Relevance of Single Particle Quantum Phenomena for Quantum Information Science

QIS: - requires control over single quantum systems
- requires possibility to construct more complex systems (bottom up approach) from single particles

⟹ new insights into the nature of quantum physics driven by
- curiosity in new regimes of physics
- new experimental techniques and methods available

↳ compare with other developments e.g. triggered by the advent of low temperatures.
- superconductivity (low T_c, high T_c)
- Hall effect (quantum & fractional)
- superfluidity

State of the art of Quantum Computing

- many physical systems have been investigated:
  - NMR
  - ion traps
  - charges & fluxes in supercond.
  - charges & spins in semicond.
  - neutral atoms
  - NV centers

- level reached:
  - factored 15 = 3 × 5 (NMR)
  - realized quantum byte (ions)
  - several 10s to 100s of manipulations possible (ions, supercond.)
  - algorithms realized (NMR, ions, s.c. qubits)

- the challenge:
  - realize larger systems

Computer Science Perspective

1936 Beginning of modern computer science

- Turing provides abstract definition of a programmable computer
- Universal Turing Machine: provides full description of any classical algorithmic information processing machine
- Church - Turing - Thesis (strong version): Any algorithmic process that can be executed on any hardware can be simulated efficiently on a Turing Machine.

  ↳ basis for theory of computer science

- John von Neumann defines components necessary for realizing a computer

1947 - Bardeen, Brattain, Shockley develop transistor at Bell Labs (USA)

⇒ beginning of efficient and low cost realization of computers in electronic circuits

Show slides on transistor and Moores law!

1947 – Now : Development of Computer Hardware follows Moore's Law

"Doubling of number of transistors on a processor every two years at constant cost"

=> show slide.

=> amazing success of technology !

- BUT how much longer can Moore's Law continue to be valid ?
- What are the consequences of continuing miniaturization ?
- What to do in post Moore's Law era ?

Estimate how much longer Moores law can continue ?
What is the role of quantum mechanics ?

$2 \log_2 650$
$= 18.5$ years
=> linear

$2 \log_2 (650^2)$
=> 37 years

Quantum Computation

- a new paradigm of computing
- uses theory of quantum mechanics for performing computations
- has speed (efficiency) advantage that cannot be overcome by any conceived classical computing scheme
- quantum computers can be simulated on classical computers BUT NOT EFFICIENTLY (c.f. Strong Church-Turing Thesis)

What could efficiently mean in the context of computation?

efficient:

computer running in time polynomial in the size of the problem

inefficient:

computer running in time super-polynomial (typically exponential) in the size of the problem

Important Developments in the Theory of Quantum Computation

- **1985 Deutsch**:
  - starts search for device to simulate efficiently any arbitrary physical (quantum) system
  - seeks a challenge for strong Church-Turing-Thesis
  - proposes to use device that is quantum mechanical
  - presents first example algorithm now known as the Deutsch Algorithm

How many steps are required classically to solve Deutsch's problem?

- **1994 Shor**:
  - develops algorithm that efficiently finds prime factors
  - no efficient classical algorithm exists (no proof though)
- **1995 Grover**:
  - finds more efficient quantum algorithm to search in unstructured data bases
- **1982 Feynman**:
  - proposes to efficiently simulate physical quantum systems on computers based on the principles of quantum mechanics

Other Algorithms ?

- finding quantum algorithms is difficult
  - adverse to intuition based on classical world
- quantum algorithms need to be better than classical ones
  - Note: all classical algorithms can be run on a quantum computer (universality)
- it is not fully understood what makes a quantum computer more powerful than a classical one
  - superposition ?
  - entanglement ?

  ↳ big challenge

Why could it be difficult to find good quantum algorithms?

Topic: Basic Elements of a Quantum Computer

Key Questions: Which components (or features) does a generic quantum computer have?

> Example: Memory to store information.

How are those different from a classical computer?

Your Background:
- General knowledge of how computers work
- Knowledge of quantum mechanics

} COMBINE

Goal:
- Understand basic structure and operation of a quantum computer.
- Make use of this 'hardware independent' knowledge about quantum computers to evaluate different physical realizations

Outline:

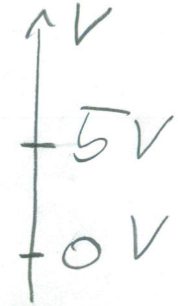
Aspect of Classical Information Processing → Structure of a quantum computer → Features of its components

Classical Information Processing

- Carrier of information in binary representation

example:

BIT — possible values: 1, 0 — physical representation:



voltage level in a circuit

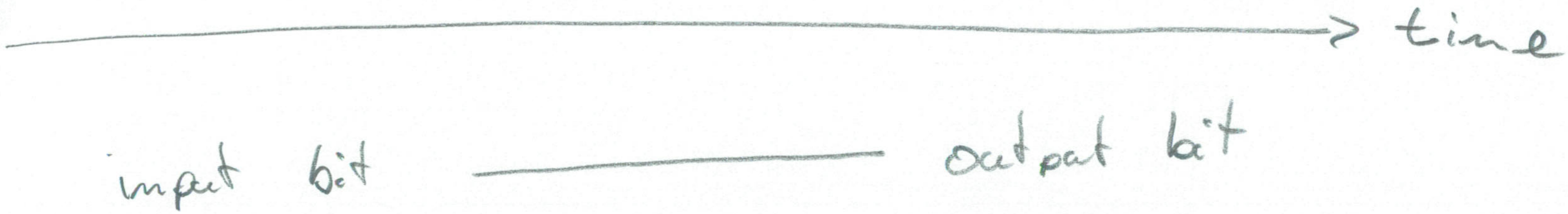
- modification of information in BIT by operating with a <u>physical</u> <u>process</u> on the BIT

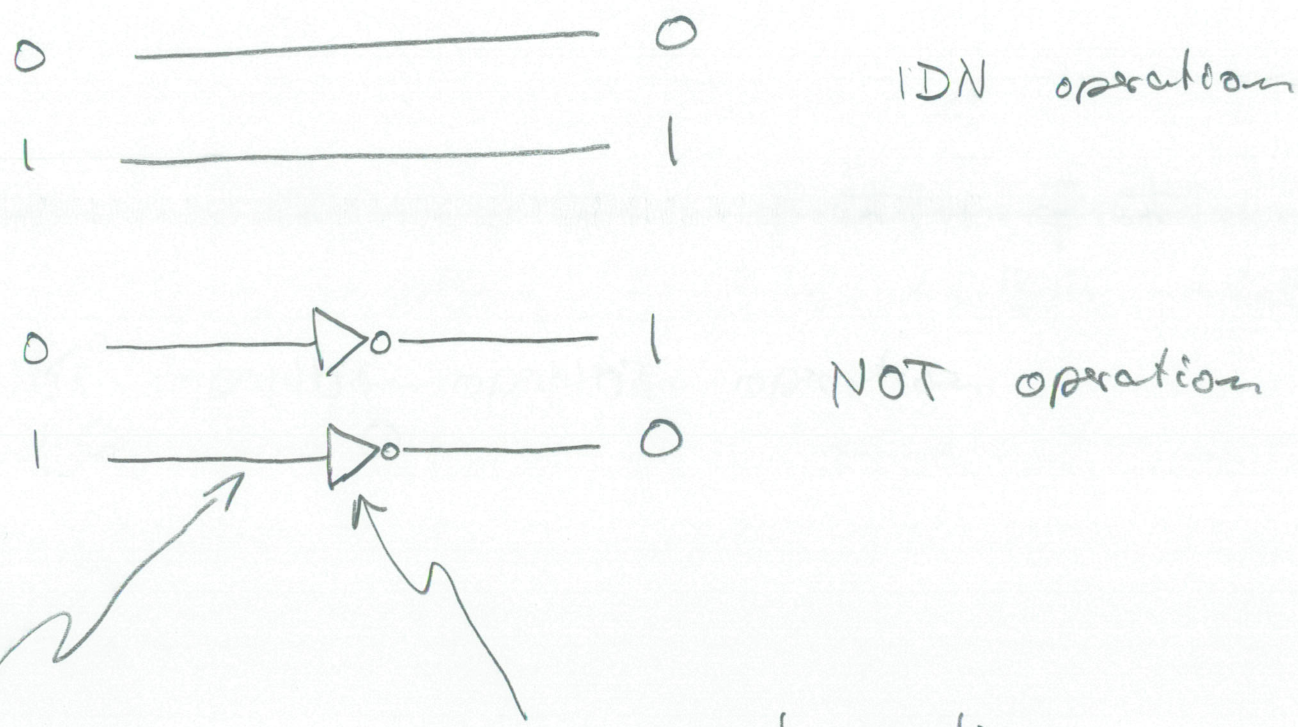
> How is information physically modified in physical realizations of bits? Give examples.
>
> - CD
> - hard disk
> - RAM
>
> The same question ⇒ will be important in quantum computers!

- any logical operation on bits can be decomposed into single and two-bit operations

> - Why is this useful?
> - You may want to think about why this is possible at all!
> - Same is true for quantum computers.

Single Bit Operation





Would you think single qubit operations are simple to realize in a quantum computer?

What could be potential problems?
- decoherence
- spurious interactions
- ...

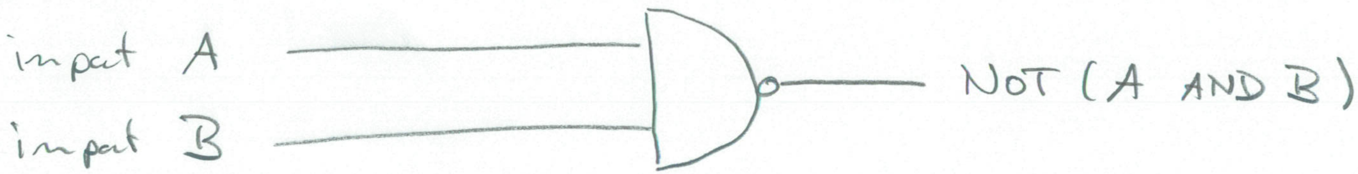
- wire represents bit
- preserves state

- symbol represents operation
- changes state

Same representation of information in circuit model for quantum computation

Two Bit Operations

The NAND gate



truth table

A	B	NOT (A AND B)
0	0	1
0	1	1
1	0	1
1	1	0

properties: – Universal logic gate

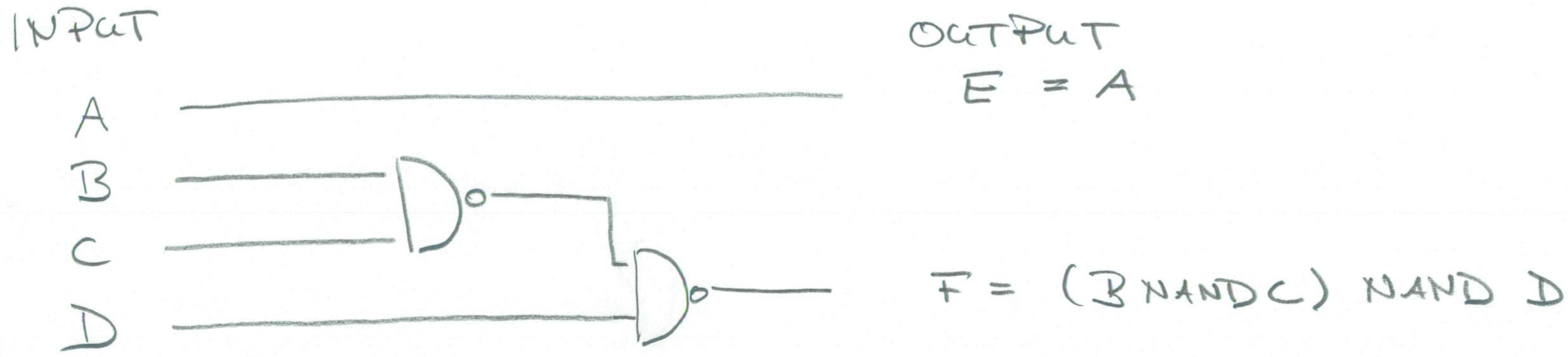
⇝ any function operating on bits can be computed using NAND gates

⇝ examples: AND, OR, XOR, NOR
(ancilla bits & possibility to make copies are required)

Why do you think universality is useful!

NOTE: Universal logic gates also do exist for quantum computers!

Circuit Representation



Any computable function can be represented as a circuit composed of logic universal gates acting on a set of input bits generating a set of output bits

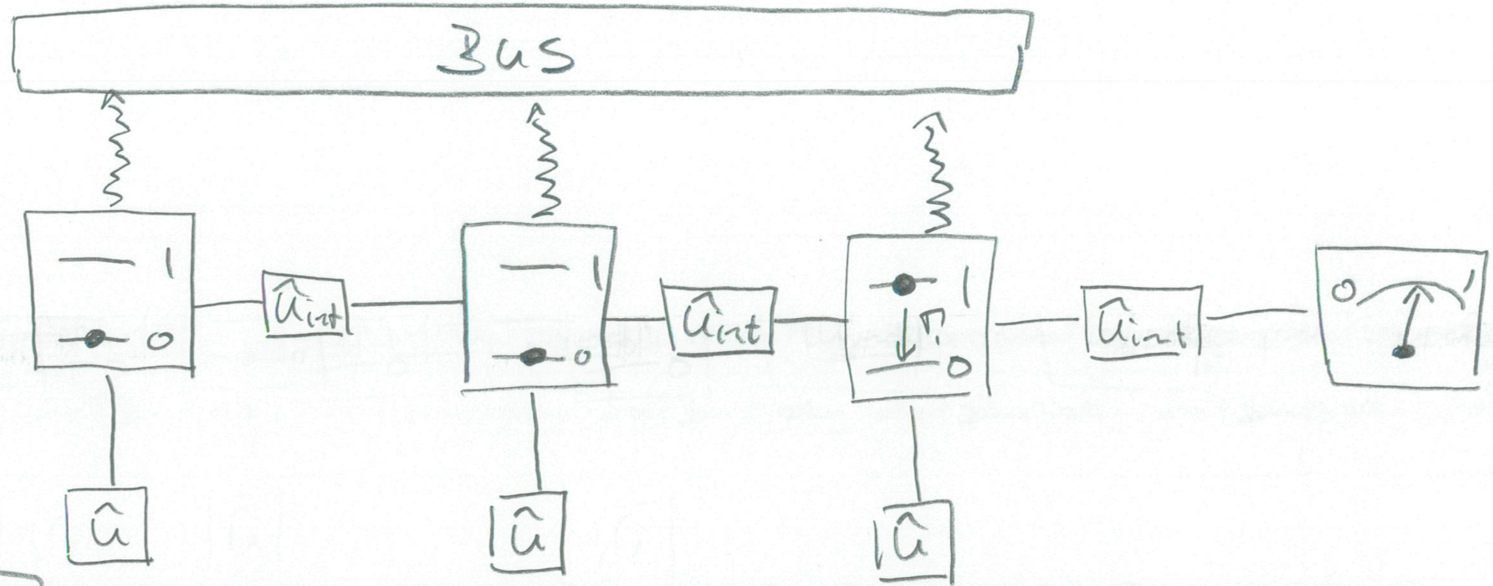
Circuit properties :
- bits can be copied (FAN OUT)
- additional working bits are allowed (ANCILLAS)
- values of bits can be interchanged (CROSSOVER)
- number of output bits may be smaller than number of input bits
- loops are not allowed

In your opinion, which of these properties might be easy / hard to realize in a quantum computer ?
- reversibility ?
- preservation of state ?
- copying of information
- universality ?

A Generic Quantum Processor

features:

1. quantum bits
2. initialization
3. coherence
4. universal gates
5. read-out



DiVincenzo Criteria

6. conversion
7. transfer

In your opinion, what kind of components are absolutely essential to realize any quantum computer?

The Quantum Bit

Give examples of systems with this energy level spectrum.
Under which conditions would you be allowed to approximate a given quantum system as a two-level system?

- quantum mechanical system with two distinct states

- representation of qubit basis states

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

vectors in 2D Hilbert space

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(1st QM postulate)

- general qubit state

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \qquad \text{with } \alpha, \beta \in \mathbb{C}$$

Properties:
- qubit can be in superposition of states $|0\rangle$ and $|1\rangle$
- probability of states

$$P_0 = |\alpha|^2$$

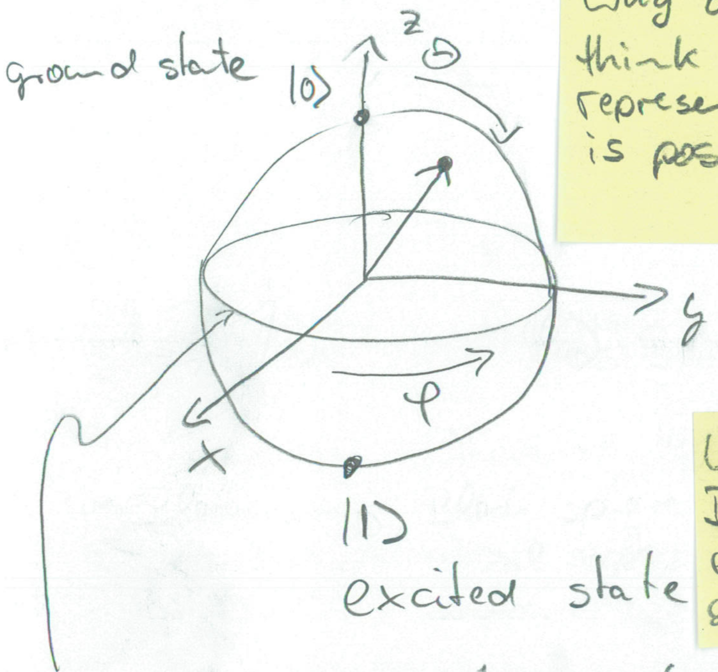
$$P_1 = |\beta|^2$$

$$\text{with } P_1 + P_2 = 1$$

What is the main difference in comp. to a classical bit?

The Bloch Sphere

representation of pure single qubit state as vector do the surface of a sphere



Why do you think this representation is possible?

- general state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
- 4 parameters $\mathrm{Re}\,\alpha,\ \mathrm{Im}\,\alpha,\ \mathrm{Re}\,\beta,\ \mathrm{Im}\,\beta$

  + 1 normalization constraint $|\alpha|^2 + |\beta|^2 = 1$

- $|\psi\rangle = e^{i\gamma}\left[\cos\frac{\Theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\Theta}{2}|1\rangle\right]$

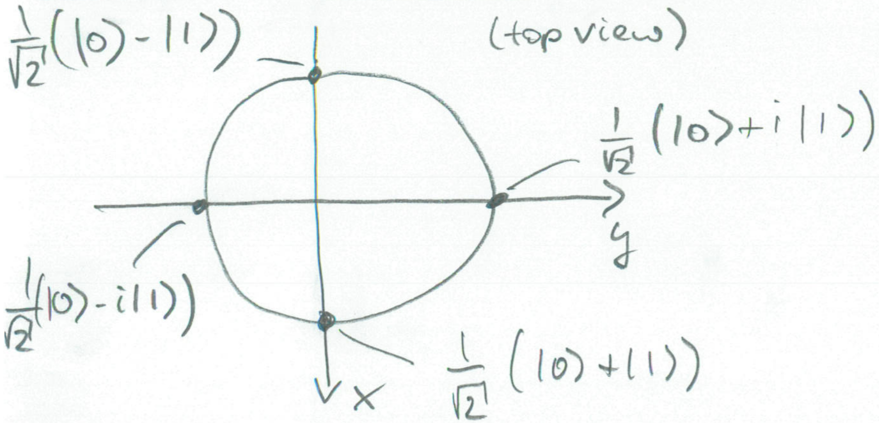
with γ : global phase factor

Θ : polar angle

φ : azimuthal angle

Where on the Bloch sphere are equal superposition states?

equal superposition states:



equivalent representation

- $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\varphi}|1\rangle\right)$

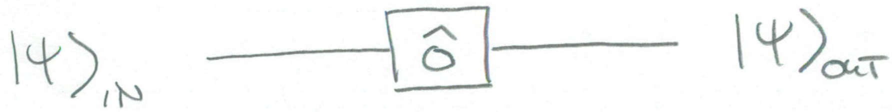
$\Rightarrow$ $\Theta = \frac{\pi}{2}$: defines equator

$\varphi = 0, \pi/2, \pi, \frac{3}{2}\pi$ defines phase angle

Single Qubit Gates

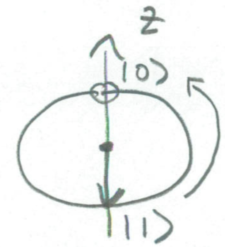
Work out the effect of the single qubit operations on some simple state on the Bloch sphere!

circuit representation



- $\hat{I} = \hat{\mathbb{1}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Identity

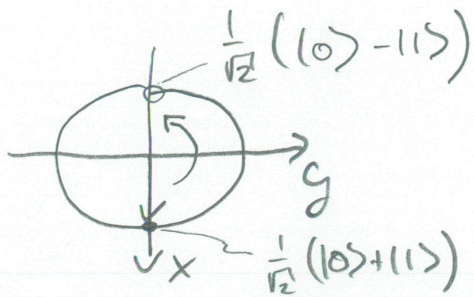
- $\hat{X} = \hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Bit flip



- $\hat{Y} = \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ Conjugate bit flip

$|0\rangle \to -i\,|1\rangle$
$|1\rangle \to i\,|0\rangle$

- $\hat{Z} = \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Phase flip



↑ Pauli matrices