QSIT 2011 - Questions 7

16. November 2011

1. **Measurement of relaxation and dephasing**
   Typically all experiments starts from the qubit being in its ground state $|0\rangle$. The state of the qubit can be manipulated with the help of the microwave pulses that can induce rotations by an angle ϕ about any of the axis x, y according to $R_{x,y}(\phi) = \exp[-i(\phi/2)\sigma_{x,y}]$ (note that the manipulation are performed in the rotating frame). In order to extract the relaxation time T_1 and T_2 we can use the following schemes:

   (a) Make a π rotation (π-pulse) about x-axis, wait a time interval of Δt and measure the population of the excited state $|1\rangle$. Repeat experiment for different Δt.

   (b) Make a $\pi/2$ rotation ($\pi/2$-pulse) about x-axis, wait a time interval of Δt, make another $\pi/2$-pulse and measure the population of the excited state $|1\rangle$. Repeat experiment for different Δt.

   Using Bloch equations answer the following questions:

   (a) What is the evolution of the Bloch vector for these experiments?

   (b) How do we extract the timescales T_1, T_2?

   (c) What happens if the microwave pulses are detuned from the frequency of the qubit by $\delta\omega$ for the second experiment. (Ramsey experiment)

   (d) What happens if the one insert π pulse exactly in the middle between $\pi/2$ pulses for the second experiment (Echo experiment).

2. **CNOT and CPHASE gates**

CNOT gate is described by the matrix:

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Show that CNOT gate can be obtained by CPHASE gate:

$$CPHASE = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$