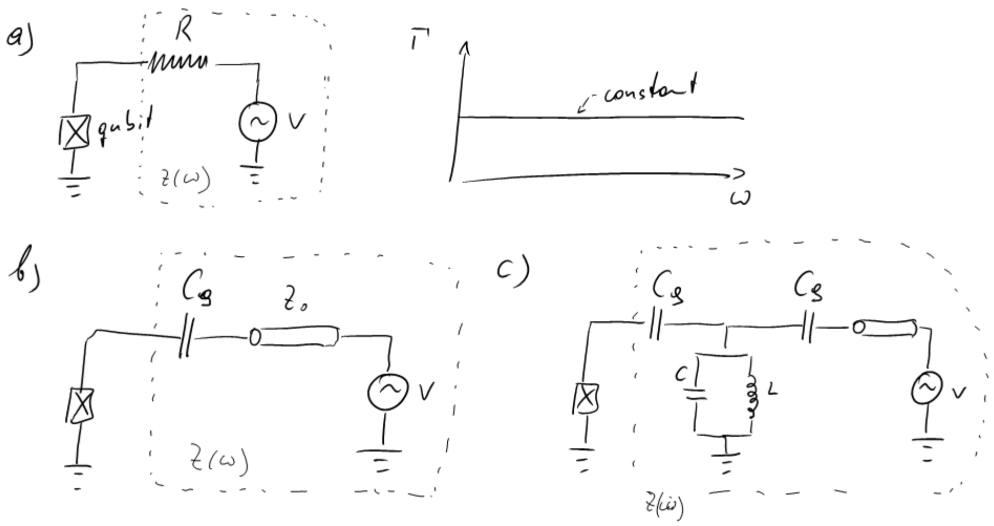
QSIT 2010 - Questions 5

2. November 2011

1. **Energy relaxation of a qubit**
   In analogy to the harmonic oscillator, the energy decay time of a qubit is given by $T_1 = RC$, where C is the intrinsic capacitance of the qubit. R denotes the effective resistance $R = 1/\mathrm{Re}[Y(\omega)]$ obtained from the impedance of the environment $Z(\omega) = 1/Y(\omega)$ as seen from the position of the qubit.



   If the impedance of the environment is purely resistive, e.g. $Z(\omega) = 50\ \Omega$, the decay rate $\Gamma = 1/T_1$ is frequency independent (see Figure a).

   (a) Derive the impedance of a Cooper-pair box qubit that is capacitively coupled to a transmission line ($Z_0 = 50\ \Omega$) via a gate capacitance C_g (Figure b). Sketch the decay rate Γ as a function of frequency.

   (b) What is the spectral shape of Γ for a coupling to an LC oscillator (Figure c)?

2. **Two-level approximation for a Cooper-pair box**
   The Hamiltonian for a Cooper-pair box is given by

$$H_{CPB} = \sum_n \left[E_C(\hat{n} - n_g)^2 |n\rangle\langle n| - \frac{E_J}{2} \left(|n\rangle\langle n+1| + |n+1\rangle\langle n| \right) \right].$$

   Write down the Hamiltonian for the two-dimensional qubit subspace in terms of the Pauli matrices σ_x and σ_z by restricting the quantum states to $n = 0, 1$. What is the transition frequency between ground and excited state?