Violation of Bell Inequalities

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Quantum Systems for
Information Technology

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Einstein-Podolsky-Rosen paradox (1935)

- Goal: prove that quantum mechanics is incomplete
- Completeness:

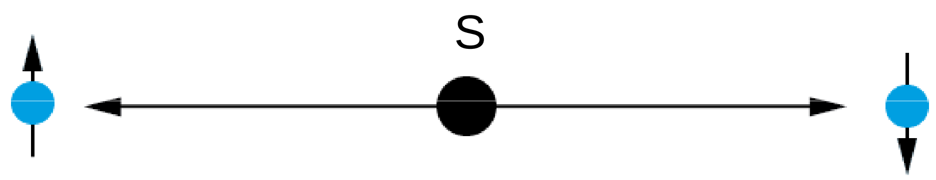
  every element of reality must be represented in a complete physical theory
- Element of reality:

  value can be predicted with certainty (exists independently of a later measurement)
- Locality:

  It is possible to separate physical systems so that they do not influence each other as they cannot transmit information with $v>c$ (space-like separation)

Thought experiment

- Setup: Two spin 1/2 particles in an entangled singlet state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$
- Perfect anticorrelations



→ Spin value is an element of physical reality

- But QM cannot predict the value

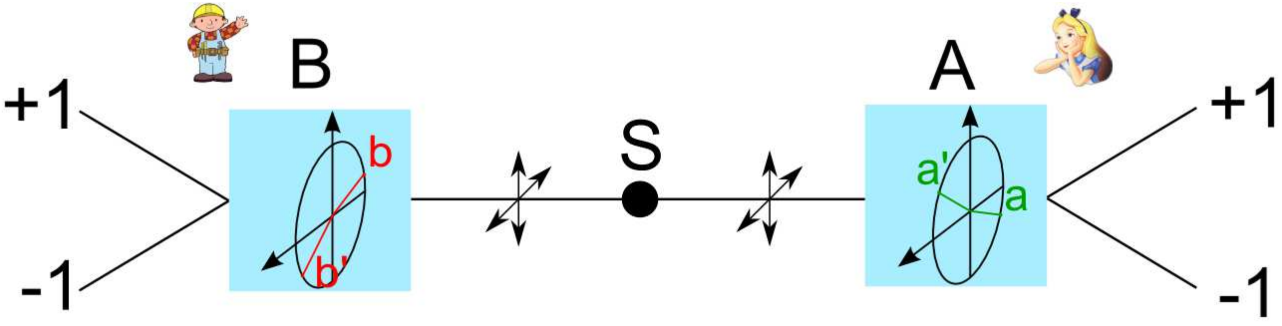
→ QM cannot be a complete local realistic theory

→ Solution: hidden variables

Outline

- Bell inequality
  - CHSH classical
  - CHSH quantum mechanical
- Loopholes
  - Detection loophole
  - Locality/Causality loophole
- Experimental violations
  - Optical experiments
  - Josephson phase qubits
- Importance of Bell inequalities

Bell – CHSH classical



$$A(a)B(b) - A(a)B(b') + A(a')B(b) + A(a')B(b') =$$

[redacted]

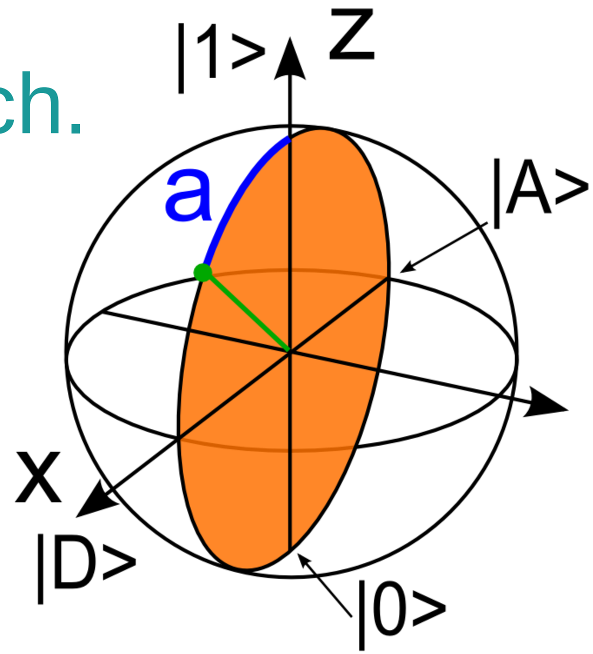
$$\pm 2 \qquad 0$$

$$\pm 1 \qquad 0 \qquad \pm 1 \qquad \pm 2$$

$$-2 \qquad 0$$

$$S = E(a,b) - E(a,b') + E(a',b) + E(a',b') \leq \pm 2$$

Bell – CHSH quantum-mech.



- for a maximal entangled state:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

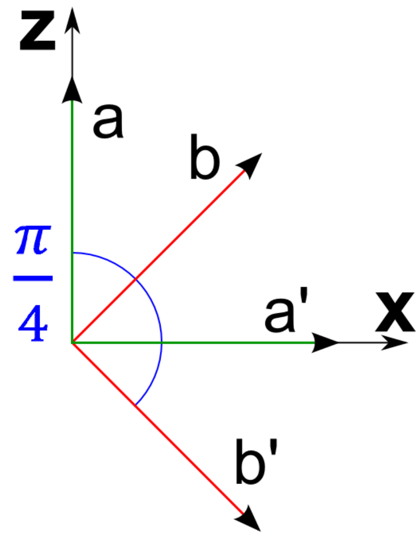
- and an operator:

$$\widehat{A}(a) = \vec{r}(a)\ \vec{\widehat{\sigma}} = \sin(a)\widehat{\sigma}_x + \cos(a)\widehat{\sigma}_z \qquad (\phi = 0)$$

$$E_{QM}(a,b) = \langle\phi^+|\widehat{A}(a) \otimes \widehat{B}(b)|\phi^+\rangle =$$

$$= \langle\phi^+|\vec{r}(a)\ \vec{\widehat{\sigma}} \otimes \vec{r}(b)\ \vec{\widehat{\sigma}}|\phi^+\rangle = \cos(a-b)$$

Bell – CHSH quantum-mech.



- for a specific set of measurement angles:

$$a = \pi/2 \quad a' = 0 \quad b = \pi/4 \quad b' = -\pi/4$$

$$E(a,b) = E(a',b) = E(a',b') = -E(a,b') = \cos(\pi/4) = \frac{1}{\sqrt{2}}$$

$$S = E(a,b) - E(a,b') + E(a',b) + E(a',b') \leq 2\sqrt{2} \approx 2.8284$$

Implication (if quantum mechanics is correct):

- 'spooky action at a distance' (non local) or
- state is not determined before its measurement (not real)

→ Experiments are needed

Loopholes

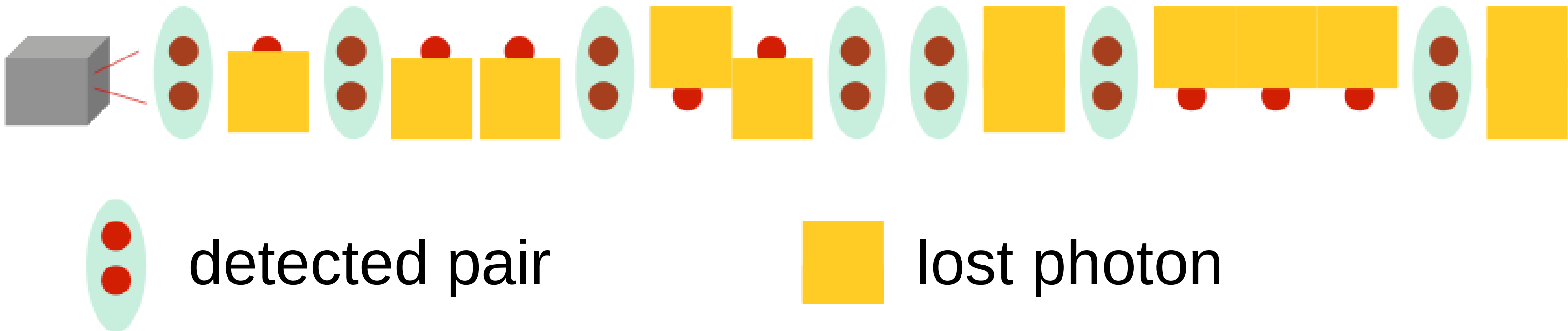
- Two EPR requirements not fulfilled in typical experiments:
  - Detection loophole
  - Locality/Causality loophole

→ Room for the local realistic interpretation

- Goal: close both loopholes in one experiment

Detection loophole

- Mainly a problem in photon experiments
- Experiments detect only a subset of the created pairs



- Fair-sampling assumption: the detected pairs constitute a representative sample of all created pairs

Locality/Causality Loophole

- Space-like separation must be ensured

  $d \geq ct_{meas}$

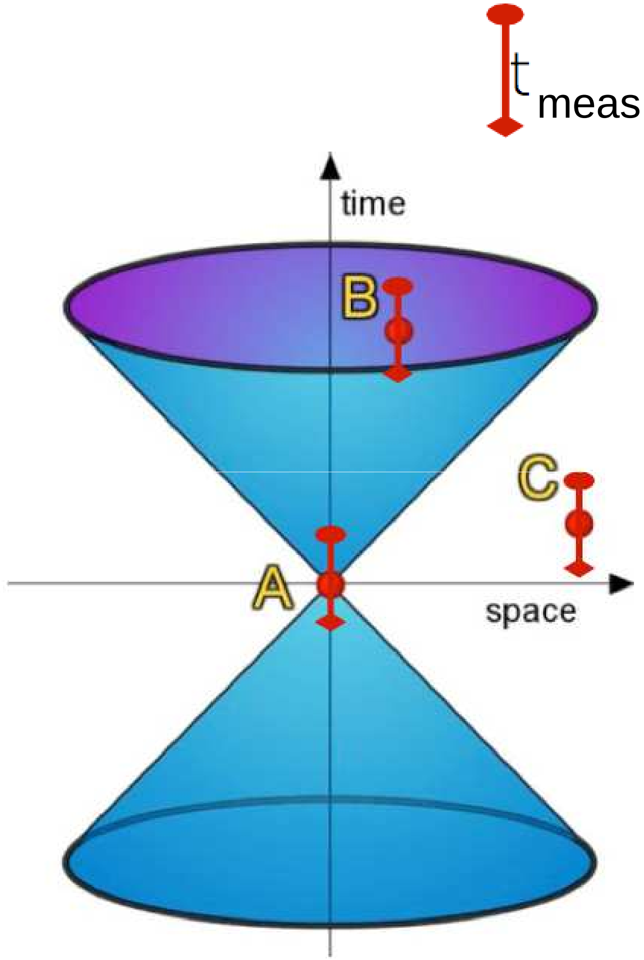
  d: distance between particles

  c: speed of light

  t_{meas}: first point which influences the measurement direction → final registration of the photon

- A and C are space-like separated

  A and B are not



Experiments

Photons:

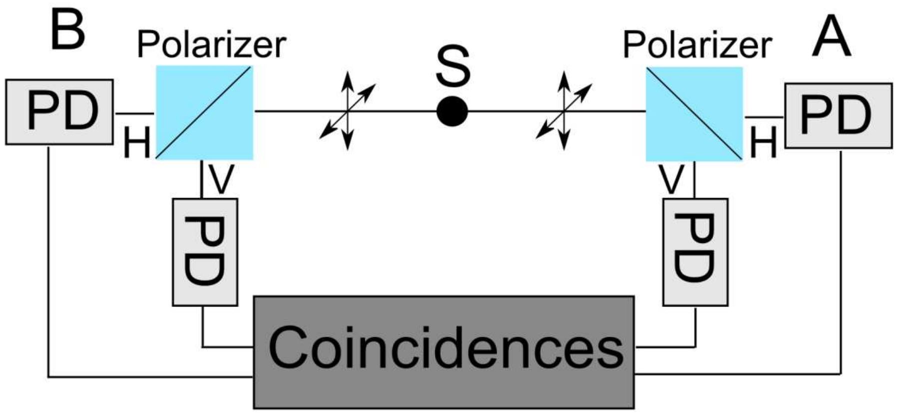
- Aspect *et al.*
- Weihs *et al.* } closed locality loophole

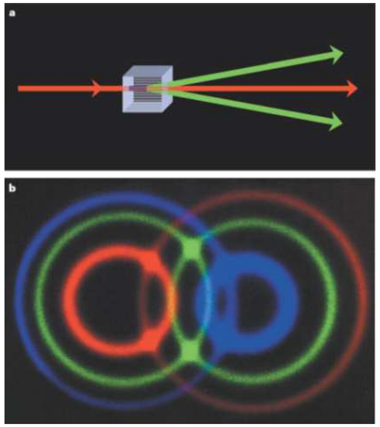
Josephson phase qubits:

- Ansmann *et al.* } closed detection loophole

. . .

Optical experiments - exemplary setup





C. Monroe Nature 416, 238–246 (2002)

PDC type II

Common Source:

•Parametric down-conversion

Measurement:

•Using a fourfold coincidence technique

→ directly measure E(a,b) in one run (Fair-sampling assum.)

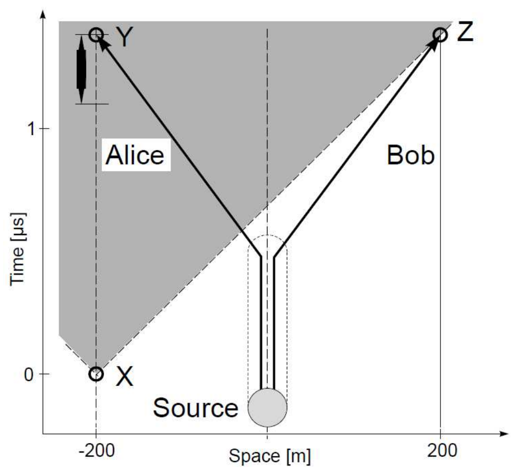
$$E(a,b) = \frac{C_{HH}(a,b) + C_{VV}(a,b) - C_{HV}(a,b) - C_{VH}(a,b)}{C_{HH}(a,b) + C_{VV}(a,b) + C_{HV}(a,b) + C_{VH}(a,b)}$$

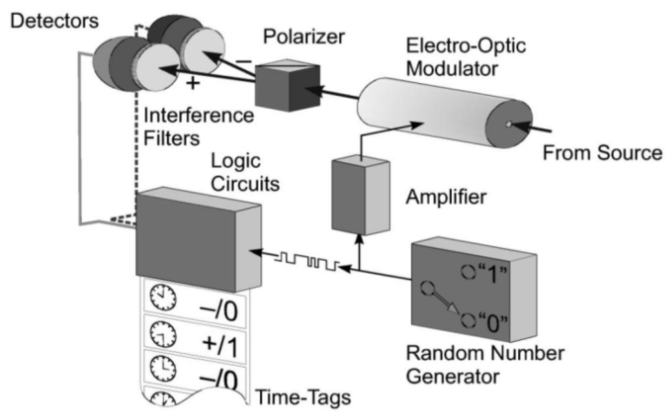
Violation of Bell inequality under strict Einstein locality conditions

Weihs et al. Phys. Rev. Lett. 81, 5039 (1998)

special feature for closing the locality loophole:

- sufficient physical distance between the observers
- ultrafast and random setting of the analyzers
- completely independent data registration



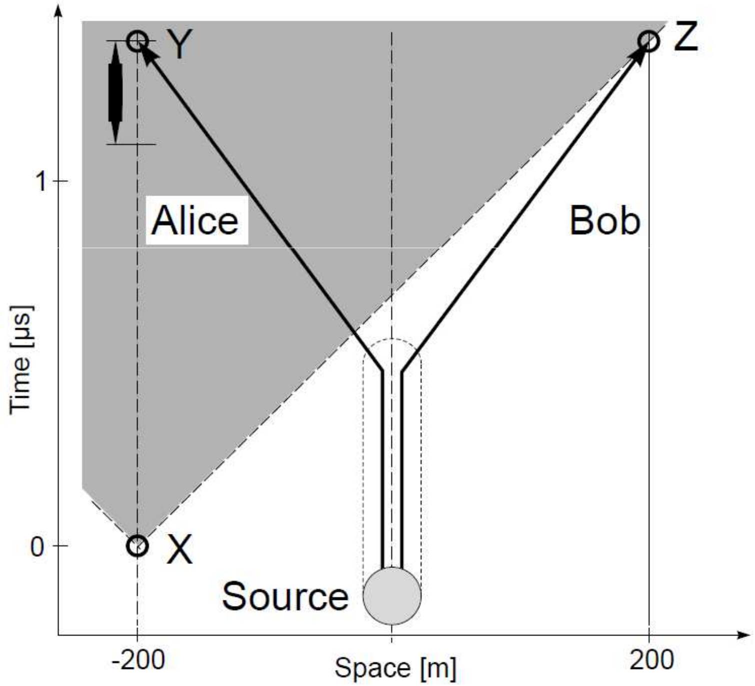


Spatial separation

- observers spatially separated by 400 m

  ≈ **1.3 μs** until registration of photons

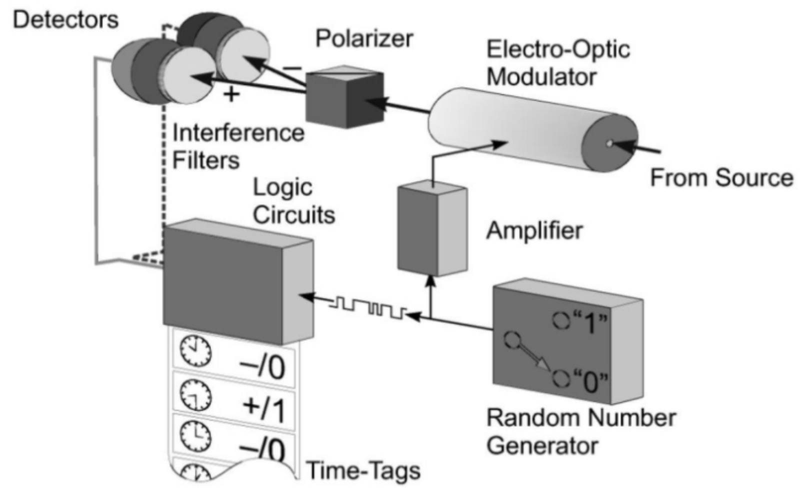
←→ whole measurement process takes **100 ns**



Measurement process

- analyzer directions are completely unpredictable
  - physical random number generators
  - fast electro-optic modulators
- independent data registration
  - events registered individually (synchronized by a atomic clock before measurement)
  - compared after the measurements

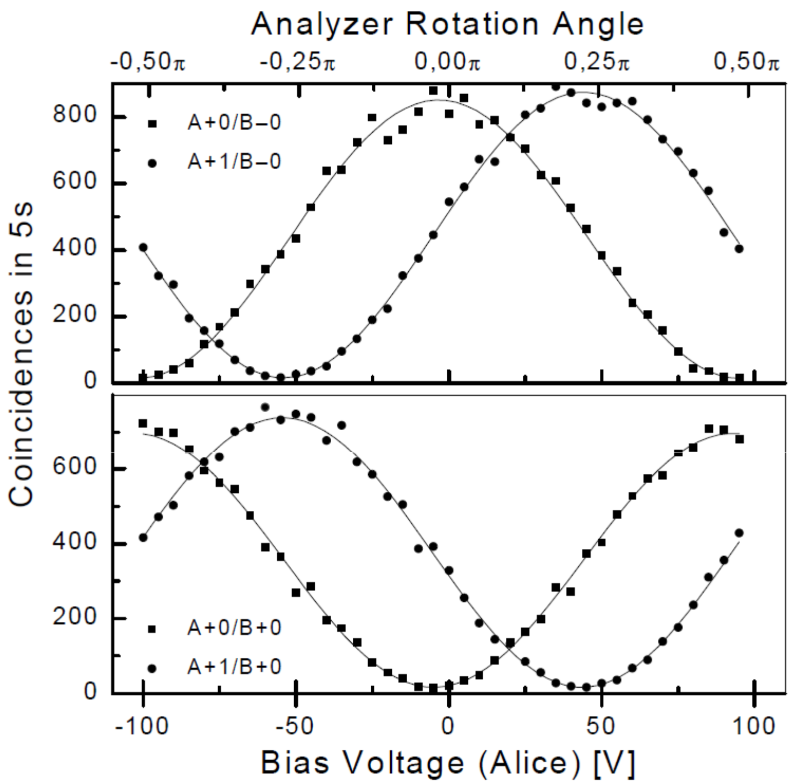
→ strict locality conditions



Results

- Visibility 97%→ S ≈ 2.74
- Measurement:

  2.73 ± 0.02
- violation of **30 standard deviations**



- **but:**

"ultimate experiment should also have higher detection efficiency, which was 5% in our experiment."

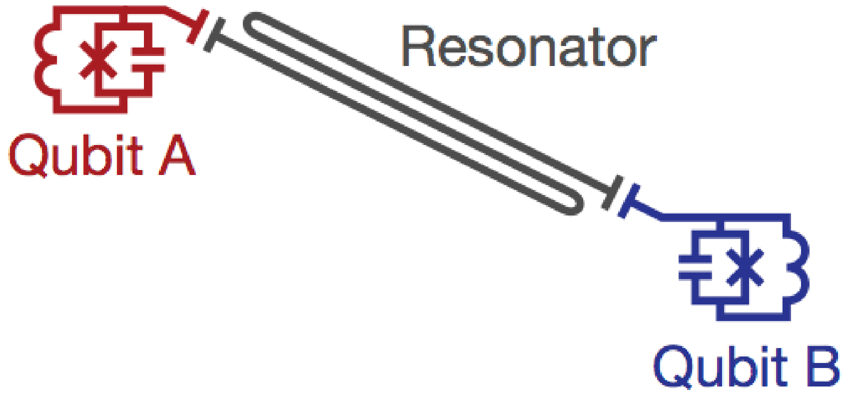
Violation in Josephson Phase Qubits

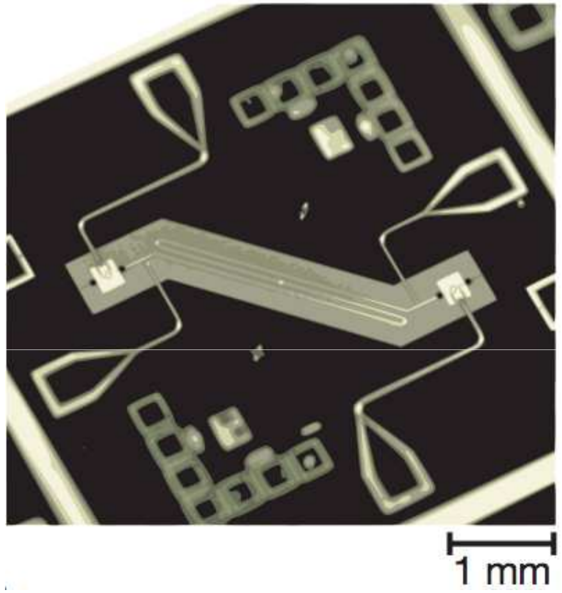
Ansmann *et al.* Nature 461, 504 (2009)

- Two qubits in the Bell singlet state

$$|\psi_s\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

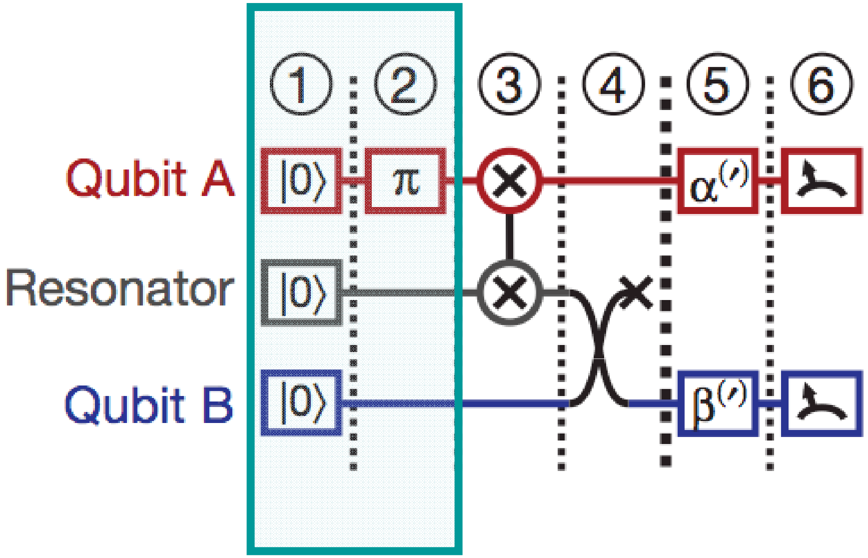
- Josephson phase qubits coupled via a resonator





Qubit control sequence

① & ② $|00\rangle$ ⌫ $|10\rangle$

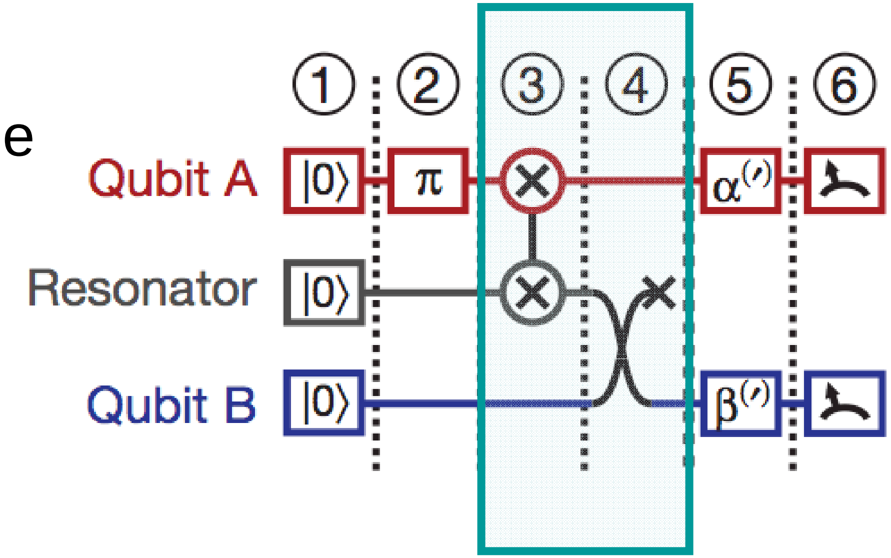


Qubit control sequence

① & ② $|00\rangle$⌫$|10\rangle$

③ & ④ Entanglement via the resonator

$$\frac{1}{\sqrt{2}}\left(|01\rangle - e^{i\theta}\,|10\rangle\right)$$



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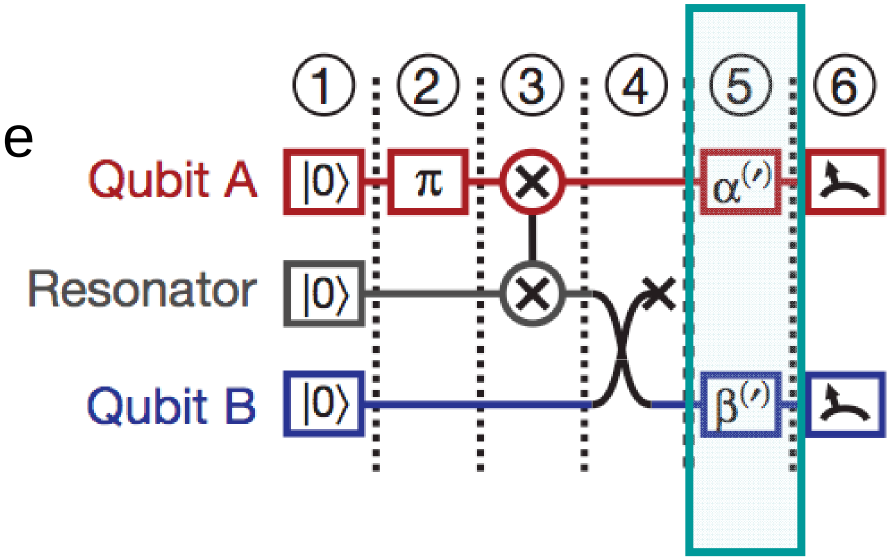
⑤ Rotation to change the measurement axis



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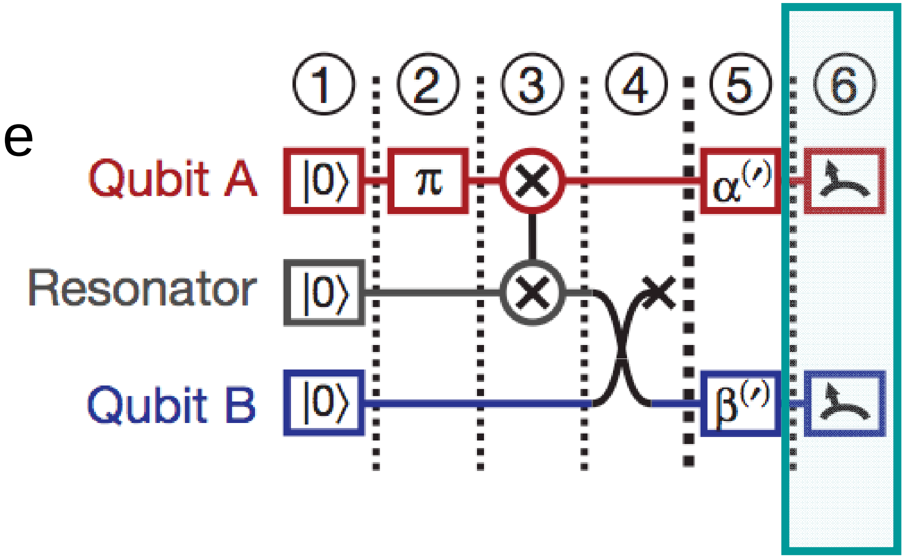


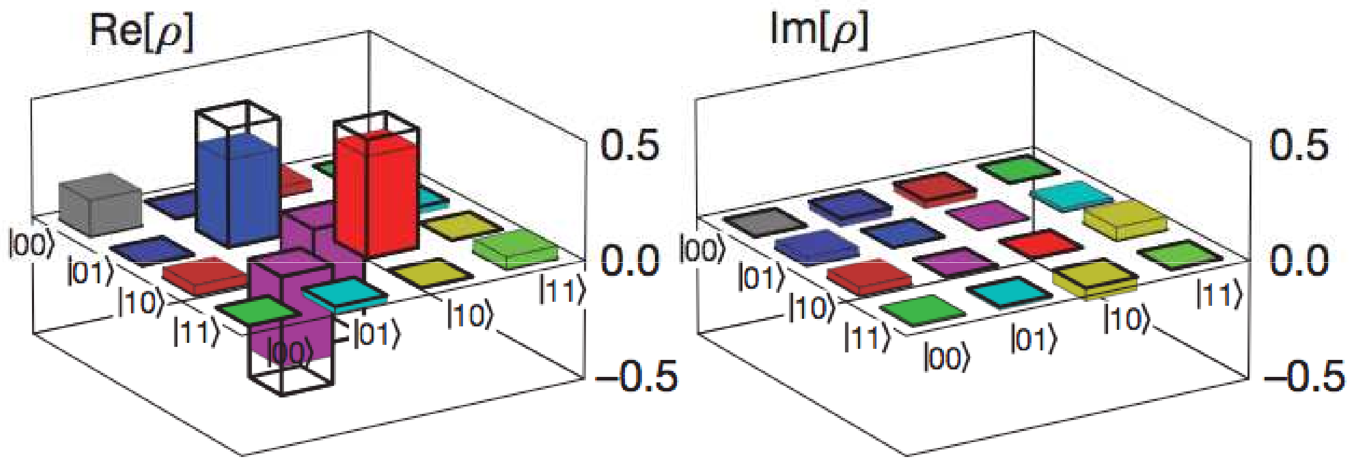
⑤ Rotation to change the measurement axis

⑥ Measurement along the z-axis

Single shot read-out
Measurement fidelity: 94.6 %

Entanglement analysis

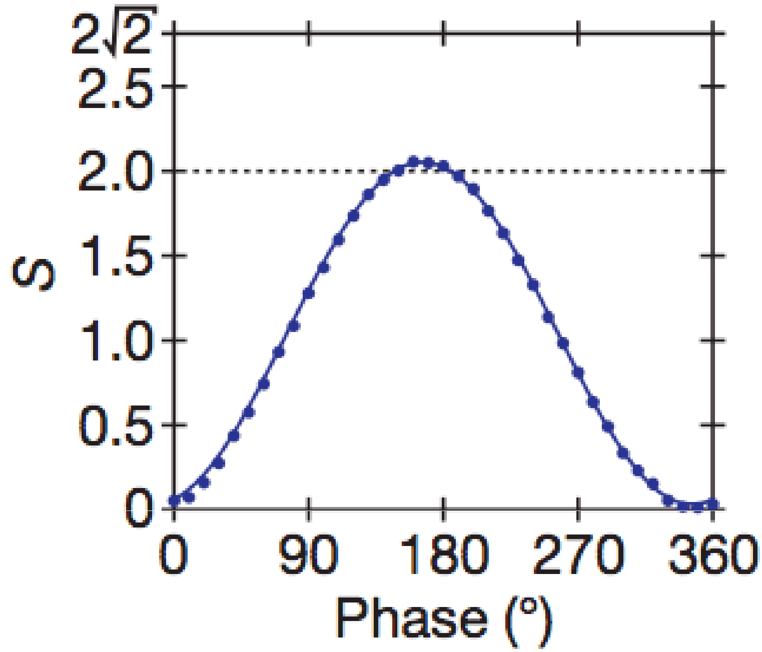
- Measured density matrix of the entangled state



- Fidelity of the entangled state with respect to the Bell singlet: $F(\rho) = \langle \psi_s | \rho | \psi_s \rangle = 88.3\%$

Results

- S depends on angle between (a‘,a) and (b‘,b) plane



- Maximal violation: S=2.0732±0.0003

- Violation of 244 standard deviations

- Creation and measurement of the entangled pair with certainty → detection loophole closed

- No true space-like separation → locality loophole remains open

Importance of Bell inequalities

- Bell inequality classical: $S \leq \pm 2$

  QM: $S \leq \pm 2\sqrt{2}$

  → Fundamental test whether QM is complete

- Experimental violations
  - Weihs *et al.:* S=2.73±0.02
    - closed locality loophole
  - Ansmann *et al.*: S=2.0732±0.0003
    - closed detection loophole

- Useful benchmark for the comparison of different quantum computational architectures

References

- **Aspect, A; Grangier, P; Roger, G**
  Experimental Realization of EPR-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities
  Phys. Rev. Lett. 49, 91 (1982)

- **Weihs, G; Jennewein, T; Simon, C; et al.**
  Violation of Bell inequality under strict Einstein locality conditions
  Phys. Rev. Lett. 81, 5039 (1998)

- **Ansmann, M; Wang, H; Bialczak, RC; et al.**
  Violation of Bell's inequality in Josephson phase qubits
  Nature 461, 504 (2009)