Realization of the quantum Toffoli gate

Based on:

Monz, T; Kim, K; Haensel, W; et al
Realization of the quantum Toffoli gate with trapped ions
Phys. Rev. Lett. **102**, 040501 (2009)

Lanyon, BP; Barbieri, M; Almeida, MP; et al.
Simplifying quantum logic using higher dimensional Hilbert spaces
Nat. Phys. **5**, 134 (2009)

Outline

1. Motivation
2. Principles of the quantum Toffoli gate
3. Implementation with trapped ions
4. Implementation with photons
5. Comparison and conclusion
6. Summary

1. Motivation

- Universal quantum logic gate sets are needed to implement algorithms
- Implementation of algorithms is difficult due to the finite fidelity and large amount of gates
- Use of other degrees of freedom to store information
- Reduction of complexity and runtime

2. Principles of the Toffoli gate

- Three-qubit gate (C_1, C_2, T)
- Logic flip of T depending on (C_1 AND C_2)

Truth table

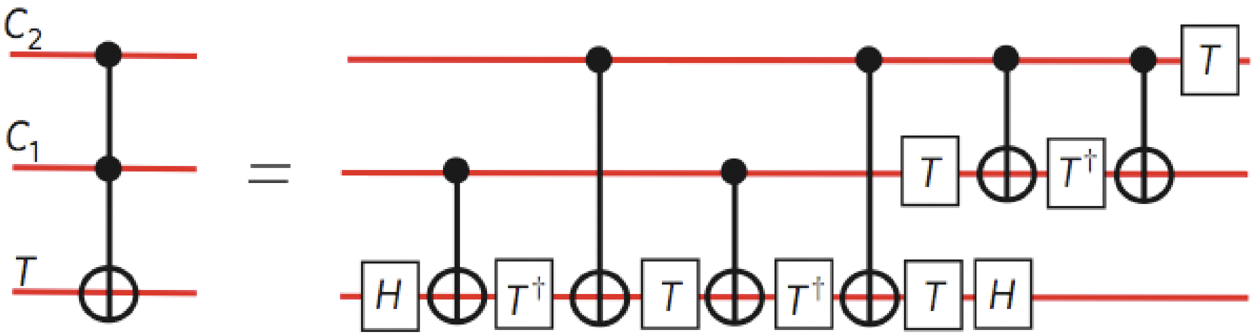
Input			Output		
C_1	C_2	T	C_1	C_2	T
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

Matrix form

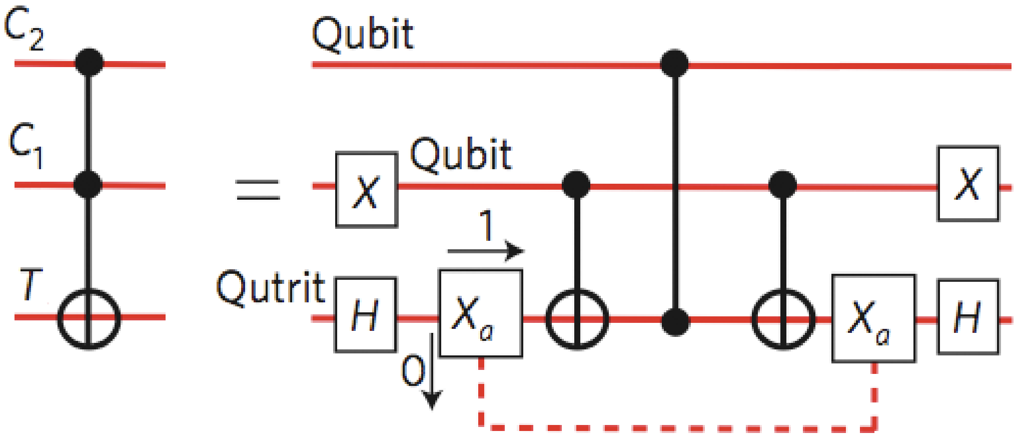
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

2. Principles of the Toffoli gate

- Qubit Implementation



- Qutrit Implementation

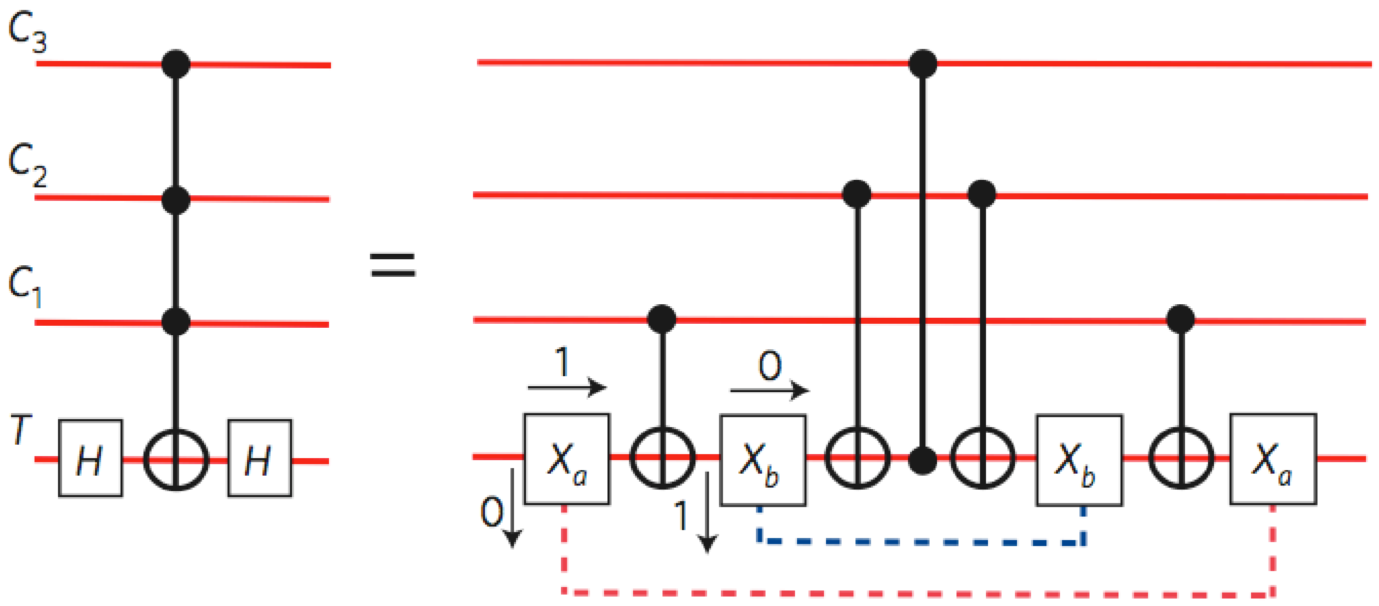


Qutrit states:

$|0\rangle, |1\rangle$ and $|2\rangle$

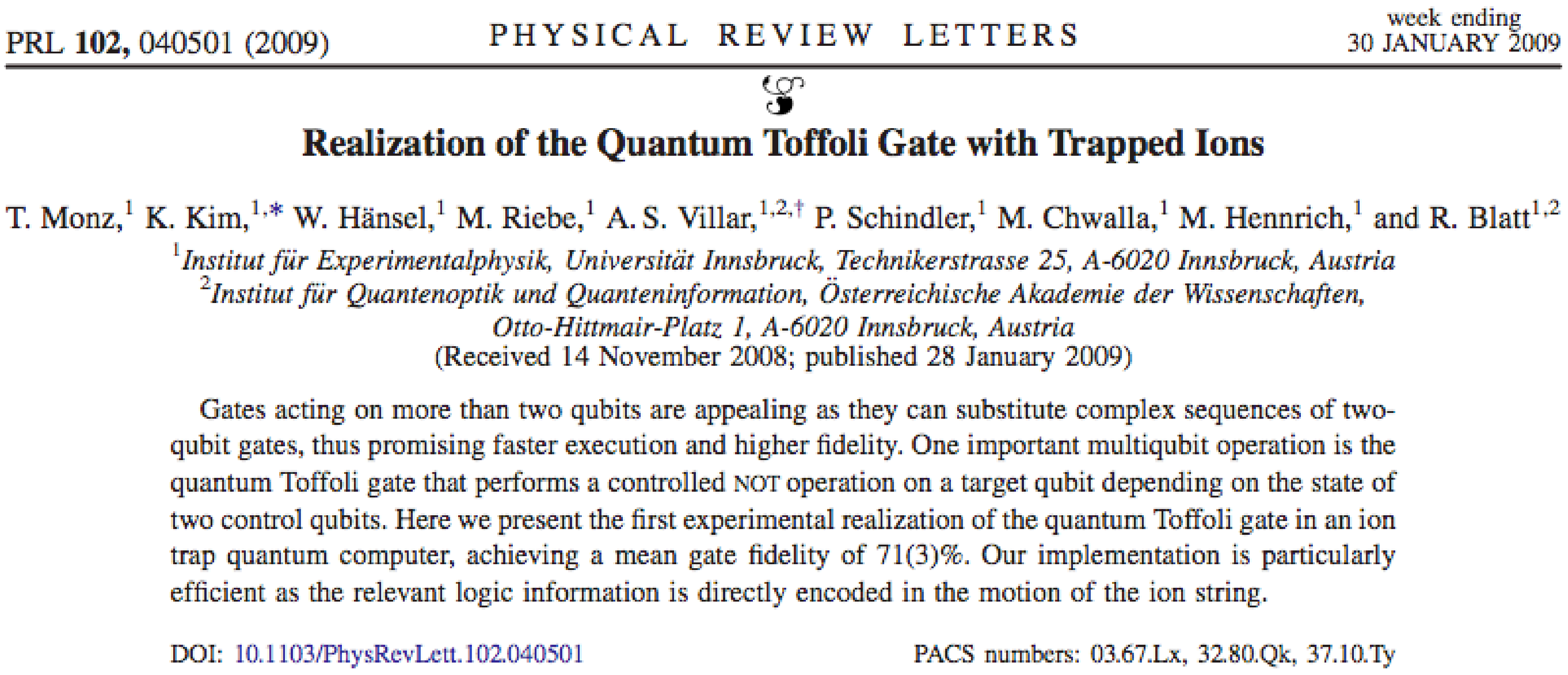
2. Principles of the Toffoli gate

- Higher order Toffoli gates easily implementable



- Gates needed: 2n-1, prior 12n-11 plus n-1 ancilla qubits

3. Implementation with trapped ions



PRL **102,** 040501 (2009) PHYSICAL REVIEW LETTERS week ending 30 JANUARY 2009

Realization of the Quantum Toffoli Gate with Trapped Ions

T. Monz,[1] K. Kim,[1,*] W. Hänsel,[1] M. Riebe,[1] A. S. Villar,[1,2,†] P. Schindler,[1] M. Chwalla,[1] M. Hennrich,[1] and R. Blatt[1,2]
[1]*Institut für Experimentalphysik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria*
[2]*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften,*
Otto-Hittmair-Platz 1, A-6020 Innsbruck, Austria

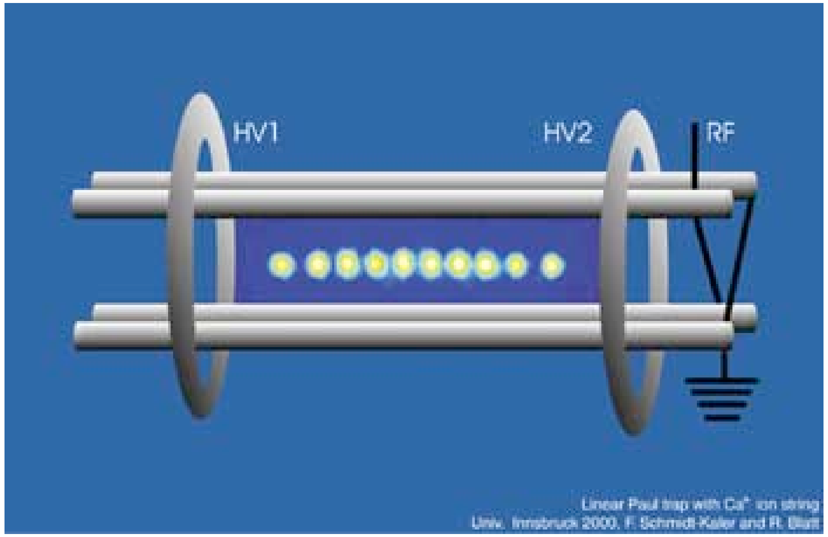
(Received 14 November 2008; published 28 January 2009)

Gates acting on more than two qubits are appealing as they can substitute complex sequences of two-qubit gates, thus promising faster execution and higher fidelity. One important multiqubit operation is the quantum Toffoli gate that performs a controlled NOT operation on a target qubit depending on the state of two control qubits. Here we present the first experimental realization of the quantum Toffoli gate in an ion trap quantum computer, achieving a mean gate fidelity of 71(3)%. Our implementation is particularly efficient as the relevant logic information is directly encoded in the motion of the ion string.

DOI: 10.1103/PhysRevLett.102.040501 PACS numbers: 03.67.Lx, 32.80.Qk, 37.10.Ty

3. Implementation with trapped ions

- String of $^{40}\mathrm{Ca}^{+}$ ions in linear ion trap
- Ground state: $\mathrm{S}_{1/2}(\mathrm{m}{=}-1/2) = |S\rangle \equiv |1\rangle$
- Excited state: $\mathrm{D}_{5/2}(\mathrm{m}{=}-1/2) = |D\rangle \equiv |0\rangle$
- Usage of COM vibratinal modes (phonons)

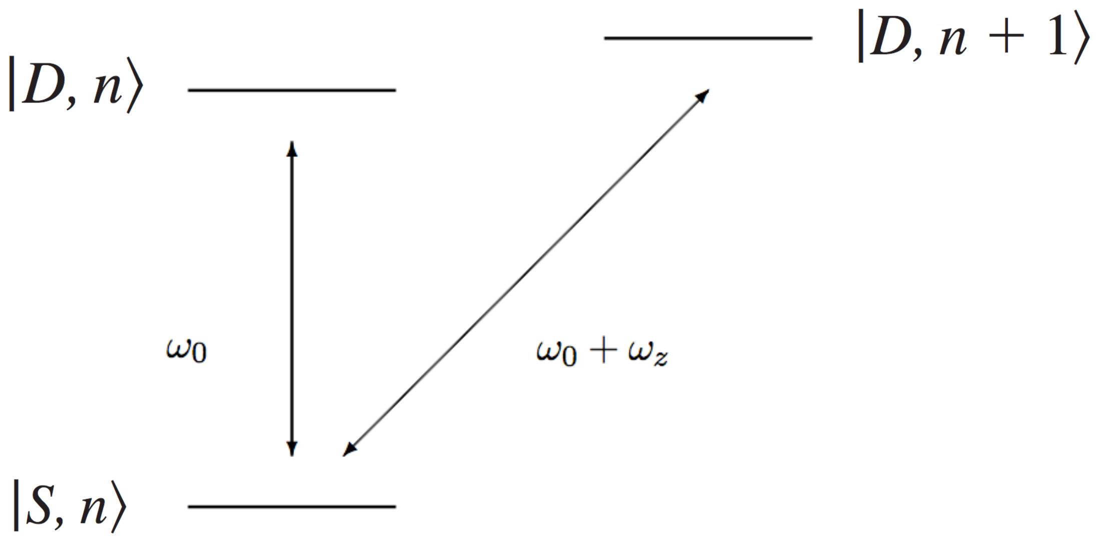


Picture: Linear Paul trap with Ca^{+} on string
University Innsbruck, 2000: F. Schmidt-Kaler and R. Blatt,

3. Implementation with trapped ions

Used Transitions

- First type of transition: $|S, n\rangle \leftrightarrow |D, n\rangle$
- Second type of transition: $|S, n\rangle \leftrightarrow |D, n+1\rangle$



3. Implementation with trapped ions

Implementation

- Three steps
  - Encoding of the information in the vibrational COM mode
  - NOT operation, depending on C_1 and C_2
  - Reversal encoding and readout

3. Implementation with trapped ions

- Laser pulses prepare C_1 and C_2 by type 1 transition
- Phonon excitation by type 2 transition

$$|c_1 c_2, 0\rangle = |SS, 0\rangle \rightarrow |DD, 2\rangle,$$

$$|c_1 c_2, 0\rangle = |DS, 0\rangle \rightarrow \sin\frac{\pi}{2\sqrt{2}}|DD, 1\rangle + \cos\frac{\pi}{2\sqrt{2}}|DS, 0\rangle$$

$$|c_1 c_2, 0\rangle = |SD, 0\rangle \rightarrow \cos\frac{\pi}{2\sqrt{2}}|DD, 1\rangle - \sin\frac{\pi}{2\sqrt{2}}|DS, 0\rangle$$

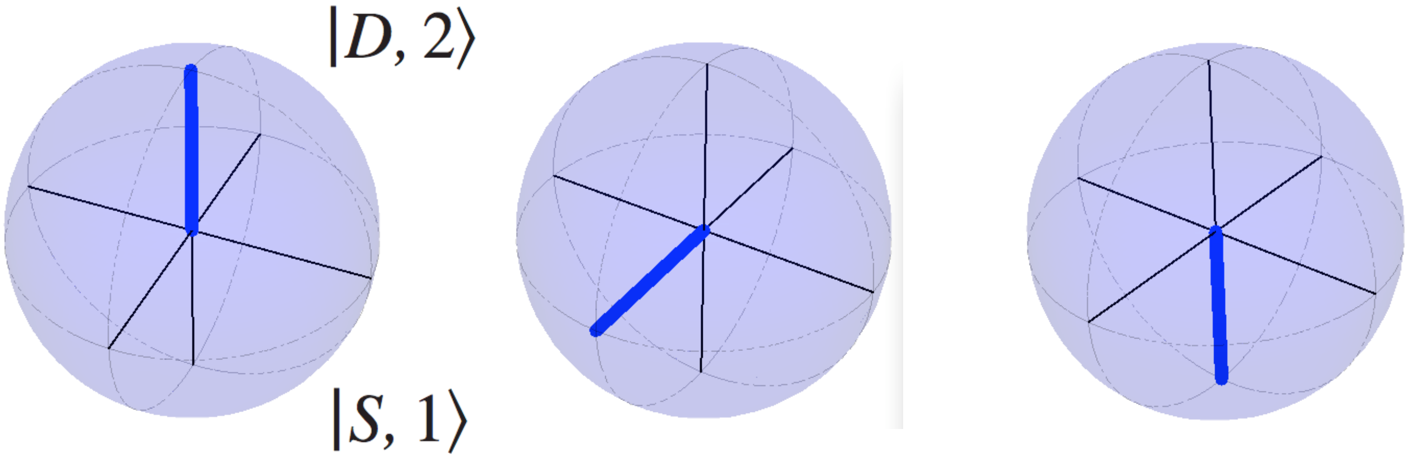
$$|c_1 c_2, 0\rangle = |DD, 0\rangle \rightarrow |DD, 0\rangle.$$

- From $|SS\rangle$ state we get the phonon in state 2
- After type 2 transition, C_1 is always in the D state

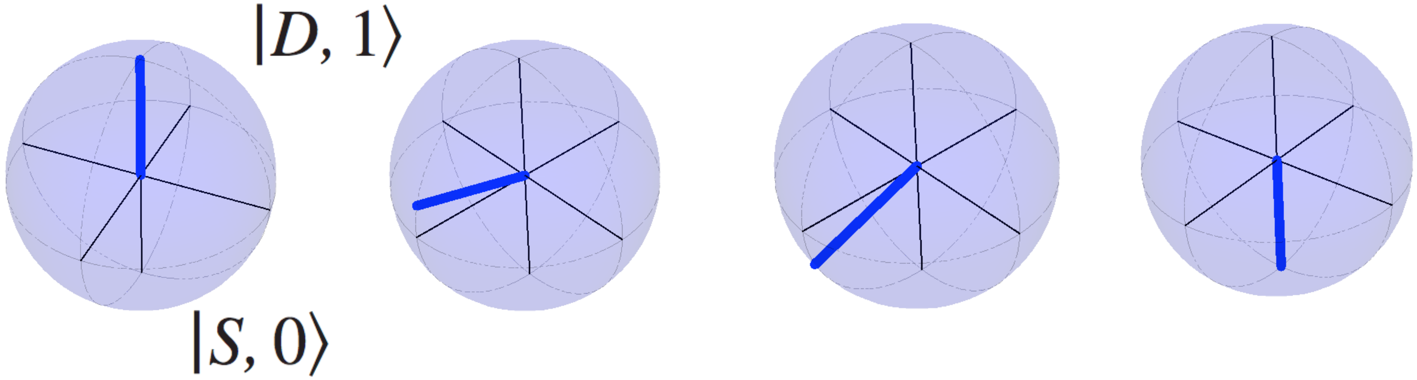
3. Implementation with trapped ions

Implementation

$|D, 2\rangle$



$|D, 1\rangle$



3. Implementation with trapped ions

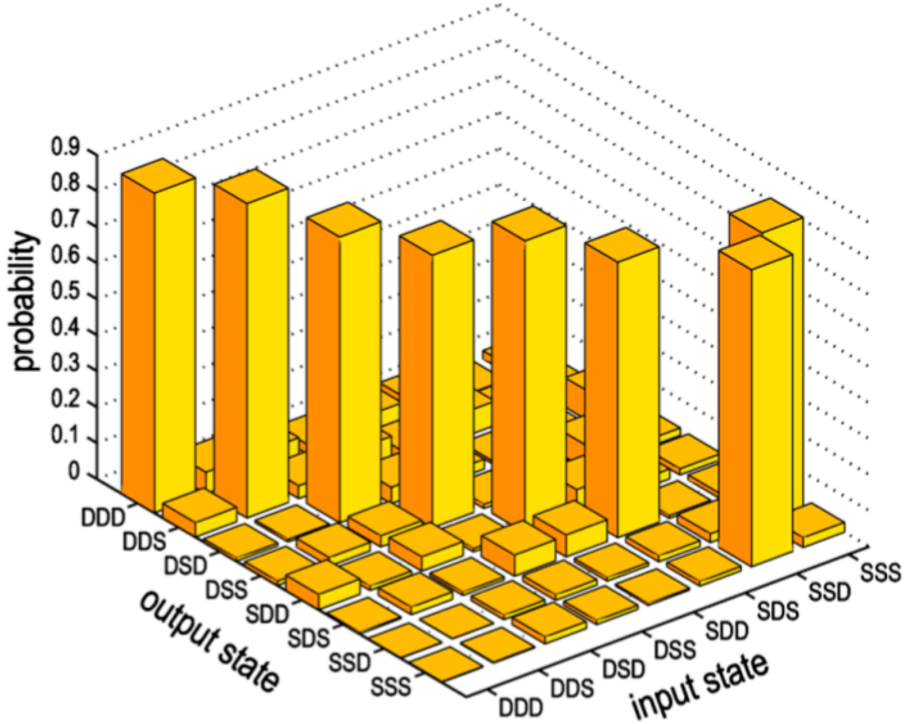
Implementatin

- $|SS\rangle$ is the only state with phonon left
- C-NOT operation depending on the existance of phonon
- Undo encoding for C_1, C_2 (control qubits remain unchanged)
- Readout by measuring the $S_{1/2} \leftrightarrow P_{1/2}$ transition

3. Implementation with trapped ions

Results

$$U_T = \exp\left(-i\pi\frac{1}{2\sqrt{2}}\sigma_{Z,t}\right)\begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&i\\0&0&0&0&0&0&-i&0\end{pmatrix}$$

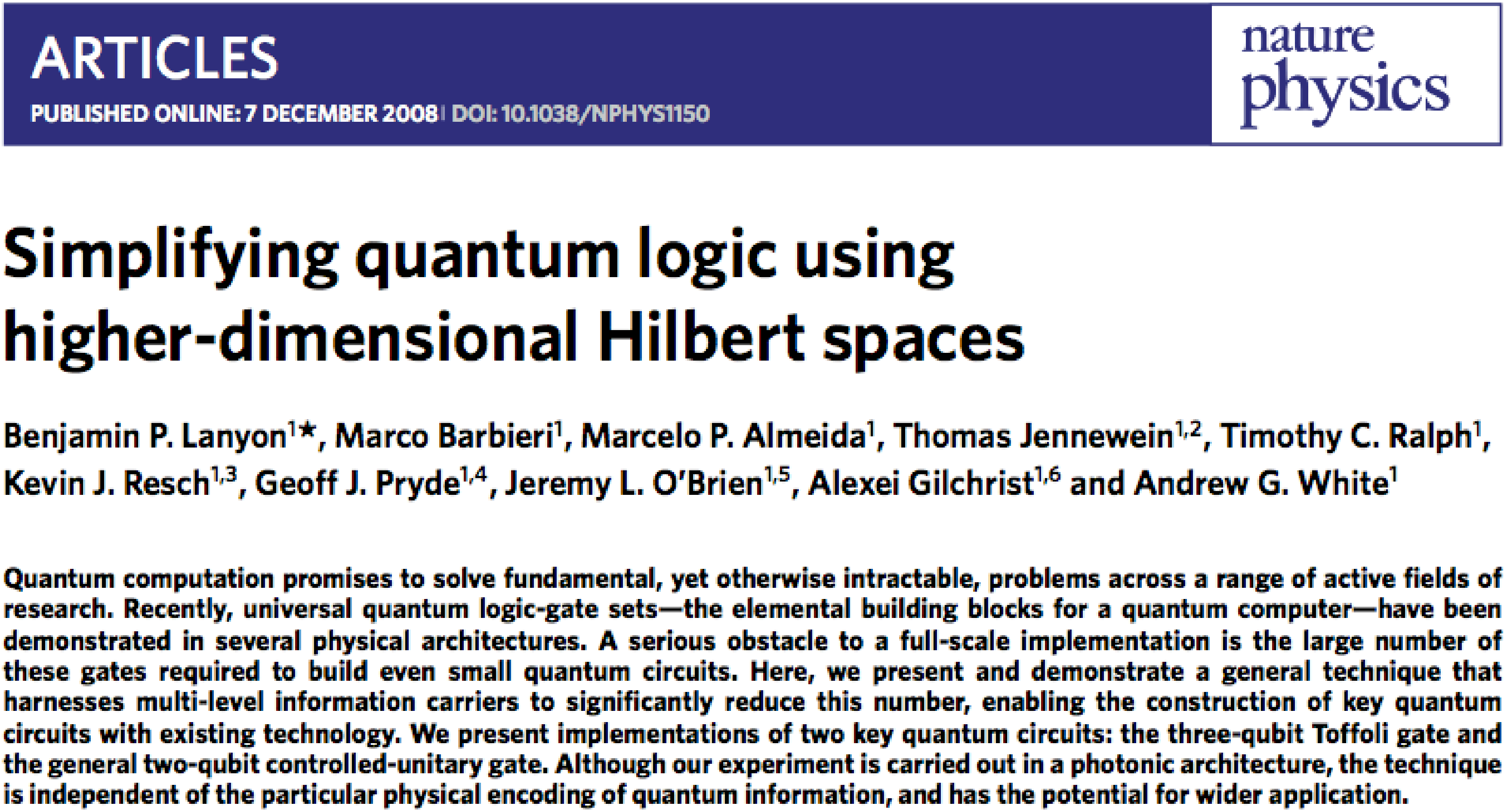


3. Implementation with trapped ions

Measurements

- Enhanced fidelity from 63% to 71%
- Errors dominated by Rabi frequency (infidelity of 12%) and temperature changes plus voltage fluctuations (7%)
- Runtime 1.5ms vs 4.2 ms
- Runtime determined by coupling strength of the second transition

4. Implementation with photons



ARTICLES

PUBLISHED ONLINE: 7 DECEMBER 2008 | DOI: 10.1038/NPHYS1150

nature physics

Simplifying quantum logic using higher-dimensional Hilbert spaces

Benjamin P. Lanyon[1]*, Marco Barbieri[1], Marcelo P. Almeida[1], Thomas Jennewein[1,2], Timothy C. Ralph[1], Kevin J. Resch[1,3], Geoff J. Pryde[1,4], Jeremy L. O'Brien[1,5], Alexei Gilchrist[1,6] and Andrew G. White[1]

Quantum computation promises to solve fundamental, yet otherwise intractable, problems across a range of active fields of research. Recently, universal quantum logic-gate sets—the elemental building blocks for a quantum computer—have been demonstrated in several physical architectures. A serious obstacle to a full-scale implementation is the large number of these gates required to build even small quantum circuits. Here, we present and demonstrate a general technique that harnesses multi-level information carriers to significantly reduce this number, enabling the construction of key quantum circuits with existing technology. We present implementations of two key quantum circuits: the three-qubit Toffoli gate and the general two-qubit controlled-unitary gate. Although our experiment is carried out in a photonic architecture, the technique is independent of the particular physical encoding of quantum information, and has the potential for wider application.

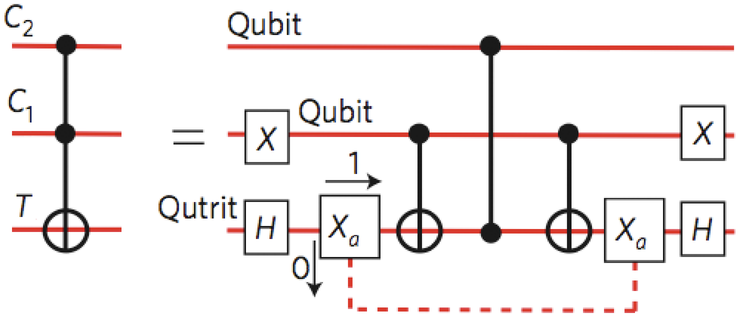
4. Implementation with photons

Theory

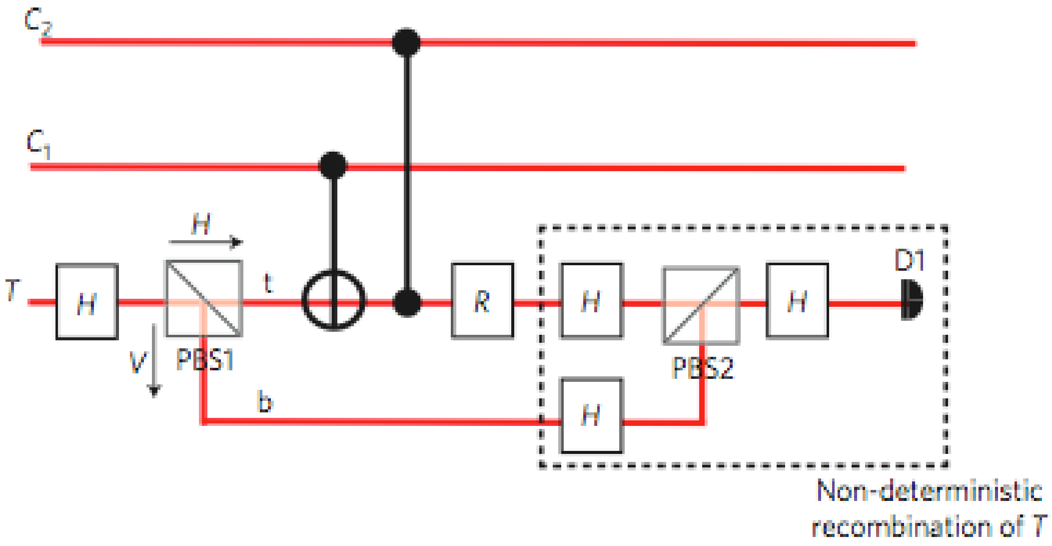
- Photons offer fast gate speeds
- Photons have a large number of d.o.f. (polarization, frequency)
- Qubit states realized by horizontal $|\mathbf{H}\rangle$ and vertical polarization $|\mathbf{V}\rangle$
- Two additional levels by beam splitting ($|\mathbf{H},\mathbf{t}\rangle$, $|\mathbf{V},\mathbf{t}\rangle$, $|\mathbf{H},\mathbf{b}\rangle$ and $|\mathbf{V},\mathbf{b}\rangle$)

4. Implementation with photons

- Theoretical Implementation

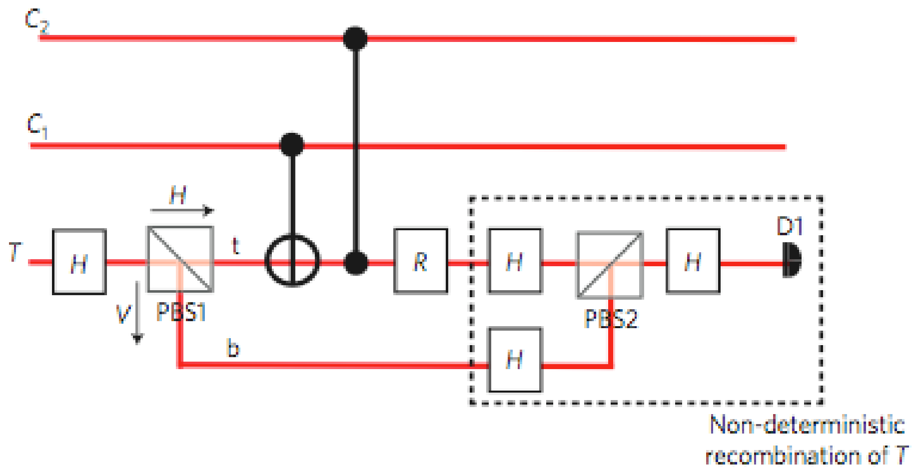


- Probabilistic / Experimental Implementation



4. Implementation with photons

Implementation



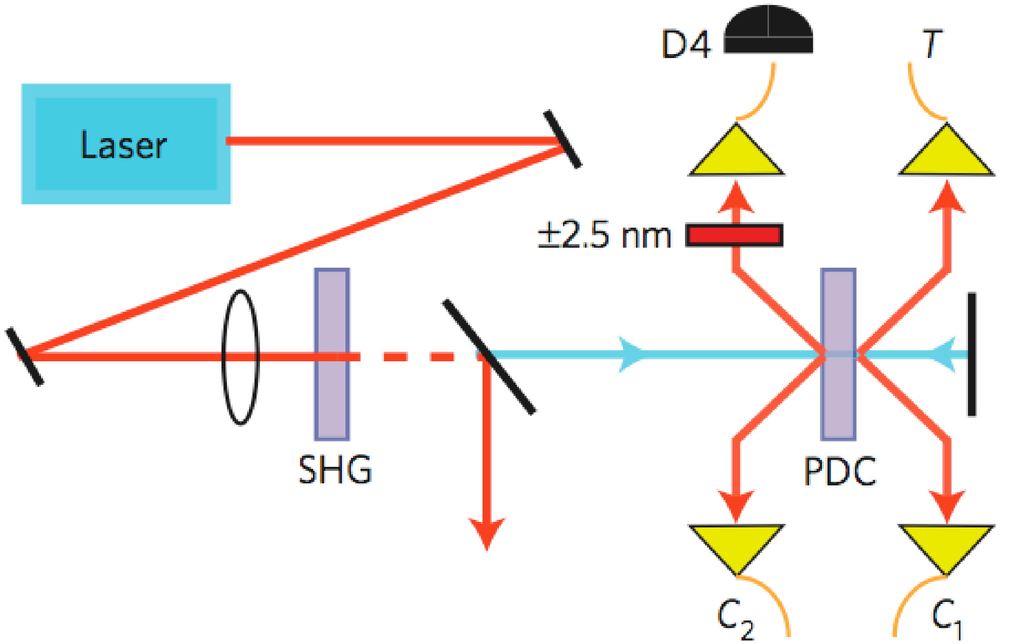
- Beam splitting (PBS1)
- Two-qubit operations on top Qubit
- Non-deterministic recombination of T (PBS2)

↳ Second C-NOT operation is displaced

4. Implemenation with photons

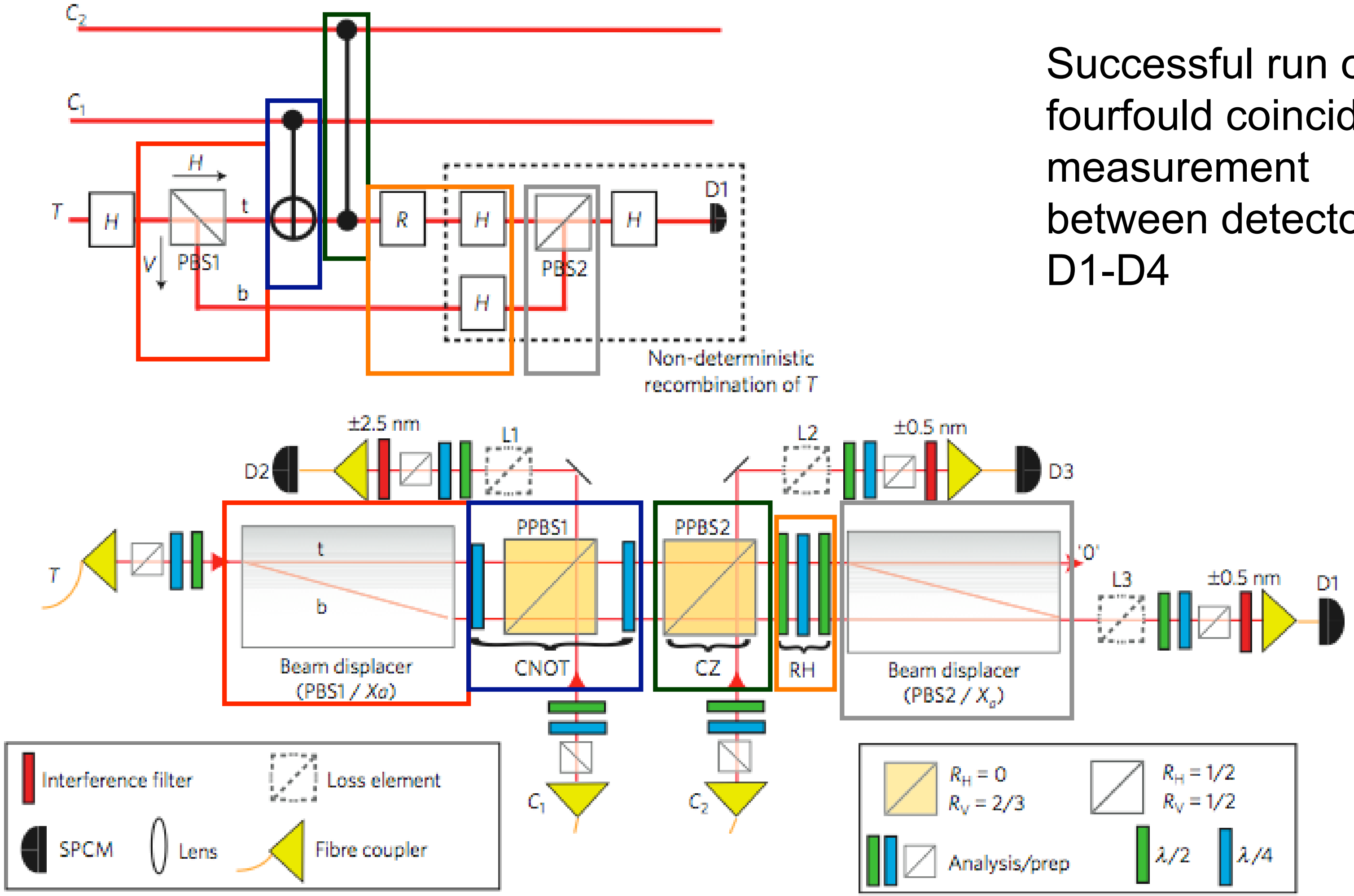
Implementation

- Photon source
- Photons detected by non-number-resolving photon-counting modules



4. Implementation with photons

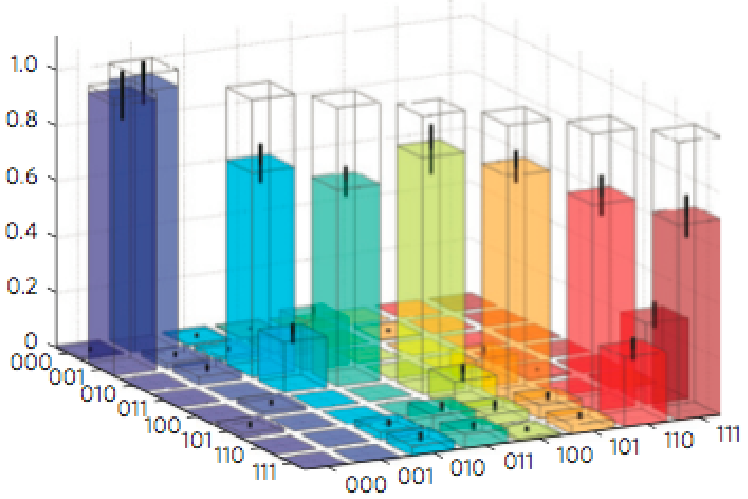
Implementation



Successful run on a fourfould coincident measurement between detectors D1-D4

4. Implementation with photons

Measurement



Toffoli logical truth table
Ability to apply the correct operation to all eight logical input states.

- Target swapped on $|C_2, C_1\rangle = |\mathbf{0},\mathbf{0}\rangle$
- 81% overlap with ideal case (wire grid)
- Comparable with two qubit fidelity (84%)

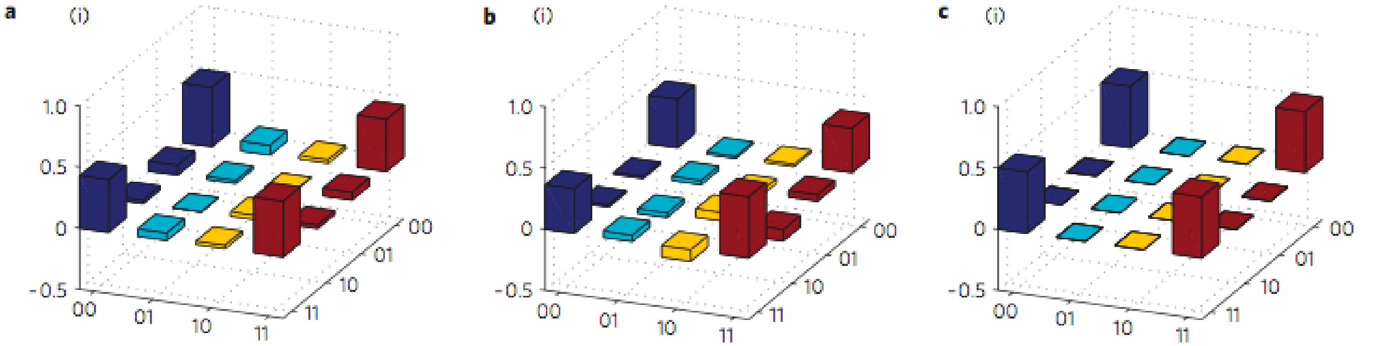
4. Implementation with photons

Output of C_1 and T

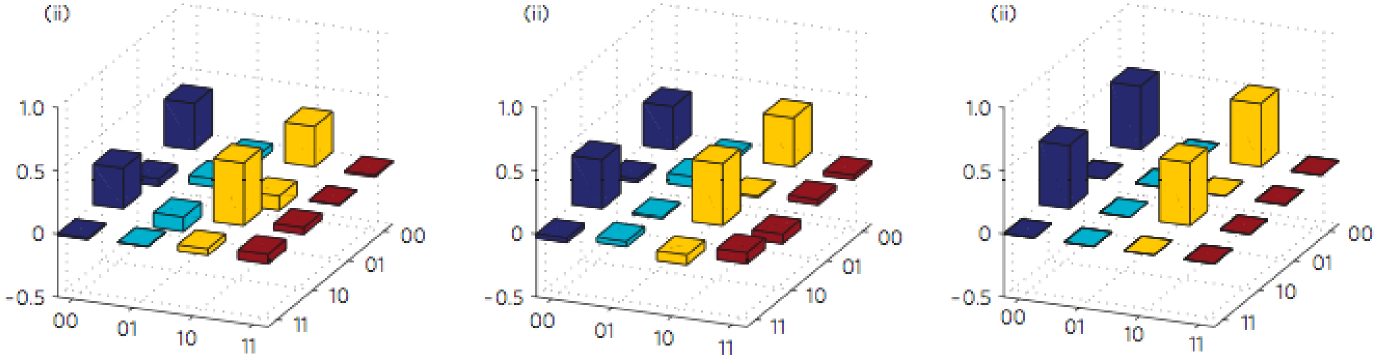
Output of C_2 and T

Ideal density matrices

Input $|\mathbf{1},(\mathbf{0}+\mathbf{1}),\mathbf{0}\rangle/\sqrt{2}$ $|(\mathbf{0}+\mathbf{1}),\mathbf{1},\mathbf{0}\rangle/\sqrt{2}$

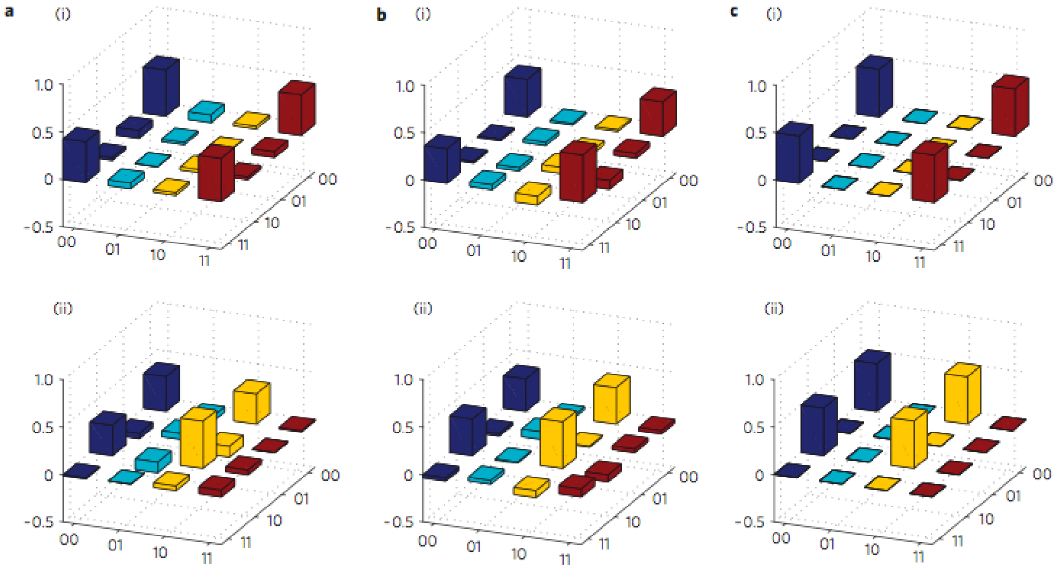


Input $|\mathbf{0},(\mathbf{0}+\mathbf{1}),\mathbf{0}\rangle/\sqrt{2}$ $|(\mathbf{0}+\mathbf{1}),\mathbf{0},\mathbf{0}\rangle/\sqrt{2}$



4. Implementation with photons

Results



- Fidelity:

  i) 0.90 ± 0.04

  0.81 ± 0.02

  ii) 0.75 ± 0.06

  0.80 ± 0.03
- Entangled states can be observed

5. Comparison and Conclusion

- Information stored in multilevel qubits
- The additional level is used in the target for the photon procedure and in the control qubits for the ion procedure
- Significant practical advantages
  - Fidelity
  - Duration
- Method might be used for other gates in the future

6. Summary

- Toffoli gate flips target, depending on C_1 and C_2
- Reduction of 2-qubit gates with multilevel qubits
- Higher level stores information temporally
- Realized with trapped ions
- Realized with photons
- Reduction of runtime and higher fidelity could be achieved
- Entanglement could be shown

Questions

?