# Superconducting circuits I

**letters to nature**

## Demonstration of conditional gate operation using superconducting charge qubits

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LETTERS

## Demonstration of controlled-NOT quantum gates on a pair of superconducting quantum bits

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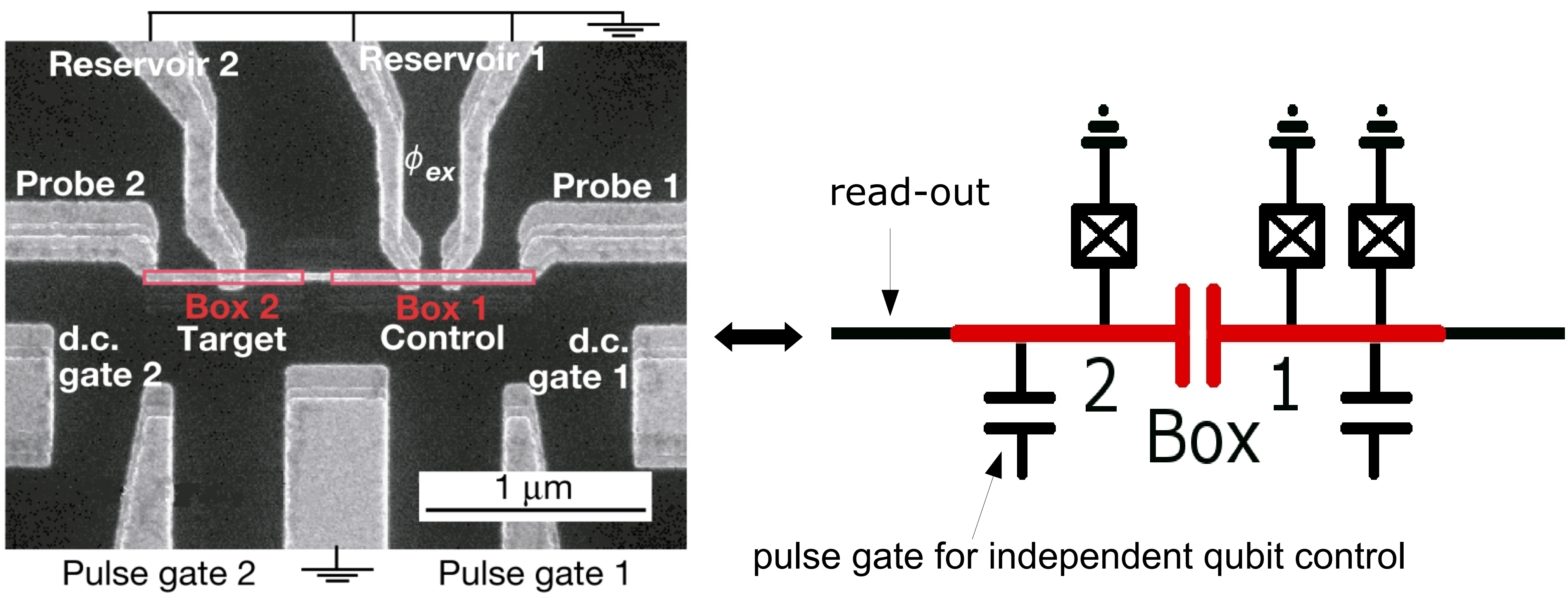
# Motivation

- Universal set of gates: single qubit operation & any entangling gate can be realized with this gates
- Most known entangling gate: CNOT gate
- Definition: Target qubit is flipped if an only if control qubit is in state 1.

$$M_{\mathrm{CNOT}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

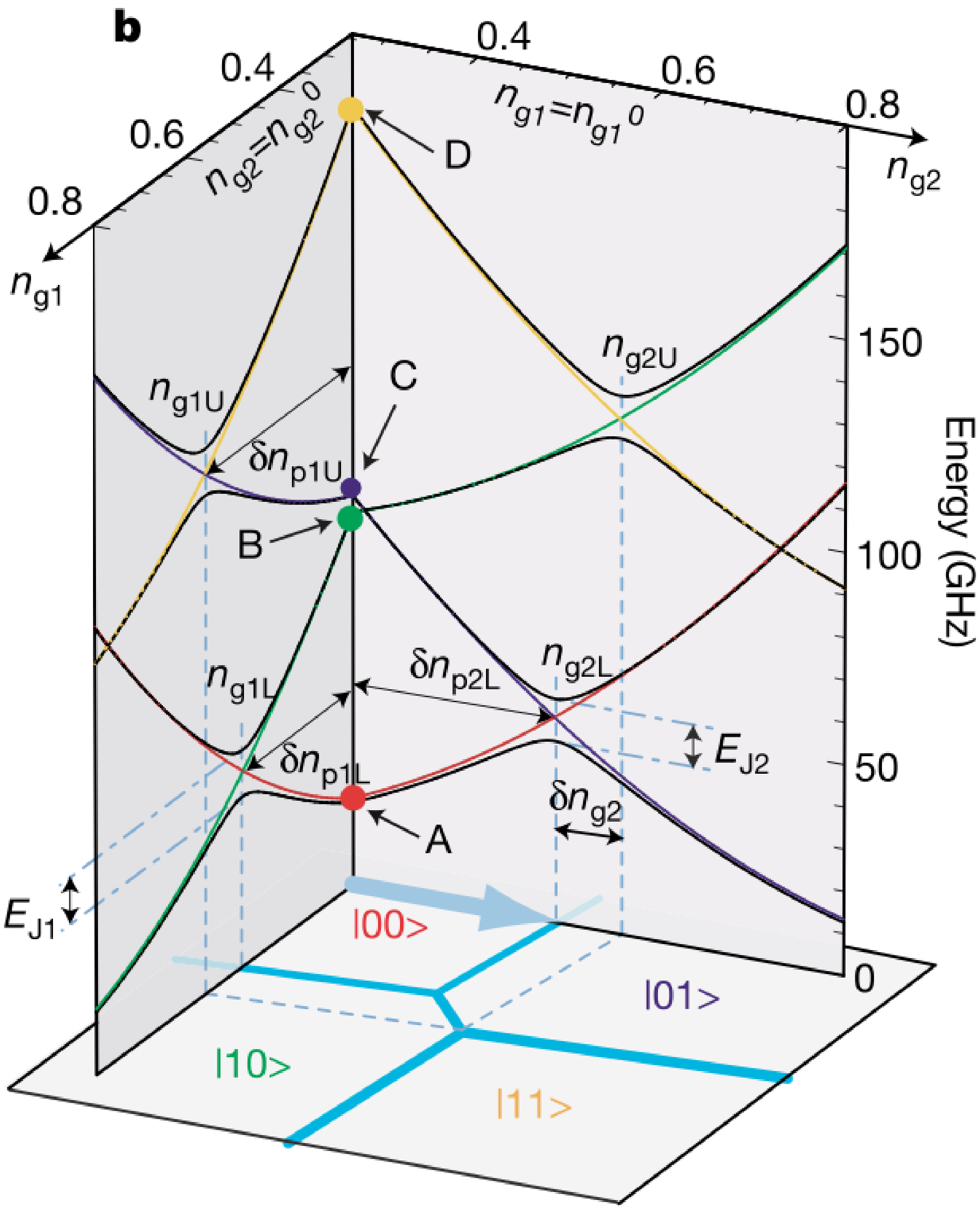
- Desire high fidelity

# Set-up of charge qubit




- 40 mK in a dilution refringerator
- evaporation of Al on a $SiN_x$ insulating layer

# Energy Band Diagram



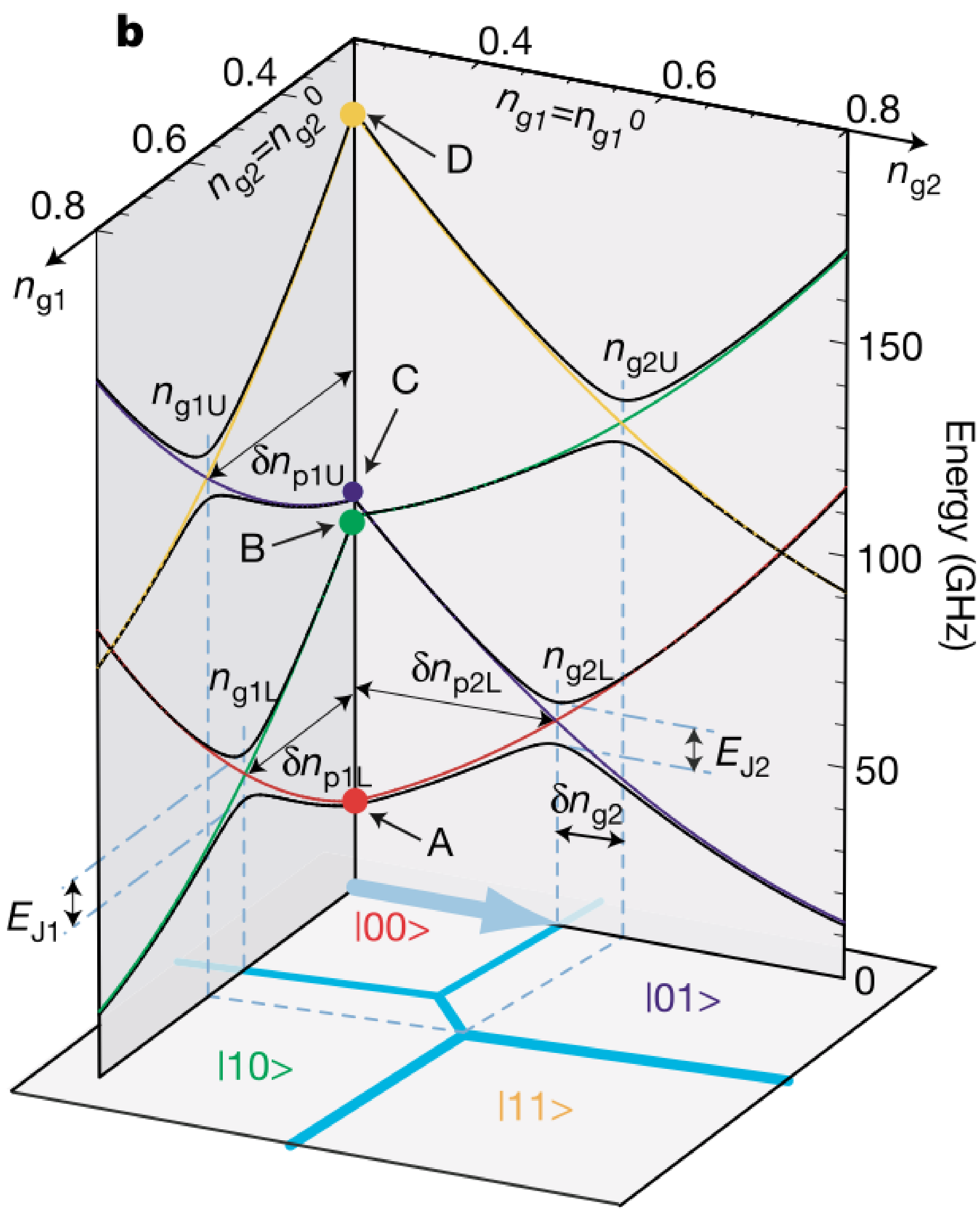

total electrostatic energy

$$H = \sum_{n_1,n_2=0,1} E_{n_1 n_2}|n_1,n_2\rangle\langle n_1,n_2| - \frac{E_{J1}}{2}\sum_{n_2=0,1}(|0\rangle\langle 1| + |1\rangle\langle 0|)\otimes|n_2\rangle\langle n_2| - \frac{E_{J2}}{2}\sum_{n_1=0,1}|n_1\rangle\langle n_1|\otimes(|0\rangle\langle 1| + |1\rangle\langle 0|)$$

Josephson coupling energy (box 2)

- $E_{Ji}$ is the Josephson couplin energy of the i-th box
- $E_{n_1 n_2}$ is the total electrostatic energy

# Energy Band Diagram




$|00\rangle \rightarrow$ A  $|10\rangle \rightarrow$ B

$|01\rangle \rightarrow$ C  $|11\rangle \rightarrow$ D

- time dependent oscillation between A and C

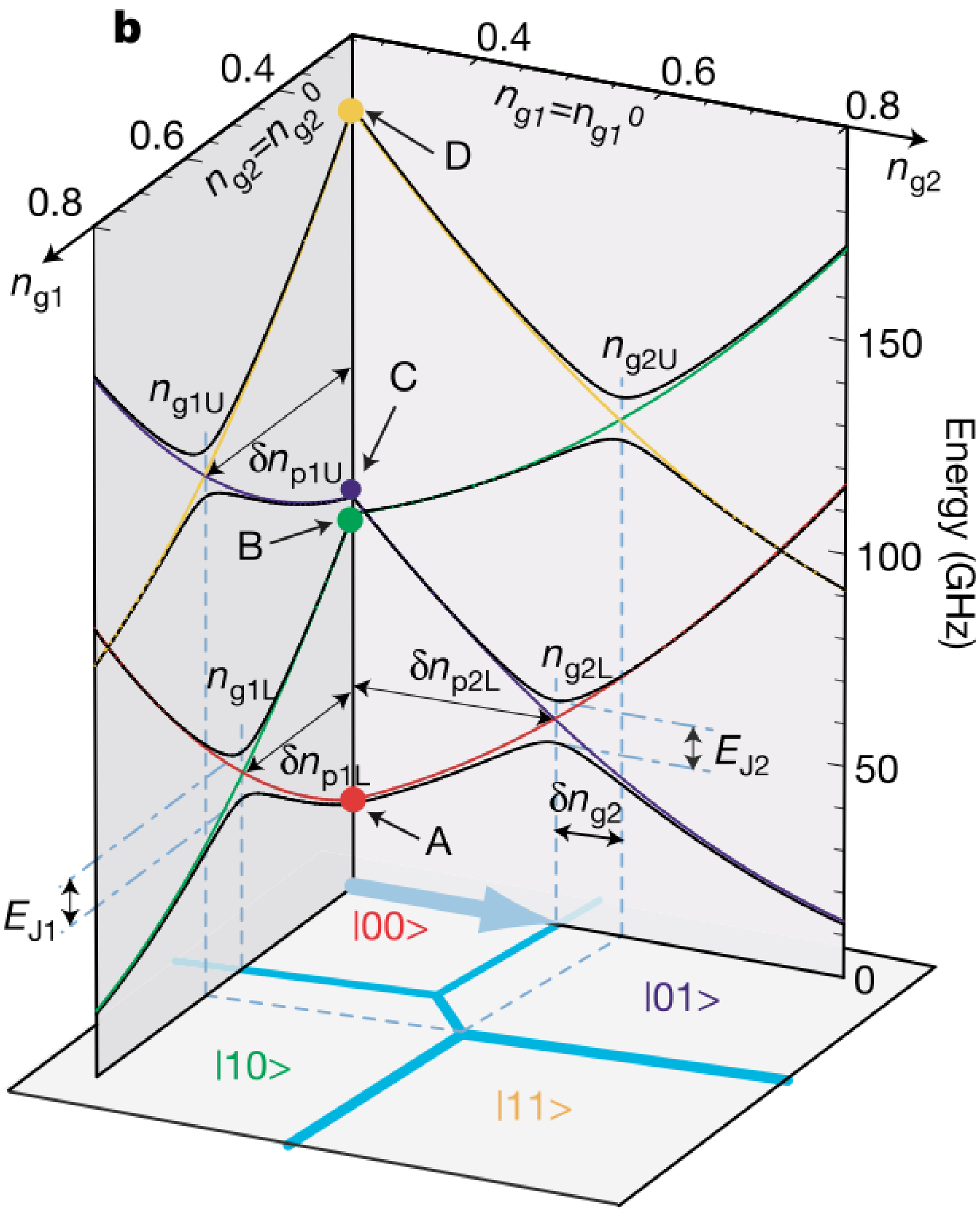
  $\cos(\Omega\Delta t/2)|00\rangle + \sin(\Omega\Delta t/2)\,|01\rangle$

  $\Omega = E_{J2}/\hbar$

- $\Omega\Delta t = \pi$ ($\pi$-pulse)

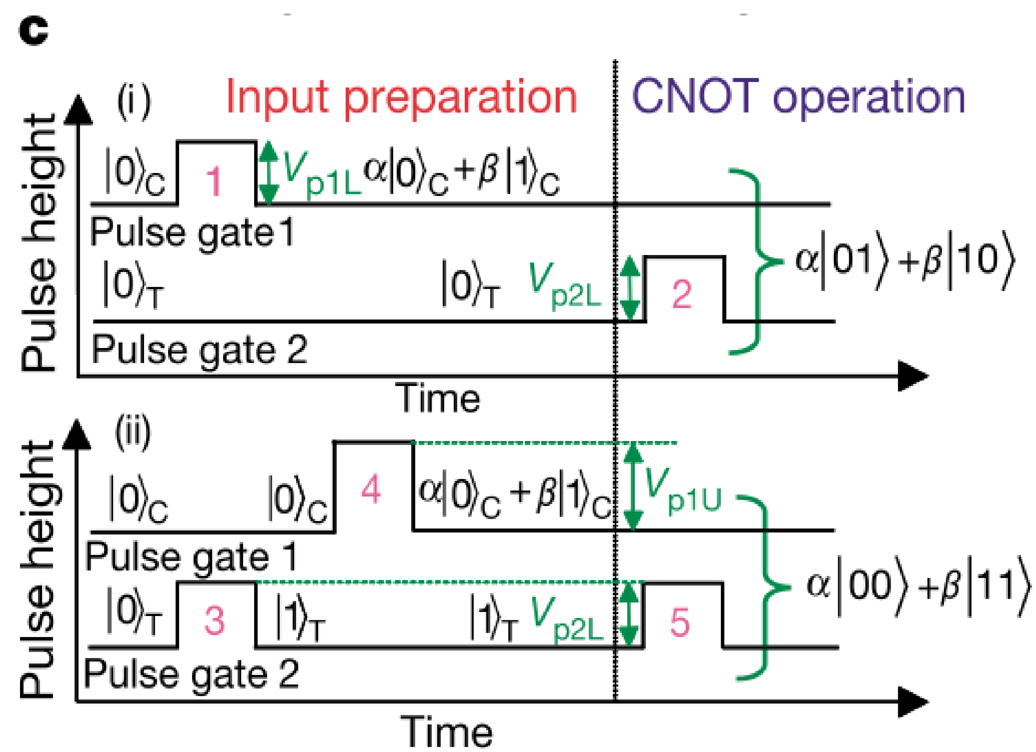
- CNOT-Gate by aplying π-pulses

# Energy Band Diagram

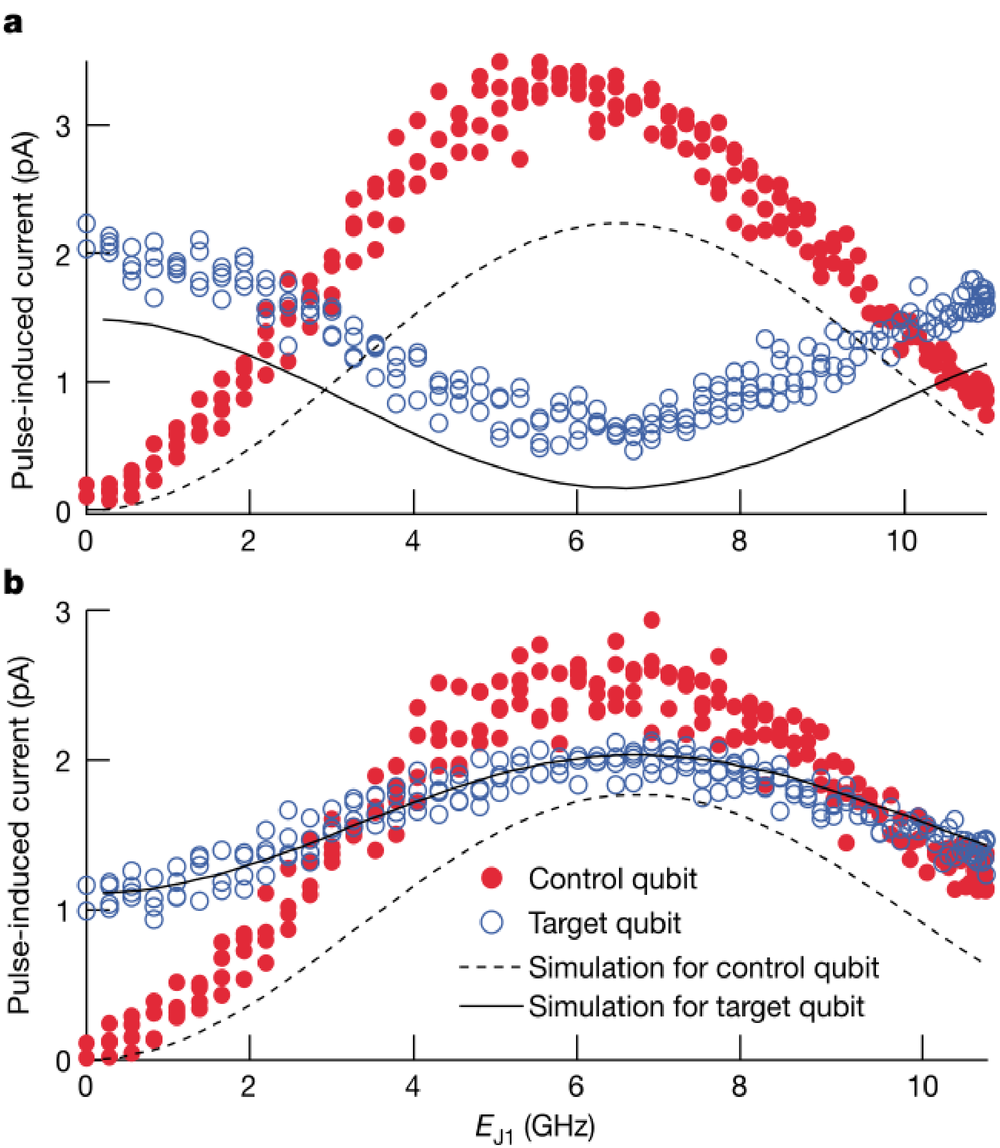


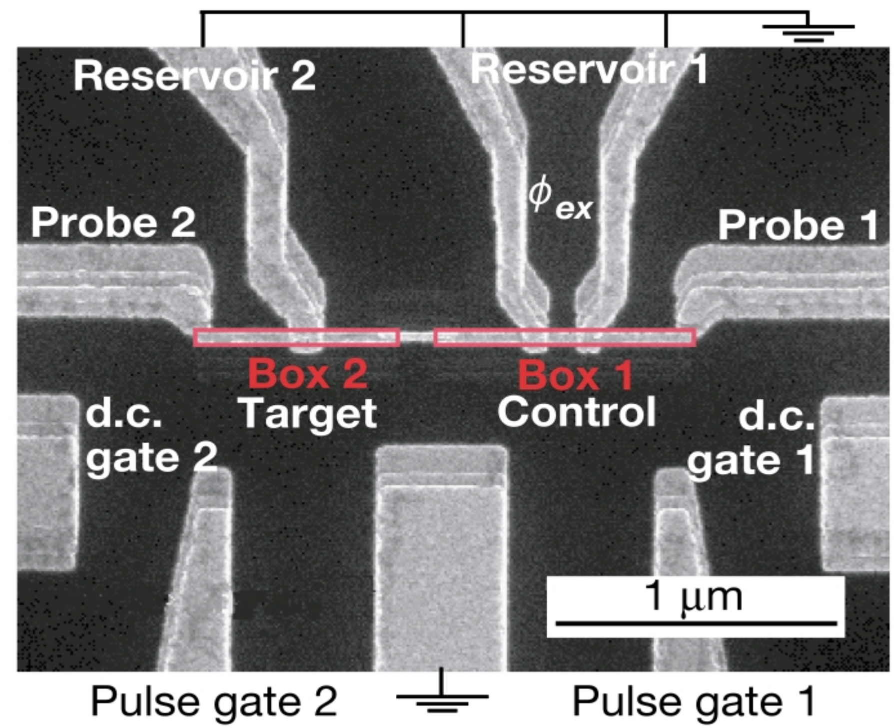

$|00\rangle \rightarrow A$ $\quad$ $|10\rangle \rightarrow B$

$|01\rangle \rightarrow C$ $\quad$ $|11\rangle \rightarrow D$



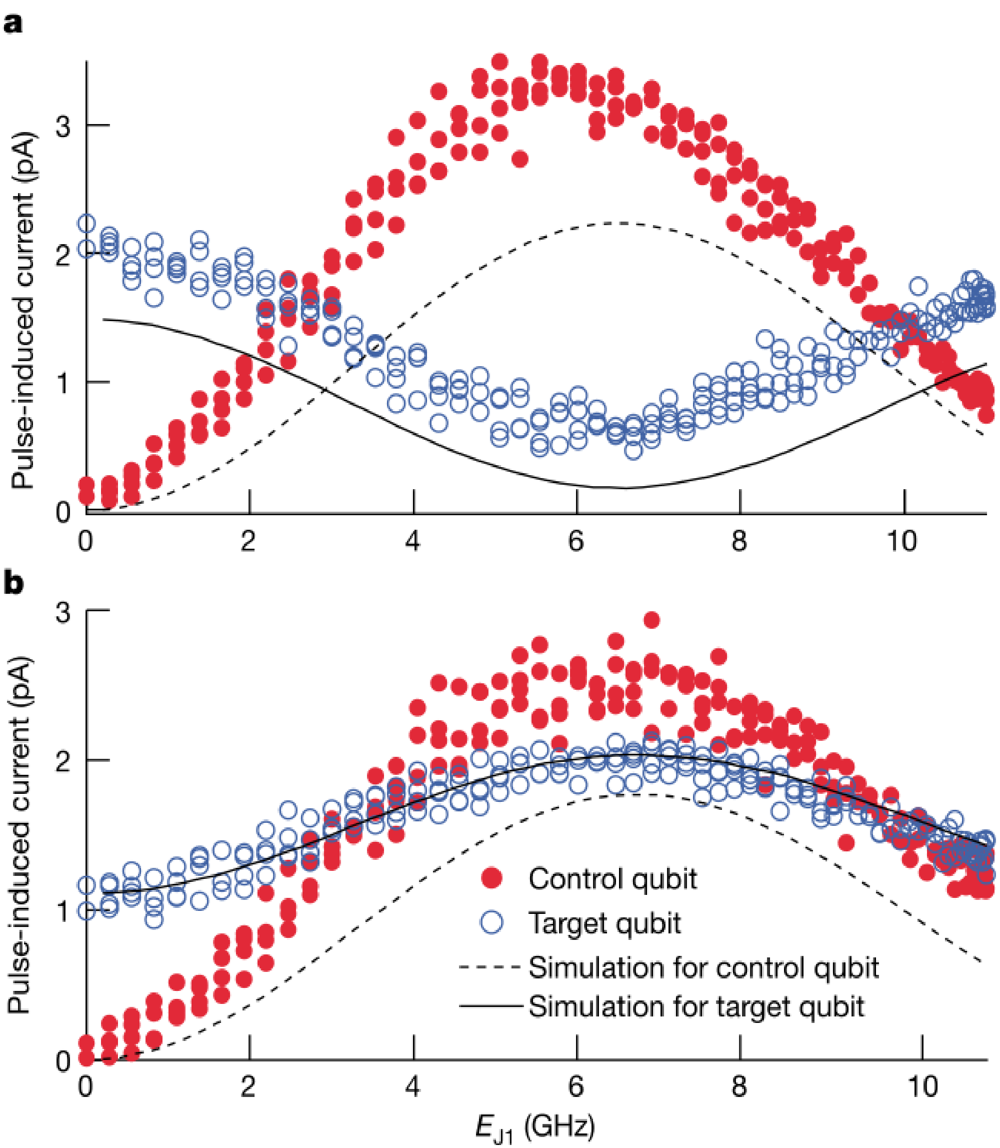
# Pulse induced current

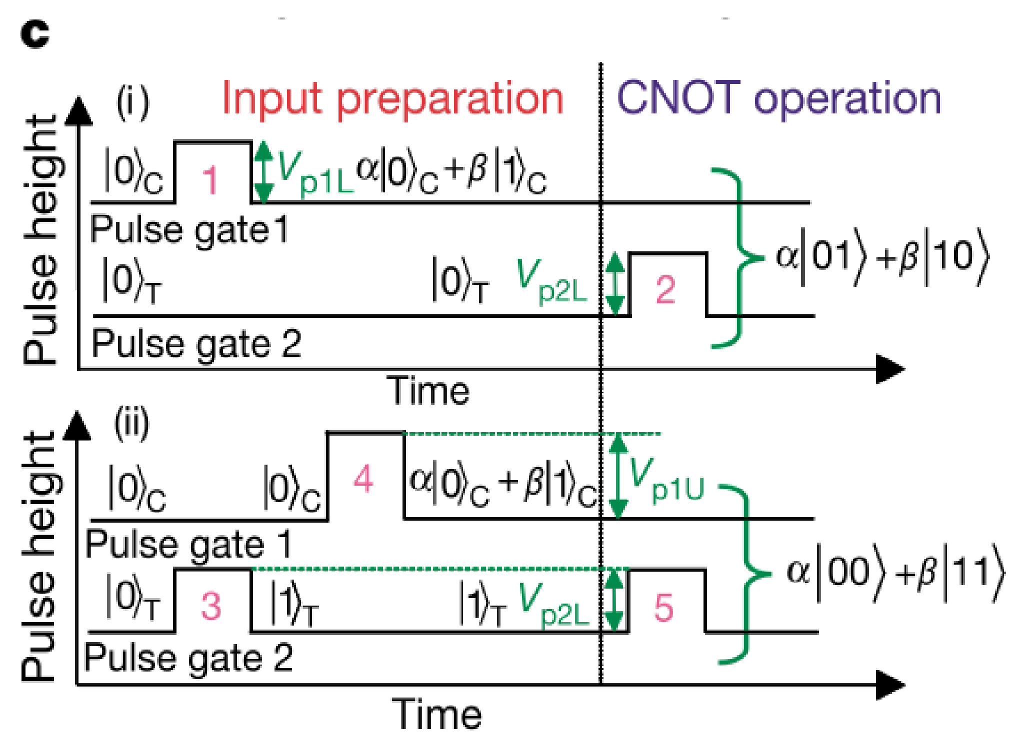



- flux ⟶ Josephson Energy
  ⟶ eigenstates of qubits
  ⟶ output current

# Pulse induced current




- $E_{J1} = E_{J1max}|\cos(\pi\ \phi_{ex}/\phi_0)|$
- rise and fall time of 40 ps for simulation

# Truth table

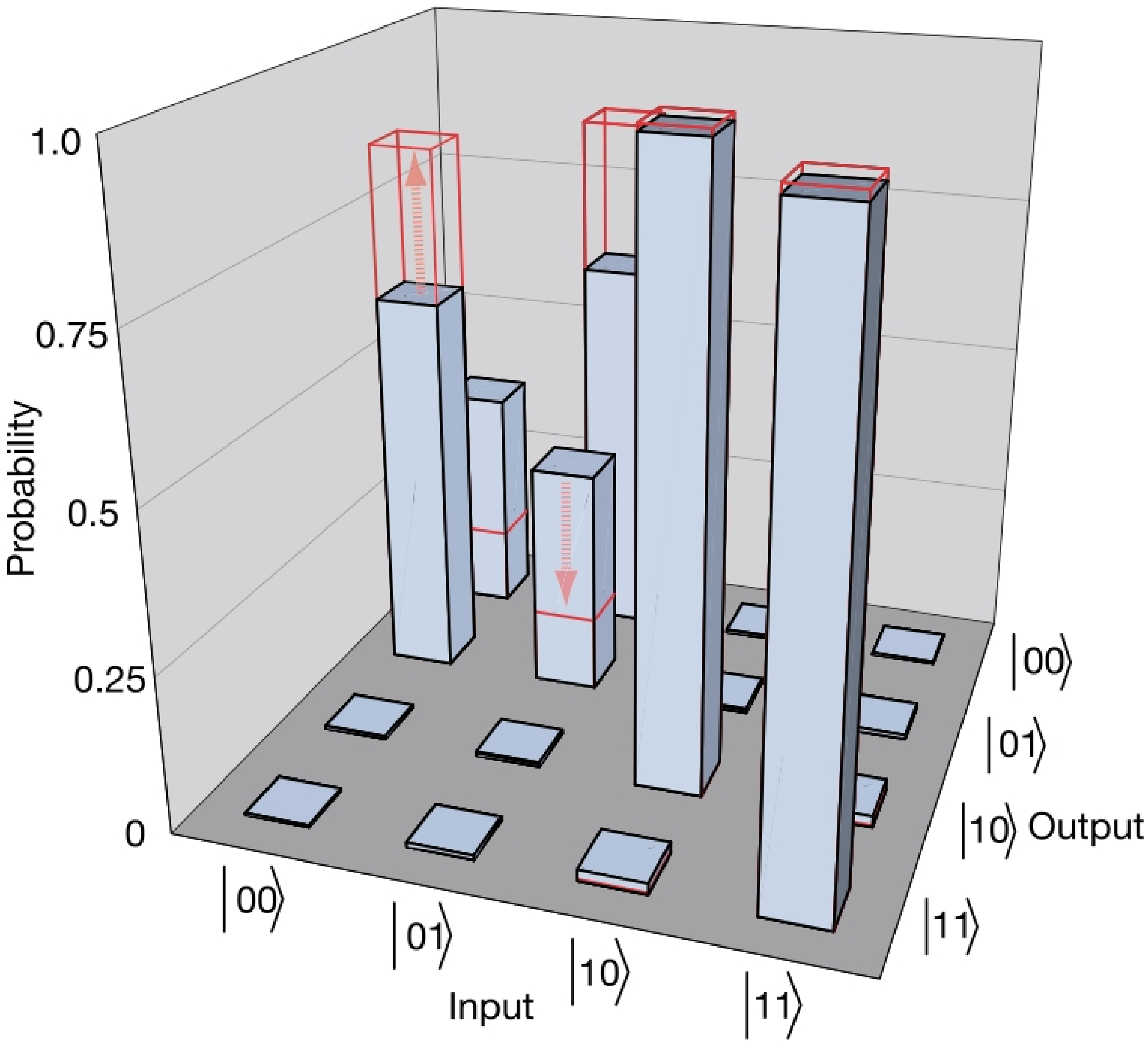
$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- ideal CNOT Operator

- numerical calculated CNOT Operator

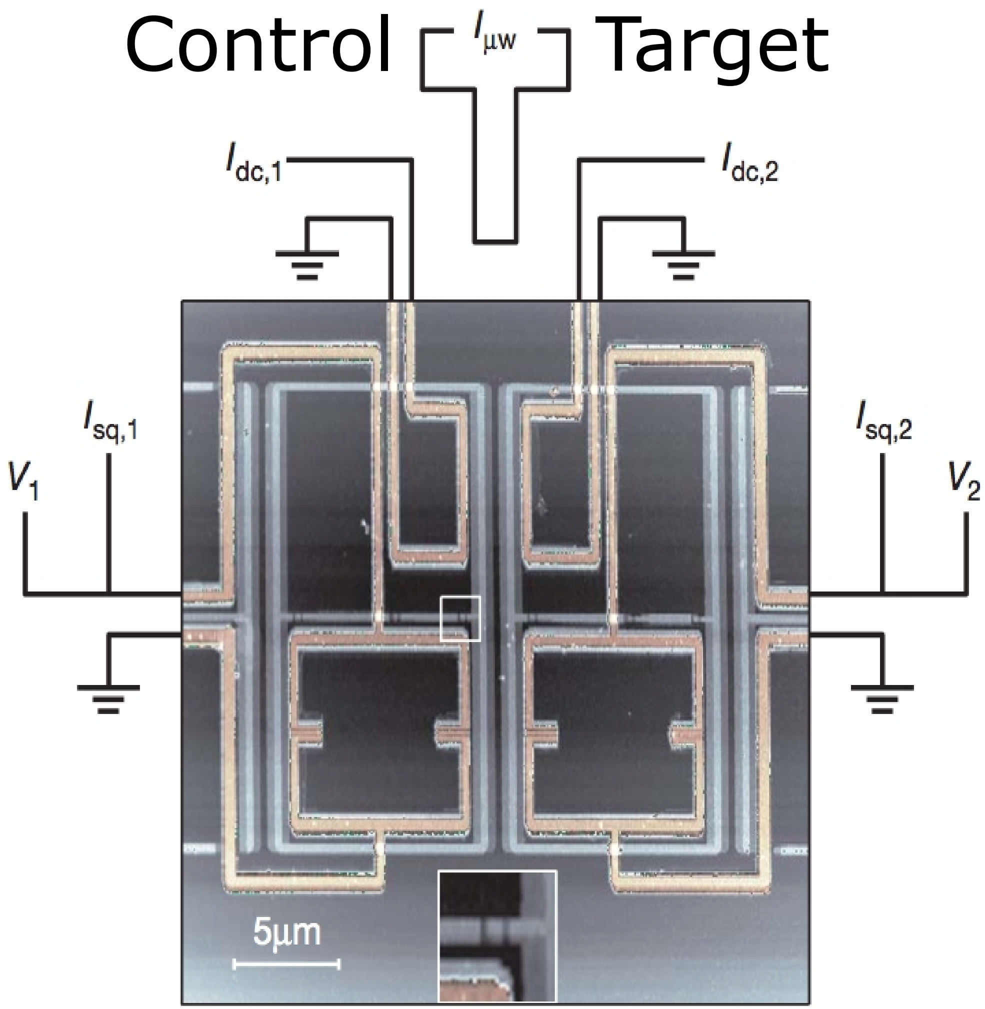
$$\begin{pmatrix} 0.37 & 0.62 & 0.004 & 0.003 \\ 0.62 & 0.37 & 0.004 & 0.007 \\ 0.004 & 0.004 & 0.97 & 0.018 \\ 0.003 & 0.007 & 0.018 & 0.97 \end{pmatrix}$$

# Truth table

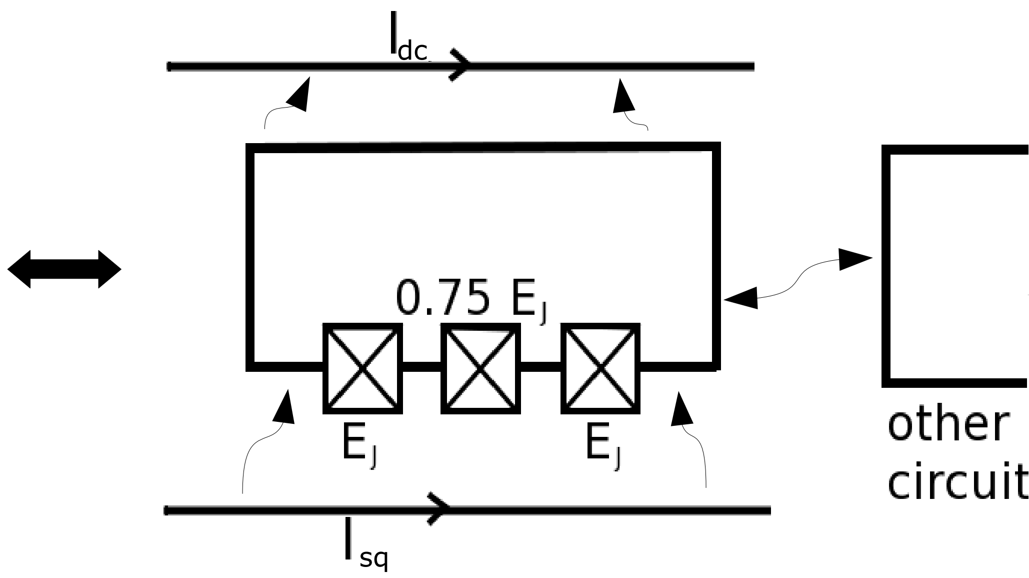



- numerical calculation for 40 ps rise / fall time
- red lines: numerical calculation for 30 ps
- finite fall time → perturbates oscillation of states

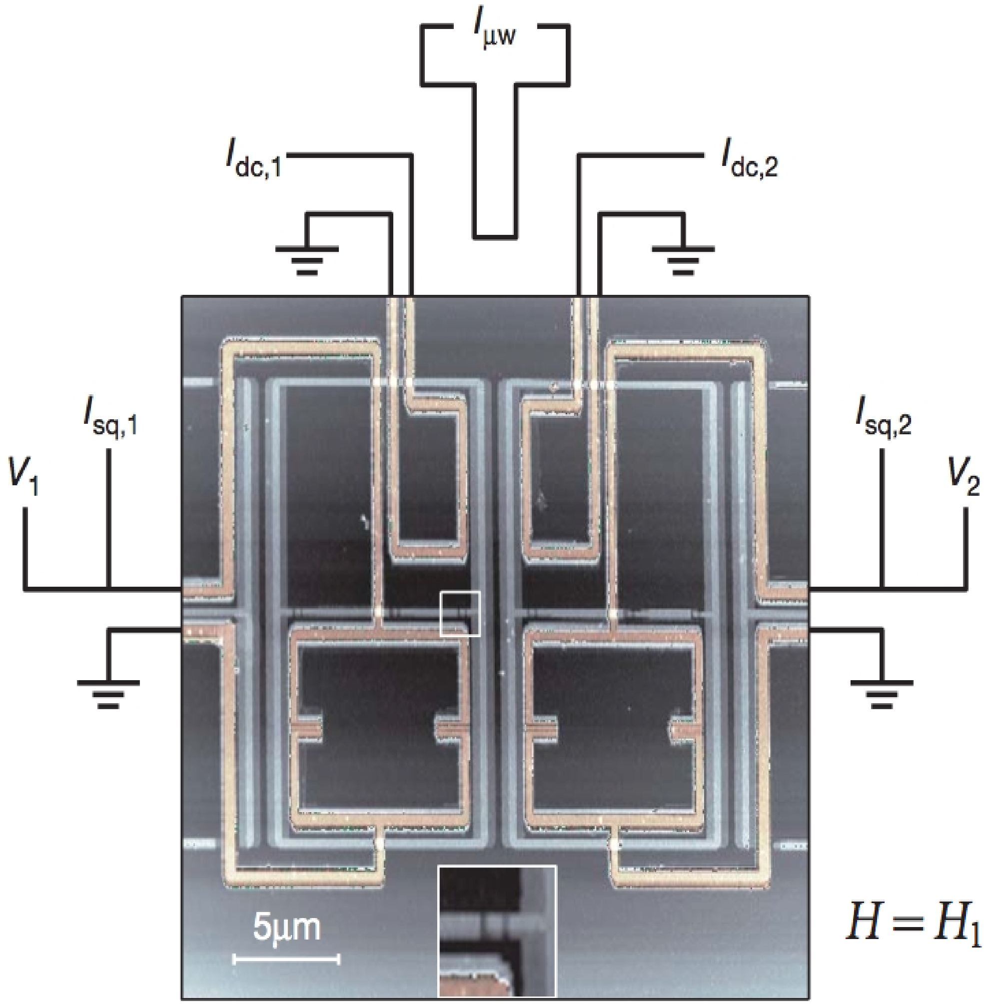
# Set-up of flux qubit




- target / control is exchangable



# Set-up of flux qubit




- qubit energies tuned by B-field of $I_{dc}$
- SQUID used for single qubit read-out
- couple two flux qubits Magnetically → four level system (tunable with flux)

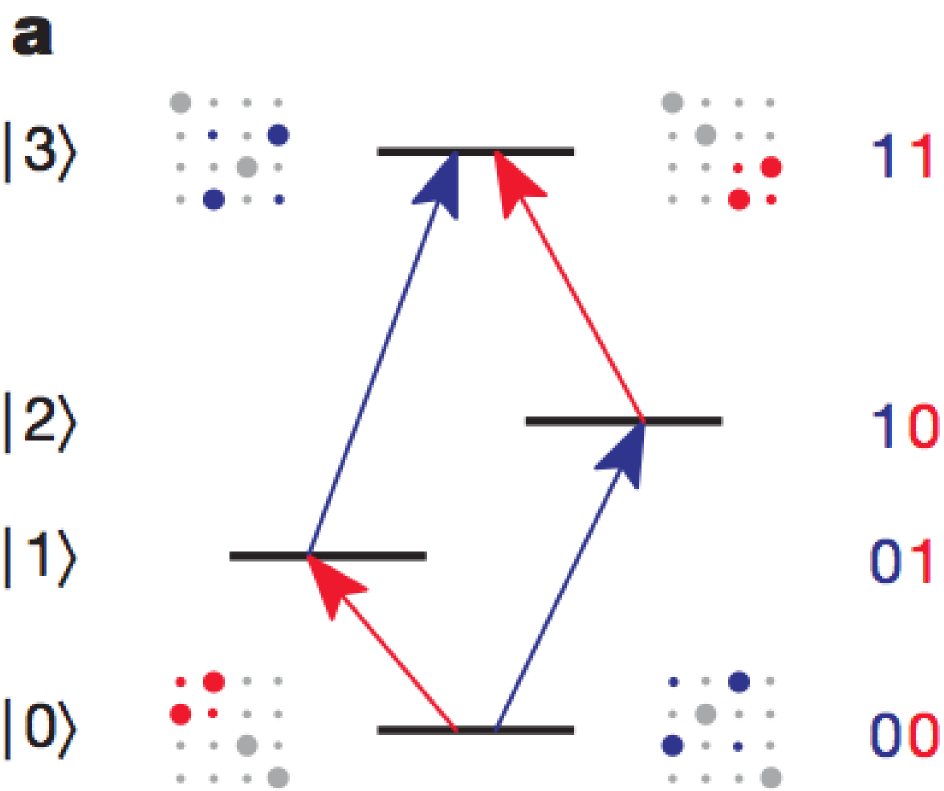
$$H = H_1 + H_2 + H_{12} = -\frac{1}{2}\left(\varepsilon_1\sigma_z^1 + \Delta_1\sigma_x^1 + \varepsilon_2\sigma_z^2 + \Delta_2\sigma_x^2\right) + J\sigma_z^1\sigma_z^2$$

magnetic energy

coupling btw two classical currents

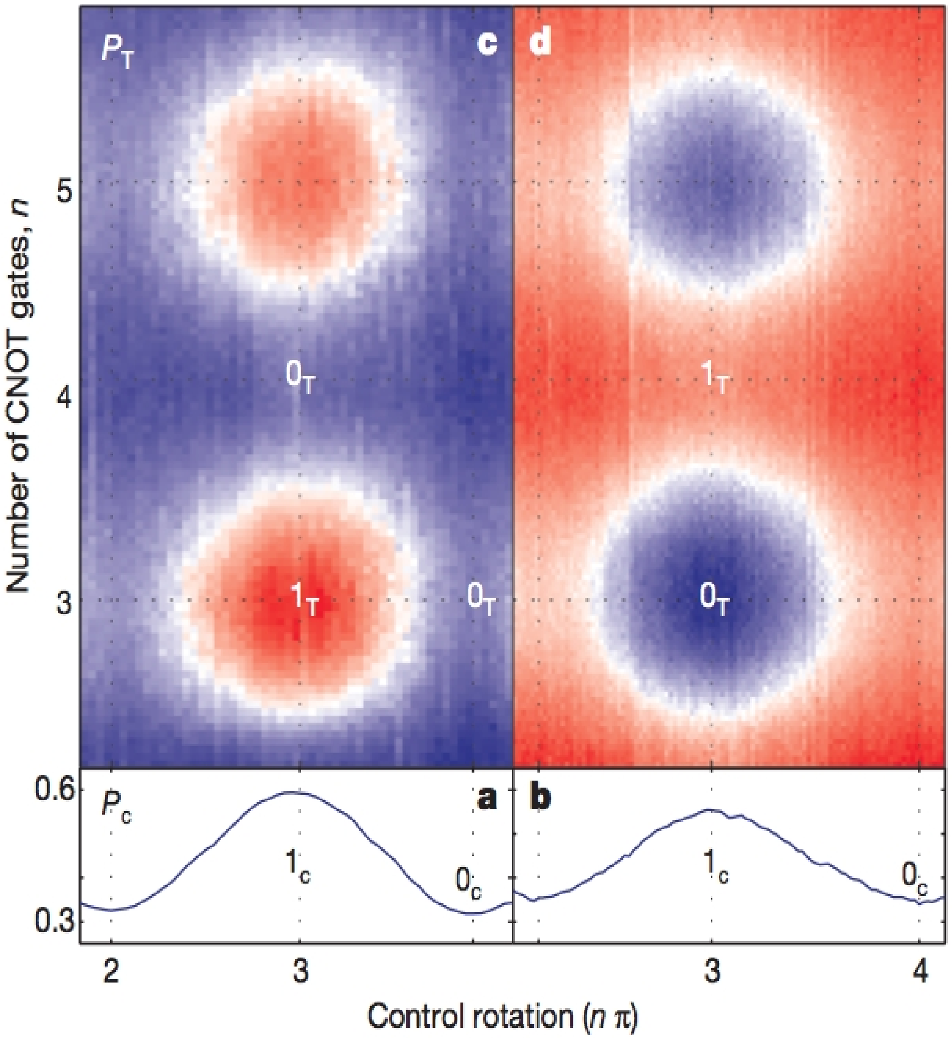
qubit-qubit coupling

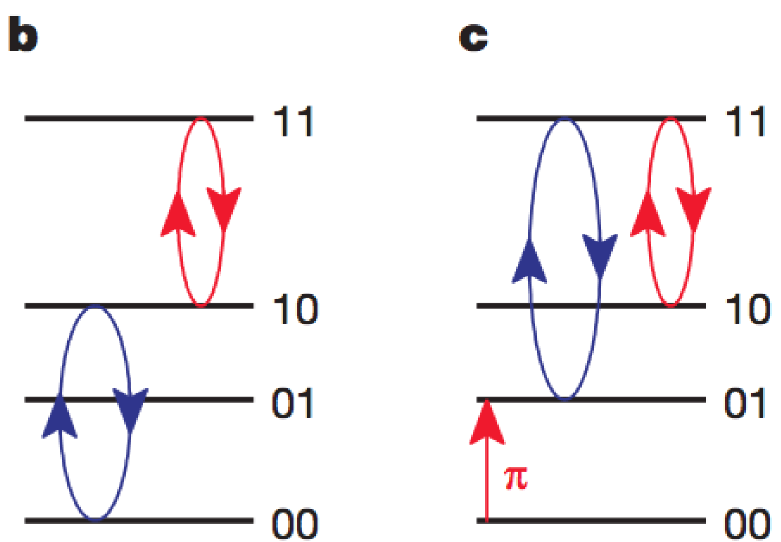
# Four Level System




- four resonance frequencies
- one qubit shifts the other resonant frequency → 1- or 0-controlled operation
- symmetrie of qubits → target and control are exchangeable

# Operation of coupled qubits




- probability of output state as a function of CNOT pulse length and control rotation

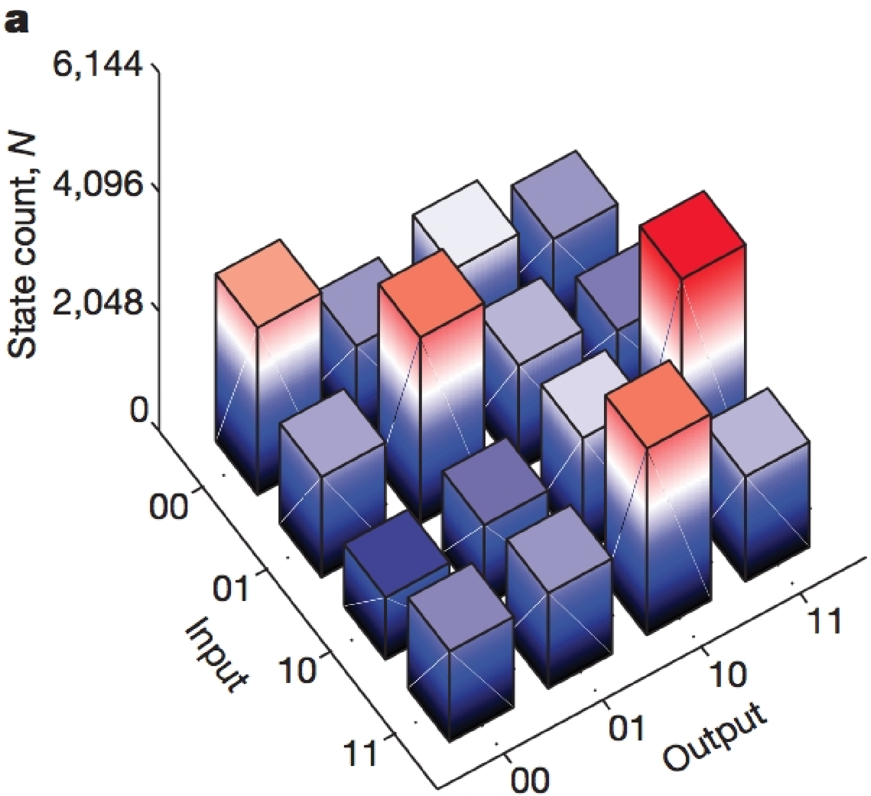
# Operation of coupled qubits

$$M_{\mathrm{CNOT}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

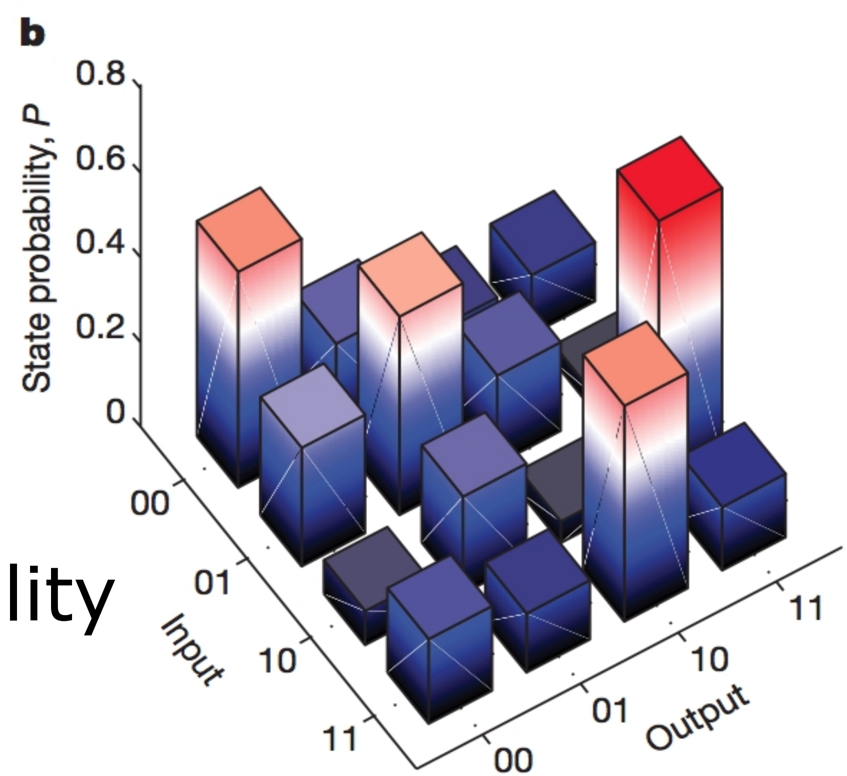
$$M_{\mathrm{exp}} = \begin{pmatrix} 0.51 & 0.22 & 0.13 & 0.14 \\ 0.28 & 0.47 & 0.21 & 0.04 \\ 0.08 & 0.23 & 0.05 & 0.64 \\ 0.20 & 0.14 & 0.51 & 0.15 \end{pmatrix}$$

- $F = \mathrm{Tr}(M_{\mathrm{exp}} M_{\mathrm{CNOT}}^{\mathrm{T}})/4$
- $F_{\mathrm{CNOT}} = 1$
- $F_{\mathrm{exp}} = 0.4$

# Truth table

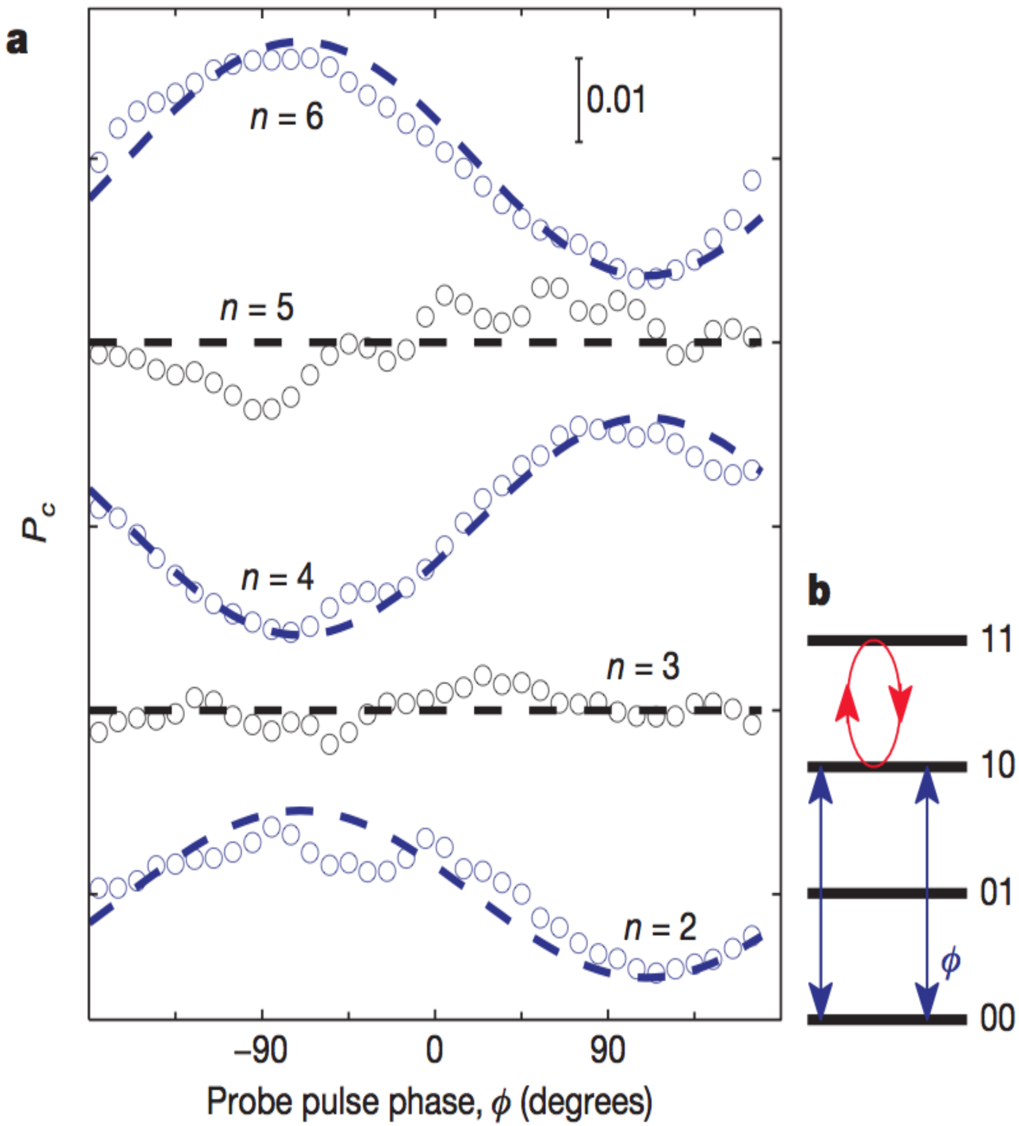



- error sources: decoherence, control errors and 40% measurement visibility




- error sources: still decoherence, control errors but 90% measurement visibility
- normalized

# Phase-shift




- Ramsey-like interference experiment, where n CNOT gates are placed within two pi half pulses on the control qubit
- Probability to obtain 0 or 1
- Quantum phase after n CNOT gates

# Summary

- charge qubit:
  - coupling through capacitance
  - asymmetric setup/configuration
  - operation by by tuning gate voltages
  - measurement of amplitude
  - truth table simulated

- flux qubit:
  - coupling through induction
  - symmetric setup/configuration
  - operation by microwave pulses
  - measurement of amplitude and phase
  - truth table measured