QSIT 2010 - Questions 3

19. Oktober 2011

1. **Rotating Wave Approximation**
   A two level system with transition frequency Ω is driven by a laser with frequency ω, phase ϕ and amplitude ϵ. The corresponding Hamiltonian is
   $$H(t) = \frac{\hbar\Omega}{2}\sigma_z + \hbar\epsilon\cos(\omega t + \phi)\sigma_x.$$
   (a) Show that in the rotating frame and after applying the rotating wave approximation the effective (static) Hamiltonian is given by
   $$H_{\text{eff}} = \Delta\sigma_z + \Omega_x\sigma_x + \Omega_y\sigma_y,$$
   where the detuning $\Delta = \Omega - \omega$, $\Omega_x = \epsilon\cos\phi$ and $\Omega_y = \epsilon\sin\phi$.
   (b) Choose values for Δ, ϵ and ϕ and a time t that lead to an equally weighted superposition state, a state on the equator of the Bloch sphere. What values lead to state on the equator of the Bloch sphere that points in a direction orthogonal to the former state.

2. **CNOT operation**

   (a) Verify the unitarity of the CNOT operation given by the transformation
   $$\text{CNOT:} \begin{array}{l} |00\rangle \to |00\rangle \\ |01\rangle \to |01\rangle \\ |10\rangle \to |11\rangle \\ |11\rangle \to |10\rangle, \end{array}$$
   where A in $|AB\rangle$ is the control qubit and B the target.
   (b) Determine the matrix for the CNOT operation, where B is the control qubit.