Lecture 6- Cooper Pair Box

Samstag, 22. Oktober 2011
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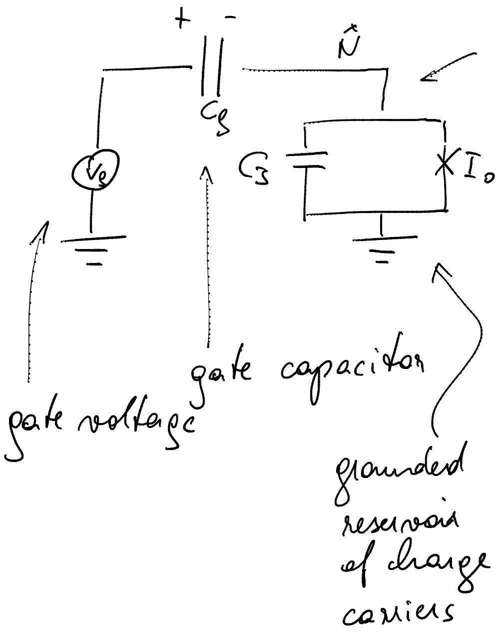


island on which charges are localized

discrete variable

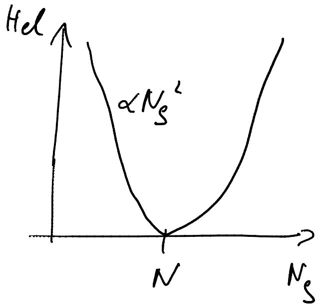
$$\boxed{\hat{N} = \frac{\hat{Q}}{2e}}$$ number Cooper pairs on island (with respect to charge neutrality)

$$\boxed{N_g = \frac{C_g V_g}{2e}}$$ polarization charge on gate capacitor

continuous variable

Hamiltonian: $H = \dfrac{Q^2}{2C} + \dfrac{\Phi^2}{2L}$ = electrostatic + magnetic energy

electrostatic energy : $H_{el} = \dfrac{Q^2}{2C} = \dfrac{(2e)^2 (N - N_g)^2}{2 C_\Sigma}$



with $C_\Sigma = C_J + C_g + \ldots$ (stray capacitances)

and $E_c = \dfrac{(2e)^2}{2C_\Sigma}$ ... charging energy

magnetic energy: $H_{mag} = - E_J \cos\delta = \dfrac{\Phi_0 I_0}{2\pi} \underbrace{\cos\delta}_{\approx 1 - \frac{\delta^2}{2}}$

$$\approx - \frac{\Phi_0 I_0}{2\pi} \left(1 - \frac{1}{2} \left(\frac{\Phi}{\Phi_0} 2\pi \right)^2 + \ldots \right)$$

$$\approx \frac{1}{2} \frac{\Phi^2}{L_{J0}}$$ (standard expression for mag. energy)

Hamilton operator:

$$\hat{H} = E_C(\hat{N}-N_g)^2 - E_J \cos\hat{\delta} \qquad [\hat{\delta},\hat{N}] = i$$

$\hat{\delta}, \hat{N}$ conjugate variables

$$\cos\hat{\delta} = \frac{1}{2}\left(e^{i\hat{\delta}} + e^{-i\hat{\delta}}\right)$$

$|N\rangle$ .. number eigenstates

$|\delta\rangle$ .. phase eigenstates

$$|\delta\rangle = \frac{1}{\sqrt{2\pi}} \sum_N e^{-iN\delta} |N\rangle \quad \text{(basis trans.)}$$

$$|N\rangle = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} d\delta\, e^{+iN\delta} |\delta\rangle$$

properties of $\hat{\delta}$ (phase) and $\hat{N}$ (number) operator:

$$[\hat{\delta},\hat{N}] = i \quad \Rightarrow \quad e^{\pm i\hat{\delta}} |N\rangle = |N \pm 1\rangle$$

$$\left[\begin{aligned} e^{\pm i\hat{\delta}} |N\rangle &= \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} d\delta\, e^{iN\delta} e^{\pm i\delta} |\delta\rangle = \\ &= \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} d\delta\, e^{i(N\pm 1)\delta} |\delta\rangle \\ &= |N \pm 1\rangle \end{aligned}\right]$$

number op.: $\hat{N}|N\rangle = N|N\rangle$

completeness: $\sum_N |N\rangle\langle N| = \mathbb{1}$ orthogonality: $\langle M|N\rangle = \delta_{M,N}$

Hamilton operator in charge basis:

$$\hat{H} = \underbrace{\sum_N E_C(\hat{N}-N_g)^2 |N\rangle\langle N|}_{\text{energy of charges on island}} - \underbrace{\frac{E_J}{2}\left(|N\rangle\langle N+1| + |N+1\rangle\langle N|\right)}_{\text{energy to add / remove charges from island}}$$

energy eigenstates:

solve time-independent Schrödingerequation to find $|\psi_n\rangle$

$$\hat{H}|\Psi_n\rangle = E_n|\Psi_n\rangle$$

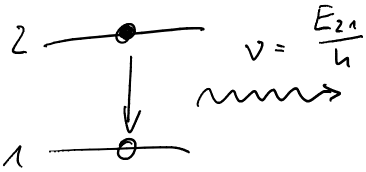
either in charge basis (see above) or in phase basis $\hat{\delta}$

$$\hat{N} = \frac{\hat{Q}}{2e} = -i\hbar\frac{1}{2e}\frac{\partial}{\partial\Phi} = -i\frac{\partial}{\partial\delta}$$

$$\hat{H} = E_C\left(-i\frac{\partial}{\partial\delta} - N_g\right)^2 - E_J\cos\delta$$

exact solutions for $H\Psi_n(\delta) = E_n\Psi_n(\delta)$ are Mathieu function

Cavity / Circuit QED



decay rate $\propto \underbrace{d_{21} E_0}_{\text{coupling strength to vacuum field } E_0} \cdot \underbrace{\rho(\omega)}_{\text{mode density}}$

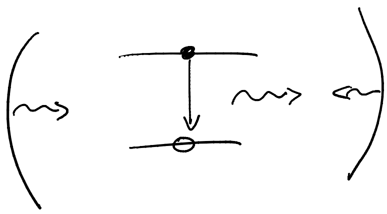
coupling strength to vacuum field E_0

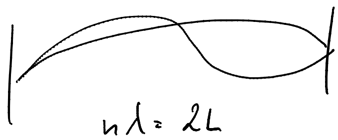
(d_{21} transition dipole)

mode density

free space:

cavity:





only specific wavelengths are allowed

$$\omega_c = \frac{2\pi c}{\lambda} = \frac{n\pi c}{L}$$

vacuum field E_0:

EM-energy in vacuum state: $\frac{1}{2}\hbar\omega_c$

($\frac{1}{2}$ magnetic, $\frac{1}{2}$ electric)

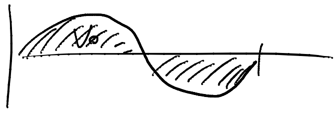
from E-dyn: $W = \frac{\varepsilon_0}{2}\int_V |E|^2 dV \overset{\text{Q.M.}}{\hat{=}} \frac{\varepsilon_0}{2}\int \langle 0 | |\hat{E}|^2 | 0 \rangle \, dV$

$$\hat{E}(x) = \mathcal{E}_0 f(x) \cdot (\hat{a}^\dagger + \hat{a})$$

$$|\hat{E}|^2 = \mathcal{E}_0^2 f(x)^2 \left(\hat{a}^\dagger\hat{a}^\dagger + \hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a} + \hat{a}\hat{a}\right)$$

$$\langle 0 | E(r)^2 | 0 \rangle = \mathcal{E}_0^2 f(r)^2 \quad \Rightarrow \quad \frac{1}{4} \cdot \hbar\omega_c = \frac{\varepsilon_0}{2} \cdot \mathcal{E}_0^2 \cdot V_0$$

mode volume: $V_0 = \int f(r)^2 \, d^3 r$



$$\boxed{\mathcal{E}_0 = \sqrt{\frac{\hbar\omega_c}{2\varepsilon_0 V_0}}}$$

$\Rightarrow$ small mode volume $\Rightarrow$ high electric fields

$\Rightarrow$ large coupling

1D cavity: $\mathcal{E}_0 \sim 0.2\,\mathrm{V/m}$