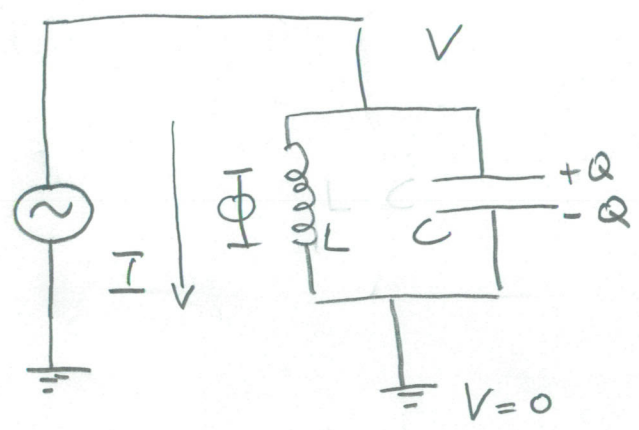


# Quantum Information Processing with Electronic Circuits

goal: learn how to construct and operate electronic circuits that behave according to the laws of quantum mechanics

- discuss:
- basic circuit elements and basic circuits
    - ↳ harmonic oscillator
    - ⇒ store photons on a chip
  - role of ⇒ discuss q.m. description of LC oscillator
  - role of dissipation
  - role of temperature

# Electronic Harmonic Oscillator



↑ V

Compare to mechanical oscillator. Which are the corresponding quantities?

- charge on capacitor

$$Q = CV$$

- flux in inductor

$$\Phi = LI$$

- Voltage across oscillator

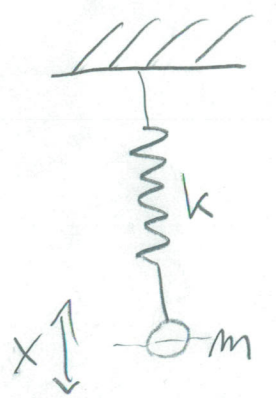
$$V = \frac{Q}{C} = -LI = -\dot{\Phi}$$

## Hamiltonian

$$H = \frac{CV^2}{2} + \frac{LI^2}{2} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

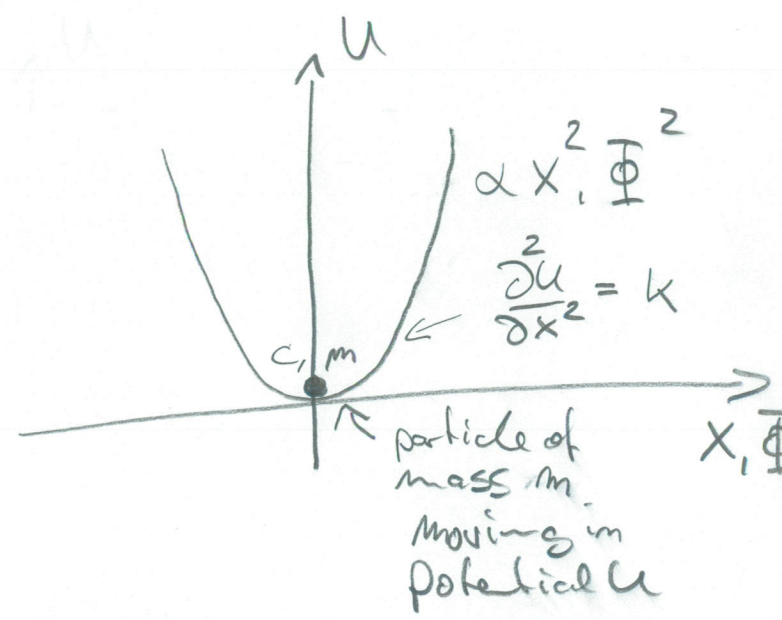
electrostatic energy      magnetic energy

compare to mechanical harmonic oscillator



$$H = \frac{p^2}{2m} + \frac{kx^2}{2}$$

kinetic energy      potential energy



# Characteristic Quantities

## mechanical

Position  $X$

Momentum  $P$

Mass  $m$

Spring constant  $k$

resonance frequency  $\omega = \sqrt{\frac{k}{m}}$

## electronic

flux  $\Phi$   
charge  $Q$

Capacitance  $C$

inverse inductance  $\frac{1}{L}$

$\omega = \frac{1}{\sqrt{LC}}$

## harmonic oscillator

conjugate variables

$$\frac{\partial H}{\partial \Phi} = \frac{\Phi}{L} = I = \dot{Q}$$

$$\frac{\partial H}{\partial Q} = \frac{Q}{C} = V = -L\dot{I} = -\dot{\Phi}$$

• quantum mechanical operators:

$$\hat{X} = X$$

$$\hat{P} = -i\hbar \frac{\partial}{\partial X}$$

$$\hat{\Phi} = \Phi$$

$$\hat{Q} = -i\hbar \frac{\partial}{\partial \Phi}$$

We know how to quantize mechanical oscillator

• Commutation relations

$$[\hat{X}, \hat{P}] = i\hbar$$

$$[\hat{\Phi}, \hat{Q}] = i\hbar$$

flux - charge

$$\Leftrightarrow \left[ 2\pi \frac{\hat{\Phi}}{\Phi_0}, \frac{\hat{Q}}{2e} \right] = [\hat{\delta}, \hat{N}] = i$$

phase - number

# Hamilton Operator

- using conjugate variables  $Q, \Phi$

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} = -\frac{\hbar^2}{2C} \frac{\partial^2}{\partial \Phi^2} + \frac{1}{2L} \hat{\Phi}^2$$

- using creation and annihilation operators

$$\hat{a}^+ = \frac{1}{\sqrt{2\hbar Z_c}} (Z_c \hat{Q}^+ - i \hat{\Phi}^+) \quad \text{creation operator}$$

$$\hat{a} = \frac{1}{\sqrt{2\hbar Z_c}} (Z_c \hat{Q} + i \hat{\Phi}) \quad \text{annihilation operator}$$

with  $Z_c = \sqrt{L/C}$  impedance of oscillator

$$\hat{H} = \hbar \omega (\hat{a}^+ \hat{a} + \frac{1}{2})$$

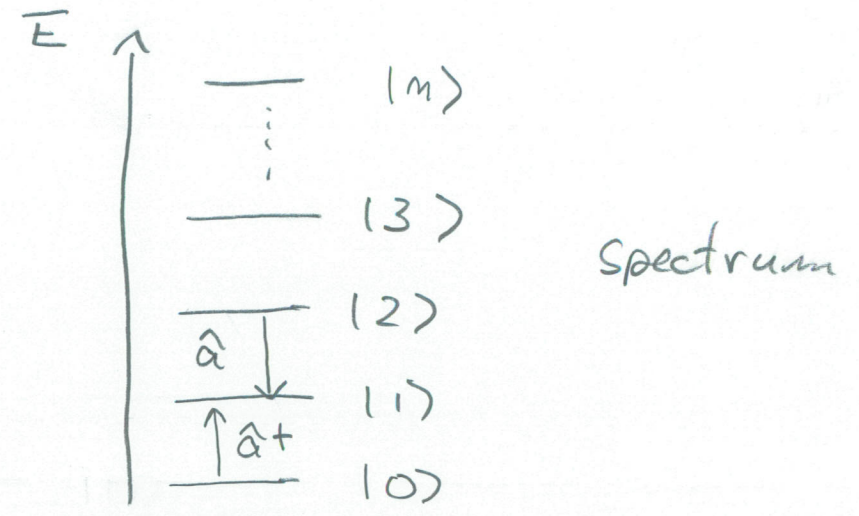
# Properties of $\hat{a}^\dagger$ and $\hat{a}$

$$\hat{a}^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle$$

$$\hat{a} |n\rangle = \sqrt{n} |n-1\rangle$$

$$\hat{a}^\dagger \hat{a} |n\rangle = n |n\rangle$$

with  $|n\rangle$  number (Fock) state of harmonic oscillator



• relation to  $\hat{Q}$  and  $\hat{\Phi}$

$$\hat{Q} = \sqrt{\frac{\hbar}{2Z_c}} (\hat{a}^\dagger + \hat{a})$$

→ relates to electric field stored on capacitor

$$\hat{\Phi} = i \sqrt{\frac{\hbar Z_c}{2}} (\hat{a}^\dagger - \hat{a})$$

→ relates to magnetic field stored in inductor

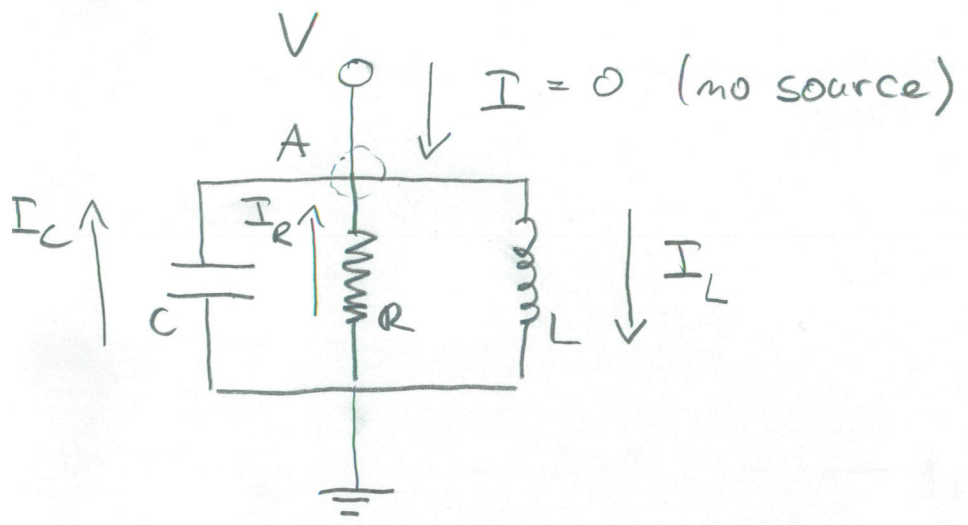
or 
$$\hat{V} = \sqrt{\frac{\hbar \omega}{2C}} (\hat{a}^\dagger + \hat{a})$$

$$\hat{I} = i \sqrt{\frac{\hbar \omega}{2L}} (\hat{a}^\dagger - \hat{a})$$

with  $\omega = \frac{1}{\sqrt{LC}}$  and  $V = \frac{Q}{C}$  and  $I = \frac{\Phi}{L}$

# Dissipation in the Harmonic Oscillator

What is the role of dissipation in an electrical oscillator? How does it arise?



with  
• current through resistor

$$I_R = V/R$$

• displacement current

$$I_C = \dot{Q}_C = C \dot{V}$$

• voltage across inductor

$$V = -L \dot{I}_L$$

• Kirchoff law at point A

$$I_L = I_R + I_C + I$$

$$\Leftrightarrow -C \dot{V} - \frac{V}{R} + I_L = 0 \quad (\text{same voltage at A})$$

$$\Leftrightarrow \boxed{\ddot{I}_L + \frac{1}{RC} \dot{I}_L + \frac{1}{LC} I_L = 0}$$

differential equation for current through inductor

• solutions

$$I_L(t) = I_L(0) e^{\lambda t} \quad \text{with} \quad \lambda_{1,2} = \frac{1}{2LC} \left( -\frac{L}{R} \pm \sqrt{\left(\frac{L}{R}\right)^2 - 4LC} \right)$$

# Energy Decay Rate

• underdamped oscillator

$$(4LC \gg L/R)$$

$$\lambda_{1,2} = -\frac{1}{2RC} \pm i \frac{1}{\sqrt{LC}} = -\alpha \pm i \omega$$

with  $\alpha = \frac{1}{2RC} = \frac{1}{\tau}$

amplitude decay constant

$$\tau = 2RC$$

amplitude decay time

$$\omega = 1/\sqrt{LC}$$

oscillator frequency

• energy decay rate

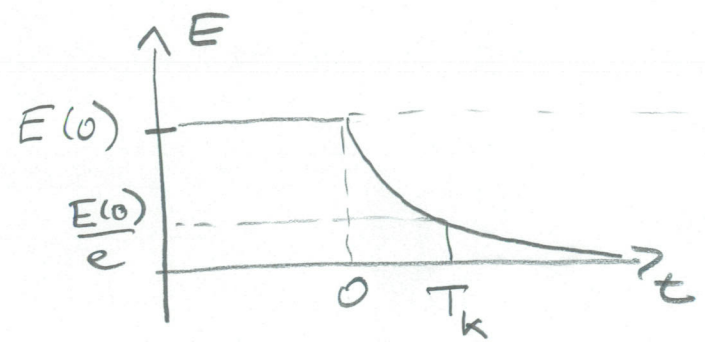
$$E \propto \frac{1}{2} L I_L^2 \propto e^{-\frac{1}{RC} t}$$

with  $\tau_k = \frac{1}{RC}$

energy decay rate

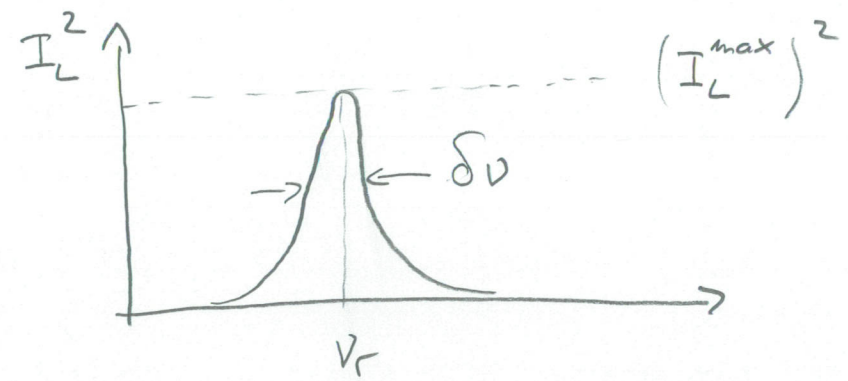
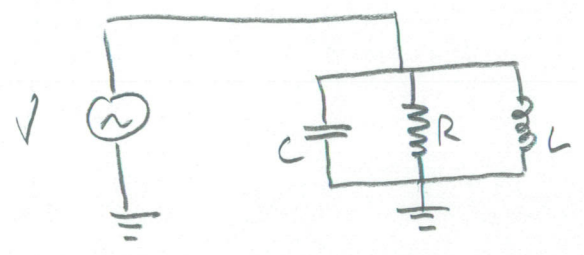
$$\tau_k = RC$$

energy decay time



# Spectral Response of Damped Harmonic Oscillator

- driven damped oscillator



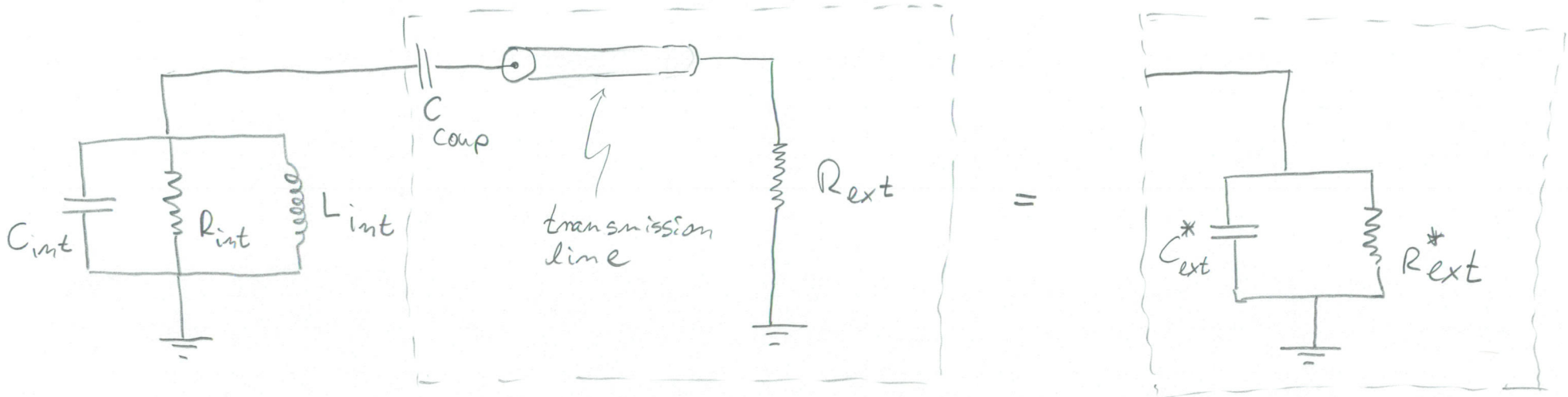
Lorentzian line shape

$$I_L^2(\nu) = (I_L^{\max})^2 \frac{\delta\nu/\pi}{(\nu - \nu_r)^2 + \delta\nu^2}$$

with  $\delta\nu$  : full width of line at half maximum



# Internal and External Dissipation



harmonic oscillator

external circuitry

- total effective resistance  $\frac{1}{R_{tot}} = \frac{1}{R_{int}} + \frac{1}{R_{ext}^*}$   $\rightarrow$  external contribution to energy decay
- total effective capacitance  $C_{tot} = C_{int} + C_{ext}^*$   $\rightarrow$  frequency shift due to external circuit
- energy decay time of combined system  $T_k = R_{tot} C_{tot}$

Show slides on Superconductivity and realizations of harmonic oscillators