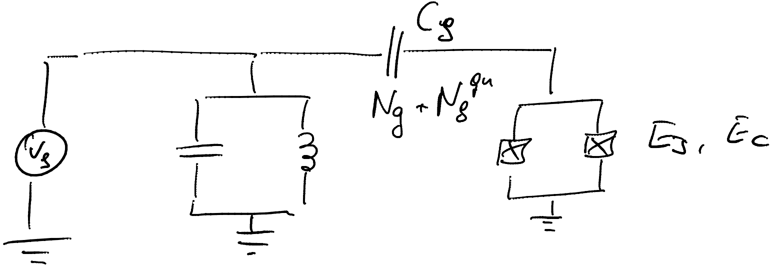
Lecture 7

Donnerstag, 27. Oktober 2011
17:33

Circuit QED <--> Jaynes Cummings Hamiltonian



N_g.. polarization charge on C_g

$$N_g = \frac{C_g V_g}{2e}$$

$$\hat{H} = \underbrace{\hbar\omega_1\left(a^\dagger a + \frac{1}{2}\right)}_{H.O.} + \underbrace{\frac{E_c}{2}\left(1 - 2\left(N_g + N_g^{qm}\right)\right)\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x}_{H\ C.P.B.}$$

charge degeneracy: $N_g = \frac{1}{2}$:

$$H_{CPB} = \frac{E_c}{2} N_g^{qm} \sigma_z - \frac{E_J}{2}\sigma_x$$

with $N_g^{qm} = \frac{\hat{Q}}{2e} = \frac{C_g}{2e}\hat{V} = \frac{C_g}{2e}\sqrt{\frac{\hbar\omega}{2C}}\left(\hat{a}^+ + \hat{a}\right)$

and change of basis: $\hat{\sigma}_z \rightarrow \hat{\sigma}_x$ and $\hat{\sigma}_x \rightarrow -\hat{\sigma}_z$

$$H_{CPB} = \frac{2E_c C_g}{2e}\sqrt{\frac{\hbar\omega}{2C}}\left(\hat{a}^+ + \hat{a}\right)\hat{\sigma}_x + \frac{E_J}{2}\hat{\sigma}_z$$

qubit raising and lowering operators: $\sigma_x = \hat{\sigma}^+ + \hat{\sigma}^-$

interaction term $\frac{2E_c C_g}{2e}\sqrt{\frac{\hbar\omega}{2C}}\left(\cancel{\hat{a}^+\hat{\sigma}^+} + \hat{a}^+\hat{\sigma}^- + \hat{a}\hat{\sigma}^+ + \cancel{\hat{a}\hat{\sigma}^-}\right)$

rotating wave approximation (RWA)

not energy conserving

$$\Rightarrow \hat{H} = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \hbar g\left(\hat{a}^\dagger\hat{\sigma}^- + \hat{a}\hat{\sigma}^+\right) + \frac{E_J}{2}\hat{\sigma}_z$$

with coupling constant $\hbar g = \frac{C_g}{C_\Sigma} e \sqrt{\frac{\hbar\omega}{2C}}$

$\frac{2g}{2\pi}$ ... vacuum Rabi frequency

Qubit drive:

apply time-dependent signal to CPB by changing N_g (classical coherent signal)

$$H = -\frac{E_C}{2}\left(1 - 2\left[N_g + \eta\cos\omega t\right]\right)\sigma_z - \frac{E_J}{2}\sigma_x$$

$$= -\frac{E_C}{2}(1 - 2N_g)\sigma_z + \underbrace{E_C\eta}_{\varepsilon}\cos\omega t\,\sigma_z - \frac{E_J}{2}\sigma_x$$

for $N_g = \frac{1}{2}$ + swap of basis:

$$\boxed{H = \frac{E_J}{2}\sigma_z + \varepsilon\cos\omega t\,\sigma_x}$$