QSIT 2011 - Questions 6

9. November 2011

1. **Bloch equations**
Let us consider a two-level system with transition frequency ω_{01} driven by external electromagnetic field (characterized by frequency ω, phase ϕ and amplitude Ω_R) according to the Hamiltonian:
$$H = \frac{1}{2}\hbar\omega_{01}\sigma_z + \hbar\Omega_R \cos(\omega t + \phi)\,\sigma_x. \tag{1}$$
The dynamics of a qubit interacting with the environment is governed by the Bloch equations:
$$\frac{d\langle\boldsymbol{\sigma}\rangle_{x,y}}{dt} = -\gamma[\langle\boldsymbol{\sigma}\rangle \times \mathbf{B}]_{x,y} - \frac{\langle\boldsymbol{\sigma}\rangle_{x,y}}{T_2}, \tag{2}$$
$$\frac{d\langle\sigma_z\rangle}{dt} = -\gamma[\langle\boldsymbol{\sigma}\rangle \times \mathbf{B}]_z - \frac{(\langle\sigma_z\rangle - \sigma_z^s)}{T_1}, \tag{3}$$
where $\boldsymbol{\sigma} = [\sigma_x, \sigma_y, \sigma_z]$ is a vector composed out of Pauli matrices, $\gamma\mathbf{B} = [\Omega_R\cos\phi, \Omega_R\sin\phi, \omega_{01} - \omega]$ is an effective magnetic field, T_1, T_2 are some relevant timescales, σ_z^s is the steady state value for $\sigma_z^s = \langle\sigma_z\rangle|_{t\to\infty}$ and $\times$ denotes the cross product of two vectors.
   (a) Derive Eqs. (2,3) from (1) under an assumption: $T_1, T_2 = \infty$.
   (b) What is the physical meaning of timescales T_1, T_2.
   (c) Sketch the trajectory of the Bloch vector dynamics for three representative examples. Decay of the excited state: $\psi(0) = |1\rangle$, $\Omega = 0$ and $\omega_{01} - \omega = 0$. Dephasing of the superposition: $\psi(0) = (1/\sqrt{2})(|0\rangle + |1\rangle)$, $\Omega = 0$ and $\omega_{01} - \omega \neq 0$, $T_2 \ll T_1$.Dephasing of the superposition: $\psi(0) = (1/\sqrt{2})(|0\rangle + |1\rangle)$, $\Omega = 0$ and $\omega_{01} - \omega \neq 0$, $T_2 \sim T_1$.
   (d) What happens to the state of the qubit if you strongly drive it by microwaves $\Omega \gg T_1, T_2$ in resonance with the qubit $\omega_{01} = \omega$ for very long time.

(e) Could you think of the experiments to measure T_1 and T_2 timescales?

2. **SQUID**

SQUID stands for Superconducting Quantum Interference Device and consists of a superconducting loop intersected by two Josephson junctions as shown in the figure below. In general the junction is described the its phase ϕ which depends on the supercurrent flowing through it according to

$$I = I_c \sin\phi, \tag{4}$$

where I_c is the critical current of the jucntion. In addition to Eq. (4) we can relate magnetic flux Φ through the loop and all currents by Kirchhoff rules:

$$I_2 + I_1 = I, \tag{5}$$

$$\Phi + \frac{\Phi_0}{2\pi}(\phi_1 - \phi_2) = 0 \tag{6}$$

(a) Show that the total current I is related to the average phase $\phi_p = (\phi_1 + \phi_2)/2$ by the current-phase relationship similar to (4) with the critical current modulated by the magnetic flux through the loop $I_c(\Phi)$. Assume both junctions to be identical.