ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Demonstration of Quantum Algorithms with Cavity Coupled Superconducting Qubits

Chris Myers and Yves Salathé

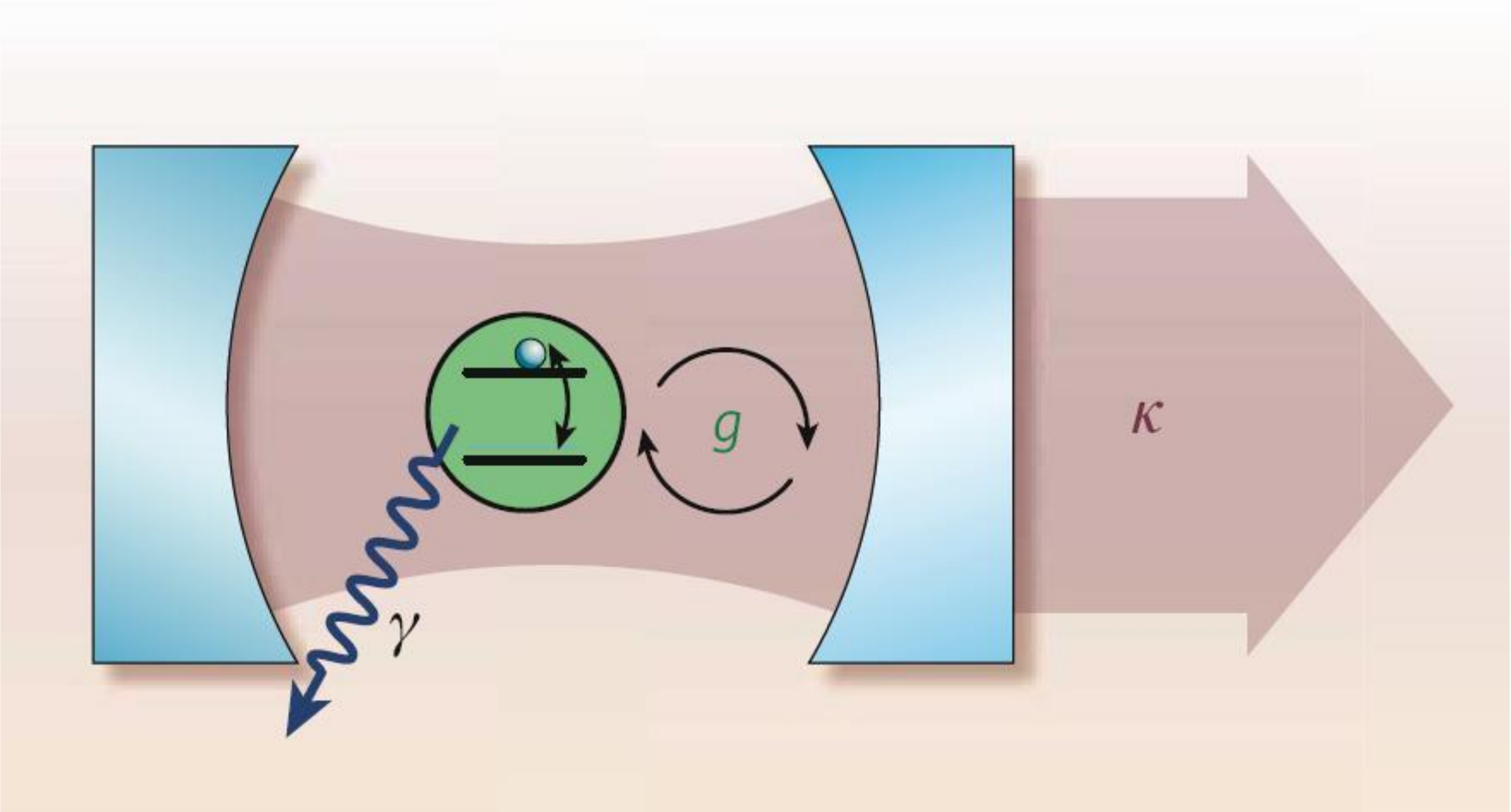
Based on paper by L.DiCarlo et al. Nature 2009

Introduction

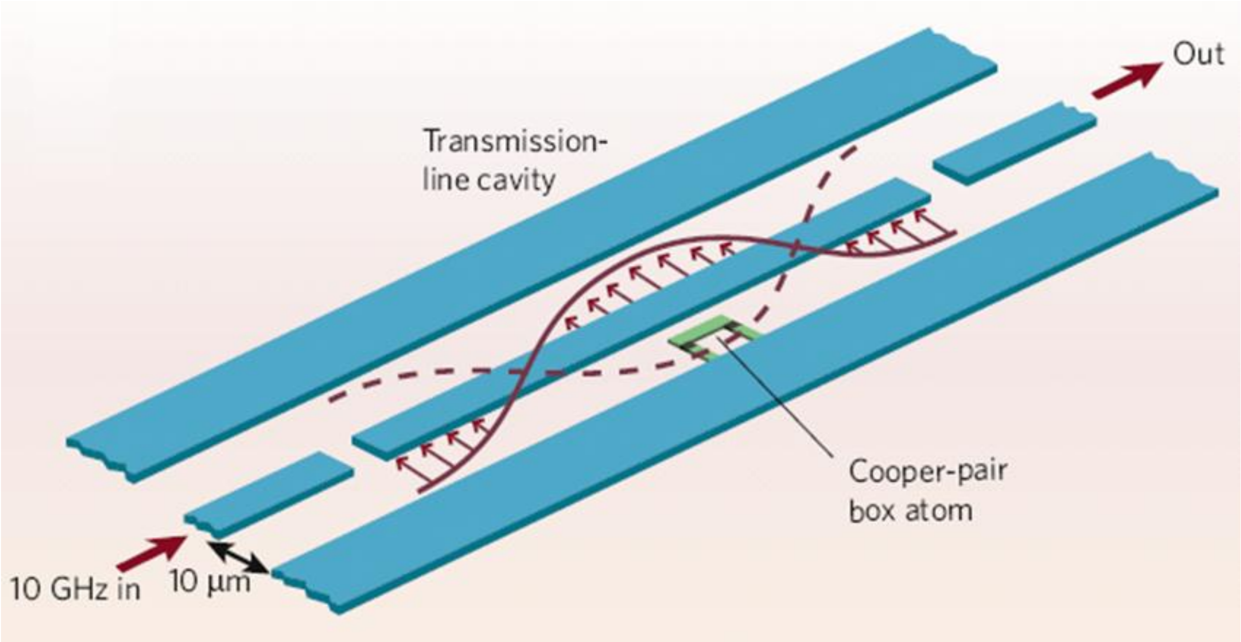
Several criteria have to be fulfilled in order to build a quantum computer (DiVincenzo).

Contents

Cavity QED



Circuit QED



Cavity QED	Circuit QED
Atom	Superconducting Qubit
Cavity	Transmission Line Resonator

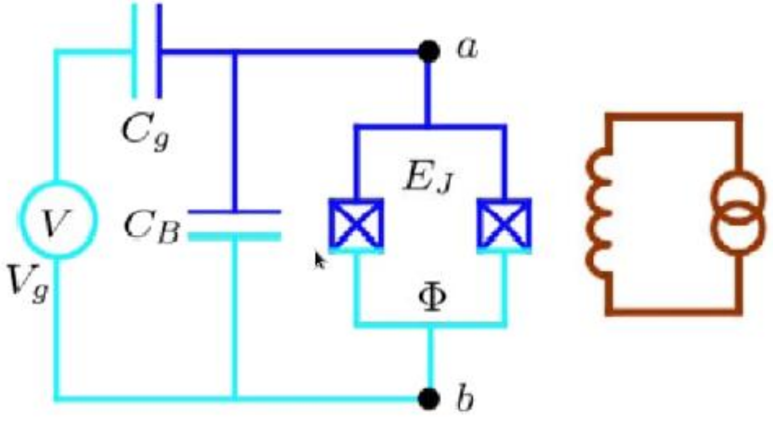
The Transmon Qubit

- Same Hamiltonian as Cooper Pair Box

$$\hat{H}=4E_C(\hat{n}-n_g)^2-E_J\cos\hat{\phi}$$

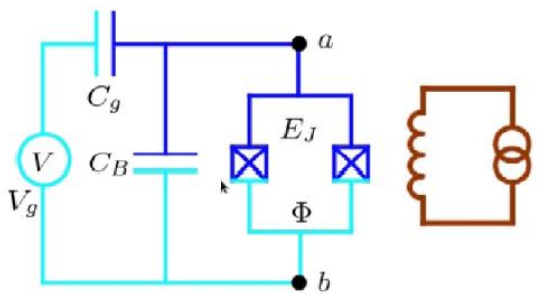
- Additional large capacitance C_B

- SQUID allows tuning of $E_J=E_{J,max}|\cos(\pi\phi/\phi_0)|$



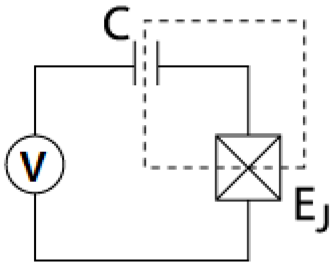
Comparison of Transmon and CPB

Transmon



$$E_J/E_C \gg 1$$

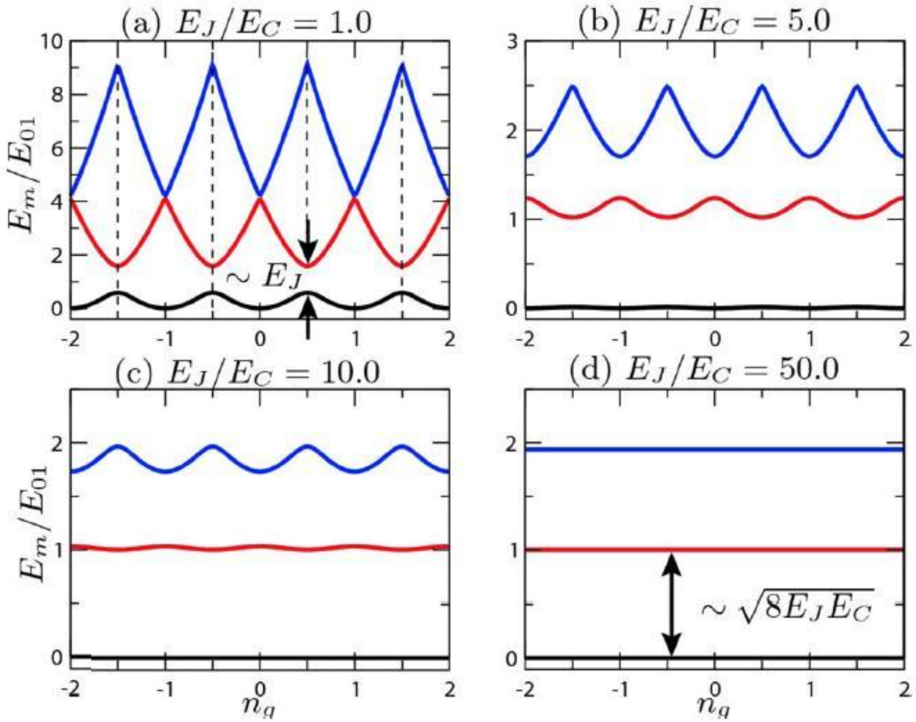
CPB



$$E_J/E_C \ll 1$$

- The increase in this ratio makes the transmon less sensitive to charge noise
- Also decreases anharmonicity

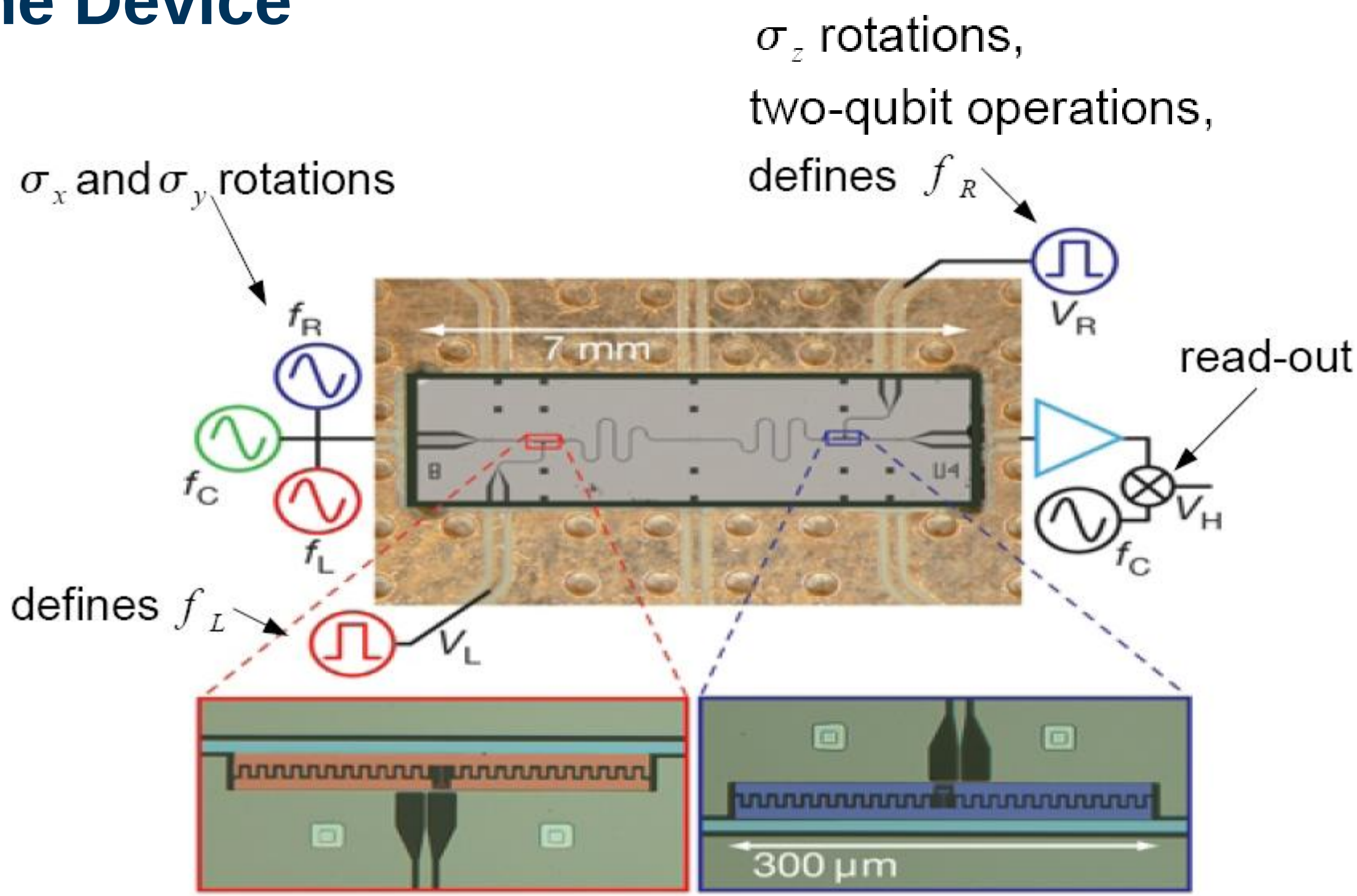
Charge Dispersion



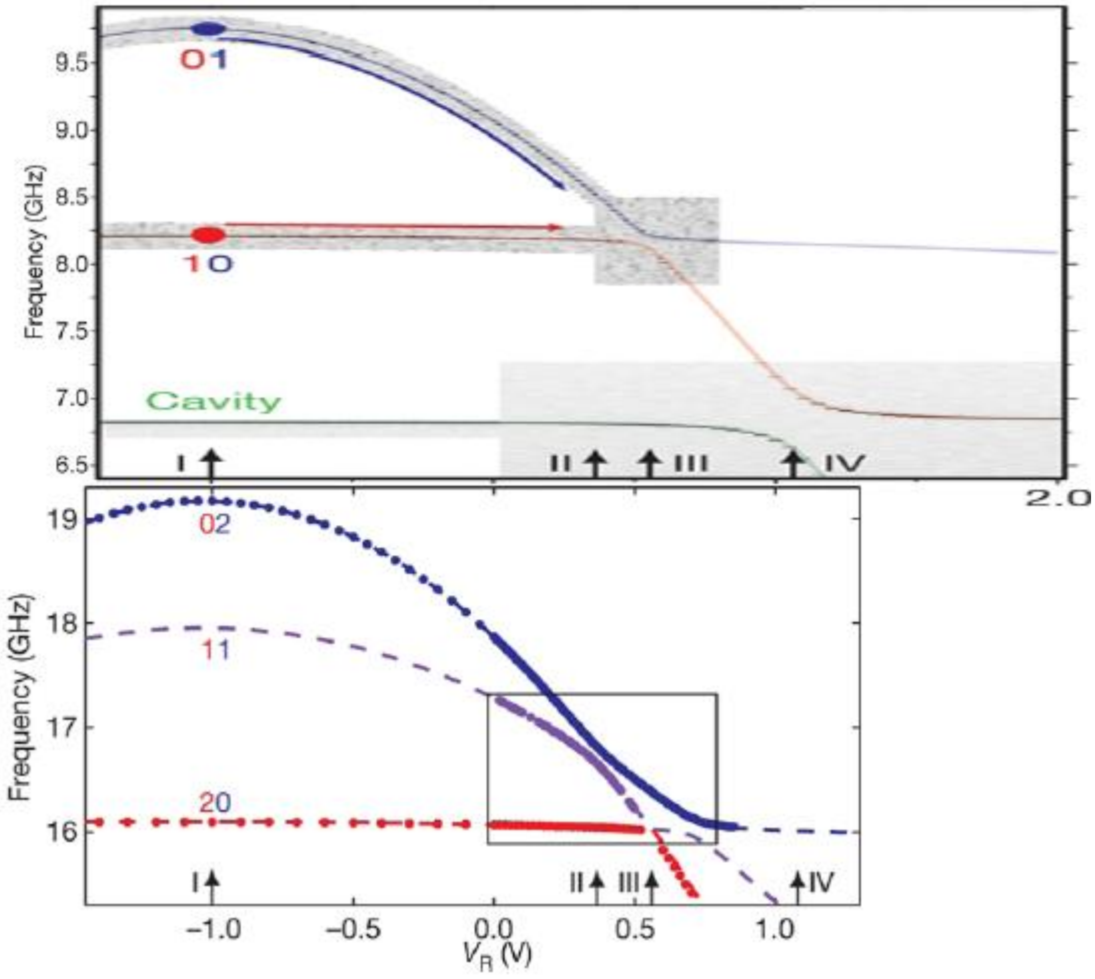
$$E_m(n_g) \simeq E_m(n_g = 1/4) + \frac{\epsilon_m}{2} \cos(2\pi n_g)$$

$$\epsilon_m \simeq (-1)^m E_C \frac{2^{4m+5}}{m!} \sqrt{\frac{2}{\pi}} \left(\frac{E_J}{2E_C}\right)^{\frac{m}{2}+\frac{3}{4}} e^{-\sqrt{8E_J/E_C}}$$

The Device


σ_z rotations,
two-qubit operations,
defines f_R
σ_x and σ_y rotations
f_R
V_R
7 mm
read-out
f_C
f_L
V_H
f_C
defines f_L
V_L
300 µm

Qubit Spectroscopy



Interesting Points

- I : Flux sweet spot for both qubits
- II : Avoided crossing with state outside computational domain
- III : Transverse coupling
- IV : Vacuum Rabi splitting

Single-Qubit Operations

- Apply microwave pulses through the cavity with frequencies close to either f_L or f_R.

- In the frame of the pulse, the Hamiltonian close to resonance for the corresponding qubit is:

$$\hat{H}(t) = \hbar\omega_{qubit}/2\,\hat{\sigma}_z + \hbar\epsilon\, cos(\omega_{pulse}t + \varphi)\hat{\sigma}_x$$

- Rotating wave approximation (see exercise 3):

$$\hat{H}_{eff} = (\omega_{qubit} - \omega_{pulse})\hat{\sigma}_z + \epsilon\, cos\,\varphi\,\hat{\sigma}_x + \epsilon\, sin\,\varphi\,\hat{\sigma}_y$$

Towards Two-Qubit Operations: The Dynamical Phase

- Recall: in general, solutions to the Schrödinger equation have to obey

$$\psi_n(t) = \sum_n c_n(t)\,\phi_n(t)\,e^{-i/\hbar \int_0^t E_n(t')\,dt'}$$

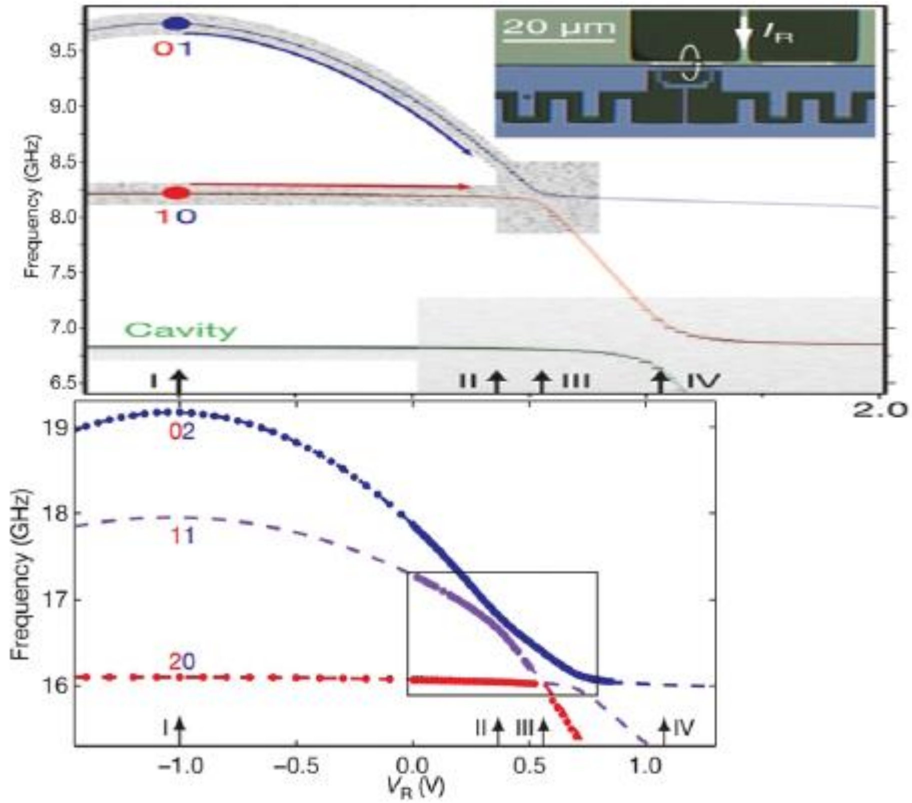
With $\hat{H}(t)\phi_n(t) = E_n(t)\phi_n(t)$.

- The exponent is called dynamical phase.

- Adiabatic evolution means tha $c_n(t)$ is constant.

Two-Qubit Operations : Spectroscopy

Adiabatically tune V_R close to point II and back:



Two-Qubit operations : The C-Phase Gate

- The avoided crossings include state-dependent dynamical phase.
- This evolution is described by the operator:

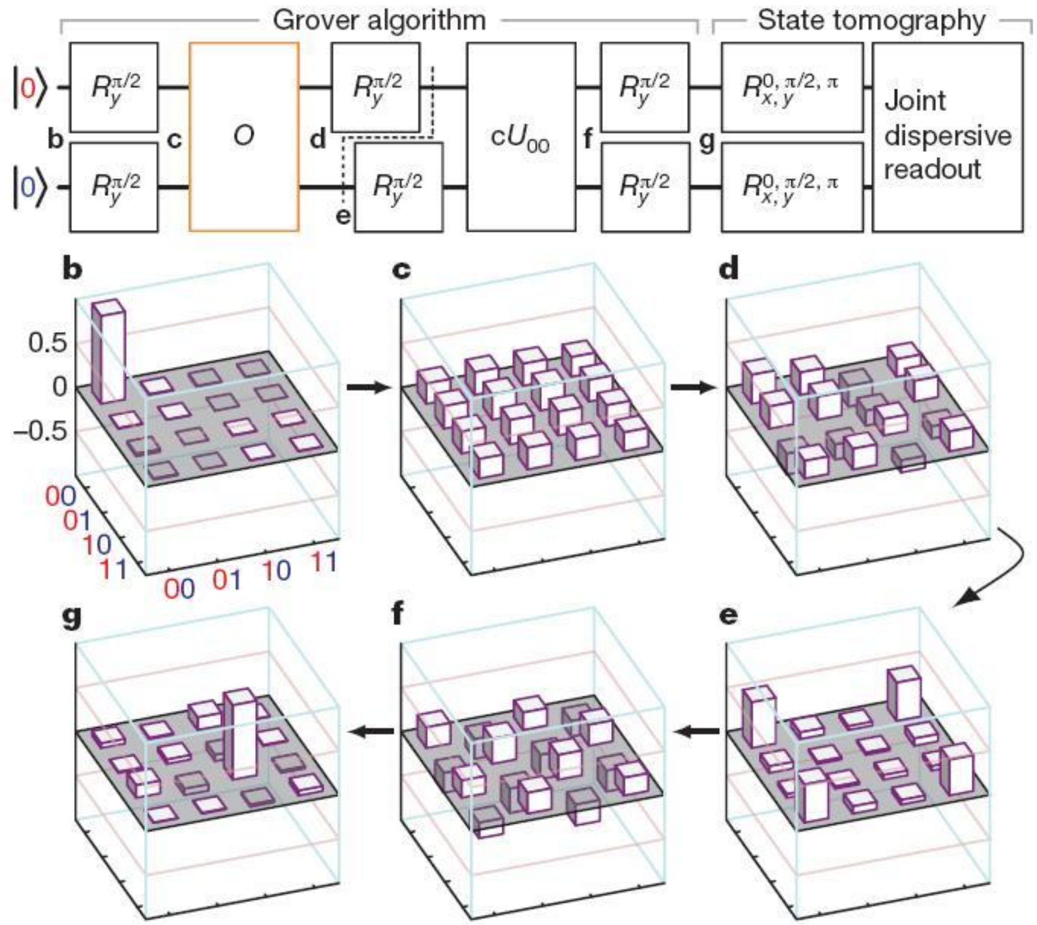
$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\phi_{01}} & 0 & 0 \\ 0 & 0 & e^{i\phi_{10}} & 0 \\ 0 & 0 & 0 & e^{i\phi_{11}} \end{pmatrix}$$

with $\phi_{lr} = 2\pi \int \delta f_{lr} dt$, where δf_{lr} is the deviation of the frequency to its value at point I.

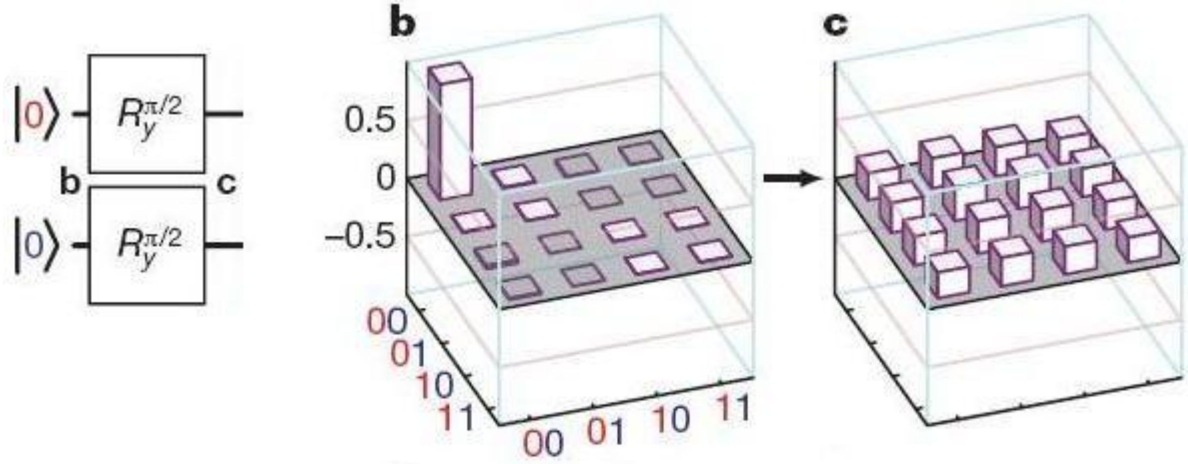
- The ground state does not acquire a dynamical phase.

Implementation of Grover Algorithm

$$O|\psi\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} |\psi\rangle$$



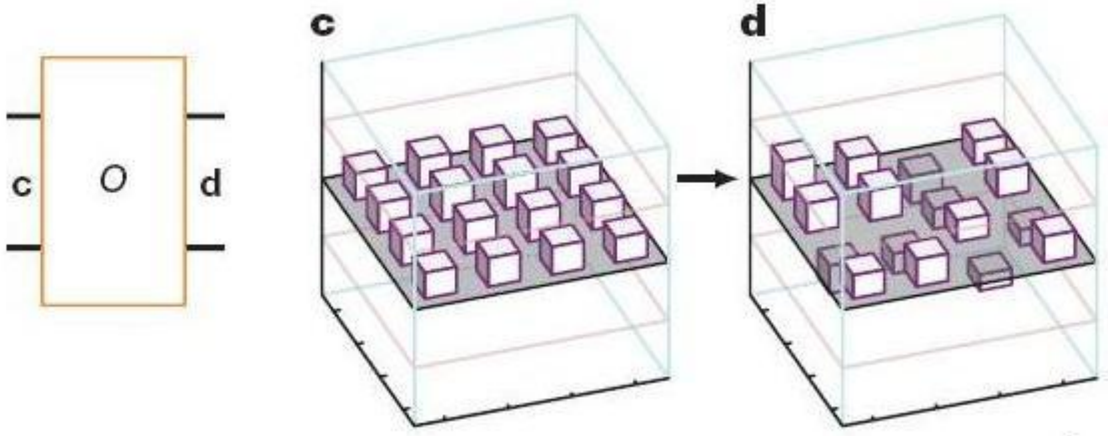
Grover Algorithm



- Start in Ground State

- Create a maximal superposition of all states

Grover Algorithm



- Apply the oracle O

$$O|\psi\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} |\psi\rangle$$

- This 'marks' the solution by inverting it's phase

$$|\psi\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle + |11\rangle\right)$$

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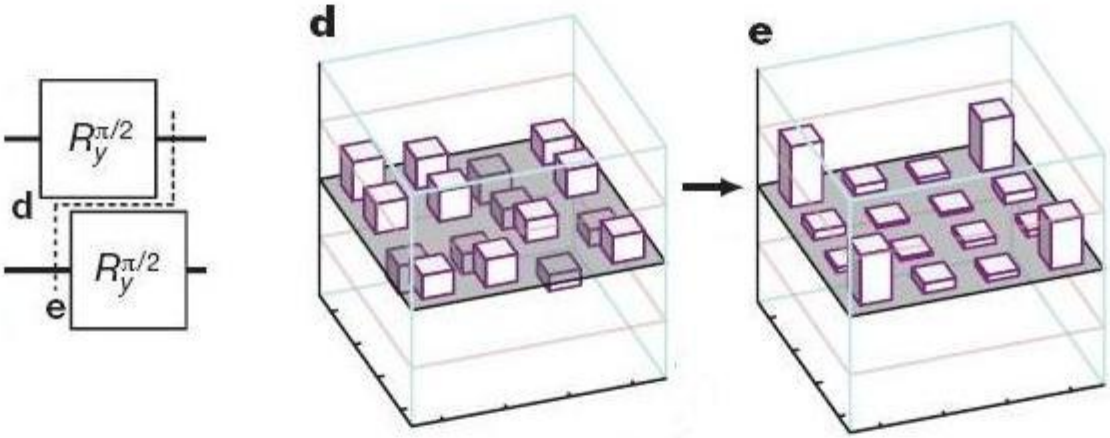
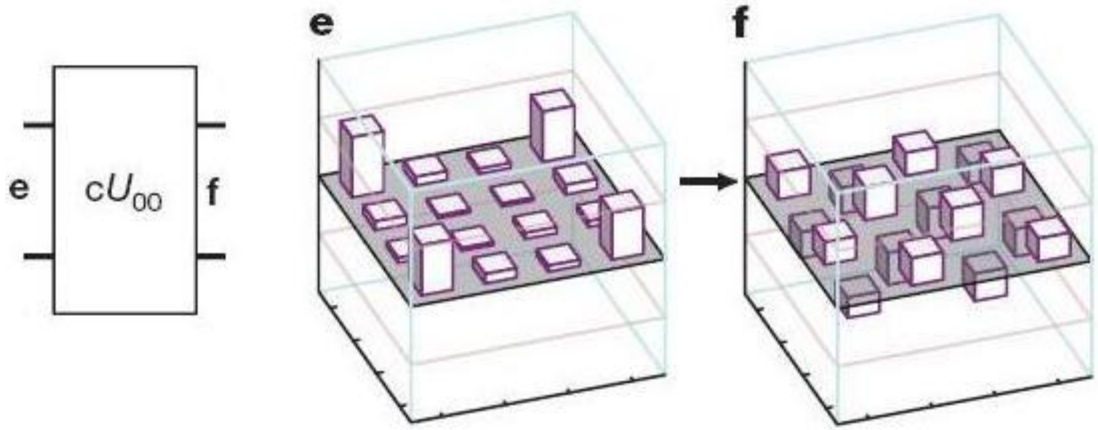
Grover Algorithm



- Apply 1 qubit rotations
- Now in one of the Bell States

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0,0\rangle + |1,1\rangle)$$

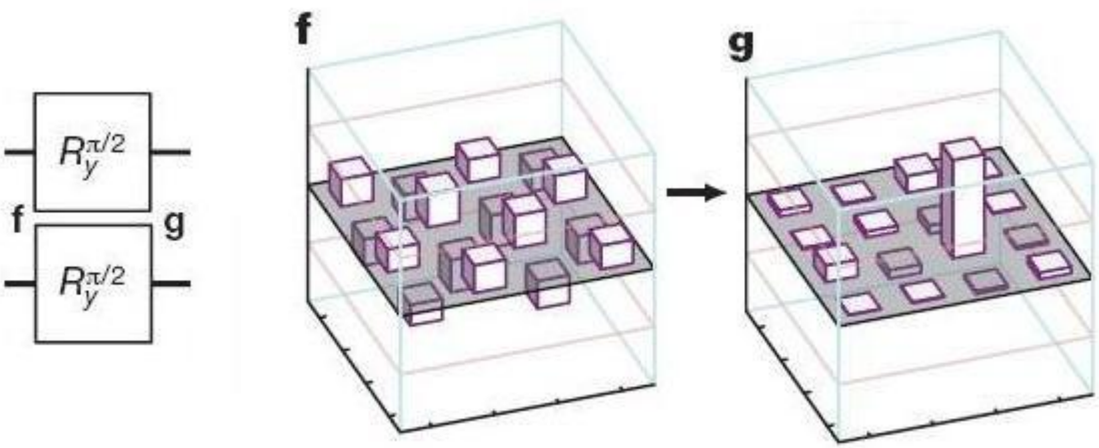
Grover Algorithm



- Apply the operation cU_{00}
- Undoes entanglement, now in equal superposition state

$$|\psi\rangle = \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle\right)$$

Grover Algorithm



- More one qubit rotations produces final state, which is the answer $|1, 0\rangle$.
- Correct state produced with a fidelity of 85%

Conclusion

- Showed the successful implementation of a two qubit system, by means of circuit QED.
- Implementation shows fulfilment of several of the DiVincenzo criteria.
- The architecture used can be expanded to produce a system of several qubits.

Thank you for your attention