

QSIT 2010 - Questions 3

19. Oktober 2010

1. Rotating Wave Approximation

A two level system with transition frequency Ω is driven by a laser with frequency ω , phase ϕ and amplitude ϵ . The corresponding Hamiltonian is

$$H(t) = \frac{\hbar\Omega}{2}\sigma_z + \hbar\epsilon \cos(\omega t + \phi)\sigma_x.$$

- (a) Show that in the rotating frame and after applying the rotating wave approximation the effective (static) Hamiltonian is given by

$$H_{\text{eff}} = \Delta\sigma_z + \Omega_x\sigma_x + \Omega_y\sigma_y,$$

where the detuning $\Delta = \Omega - \omega$, $\Omega_x = \epsilon \cos \phi$ and $\Omega_y = \epsilon \sin \phi$.

- (b) Choose values for Δ , ϵ and ϕ and a time t that lead to an equally weighted superposition state, a state on the equator of the Bloch sphere. What values lead to state on the equator of the Bloch sphere that points in a direction orthogonal to the former state.

2. CNOT operation

- (a) Verify the unitarity of the CNOT operation given by the transformation

$$\text{CNOT: } \begin{aligned} |00\rangle &\rightarrow |00\rangle \\ |01\rangle &\rightarrow |01\rangle \\ |10\rangle &\rightarrow |11\rangle \\ |11\rangle &\rightarrow |10\rangle, \end{aligned}$$

where A in $|AB\rangle$ is the control qubit and B the target.

- (b) Determine the matrix for the CNOT operation, where B is the control qubit.