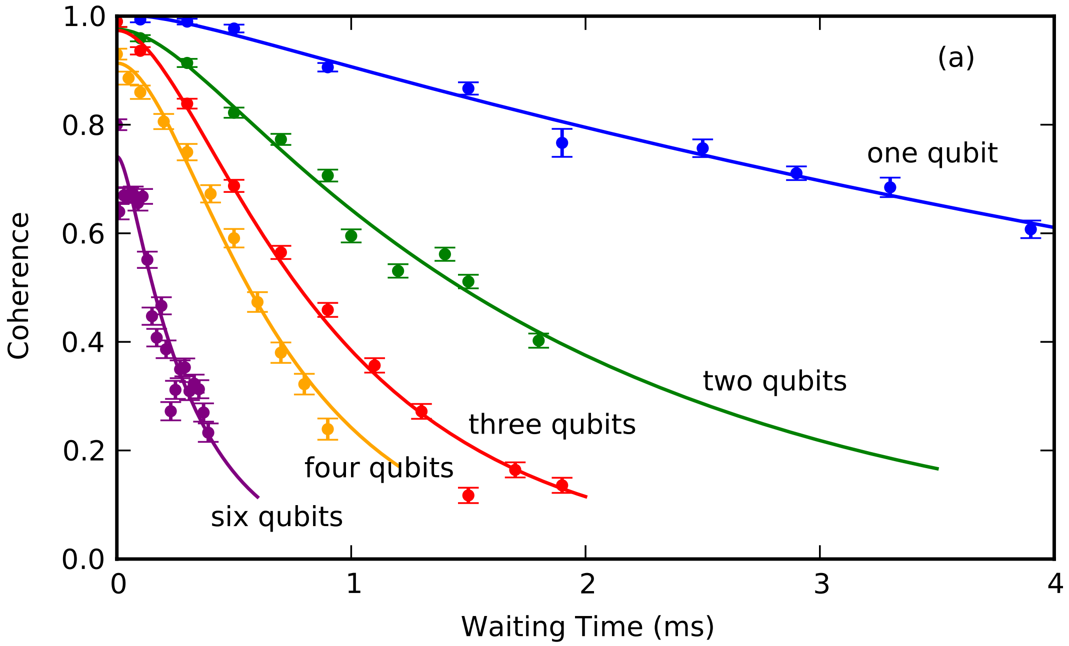
COHERANCE AS A FUNTION OF QBIT NUMBER



After the GHZ state is generated the qubit are rotated by

The amplitude of the oscillations is the coherence

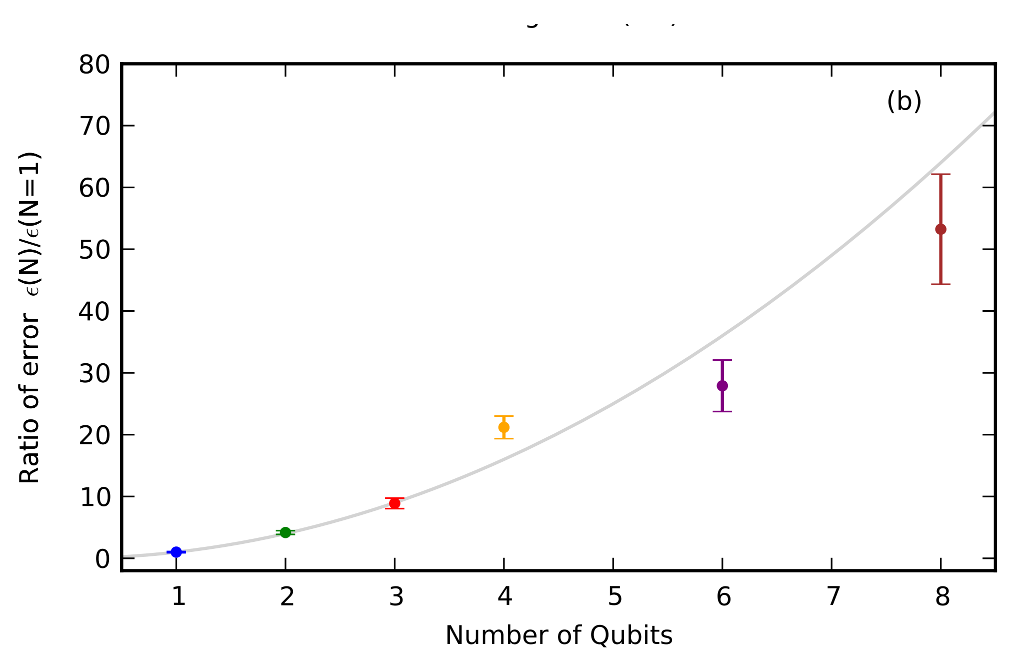
DECOHERENCE AS A FUNCTION OF TIME



A time delay is introduce between characterization and creation of state

Coherence time decreases and error increases as number if Q-bit increases

$$\epsilon(N) = N^2$$



MOST RELEVANT RESULTS

For 1 Qbit with a coherenace time of 95 ms is found

When using a base

$$|00001111\rangle + |11110000\rangle$$

a 324 ms coherence time is obtained, why??

THE MODEL FOR A SINGLE QBIT

$$H = \frac{1}{2}\sigma_z\omega_0 + \sum_k b_k^\dagger b_k + \sum_k \sigma_z (g_k b_k^\dagger + g_k^* b_k)$$

$$U(t) = \exp\{\sigma_z \frac{1}{2} \sum_k (b_k^\dagger \xi_k(t) - b_k \xi_k^*(t))\}$$

$$\sigma_z|0\rangle = -1|0\rangle$$

$$\xi_k(t) = 2g_k \frac{1 - \mathrm{e}^{i\omega_k t}}{\omega_k}$$

$$\sigma_z|1\rangle = 1|1\rangle$$

No energy exchange or spin flip

Displacement operator in Quantum optics

$$D(\alpha) = \exp\{\alpha a^\dagger - \alpha^* a\}$$

$$D(\alpha)|0\rangle = |\alpha\rangle$$

$$D(1/2\sigma_z\xi_k) = \exp\{1/2\sigma_z\xi_k b^\dagger - 1/2\sigma_z\xi_k^* b\}$$

$$U(t)|0\rangle \otimes |\Psi\rangle = \Pi_k D(-1/2\xi_k(t))$$

$$|\Psi\rangle = (c_0|0\rangle + c1|1\rangle) \otimes |0_k\rangle$$

Entanglement

$$U(t)|\Psi\rangle = c_0|0\rangle| - 1/2\xi_k(t)\rangle + c_1|1\rangle| + 1/2\xi_k(t)\rangle$$

Generalization to 2 Qbits

$$H_{int} = \sum_k \sigma_z^a(g_k^a b_k^\dagger + g_k^{*a} b_k) + \sigma_z^b(g_k^b b_k^\dagger + g_k^{*b} b_k)$$

$$U(t) = D(1/2\sigma_z^a \xi_k^a(t) + 1/2\sigma_z^b \xi_k^b(t))$$

$$|\Phi^{(-)}\rangle = (c_{10}|1_a, 0_b\rangle + c_{01}|0_a, 1_b\rangle) \otimes |0_k\rangle$$

$$|\Phi^{(+)}\rangle = (c_{00}|0_a, 0_b\rangle + c_{11}|1_a, 1_b\rangle) \otimes |0_k\rangle$$

States with 2 different Qbit configuration couple differently to the filed

$$U(t)|\Phi^{(+)}\rangle = (c_{00}|0_a, 0_b\rangle| - 1/2(\xi_k^a + \xi_k^b)\rangle + c_{11}|1_a, 1_b\rangle|1/2(\xi_k^a + \xi_k^b)\rangle$$

$$U(t)|\Phi^{(-)}\rangle = (c_{10}|1_a, 0_b\rangle| + 1/2(\xi_k^a - \xi_k^b)\rangle + c_{01}|0_a, 1_b\rangle| - 1/2(\xi_k^a - \xi_k^b)\rangle$$

If $\xi_k^a = \xi_k^b$ $|\Phi^-\rangle$ Doesn't couple to the field

general *pros* and *cons* regarding
QC with iontraps

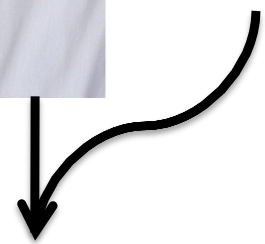
- electron and nuclear spins (Spin½ particles)

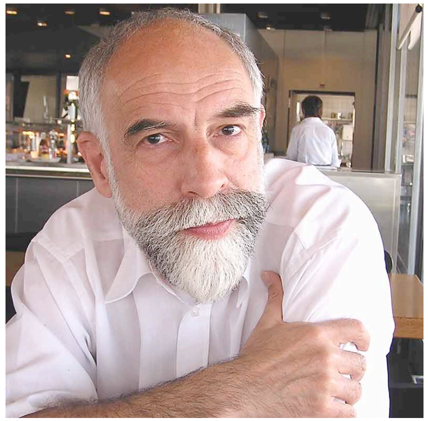
  inherently only have two states *(good!)*
- *problem*:

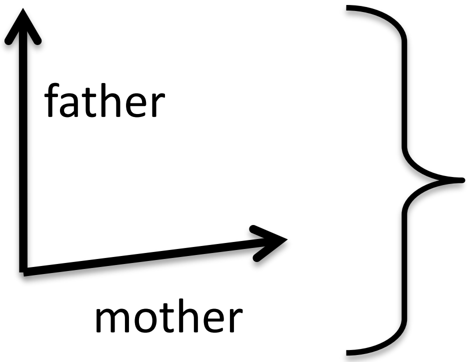
  the center of mass oscillations (phonons) have short coherence time
- *possible Solution*: strong interaction through chemical bonds -> NMR

Thanks go out to...

Anna Wanner







a 2-base eigenspace of your … soul

$$\text{soul} \cong \tfrac{5}{8}\,(\text{genome} \otimes \text{culture} \otimes \text{gods further ingredients})$$