

Relevance of Single Particle Quantum Phenomena for Quantum Information Science

- QIS:
- requires control over single quantum systems
 - requires possibility to construct more complex systems (bottom up approach) from single particles

⇒ new insights into the nature of quantum physics driven by

- curiosity in new regimes of physics
- new experimental techniques and methods available



compare with other developments e.g. triggered by the advent of low temperatures.

- superconductivity (low T_c , high T_c)
- Hall effect (quantum & fractional)
- superfluidity

Developments in the field of quantum physics in the second half of 20th century and their importance for quantum information science.

Collective quantum phenomena

- consider ensembles of quantum systems
- no control over or access to quantum state of individual particles

- e.g.: atoms: - spectroscopy in gas phase
- solid state: - electronic band structure
- superconductivity
- light: - LASERS

VS.

Single particle quantum phenomena

- isolate individual quantum systems
- control over quantum state and read-out of quantum state of individual particles

- e.g.: atoms: - ion traps
- atom dipole traps
- photons: - single photon sources & detectors
- PDC, Cavities QED
- charges: - single electron transistors
& spins - quantum dots
- spins: - superconducting qubits

What would you think were important developments in Q.M. that have allowed the field of quantum information science to develop?

State of the art of Quantum Computing

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• many physical systems have been investigated:

- NMR
- ion traps
- charges & fluxes in supercond.
- charges & spins in semicond.
- neutral atoms
- NV centers

• level reached:

- factored $15 = 3 \times 5$ (NMR)
- realized quantum byte (ions)
- several 10s to 100s of manipulations possible (ions, supercond.)
- algorithms realized (NMR, ions, s.c. qubits)

• the challenge:

- realize larger systems

Computer Science Perspective

1936 Beginning of modern computer science

- Turing provides abstract definition of a programmable computer
- Universal Turing Machine: provides full description of any classical algorithmic information processing machine
- Church-Turing-Thesis (strong version): Any algorithmic process that can be executed on any hardware can be simulated efficiently on a Turing Machine.

↳ basis for theory of computer science

- John von Neumann defines components necessary for realizing a computer

1947 - Bardeen, Brattain, Shockley develop transistor at Bell Labs (USA)

Show slides on transistor and Moore's law!

⇒ Beginning of efficient and low cost realization of computers in electronic circuits

1947 - Now : Development of Computer hardware follows

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Moore's Law

Estimate how much longer Moore's law can continue?

What is the role of quantum mechanics?

"Doubling of number of transistors on a processor every two years at constant cost"

→ show slide.

⇒ amazing success of technology!

- BUT how much longer can Moore's Law continue to be valid?
- What are the consequences of continuing miniaturization?
- What to do in post Moore's Law era?

$$2 \log_2 650$$

= 18.5 years

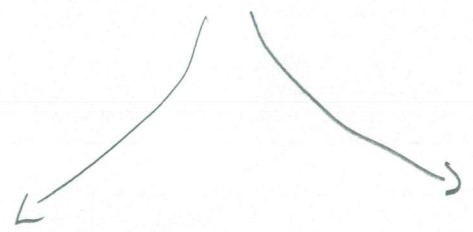
→ linear

$$2 \log_2 (650^2)$$

⇒ 37 years

Quantum Computation

- a **new paradigm** of computing
- uses theory of quantum mechanics for performing computations
- has **speed (efficiency) advantage** that cannot be overcome by any conceived classical computing scheme
- quantum computers can be simulated on classical computers **BUT NOT EFFICIENTLY** (c.f. Strong Church-Turing Thesis)



efficient:

Computer running in time polynomial in the size of the problem

inefficient:

computer running in time super-polynomial (typically exponential) in the size of the problem

What could efficiently mean in the context of computation?

Important Developments in the Theory of Quantum Computation

• 1985 Deutsch :

How many steps are required classically to solve Deutsch's problem?

- starts search for device to simulate efficiently any arbitrary physical (quantum) system
- seeks a challenge for strong Church-Turing-Thesis
- proposes to use device that is quantum mechanical
- presents first example algorithm now known as the Deutsch Algorithm

• 1994 Shor :

- develops algorithm that efficiently finds prime factors
- no efficient classical algorithm exists (no proof though)

• 1995 Grover :

- finds more efficient quantum algorithm to search in unstructured data bases

• 1982 Feynman :

- proposes to efficiently simulate physical quantum systems on computers based on the principles of quantum mechanics

Other Algorithms?

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- finding quantum algorithms is difficult
 - adverse to intuition based on classical world
- quantum algorithms need to be better than classical ones
 - Note: all classical algorithms can be run on a quantum computer (universality)
- it is not fully understood what makes a quantum computer more powerful than a classical one
 - superposition?
 - entanglement?
 - ↳ big challenge

Why could it be difficult to find good quantum algorithms?

Topic: Basic Elements of a Quantum Computer

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Key Questions:

Which components (or features) does a generic quantum computer have?

Example: Memory to store information.

How are those different from a classical computer?

Your Background:

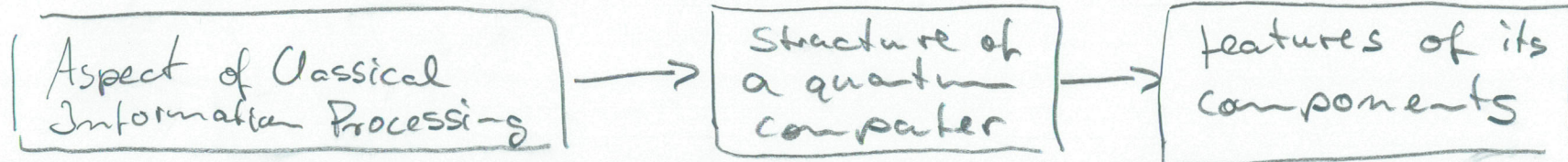
- General knowledge of how computers work
- Knowledge of quantum mechanics

COMBINE

Goal:

- Understand basic structure and operation of a quantum computer.
- Make use of this 'hardware independent' knowledge about quantum computers to evaluate different physical realizations

Outline:



Classical Information Processing

- Carrier of information in binary representation

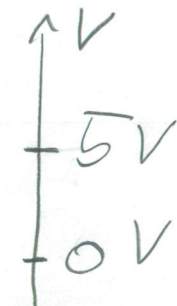
BIT

possible values

1
0

physical representation

example:



voltage level in a circuit

- modification of information in BIT by operating with a physical process on the BIT

- any logical operation on bits can be decomposed into single and two-bit operations

- Why is this useful?
- You may want to think about why this is possible at all!
- Same is true for quantum computers.

How is information physically modified in physical realizations of bits? Give examples.

- CD
- hard disk
- RAM

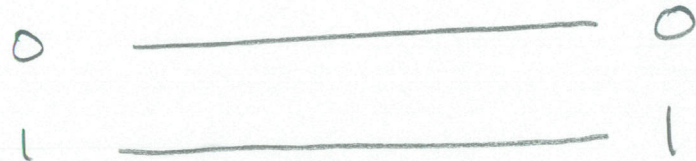
2> The same question will be important in quantum computers!

Single Bit Operation

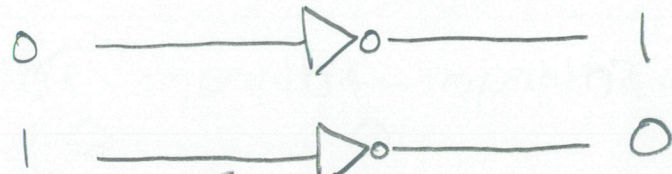
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—————→ time

input bit output bit



IDN operation



NOT operation

Would you think single qubit operations are simple to realize in a quantum computer?

What could be potential problems?

- decoherence
- spurious interactions
- ...

- Wire represents bit
- preserves state

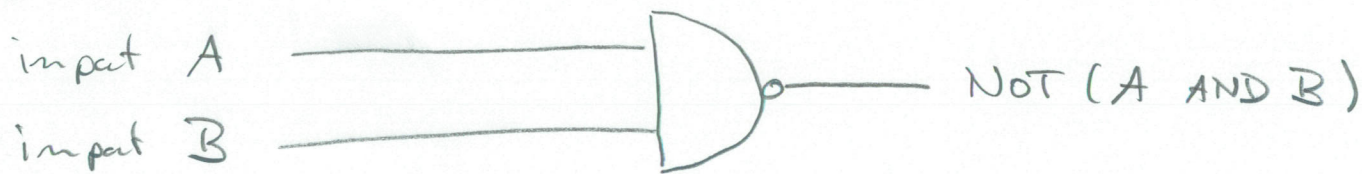
- Symbol represents operation
- changes state

Same representation of information in circuit model for quantum computation

Two Bit Operations

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The NAND gate



truth table	A	B	NOT (A AND B)
	0	0	1
	0	1	1
	1	0	1
	1	1	0

Properties:

- Universal logic gate

↳ any function operating on bits can be computed using NAND gates

↳ examples: AND, OR, XOR, NOR

(ancilla bits & possibilities to make copies are required)

Why do you think universality is useful!

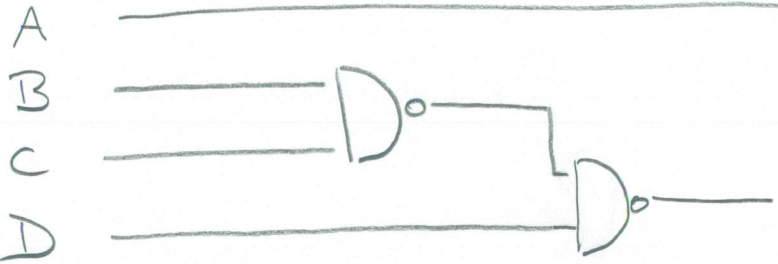
NOTE:

Universal logic gates also do exist for quantum computers!

Circuit Representation

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INPUT



OUTPUT

$$E = A$$

$$F = (B \text{ NAND } C) \text{ NAND } D$$

Any computable function can be represented as a circuit composed of logic universal gates acting on a set of input bits generating a set of output bits

Circuit properties :

- bits can be copied (FAN OUT)
- additional working bits are allowed (ANCILLAS)
- values of bits can be interchanged (CROSSOVER)
- number of output bits may be smaller than number of input bits
- loops are not allowed

In your opinion, which of these properties might be less / hard to realize in a quantum computer?

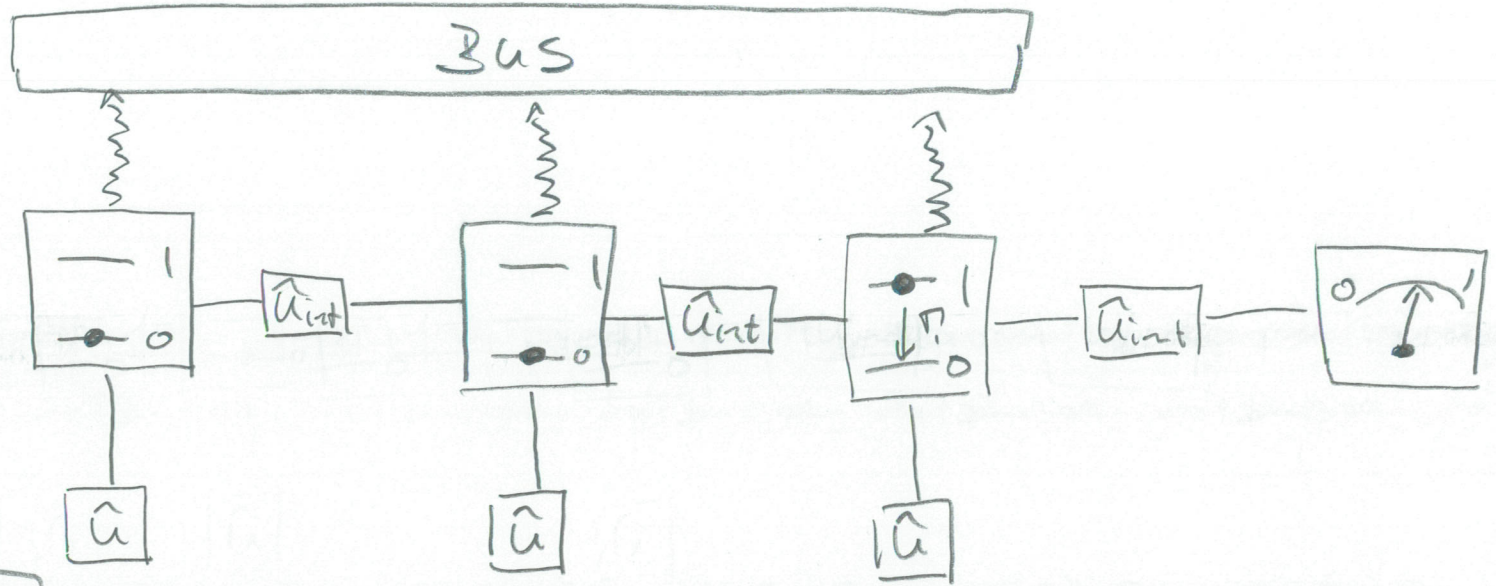
- reversibility?
- preservation of state?
- copying of information
- universality?

A Generic Quantum Processor

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features:

- (1) quantum bits
- (2) initialization
- (3) coherence
- (4) universal gates
- (5) read-out



Di Vincenzo Criteria

- (6) conversion
- (7) transfer

In your opinion, what kind of components are absolutely essential to realize any quantum computer?

The Quantum Bit

- Quantum mechanical system with two distinct states



Give examples of systems with this energy level spectrum. Under which conditions would you be allowed to approximate a given quantum system as a two-level system? (6)

- Representation of qubit basis states $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

vectors in 2D Hilbert space
(1st QM postulate)

- general qubit state

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle \quad \text{with } \alpha, \beta \in \mathbb{C}$$

properties: - qubit can be in superposition of states $|0\rangle$ and $|1\rangle$

- probability of states $P_0 = |\alpha|^2$

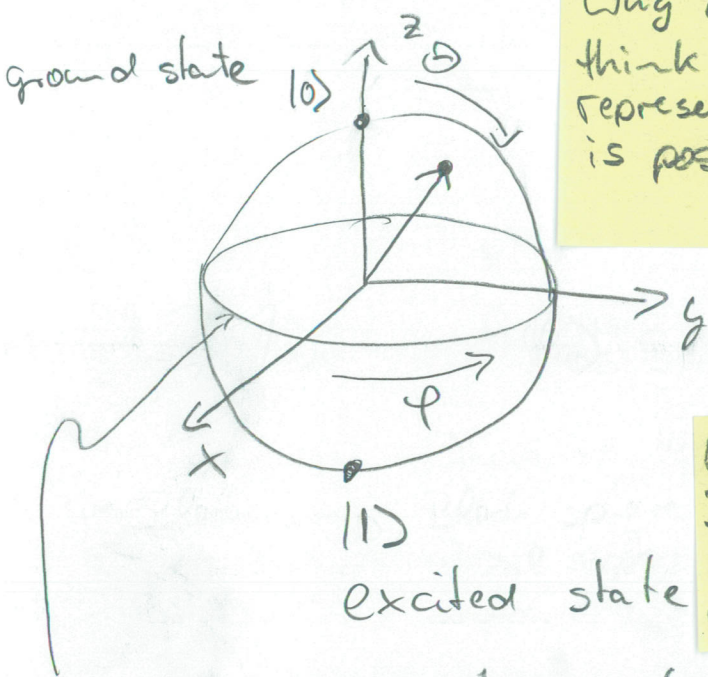
$$P_1 = |\beta|^2$$

$$\text{with } P_1 + P_2 = 1$$

What is the main difference in comp. to a classical bit?

The Bloch Sphere

Representation of pure single qubit state as vector to the surface of a sphere



Why do you think this representation is possible?

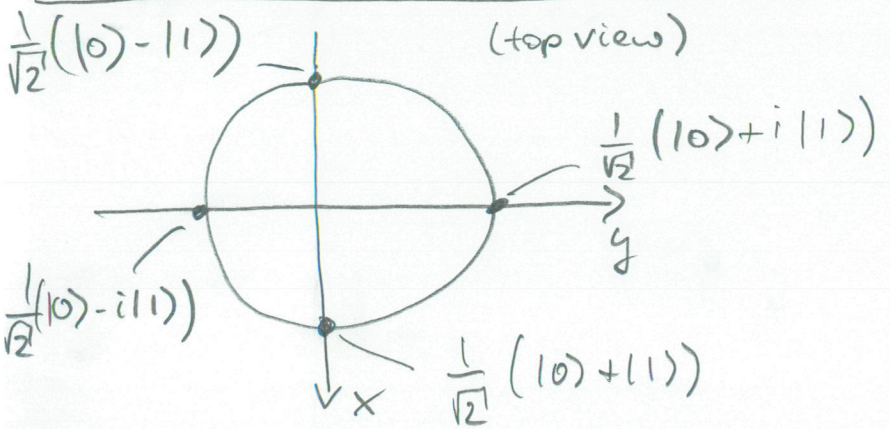
Where on the Bloch sphere are equal superposition states?

- general state $|ψ\rangle = α|0\rangle + β|1\rangle$
- 4 parameters $Re\ α, Im\ α, Re\ β, Im\ β$
- + 1 normalization constraint $|α|^2 + |β|^2 = 1$

• $|ψ\rangle = e^{iγ} \left[\cos\frac{\theta}{2} |0\rangle + e^{iφ} \sin\frac{\theta}{2} |1\rangle \right]$

with $γ$: global phase factor
 θ : polar angle
 ϕ : azimuthal angle

equal superposition states:



equivalent representation

- $|ψ\rangle = \frac{1}{\sqrt{2}} (|0\rangle + e^{iφ} |1\rangle)$
- $\Rightarrow \theta = \frac{\pi}{2}$: defines equator
- $\phi = 0, \pi/2, \pi, 3/2\pi$ defines phase angle