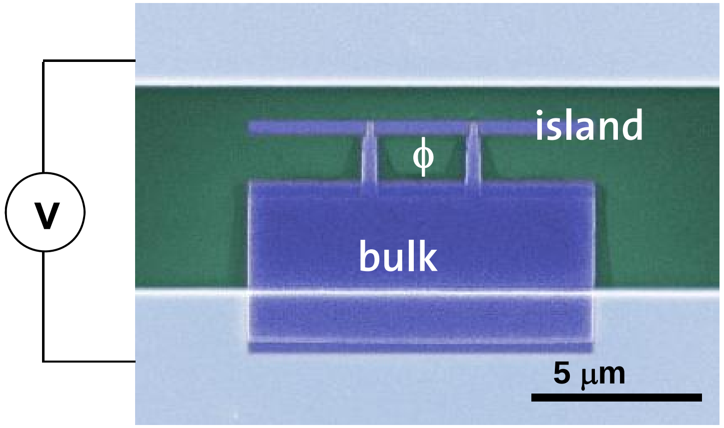
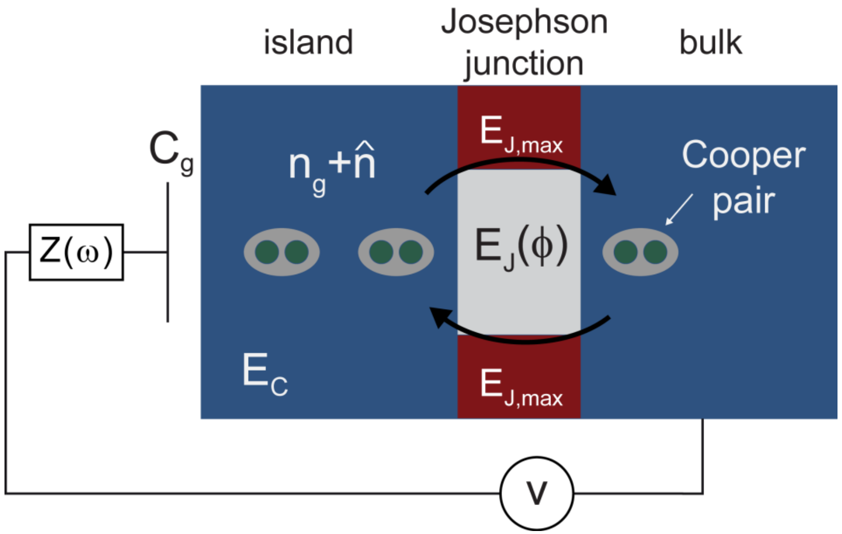
The Cooper Pair Box …

… a charge qubit.

Cooper Pair Box Qubit





- superconducting island connected via Josephson junctions to grounded reservoir (bulk)
- Cooper pairs can tunnel onto island
- relevant degree of freedom: number of Cooper pairs on island (N)
- polarization charge adjustable via voltage bias
- energy scales: charging energy E_C (energy to add another Cooper pair)
  Josephson energy E_J (coupling energy)

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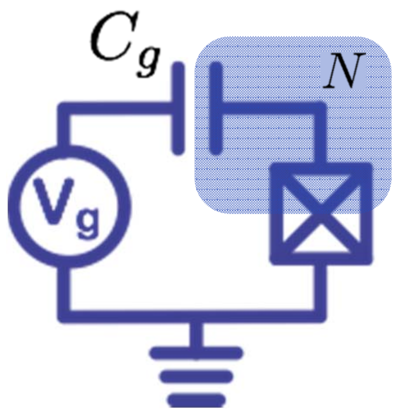
[Bouchiat, Vion, Joyez, Esteve, Devoret, *Physica Scripta* T76, 165 (1998).]

A Charge Qubit: The Cooper Pair Box

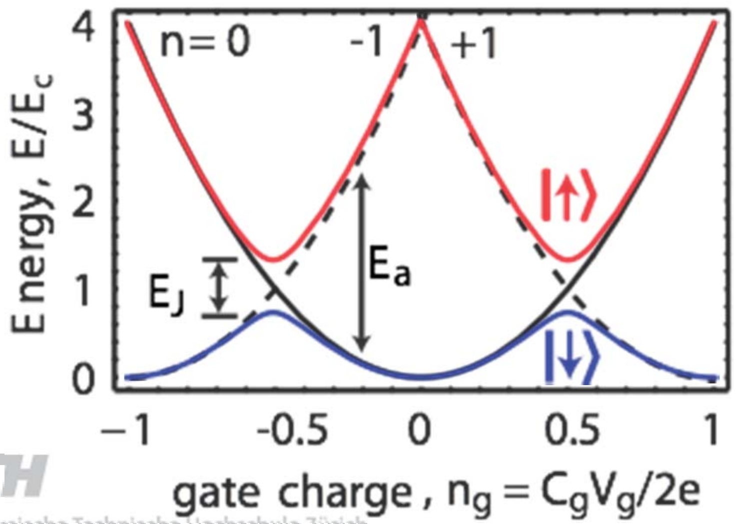


$$H_{el} = E_C N^2$$

$$H = E_C(N - N_g)^2 - E_J \cos\delta$$

$$[\delta, N] = i \quad \Rightarrow \quad e^{\pm i\delta}|n\rangle = |n \pm 1\rangle$$

$$H = \sum_N \left[E_C(N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2}\left(|N\rangle\langle N+1| + |N+1\rangle\langle N|\right) \right]$$



Charging energy: $E_C = \dfrac{(2e)^2}{2C_\Sigma}$

Gate charge: $N_g = \dfrac{C_g V_g}{2e}$

Josephson energy: $E_J = \dfrac{I_0 \Phi_0}{2\pi}$

Cooper pair box Hamiltonian:

Hamiltonian: $$\hat{H} = \underbrace{E_c(\hat{N} - N_g)^2}_{\text{electrostatic}} - \underbrace{E_J \cos\hat{\delta}}_{\text{magnetic energy}}$$

$$E_J \cos\hat{\delta} = \frac{E_J}{2}\left(e^{i\hat{\delta}} + e^{-i\hat{\delta}}\right)$$

gate charge $N_g = \frac{C_g V_g}{2e}$

charging energy $$E_c = \frac{(2e)^2}{2C_\Sigma}$$

Josephson coupling Energy $$E_J = \frac{\phi I_c}{2\pi}$$

Hamiltonian in charge representation:

$$\hat{H} = E_c(N-N_g)^2 |N\rangle\langle N| - \frac{E_J}{2}\sum_N \left(|N+1\rangle\langle N| + |N\rangle\langle N+1|\right)$$

easy to diagonalize numerically

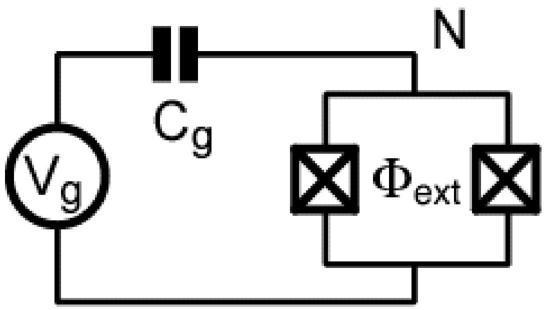
$$\hat{H} = \begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & E_c(-1-N_g)^2 & -E_J/2 & 0 & \cdots \\ \cdots & -E_J/2 & E_c(0-N_g)^2 & -E_J/2 & \cdots \\ \cdots & 0 & -E_J/2 & E_c(1-N_g)^2 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

relation between **phase** and **number basis**:

$$|\delta\rangle = \frac{1}{\sqrt{2\pi}} \sum_N e^{i\hat{N}\delta} |N\rangle \quad \text{with} \quad e^{i\hat{\delta}}|N\rangle = |N+1\rangle$$

Tuning the Josephson Energy

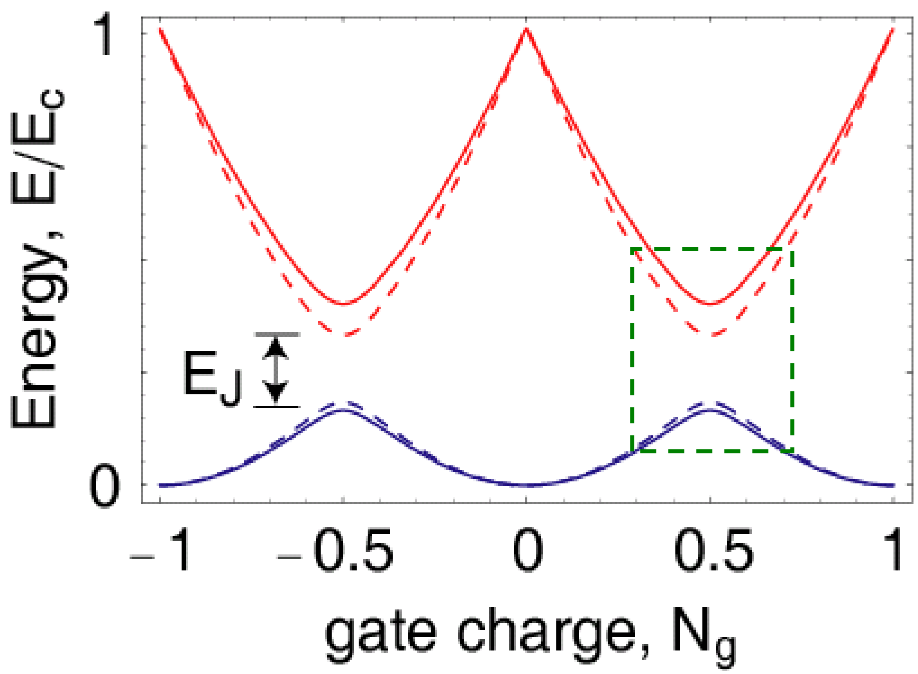
split Cooper pair box in perpendicular field



$$H = E_C\,(N - N_g)^2 - E_{J,\max}\cos\left(\pi\,\frac{\Phi_{\rm ext}}{\Phi_0}\right)\cos\hat{\delta}$$

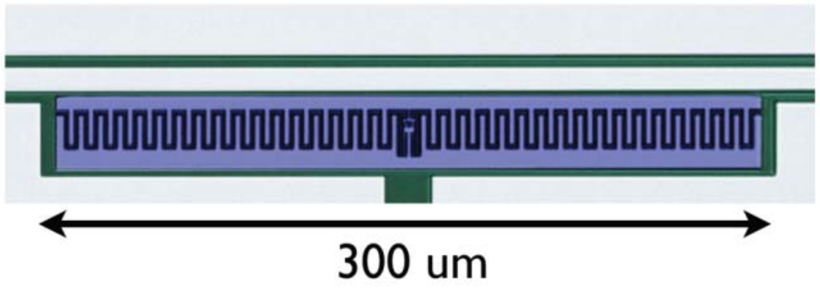
SQUID modulation of Josephson energy

$$E_J = E_{J,\max}\cos\left(\pi\,\frac{\Phi_{\rm ext}}{\Phi_0}\right)$$

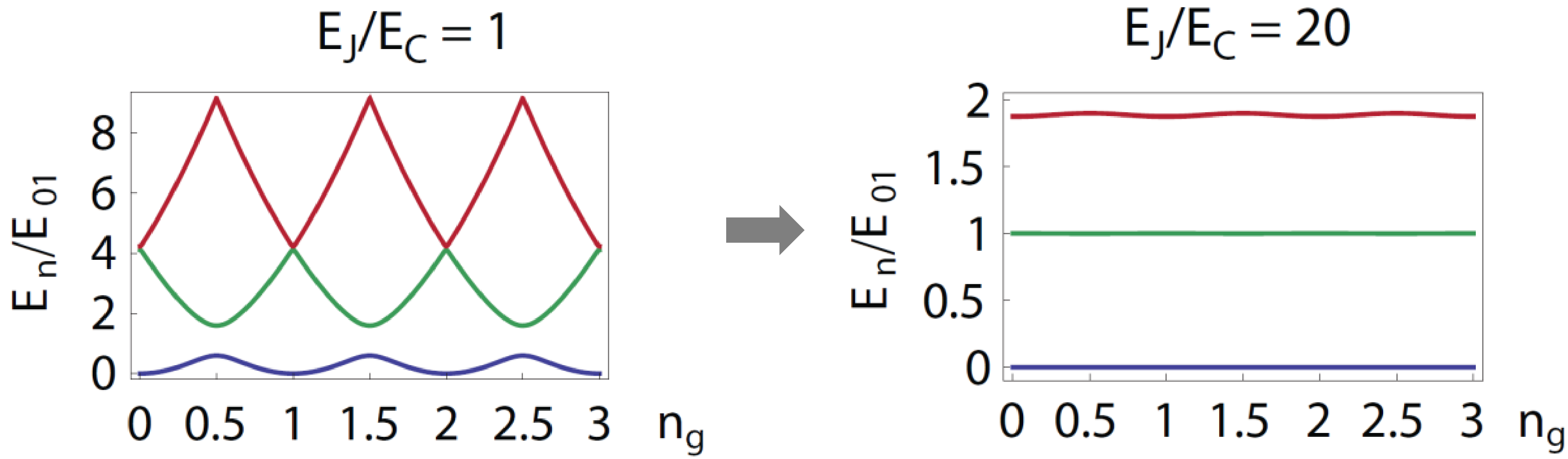


[J. Clarke, *Proc. IEEE* **77**, 1208 (1989)]

Transmon qubit – a charge noise resilient qubit



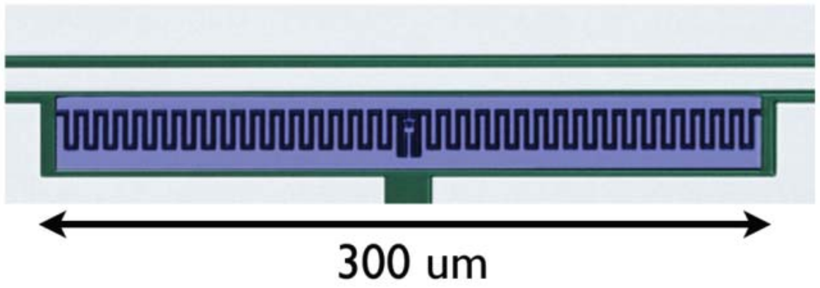
- shunting capacitor reduces E_c
- Increased E_J / E_c ratio flattens energy bands
- Less sensitivity to charge noise

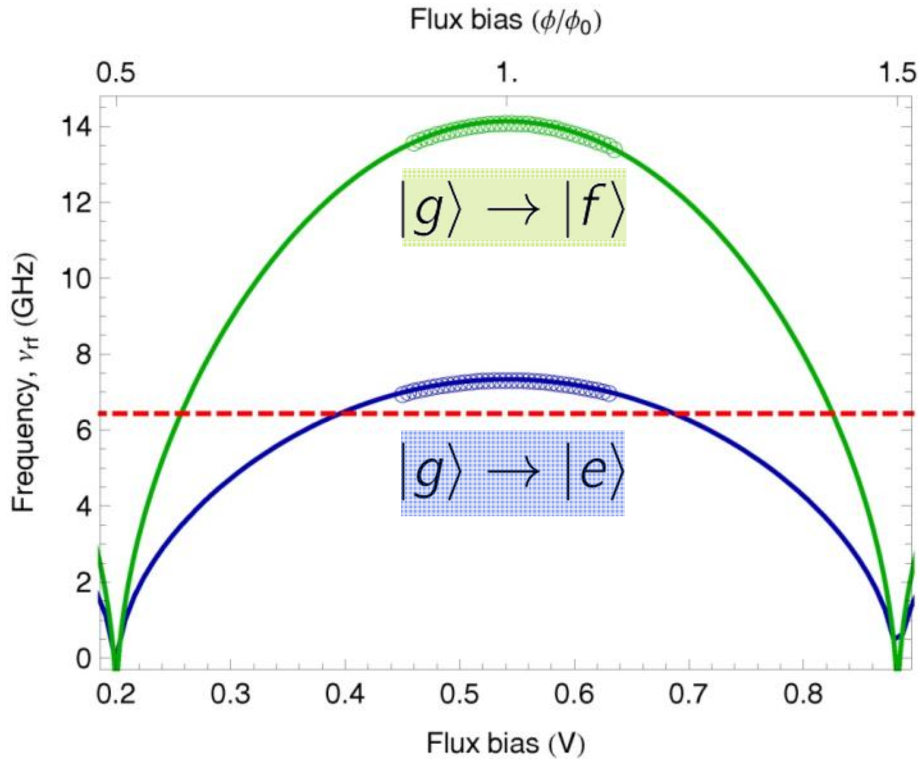


$$H = E_C(\hat{N} - N_g)^2 - E_J \cos\left(\pi \frac{\Phi_{ext}}{\Phi_0}\right) \cos\hat{\delta}$$

[Koch *et al. PRA 76, 042319 (2007)*; Schreier *et al. PRB (2008)]*

Transmon qubit – tuning of transition frequency





transition frequency can be adjusted by external flux bias

$$H = E_C(\hat{N} - N_g)^2 - E_J \cos\left(\pi \frac{\Phi_{ext}}{\Phi_0}\right) \cos\hat{\delta}$$

[Koch *et al. PRA 76, 042319 (2007);* Schreier *et al. PRB (2008)]*

Two State Approximation

$$\mathbf{H}_{\mathrm{CPB}} = \mathbf{H}_{\mathrm{el}} + \mathbf{H}_{\mathrm{J}} = E_C(N - N_g)^2 - E_J \cos\delta$$

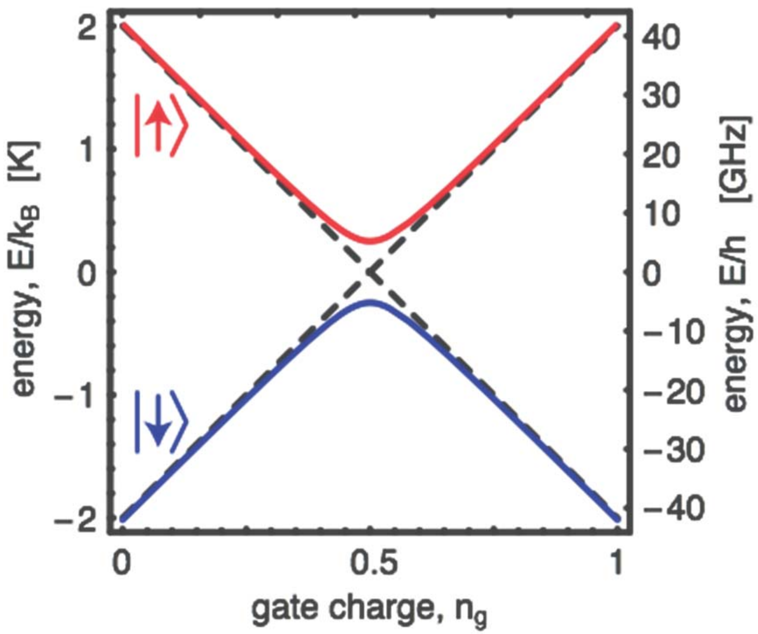
$$\mathbf{H}_{\mathrm{CPB}} = \sum_N \left[E_C(N - N_g)^2 \left|N\right\rangle \left\langle N\right| - \frac{E_{\mathrm{J}}}{2} \left(\left|N\right\rangle \left\langle N+1\right| + \left|N+1\right\rangle \left\langle N\right| \right) \right]$$

Restricting to a two-charge Hilbert space:

$$N = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1-\sigma_z}{2}$$

$$\cos\delta = \frac{\sigma_x}{2}$$

$$\begin{aligned} \mathbf{H}_{\mathrm{CPB}} &= -\frac{E_C}{2}(1 - 2N_g)\sigma_z - \frac{E_J}{2}\sigma_x \\ &= -\frac{1}{2}(E_{\mathrm{el}}\sigma_z + E_J\sigma_x) \end{aligned}$$



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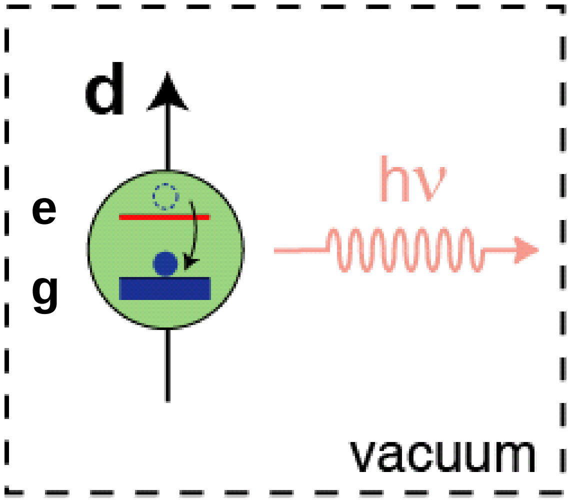
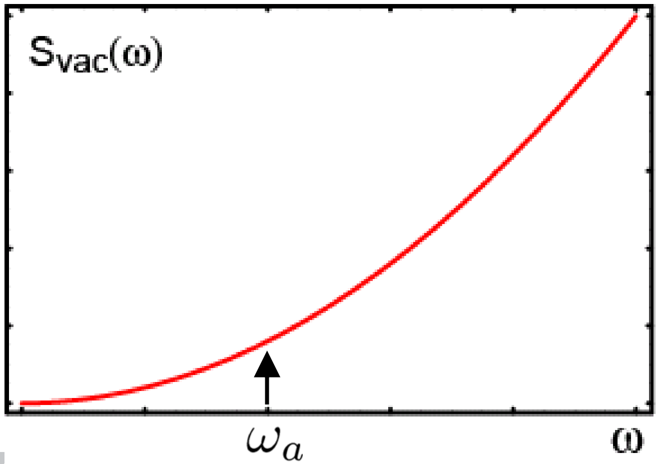
Shnirman *et al.*, *Phys. Rev. Lett.* **79**, 2371 (1997)

Cavity QED with Electronic Circuits

Spontaneous Emission



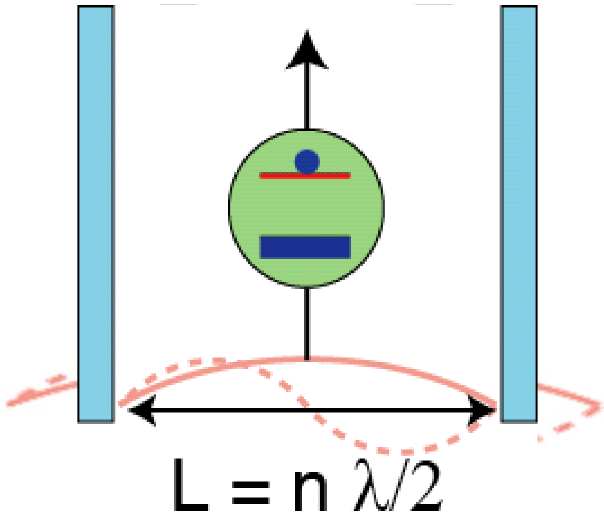


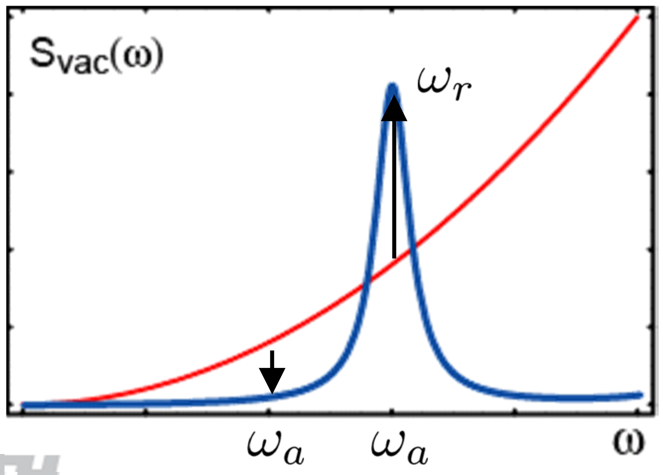
decay by dipole interaction with vacuum fluctuations

$$\gamma \sim \Omega^2 S_{\mathrm{vac}}(\omega_a)$$

Review: D. Kleppner and S. Haroche *Physics Today* **Jan.**, 24 (1989)

Suppression and Enhancement of Emission





engineering the vacuum

$$\gamma \sim \Omega^2 S_{\mathrm{vac}}(\omega_a)$$

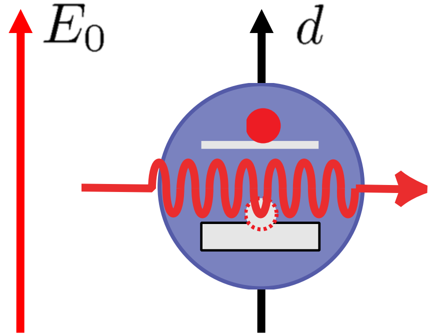
suppression of emission

enhancement of emission

Review: D. Kleppner and S. Haroche *Physics Today* **Jan.**, 24 (1989)

Free Atom

weak interaction with single photons:

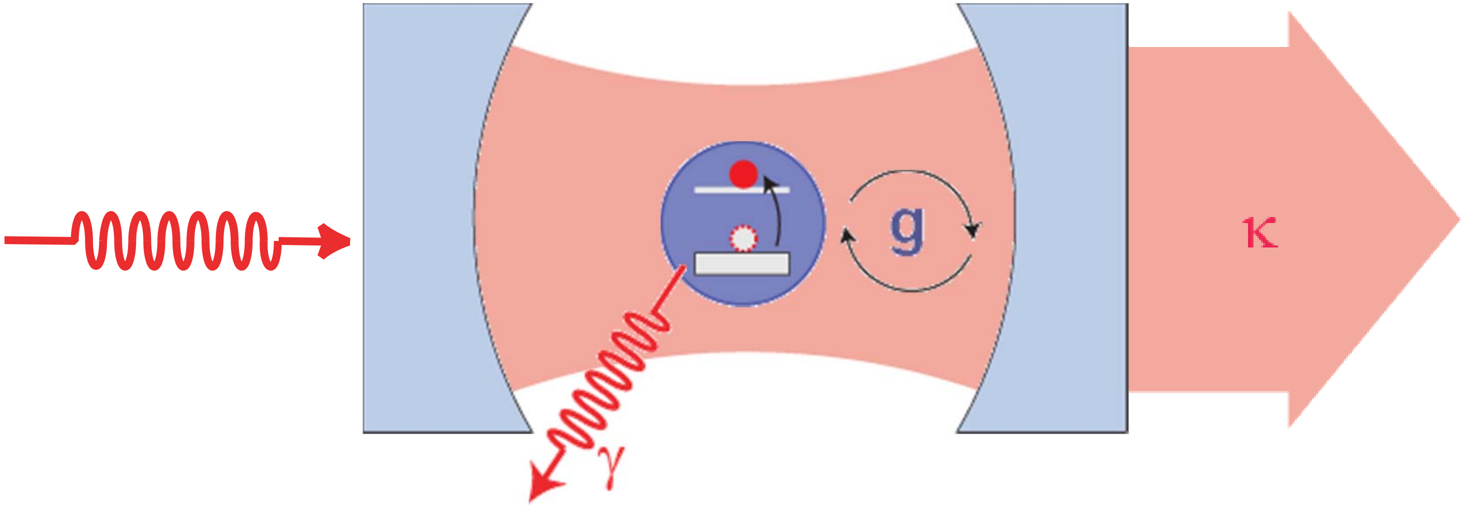


- dipole moment *d* (usually small in atoms ~ ea_o)
- single photon fields E_o (small in 3D)
- photon/atom interaction $\hbar g \sim dE_0$ (usually small)

[D. Walls, G. Milburn, Quantum Optics (Springer-Verlag, Berlin, 1994)]

Cavity Quantum Electrodynamics

interaction of atom and photon in a cavity



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit: $g = dE_0/\hbar > \gamma,\ \kappa$

[D. Walls, G. Milburn, Quantum Optics (Springer-Verlag, Berlin, 1994)]

Dressed States Energy Level Diagram

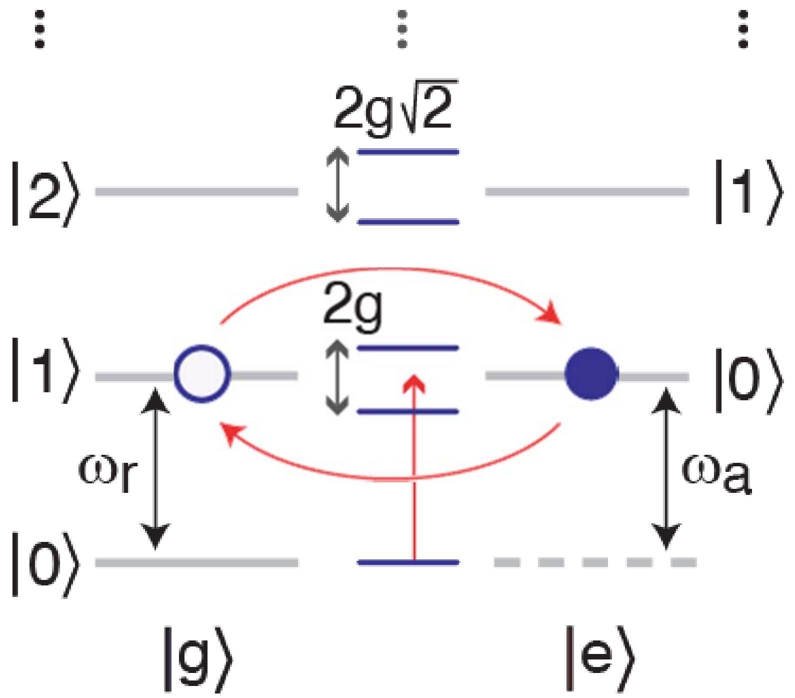
$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

$$g = \frac{dE_0}{\hbar} > \gamma,\, \kappa$$



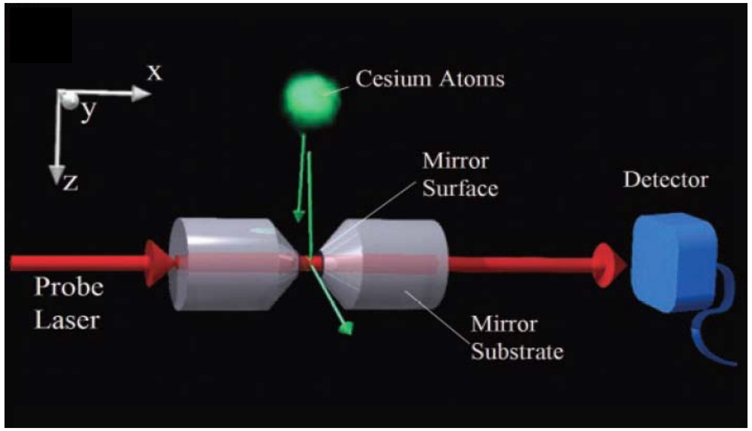
Jaynes-Cummings Ladder

Atomic cavity quantum electrodynamics reviews:

J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)

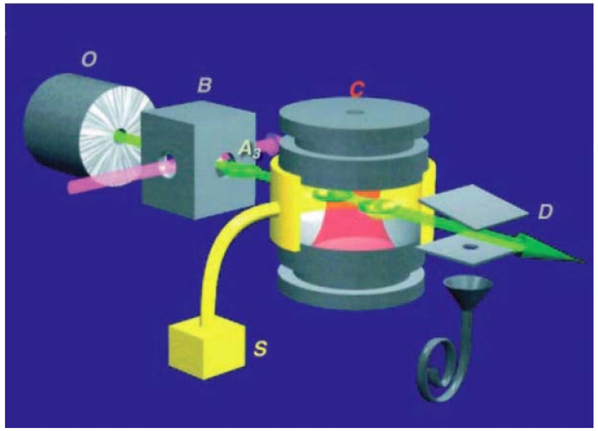
S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

Cavity Quantum Electrodynamics (QED)



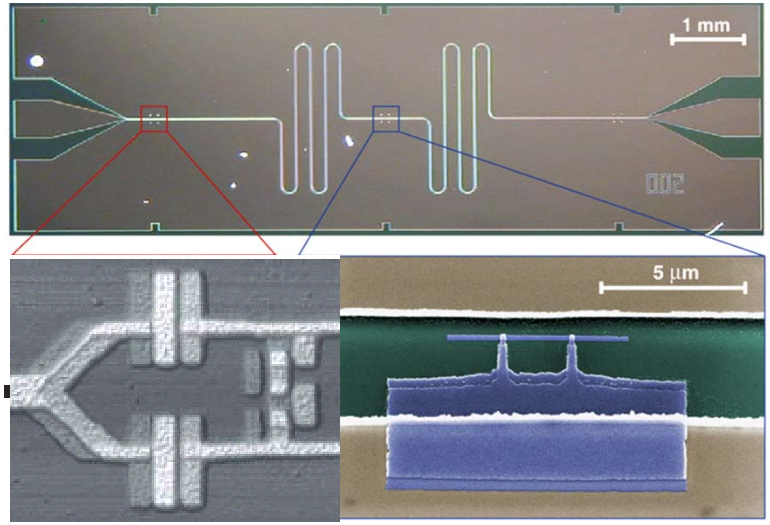
alkali atoms

MPQ, Caltech, ...



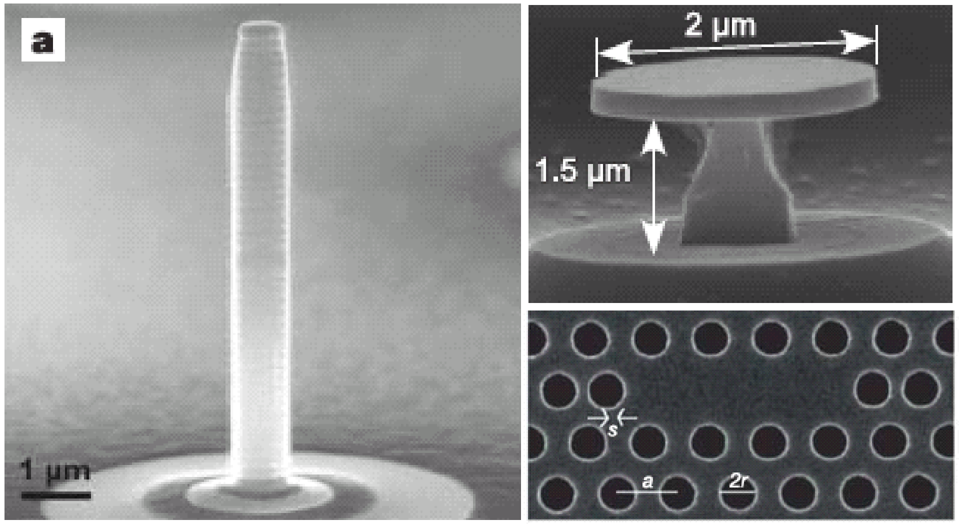
Rydberg atoms

ENS, MPQ, ...



superconductor circuits

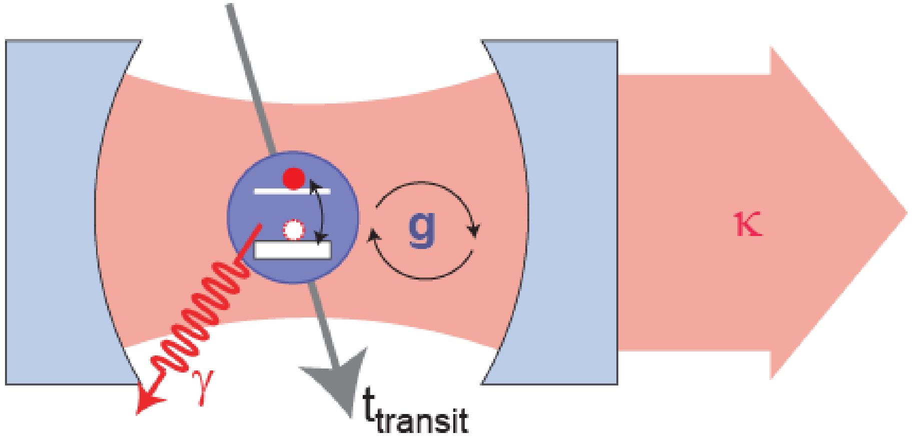
Yale, Delft, NTT, ETHZ, NIST, …



semiconductor quantum dots

Wurzburg, ETHZ, Stanford …

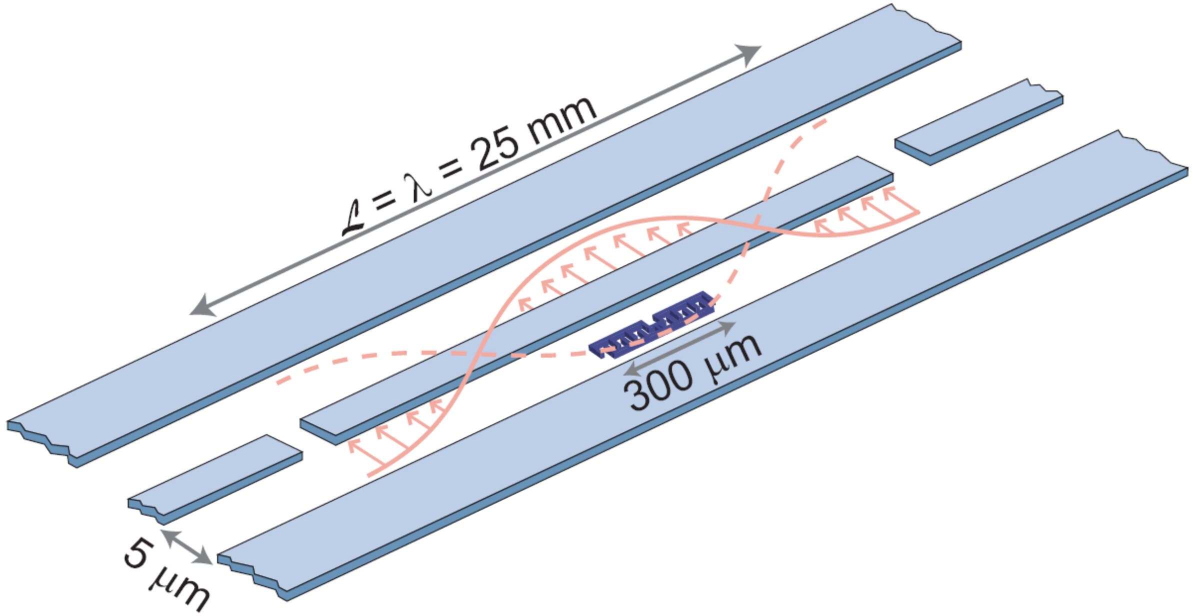
Cavity QED with Superconducting Circuits



coherent quantum mechanics with individual photons and qubits.

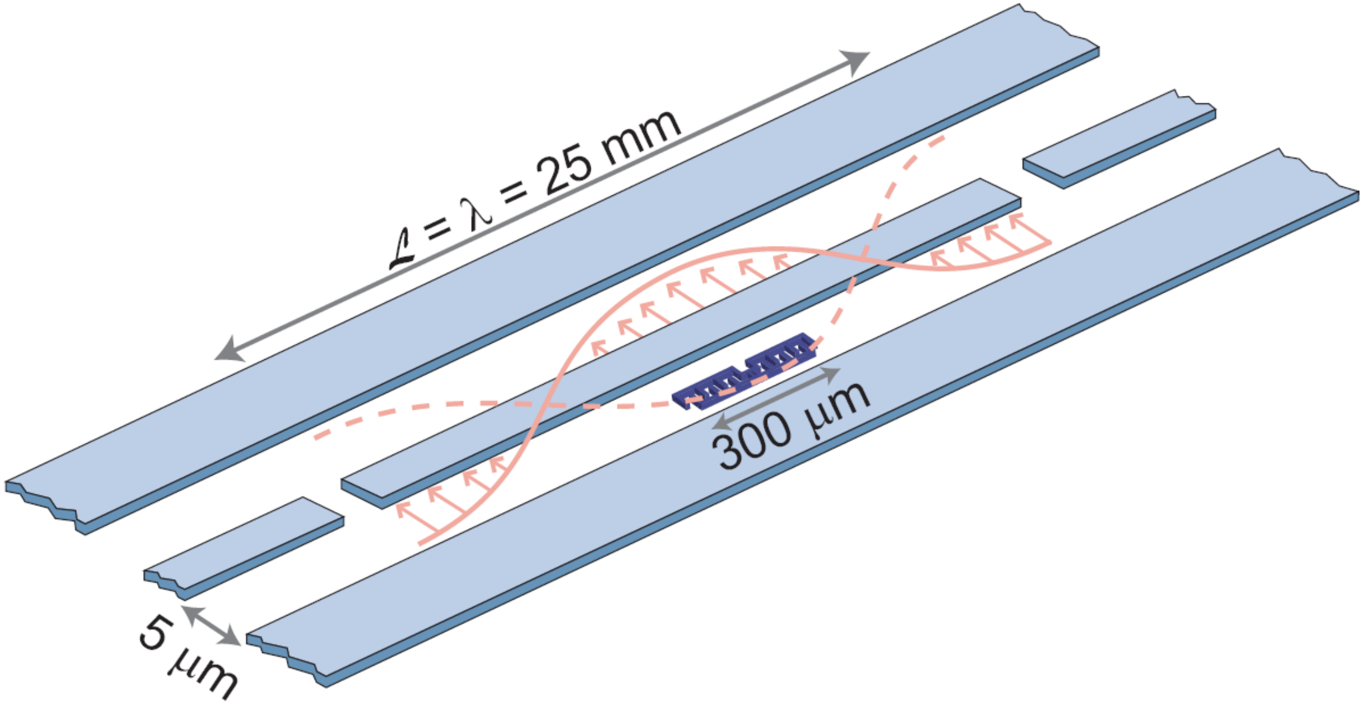
with superconducting qubits:

circuit quantum electrodynamics



[A. Blais, *et al.* , *PRA* **69,** 062320 (2004)
A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)]

Circuit Quantum Electrodynamics

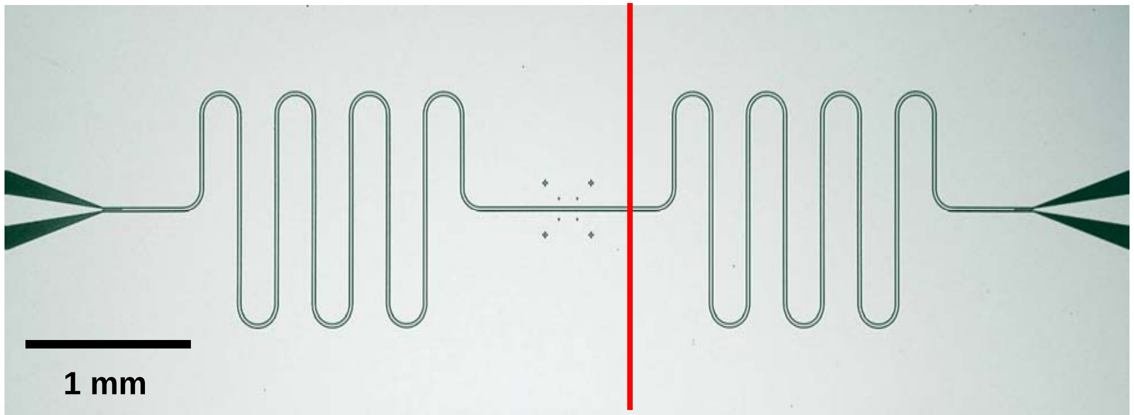


elements

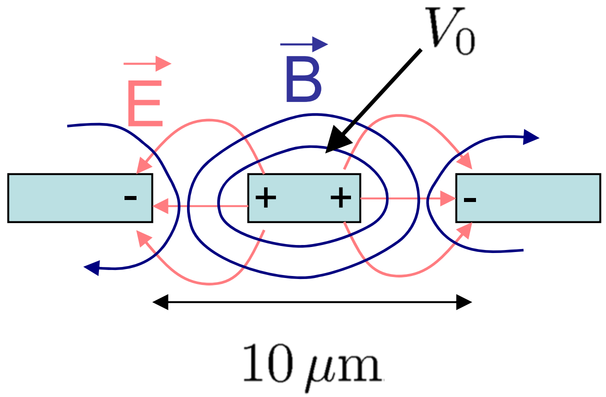
- the cavity: a superconducting 1D transmission line resonator with large vacuum field E_0 and long photon life time $1/\kappa$
- the artificial atom: a Cooper pair box (Transmon) with large dipole moment d and long coherence time $1/\gamma$

[A. Blais *et al.*, *PRA* **69,** 062320 (2004)]

Vacuum Field in 1D Cavity



cross-section of transm. line (TEM mode):



voltage across resonator in vacuum state ($n = 0$)

$$V_{0,\mathrm{rms}} = \sqrt{\frac{\hbar\omega_r}{2C}} \approx 1\,\mu\mathrm{V}$$

harmonic oscillator

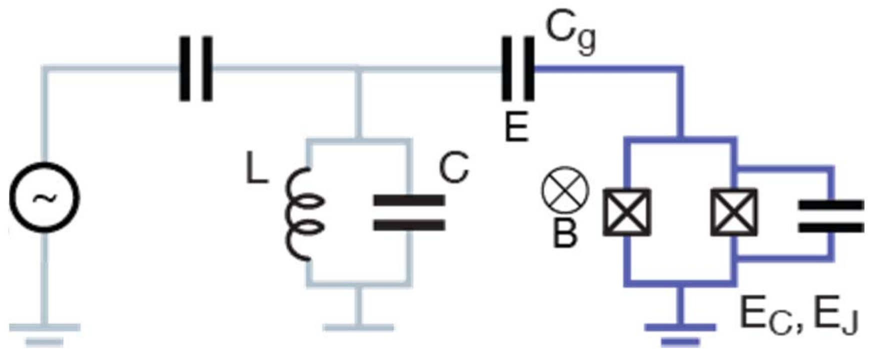
$$H_r = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right)$$

$$E_0 = \frac{V_{0,\mathrm{rms}}}{b} \approx 0.2\,\mathrm{V/m}$$

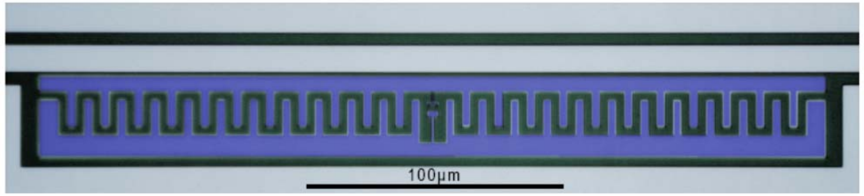
10^3 larger than in 3D cavity

for $\omega_r/2\pi \approx 6\,\mathrm{GHz}$ ($C \sim 1\,\mathrm{pF}$), $b \approx 5\,\mu\mathrm{m}$

Qubit/Photon Coupling in a Circuit



qubit coupled to resonator



coupling strength:

$$\hbar g = eV_{0,\mathrm{rms}} \frac{C_g}{C_\Sigma}$$

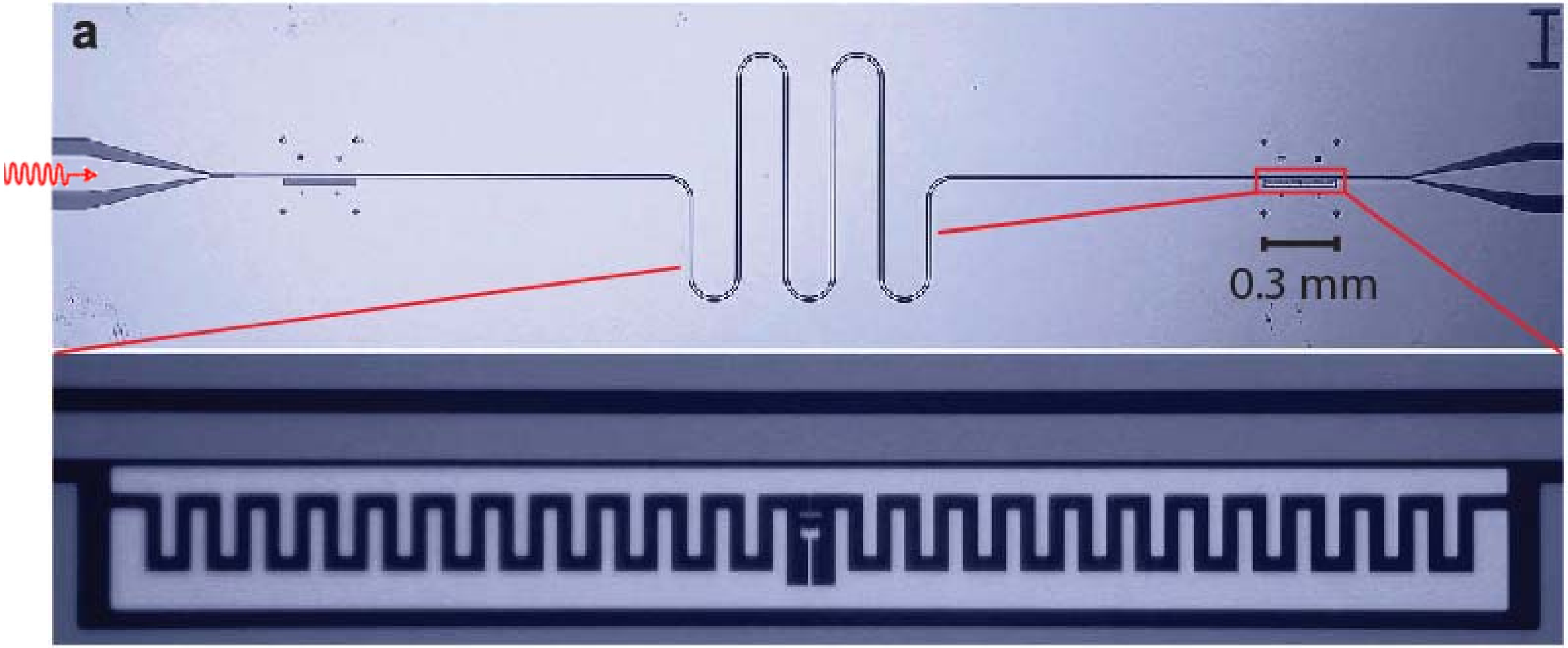
$$\Longrightarrow \nu_{\mathrm{vac}} = \frac{g}{\pi} \approx 1 \ldots 300\,\mathrm{MHz}$$

$g \gg [\kappa, \gamma]$ possible!

large effective dipole moment

$$d = \frac{\hbar g}{E_0} \sim 10^2 \ldots 10^4\, e a_0$$

Circuit QED with One Photon



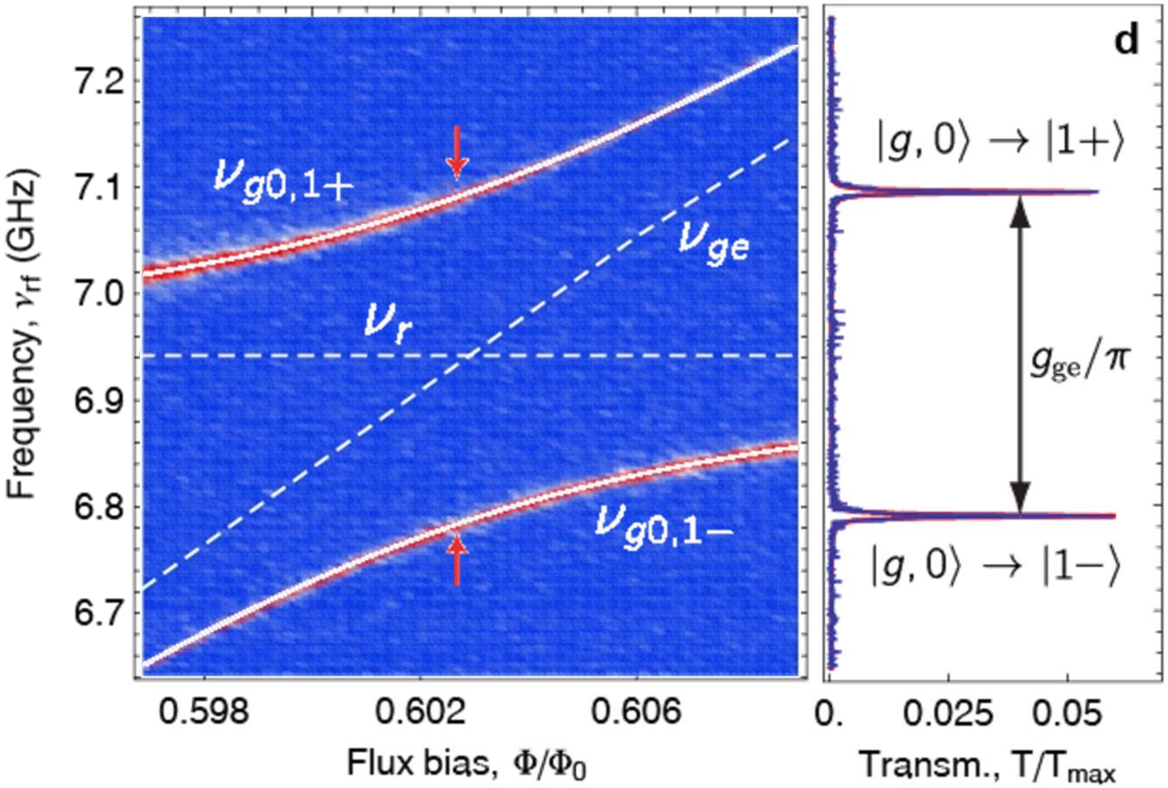
superconducting cavity QED circuit

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

A. Wallraff, …, R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004)

Resonant Vacuum Rabi Mode Splitting …

… with one photon ($n = 1$):

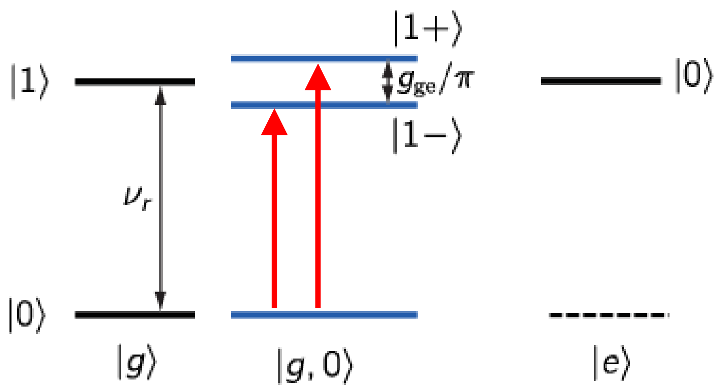


very strong coupling:

$$g_{ge}/\pi = 308\,\mathrm{MHz}$$

$$\kappa, \gamma < 1\ \mathrm{MHz}$$

$$g_{ge} \gg \kappa, \gamma$$



forming a 'molecule' of a qubit and a photon

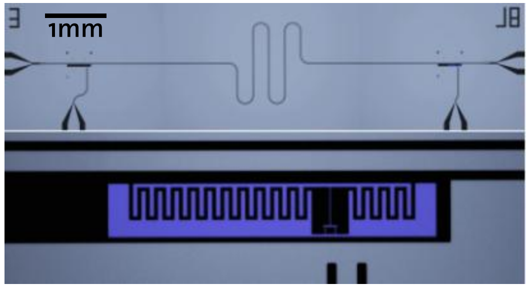
$$|1\pm\rangle = \left(|g,1\rangle \pm |e,0\rangle\right)/\sqrt{2}$$

first demonstration: A. Wallraff, … and R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004)

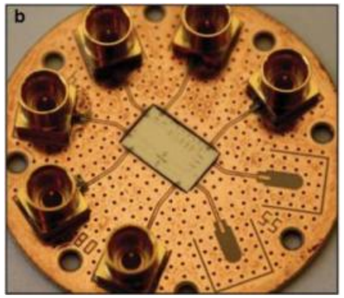
this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

Experimental Setup

Resonator+ transmon chip:



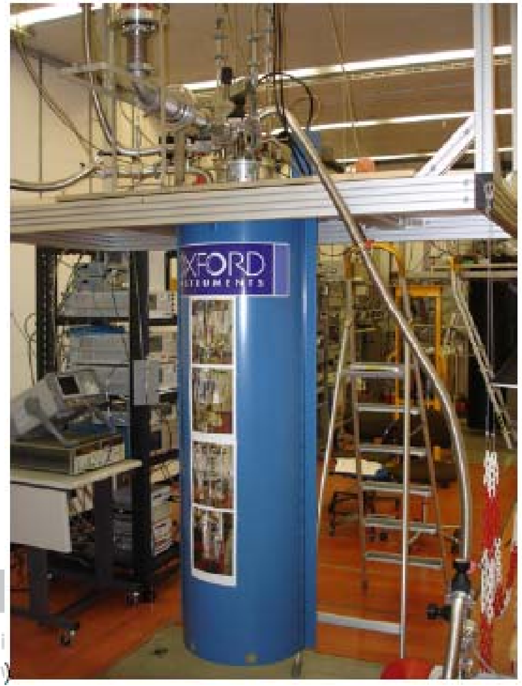
Sample holder:



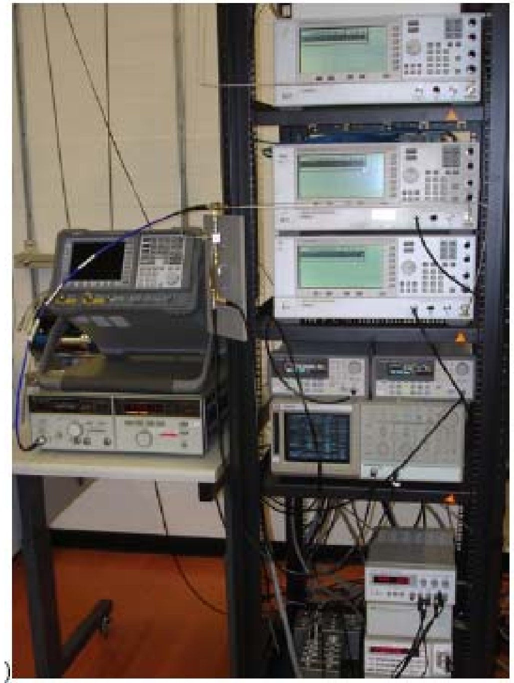
Box with B-field coils:

Cold stage 20 mK:

Dilution cryostat:



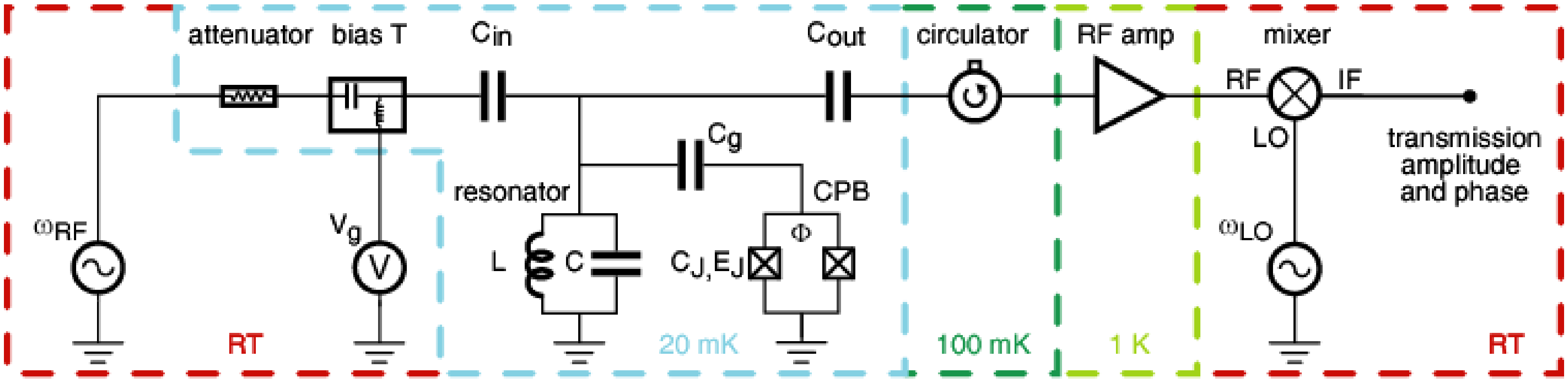
Microwave electronics:



How to Measure Single Microwave Photons

- average power to be detected

$$\rightarrow \langle n=1\rangle \hbar\omega_r\kappa/2 \approx P_{RF} = -140\,\mathrm{dBm} = 10^{-17}\,\mathrm{W}$$



- efficient with cryogenic low noise HEMT amplifier ($T_N = 6\,\mathrm{K}$)
- prevent leakage of thermal photons (cold attenuators and circulators)