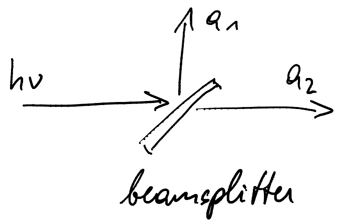
Photon qubits for Quantum Communication:
(e.g. Koh & Lovett)

e.g. using polarisation + spatial degree of freedom

spatial modes:



a_1, a_2 .. spatial modes

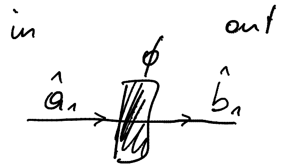
$$|0\rangle = a_1^+ |0\rangle_1 |0\rangle_2 = a_1^+ |0,0\rangle = |1,0\rangle$$

photons in mode 1

$$|1\rangle = a_2^+ |0,0\rangle = |0,1\rangle$$

operations:

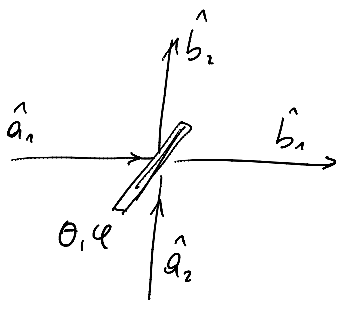
phase shifter



(dielectric with refractive index $n_1 \neq 1$)

$$\hat{a}_1 \longrightarrow \hat{b}_1 = e^{-i\phi} \hat{a}_1$$

beam splitter
(half silvered mirror)



θ... reflectivity
φ... phase

$$\left.\begin{aligned} \hat{a}_1 &\xrightarrow{BS} \hat{b}_1 = \cos\theta\, \hat{a}_1 - i e^{i\varphi} \sin\theta\, \hat{a}_2 \\ \hat{a}_2 &\xrightarrow{BS} \hat{b}_2 = -i e^{-i\varphi} \sin\theta\, \hat{a}_1 + \cos\theta\, \hat{a}_2 \end{aligned}\right\} \text{unitary transformation}$$

$$\begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -i e^{i\varphi} \sin\theta \\ -i e^{-i\varphi} \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}$$

e.g. $\theta = \frac{\pi}{4}$, $\varphi = 0$: $\begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}$

photon at input 1:

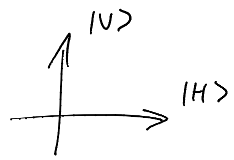
$$|0\rangle = |1,0\rangle = a_1^\dagger |0,0\rangle \longrightarrow \left(\cos\theta\, a_1^\dagger + i e^{-i\varphi} \sin\theta\, a_2^\dagger\right)|0,0\rangle$$

$$= \frac{1}{\sqrt{2}} \left(|1,0\rangle + i|0,1\rangle\right) =$$

$$= \frac{1}{\sqrt{2}} \left(|0\rangle + i|1\rangle\right)$$

$\rightarrow$ 50/50 beamsplitter: 50% probability for scattering into either output port

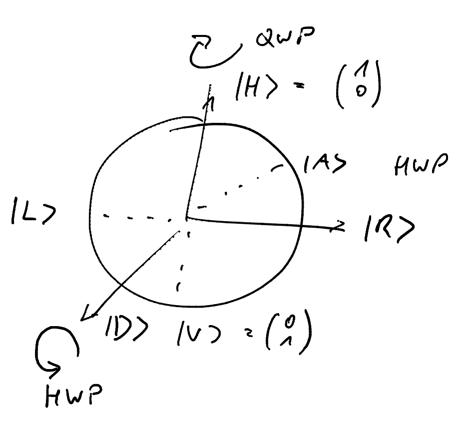
polarization basis:



$a_H^\dagger$ ... creation of photon in mode a with horizontal pol.
$a_V^\dagger$ ... ——— " ——— vertical pol.

$$|0\rangle \equiv a_H^\dagger |0,0\rangle_{HV} = |1,0\rangle_{HV} = |H\rangle$$

$$|1\rangle \equiv a_V^\dagger |0,0\rangle_{HV} = |0,1\rangle_{HV} = |V\rangle$$



$$|L\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle + i|V\rangle\right)$$

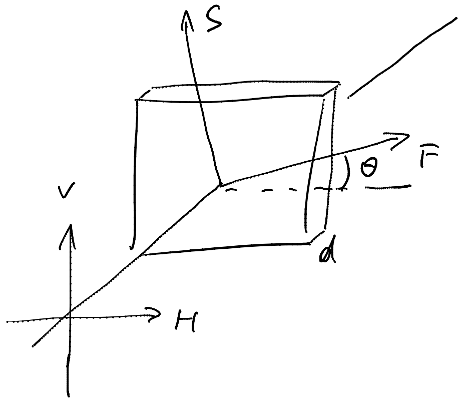
$$|R\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle - i|V\rangle\right)$$

$$|D\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle + |V\rangle\right)$$

$$|A\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle - |V\rangle\right)$$

Operations: quarter- / half wave plates

birefringent material: different polarization with different wave velocity



F/S... fast axis (parallel to optical axis)
slow axis (perp. to opt. axis)

$$\phi_i = k_i d = \frac{v_i}{c} k d = \frac{k}{n_i} d$$

↑ refractive index ($i = s, f$)

$n_s > n_f$

half-wave plate: π - phase shift between fast and slow component

$$\phi_f - \phi_s = \pi$$

$$\left[\frac{v_f - v_s}{c} \frac{2\pi}{\lambda} = \frac{\pi}{d} \qquad d = \frac{\lambda}{2} \frac{c}{v_f - v_s} \right]$$

$$|H\rangle \rightarrow \cos 2\theta\, |H\rangle + i \sin 2\theta\, |V\rangle$$
$$|V\rangle \rightarrow i \sin 2\theta\, |H\rangle + \cos 2\theta\, |V\rangle$$

$$U_{HWP}(\theta) = \begin{pmatrix} \cos 2\theta & i \sin 2\theta \\ i \sin 2\theta & \cos 2\theta \end{pmatrix}$$

=> rotation about x - axis

$$\theta = \frac{\pi}{4}: \quad U_{HWP} = e^{i\frac{\pi}{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \equiv X$$

$$|V\rangle \rightarrow |H\rangle \ ; \quad |H\rangle \rightarrow |V\rangle$$

quarter wave plate: $\phi_f - \phi_s = \frac{\pi}{2}$ $\quad \theta = \frac{\pi}{4}$

$$U_{QWP} = e^{-i\frac{\pi}{4}} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \equiv Z$$

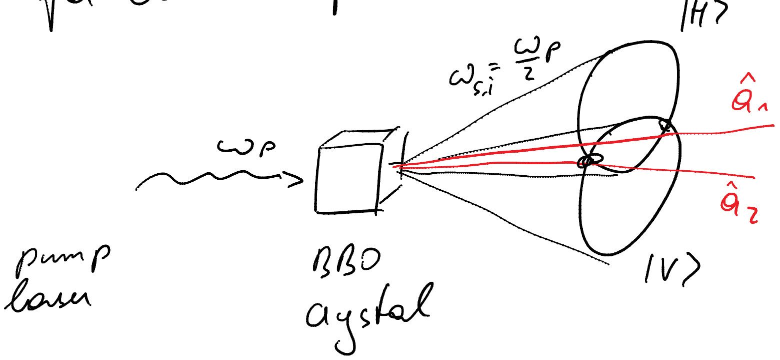
$\Rightarrow$ rotation about z-axis

$$|L\rangle = \frac{1}{\sqrt{2}} (|H\rangle + i|V\rangle) \;\hat{=}\; \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$U_{QWP}|L\rangle \longrightarrow e^{-i\frac{\pi}{4}} (|H\rangle - |V\rangle) \propto |A\rangle$$

Parametric Down Conversion

for creation of entangled photon pairs



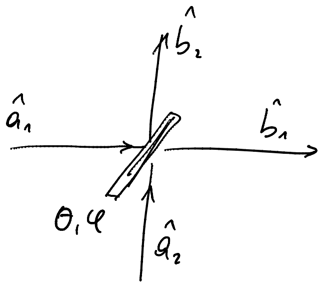
conservation of
- energy $(\omega_p = \omega_s + \omega_i)$
- momentum $(\vec{k}_p = \vec{k}_s + \vec{k}_i)$

at crossing points: $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1 |V\rangle_2 - |V\rangle_1 |H\rangle_2\right)$

anti-symmetric Bell state

single photons: attenuate laser beam such that $\langle n \rangle \ll 1$

Bell state measurement using beamsplitter



θ ... reflectivity
φ ... phase

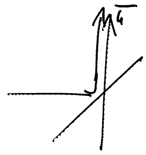
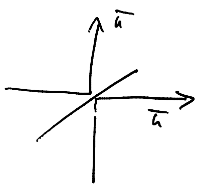
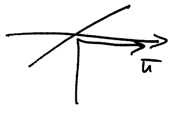
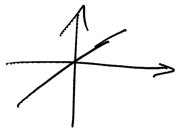
$$\left.\begin{aligned} \hat{a}_1 &\xrightarrow{BS} \hat{b}_1 = \cos\theta\, \hat{a}_1 - i e^{i\varphi} \sin\theta\, \hat{a}_2 \\ \hat{a}_2 &\xrightarrow{BS} \hat{b}_2 = -i e^{-i\varphi} \sin\theta\, \hat{a}_1 + \cos\theta\, \hat{a}_2 \end{aligned}\right\} \text{unitary transformation}$$

$$\begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -i e^{i\varphi} \sin\theta \\ -i e^{-i\varphi} \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}$$

for $\theta = \frac{\pi}{4}$ (50/50 beamsplitter) & $\varphi = 0$ (phase shift $\frac{\pi}{4}$ for reflected beam)

$$\hat{a}_1 \longrightarrow \frac{1}{\sqrt{2}} \left(\hat{a}_1 - i \hat{a}_2 \right) \qquad \hat{a}_2 \longrightarrow \frac{1}{\sqrt{2}} \left(\hat{a}_2 - i \hat{a}_1 \right)$$

two photons incident on the beamsplitter:

4 possibilities:			spatial wavefunction
(1) 	bunching		symm.
(2) 	anti-bunching		anti-symm
(3) 	bunching		symm.
(4) 	anti-bunching		anti-symm

input state: $\frac{1}{\sqrt{2}}\left(a_{1H}^{\dagger}a_{2V}^{\dagger} \mp a_{1V}^{\dagger}a_{2H}^{\dagger}\right) \xrightarrow{BS}$

anti-sym

symm.

"apply beamsplitter transformation to each mode"

$$\frac{1}{2}\frac{1}{\sqrt{2}}\left[\left(a_{1H}^{\dagger}+ia_{2H}^{\dagger}\right)\left(a_{2V}^{\dagger}+ia_{1V}^{\dagger}\right) \mp \left(a_{1V}^{\dagger}+ia_{2V}^{\dagger}\right)\left(a_{2H}^{\dagger}+ia_{1H}^{\dagger}\right)\right] =$$

$$= \frac{1}{2\sqrt{2}}\left[a_{1H}^{\dagger}a_{2V}^{\dagger} + ia_{2H}^{\dagger}a_{2V}^{\dagger} + ia_{1H}^{\dagger}a_{1V}^{\dagger} - a_{2H}^{\dagger}a_{1V}^{\dagger} \mp a_{1V}^{\dagger}a_{2H}^{\dagger} \mp ia_{2V}^{\dagger}a_{2H}^{\dagger} \mp ia_{1V}^{\dagger}a_{1H}^{\dagger} \pm a_{2V}^{\dagger}a_{1H}^{\dagger}\right]$$

for anti-symmetric spatial wavefunction $(-)$: " "

$$= \frac{1}{\sqrt{2}}\left(a_{1H}^{\dagger}a_{2V}^{\dagger} - a_{1V}^{\dagger}a_{2H}^{\dagger}\right) \Rightarrow \text{anti-bunching}$$

for symmetric spatial wavefunction $(+)$: " "

$$= \frac{1}{\sqrt{2}}\left(a_{1H}^{\dagger}a_{1V}^{\dagger} + ia_{2H}^{\dagger}a_{2V}^{\dagger}\right) \Rightarrow \text{bunching}$$

similar for other symmetric spatial wavefunctions

$$\left[\frac{1}{\sqrt{2}}\left(a_{1H}^{\dagger}a_{2H}^{\dagger} \pm a_{1V}^{\dagger}a_{2V}^{\dagger}\right)\right]$$

Super Dense Coding:

1) Preparation of initial entangled state (PDC)

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle + |VH\rangle\right)$$

2) Generation of 4 maximally entangled 2 photon polarization states

$$|\psi^+\rangle \xrightarrow{\mathbb{1}_2} |\psi^+\rangle$$

$$|\psi^+\rangle \xrightarrow{X_2} \frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right) = |\phi^+\rangle \quad (HWP)$$

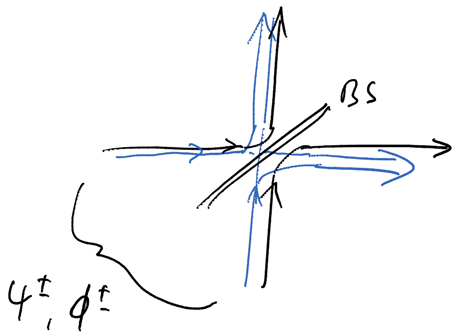
(i) ... omitted

$$|\psi^+\rangle \xrightarrow{Z_2} \frac{1}{\sqrt{2}}\left(|HV\rangle - {}^{(i)}|VH\rangle\right) = |\psi^-\rangle \quad (QWP)$$

$$|\psi^+\rangle \xrightarrow{Z_2 X_2} \frac{1}{\sqrt{2}}\left(|HH\rangle - {}^{(i)}|VV\rangle\right) = |\phi^-\rangle \quad (HWP + QWP)$$

3) Bell State measurement using beam splitter

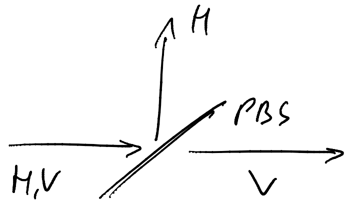
A) distinguish symmetric ($|\psi^+\rangle, |\phi^+\rangle, |\phi^-\rangle$) from antisymmetric ($|\psi^-\rangle$) state using a beamsplitter (BS)



anti-bunching for ψ^-

bunching for symm. states

B) distinguish polarization states using polarizing beam splitter



outcomes: $|\psi^+\rangle$: coincidence D_H & D_V or $D_{H'}$ & $D_{V'}$

$|\psi^-\rangle$: coincidence $D_{H'}$ & D_V or D_H & $D_{V'}$

$|\phi^+\rangle, |\phi^-\rangle$: 2 photons in $D_H, D_V, D_{H'}, D_{V'}$

(cannot be distinguished)