Quantum information processing with trapped ions

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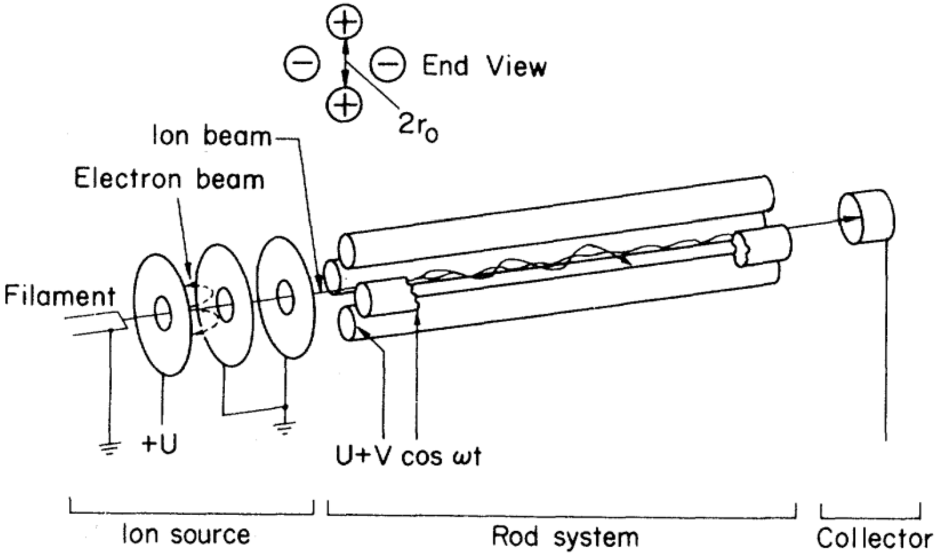
Lecture program:

- Ion traps, linear ion crystals
- Encoding of qubits in trapped ions
- Initialization, manipulation and detection
- Single qubit and entangling quantum gates
- Some recent experiments

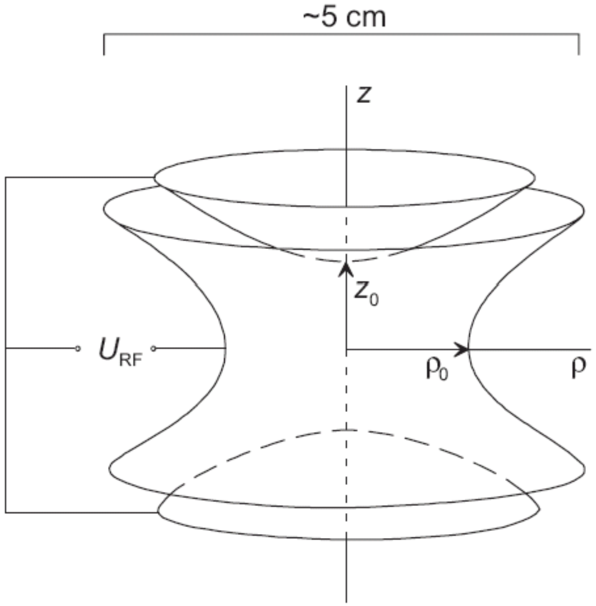
ETH Zürich, 23.11. 2009

Paul traps: Historical development

Predecessor of the ion trap (1953, Wolfgang Paul and co-workers):
Quadrupole mass filter: Mass-dependent confinement of charged particles in two dimensions.



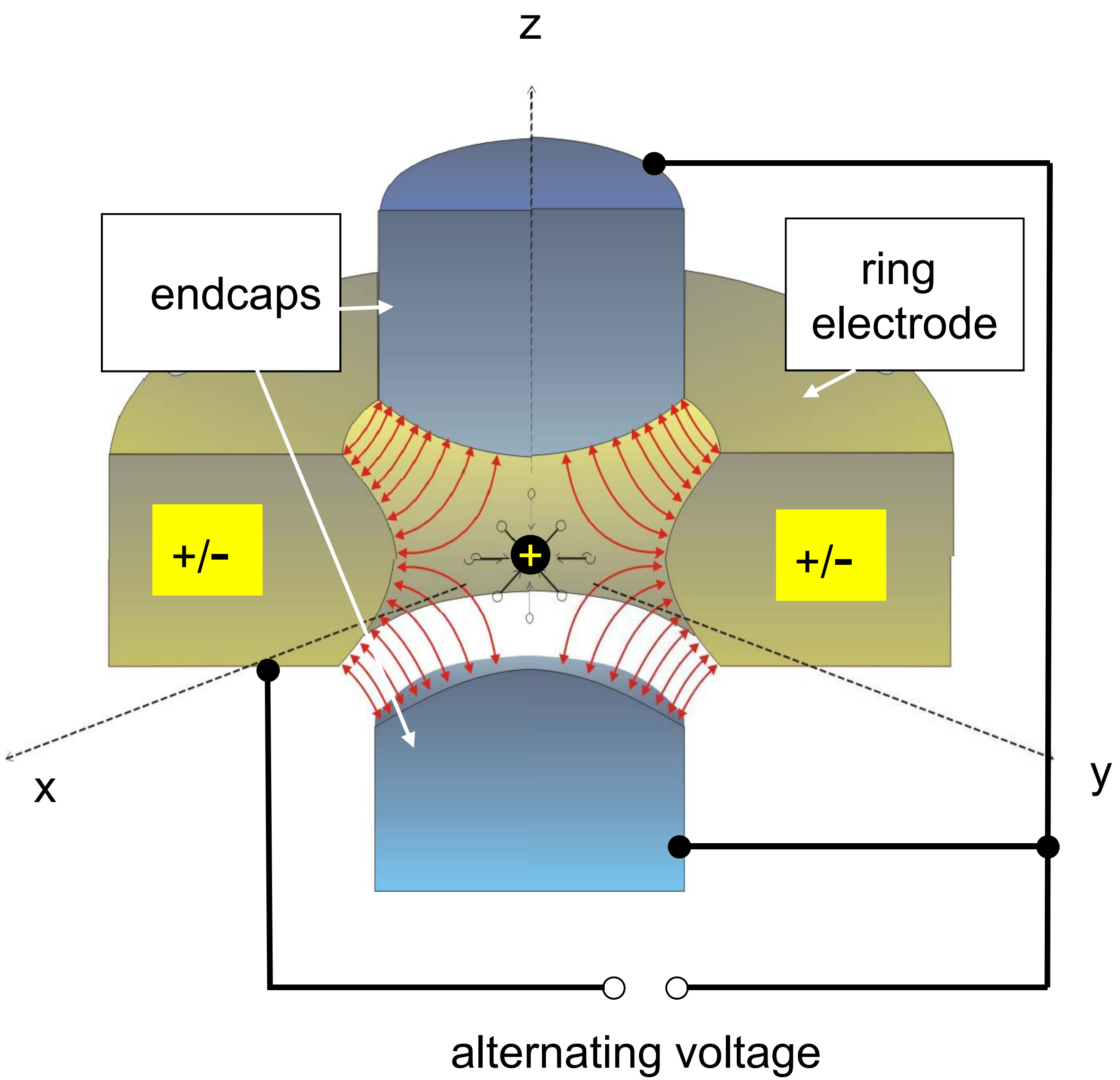
Mass spectrometry:
Measurement of e/m



Paul trap, „Ionenkäfig“ (1955-1958):
Confinement of particles in a radio-frequency 3D-quadrupole field

Very sensitive mass analyzer

Paul traps: Historical development



Trapped microspheres

trapped ion group, JQI, University of Maryland

http://www.iontrap.umd.edu/research/microspheres/MVI_01731.wmv

Mechanical analogue of the Paul trap

Rotating saddle potential

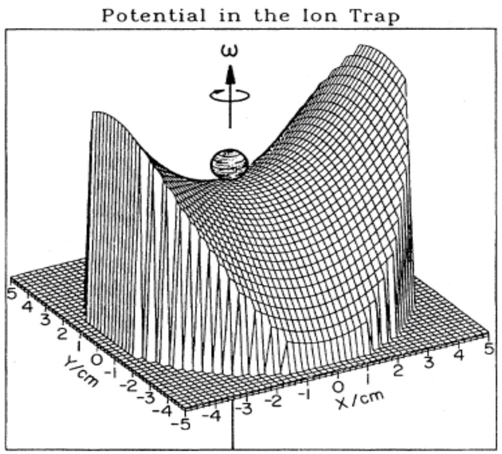
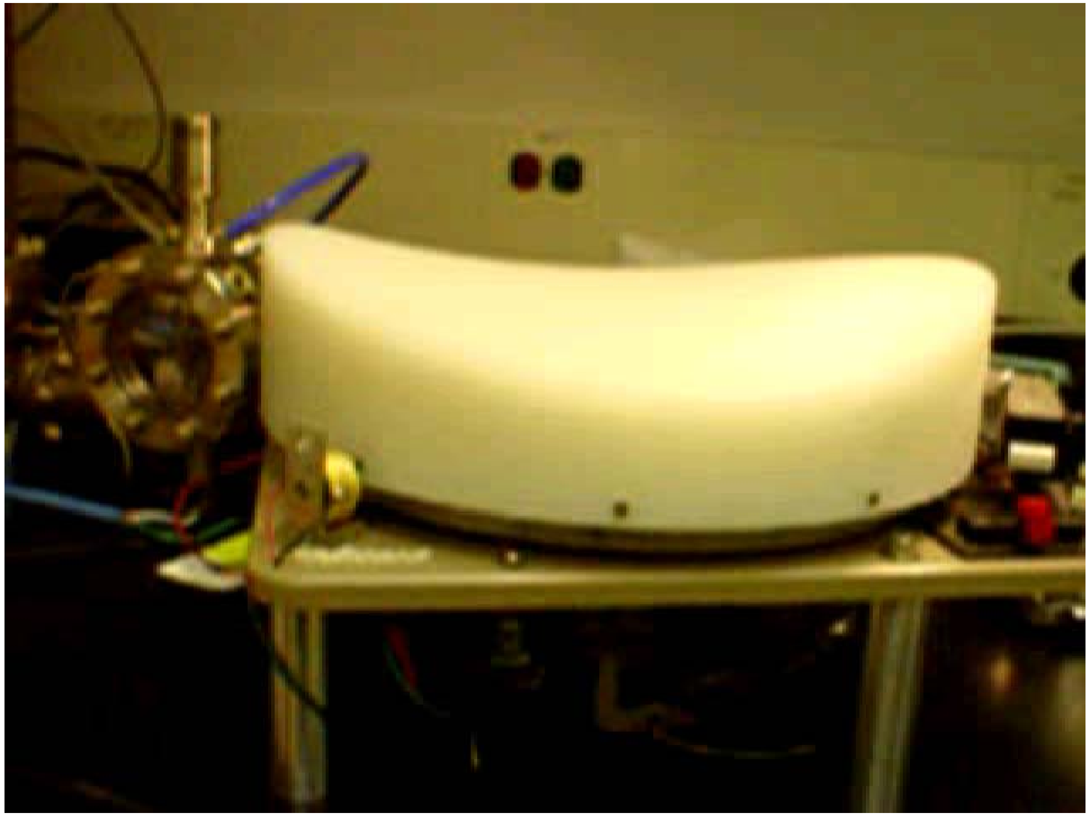


FIG. 8. Mechanical analogue model for the ion trap with steel-ball as "particle."



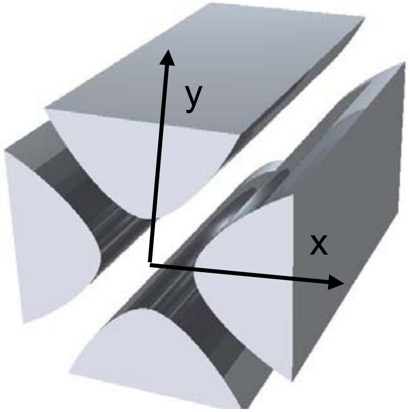
D. Hucul, trapped ion group, JQI, University of Maryland

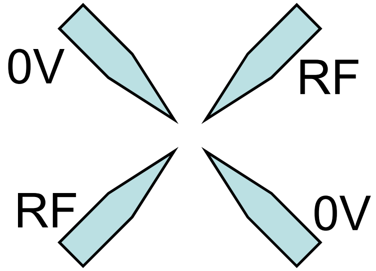
http://www.iontrap.umd.edu/research/microspheres/HUCUL!.WMV

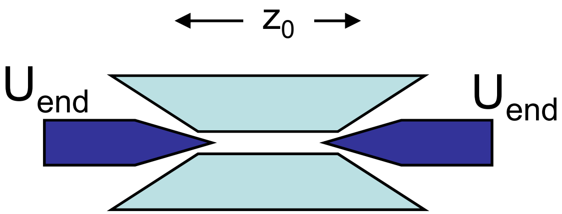
Linear ion trap

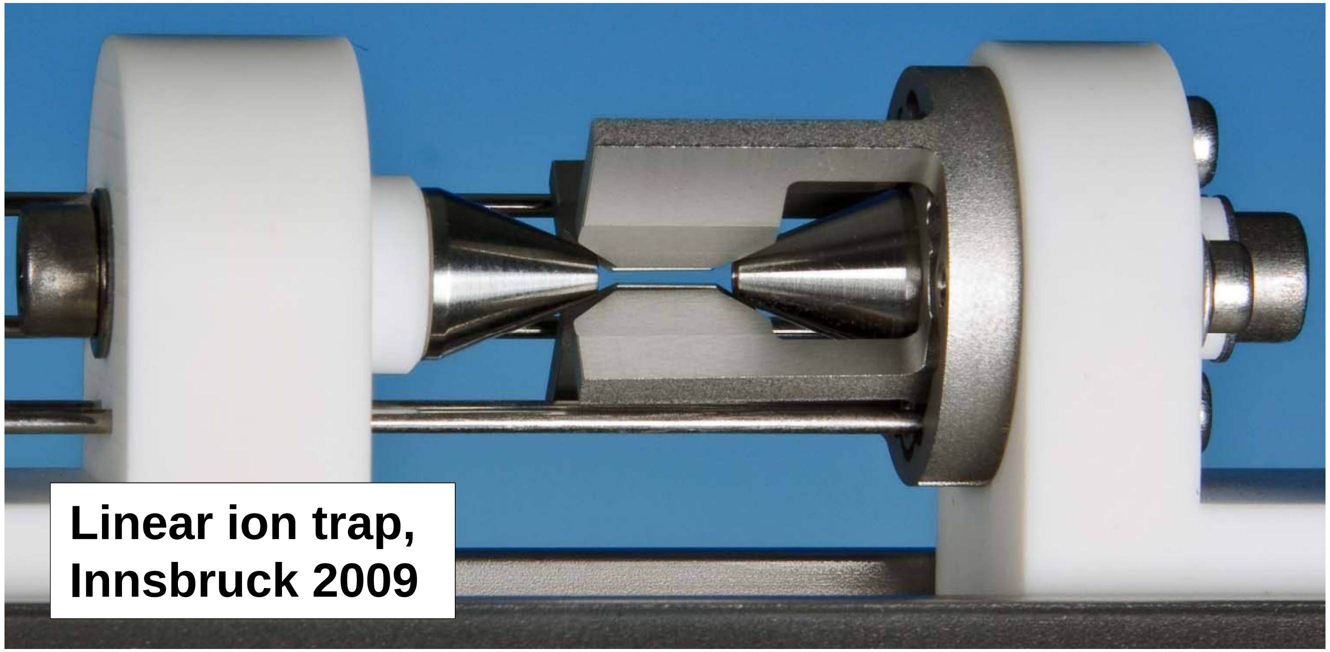
2d rf-quadrupole + static potential

PRL **68**, 2007 (1992), PRA **45**, 6493 (1992)



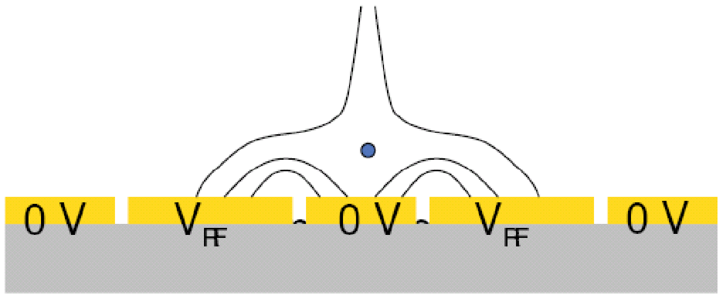




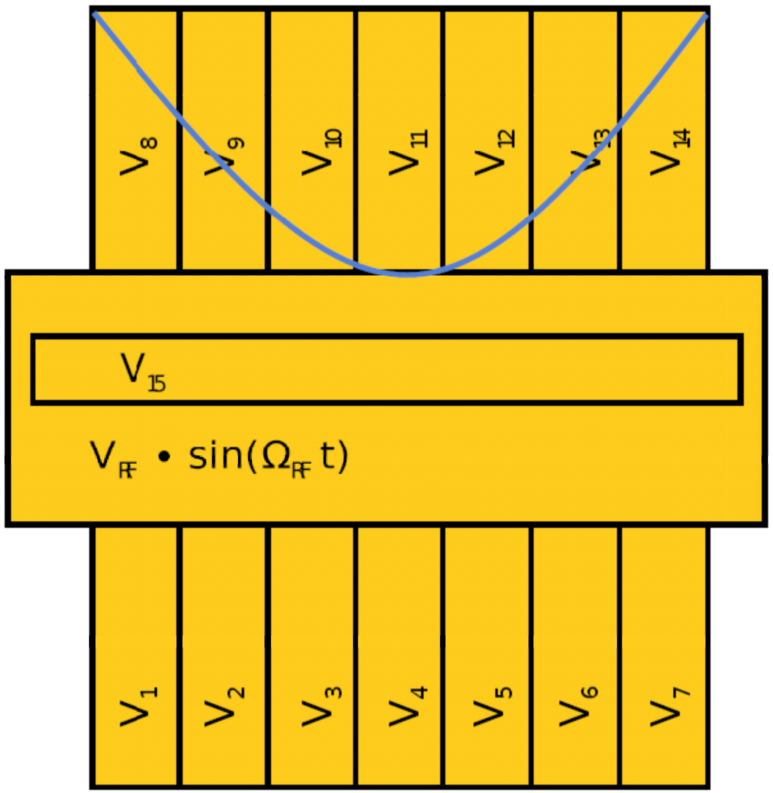


Linear ion trap, Innsbruck 2009

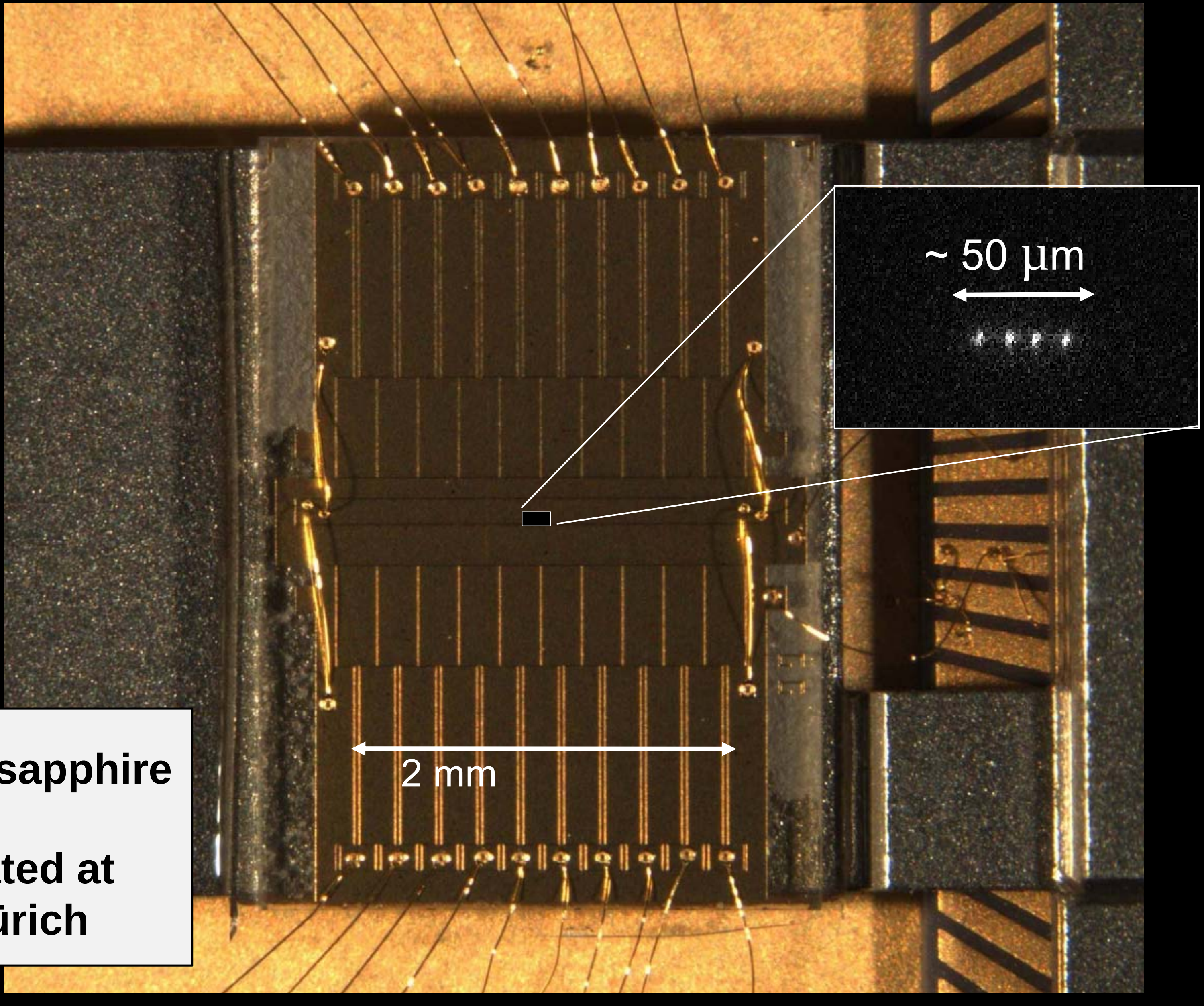
Microfabricated segmented linear traps



Ion height ≈ 220 µm



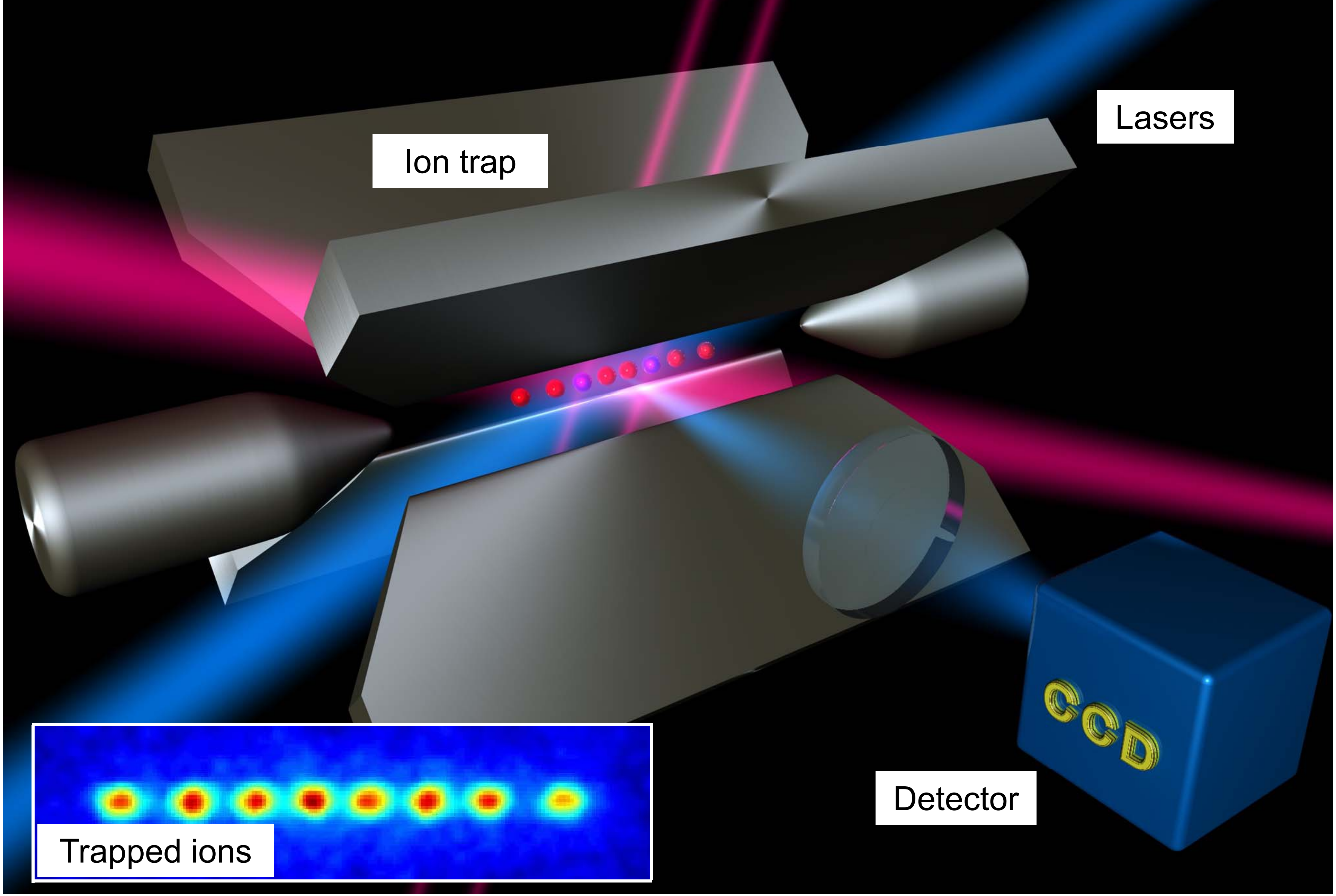
Microfabricated segmented linear traps



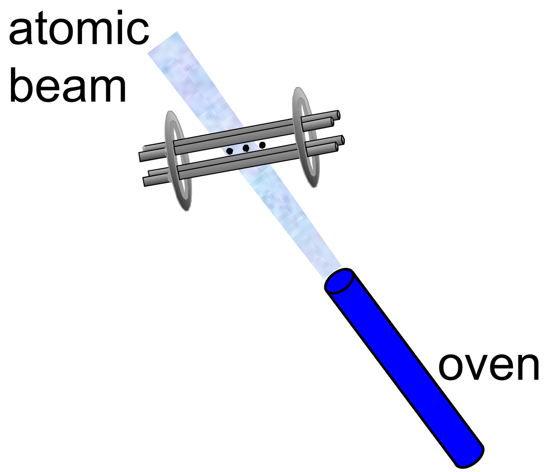
Au on sapphire

**fabricated at
ETH Zürich**

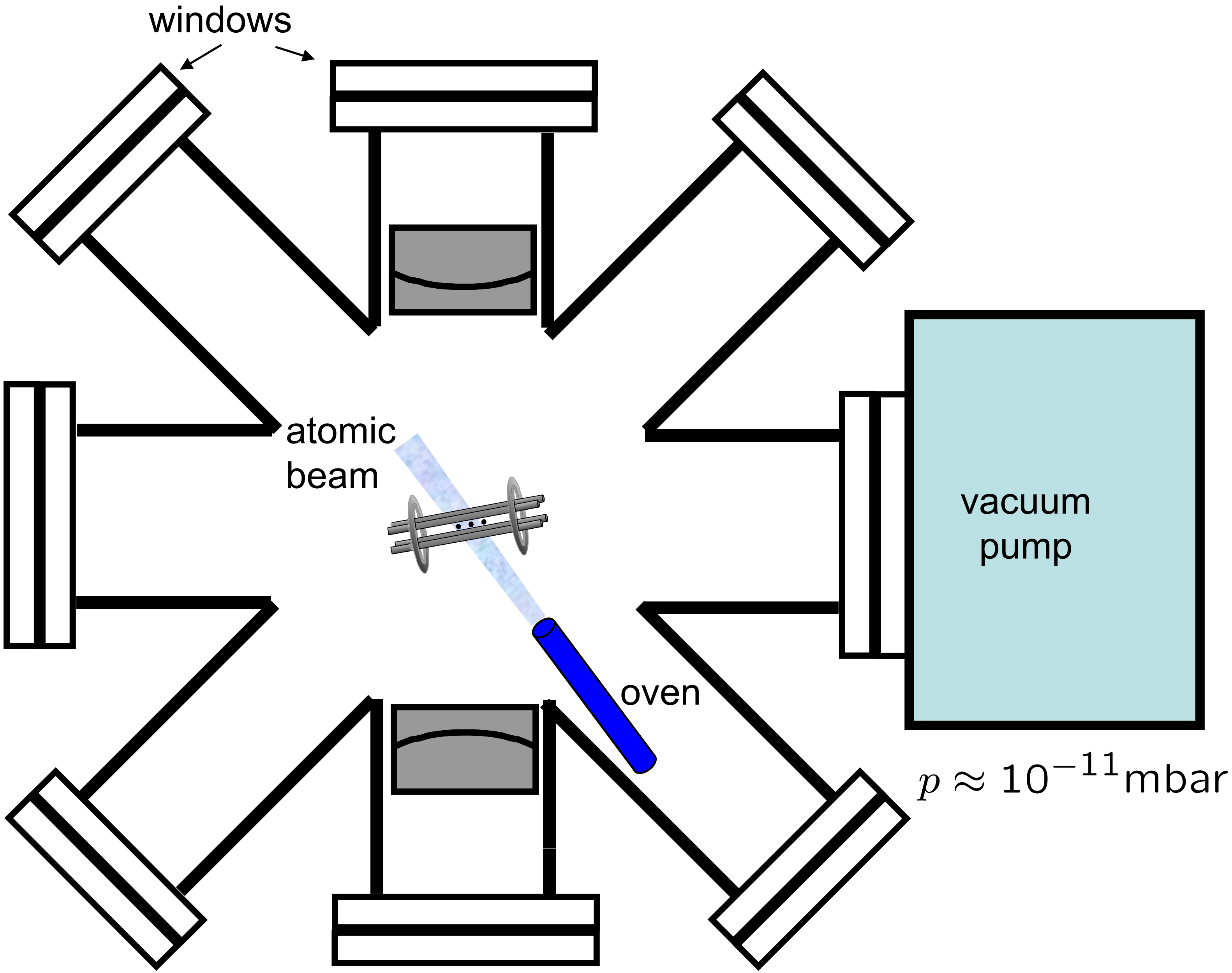
Quantum physics with trapped ions: the experimental tools



How to trap single ions



How to trap single ions

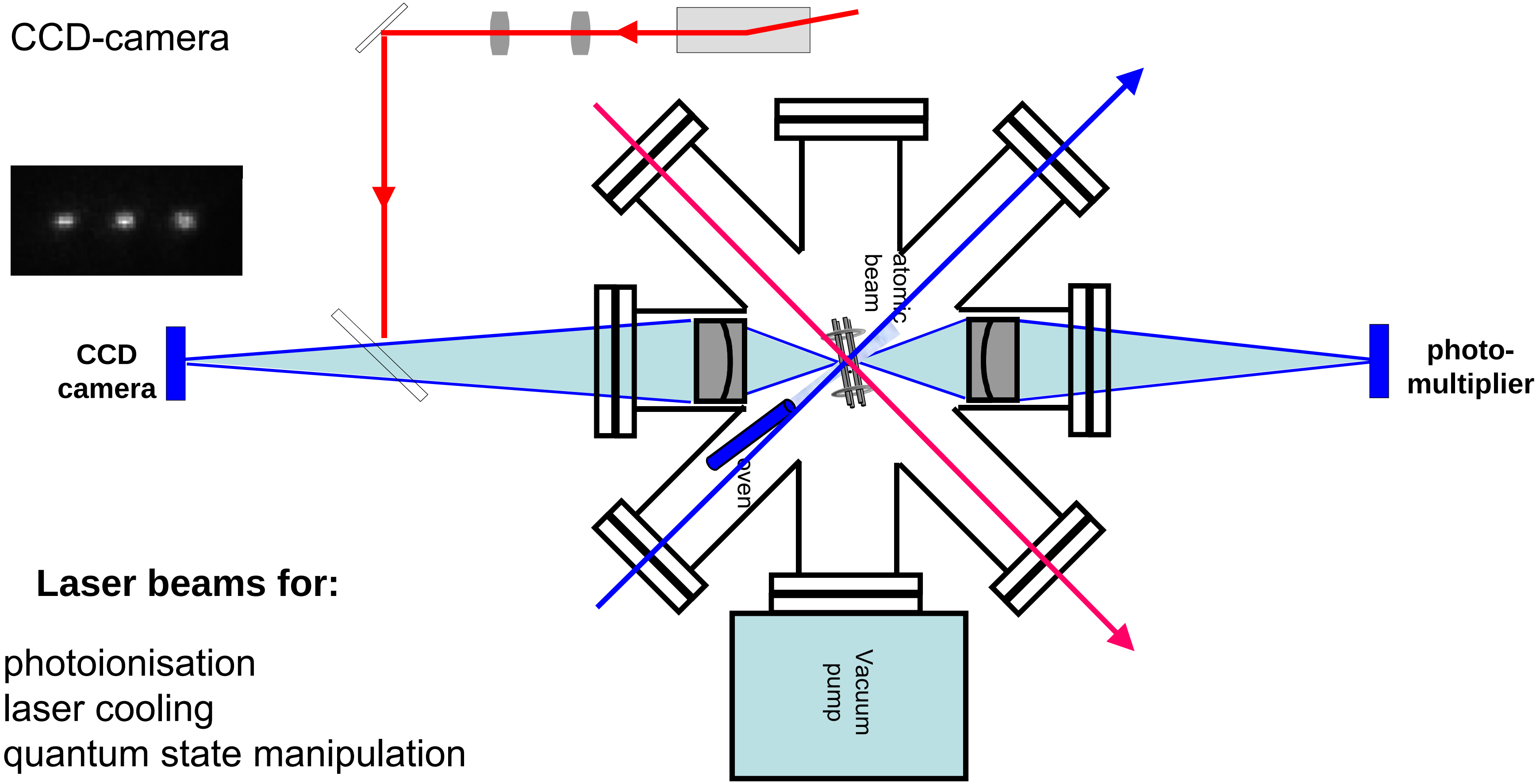


$p \approx 10^{-11}$mbar

How to trap single ions

fluorescence detection with

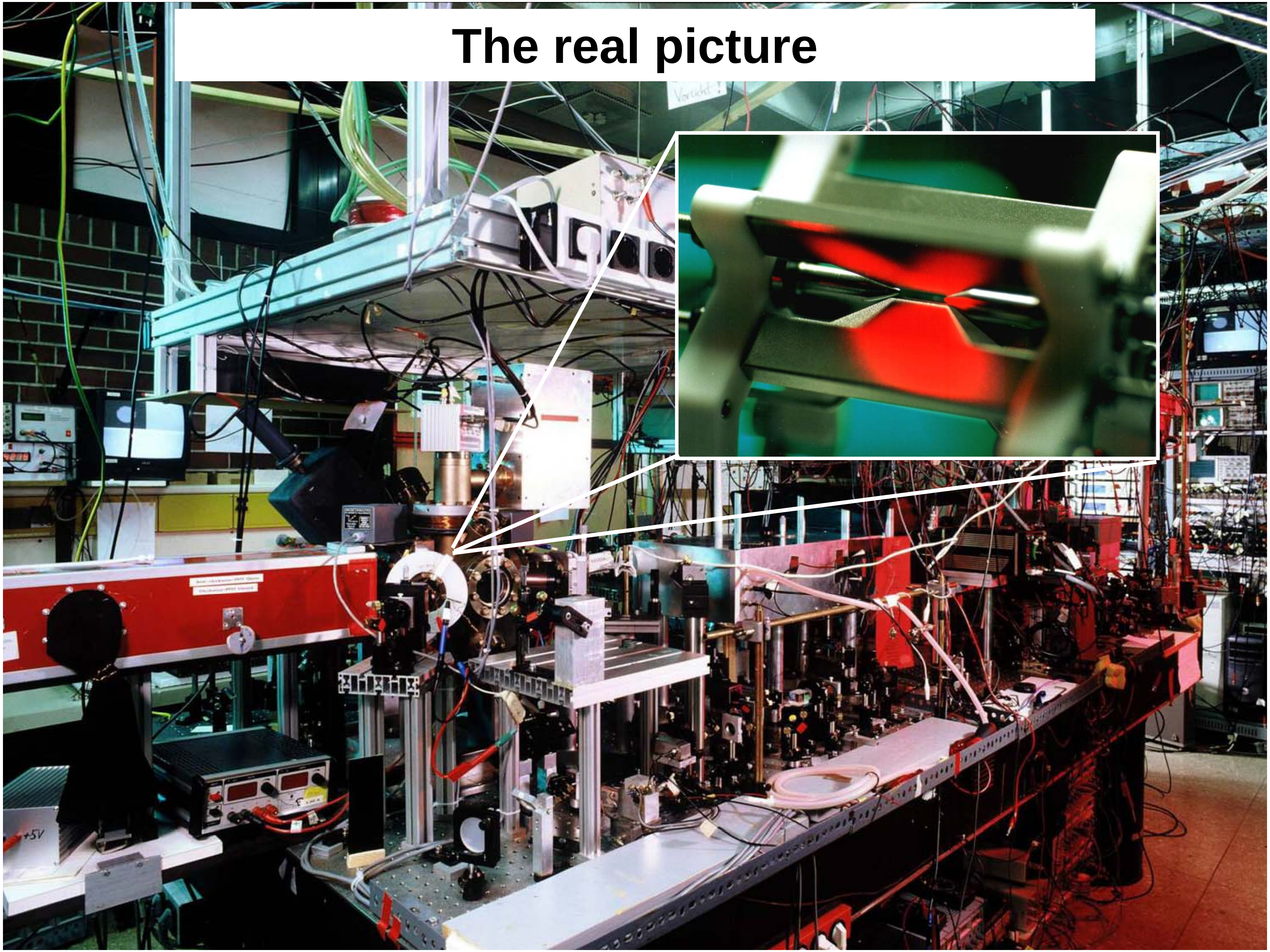
CCD-camera



Laser beams for:

photoionisation
laser cooling
quantum state manipulation
fluorescence detection

The real picture



Quantum physics with trapped ions

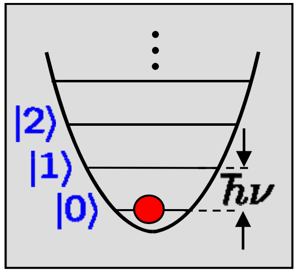
„ …we *never* experiment with just *one* electron or atom or (small) molecule. In thought-experiments we sometimes assume that we do; this invariably entails ridiculous consequences.“

Erwin Schrödinger , 1952

Quantum aspects of trapped ion experiments

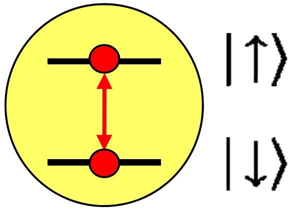
A single trapped ion: Realization of a quantum harmonic oscillator

Motional degrees of freedom



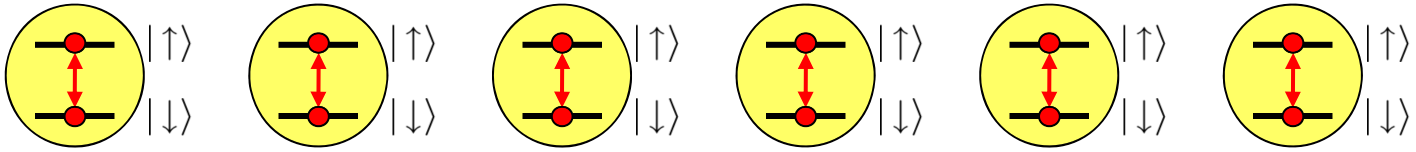
A single trapped ion: Realization of a quantum bit

Internal degrees of freedom

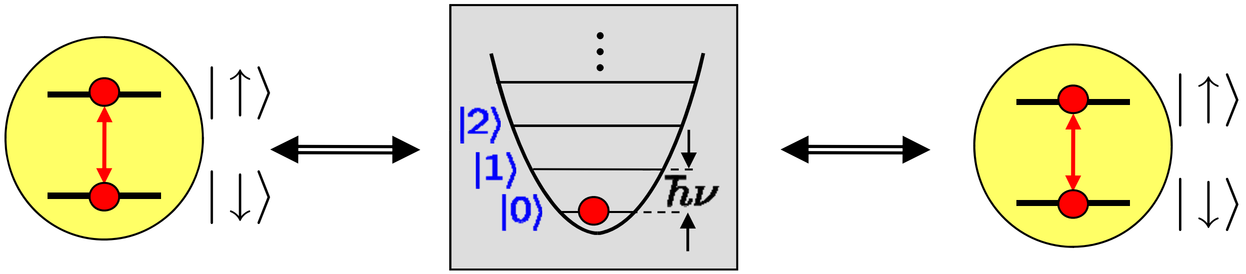


Quantum information processing with trapped ions

Strings of trapped ions: Each ion encodes a qubit



Coupling of internal states via motional degrees of freedom



Effective spin-spin interaction

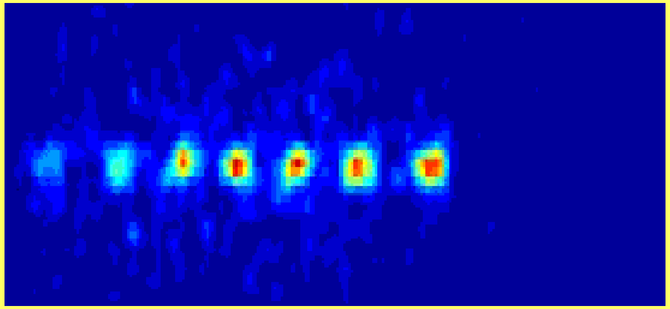
- Generation of entangled states
- Realization of quantum gates

Ion strings: Collective modes of motion

Electronic excitation of motional modes + stroboscopic illumination:

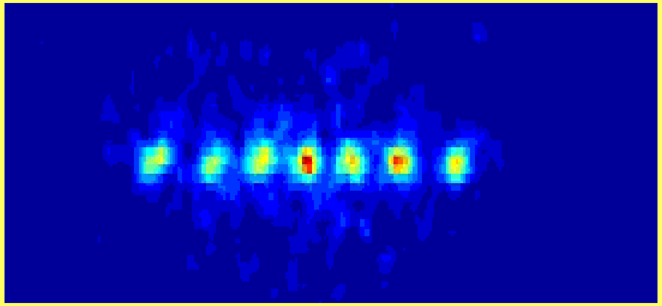
„center-of-mass mode“

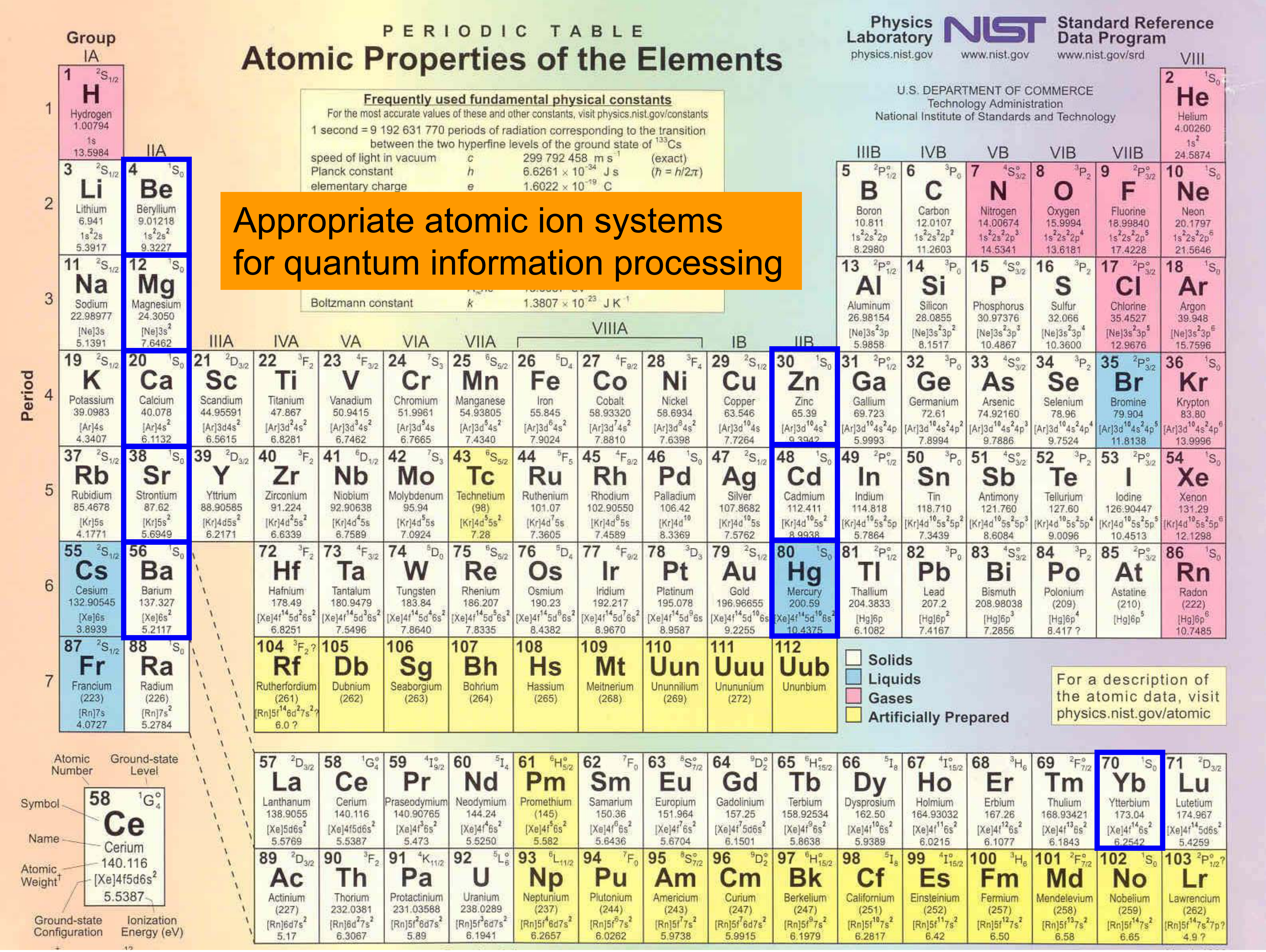
$$\nu = \nu_z$$



„stretch mode“

$$\nu = \sqrt{3}\,\nu_z$$





Appropriate atomic ion systems
for quantum information processing

PERIODIC TABLE

Atomic Properties of the Elements

Physics Laboratory physics.nist.gov
NIST www.nist.gov
Standard Reference Data Program www.nist.gov/srd

U.S. DEPARTMENT OF COMMERCE
Technology Administration
National Institute of Standards and Technology

Frequently used fundamental physical constants

For the most accurate values of these and other constants, visit physics.nist.gov/constants

1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs

speed of light in vacuum	c	299 792 458 m s^{-1}	(exact)
Planck constant	h	6.6261×10^{-34} J s	($\hbar = h/2\pi$)
elementary charge	e	1.6022×10^{-19} C	
Boltzmann constant	k	1.3807×10^{-23} J K^{-1}	

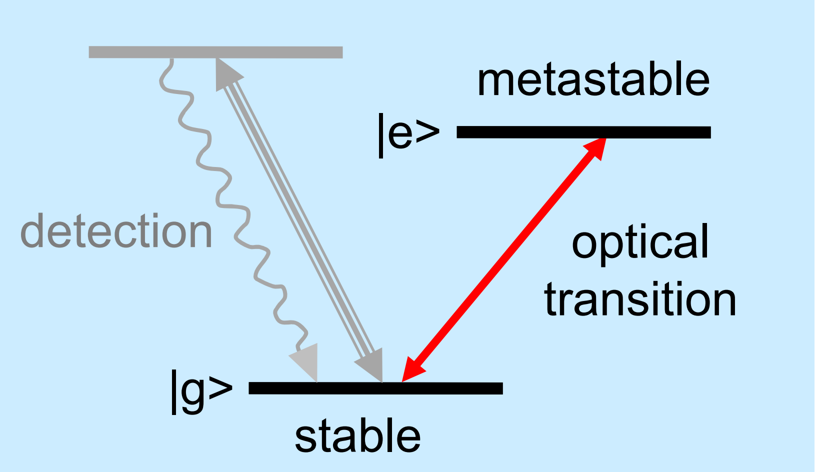
Atomic Number	Symbol	Name	Atomic Weight	Ground-state Configuration	Ionization Energy (eV)	Ground-state Level
1	H	Hydrogen	1.00794	$1s$	13.5984	${}^2S_{1/2}$
2	He	Helium	4.00260	$1s^2$	24.5874	${}^1S_0$
3	Li	Lithium	6.941	$1s^22s$	5.3917	${}^2S_{1/2}$
4	Be	Beryllium	9.01218	$1s^22s^2$	9.3227	${}^1S_0$
5	B	Boron	10.811	$1s^22s^22p$	8.2980	${}^2P^\circ_{1/2}$
6	C	Carbon	12.0107	$1s^22s^22p^2$	11.2603	${}^3P_0$
7	N	Nitrogen	14.00674	$1s^22s^22p^3$	14.5341	${}^4S^\circ_{3/2}$
8	O	Oxygen	15.9994	$1s^22s^22p^4$	13.6181	${}^3P_2$
9	F	Fluorine	18.99840	$1s^22s^22p^5$	17.4228	${}^2P^\circ_{3/2}$
10	Ne	Neon	20.1797	$1s^22s^22p^6$	21.5646	${}^1S_0$
11	Na	Sodium	22.98977	[Ne]$3s$	5.1391	${}^2S_{1/2}$
12	Mg	Magnesium	24.3050	[Ne]$3s^2$	7.6462	${}^1S_0$
13	Al	Aluminum	26.98154	[Ne]$3s^23p$	5.9858	${}^2P^\circ_{1/2}$
14	Si	Silicon	28.0855	[Ne]$3s^23p^2$	8.1517	${}^3P_0$
15	P	Phosphorus	30.97376	[Ne]$3s^23p^3$	10.4867	${}^4S^\circ_{3/2}$
16	S	Sulfur	32.066	[Ne]$3s^23p^4$	10.3600	${}^3P_2$
17	Cl	Chlorine	35.4527	[Ne]$3s^23p^5$	12.9676	${}^2P^\circ_{3/2}$
18	Ar	Argon	39.948	[Ne]$3s^23p^6$	15.7596	${}^1S_0$
19	K	Potassium	39.0983	[Ar]$4s$	4.3407	${}^2S_{1/2}$
20	Ca	Calcium	40.078	[Ar]$4s^2$	6.1132	${}^1S_0$
21	Sc	Scandium	44.95591	[Ar]$3d4s^2$	6.5615	${}^2D_{3/2}$
22	Ti	Titanium	47.867	[Ar]$3d^24s^2$	6.8281	${}^3F_2$
23	V	Vanadium	50.9415	[Ar]$3d^34s^2$	6.7462	${}^4F_{3/2}$
24	Cr	Chromium	51.9961	[Ar]$3d^54s$	6.7665	${}^7S_3$
25	Mn	Manganese	54.93805	[Ar]$3d^54s^2$	7.4340	${}^6S_{5/2}$
26	Fe	Iron	55.845	[Ar]$3d^64s^2$	7.9024	${}^5D_4$
27	Co	Cobalt	58.93320	[Ar]$3d^74s^2$	7.8810	${}^4F_{9/2}$
28	Ni	Nickel	58.6934	[Ar]$3d^84s^2$	7.6398	${}^3F_4$
29	Cu	Copper	63.546	[Ar]$3d^{10}4s$	7.7264	${}^2S_{1/2}$
30	Zn	Zinc	65.39	[Ar]$3d^{10}4s^2$	9.3942	${}^1S_0$
31	Ga	Gallium	69.723	[Ar]$3d^{10}4s^24p$	5.9993	${}^2P^\circ_{1/2}$
32	Ge	Germanium	72.61	[Ar]$3d^{10}4s^24p^2$	7.8994	${}^3P_0$
33	As	Arsenic	74.92160	[Ar]$3d^{10}4s^24p^3$	9.7886	${}^4S^\circ_{3/2}$
34	Se	Selenium	78.96	[Ar]$3d^{10}4s^24p^4$	9.7524	${}^3P_2$
35	Br	Bromine	79.904	[Ar]$3d^{10}4s^24p^5$	11.8138	${}^2P^\circ_{3/2}$
36	Kr	Krypton	83.80	[Ar]$3d^{10}4s^24p^6$	13.9996	${}^1S_0$
37	Rb	Rubidium	85.4678	[Kr]$5s$	4.1771	${}^2S_{1/2}$
38	Sr	Strontium	87.62	[Kr]$5s^2$	5.6949	${}^1S_0$
39	Y	Yttrium	88.90585	[Kr]$4d5s^2$	6.2171	${}^2D_{3/2}$
40	Zr	Zirconium	91.224	[Kr]$4d^25s^2$	6.6339	${}^3F_2$
41	Nb	Niobium	92.90638	[Kr]$4d^45s$	6.7589	${}^6D_{1/2}$
42	Mo	Molybdenum	95.94	[Kr]$4d^55s$	7.0924	${}^7S_3$
43	Tc	Technetium	(98)	[Kr]$4d^55s^2$	7.28	${}^6S_{5/2}$
44	Ru	Ruthenium	101.07	[Kr]$4d^75s$	7.3605	${}^5F_5$
45	Rh	Rhodium	102.90550	[Kr]$4d^85s$	7.4589	${}^4F_{9/2}$
46	Pd	Palladium	106.42	[Kr]$4d^{10}$	8.3369	${}^1S_0$
47	Ag	Silver	107.8682	[Kr]$4d^{10}5s$	7.5762	${}^2S_{1/2}$
48	Cd	Cadmium	112.411	[Kr]$4d^{10}5s^2$	8.9938	${}^1S_0$
49	In	Indium	114.818	[Kr]$4d^{10}5s^25p$	5.7864	${}^2P^\circ_{1/2}$
50	Sn	Tin	118.710	[Kr]$4d^{10}5s^25p^2$	7.3439	${}^3P_0$
51	Sb	Antimony	121.760	[Kr]$4d^{10}5s^25p^3$	8.6084	${}^4S^\circ_{3/2}$
52	Te	Tellurium	127.60	[Kr]$4d^{10}5s^25p^4$	9.0096	${}^3P_2$
53	I	Iodine	126.90447	[Kr]$4d^{10}5s^25p^5$	10.4513	${}^2P^\circ_{3/2}$
54	Xe	Xenon	131.29	[Kr]$4d^{10}5s^25p^6$	12.1298	${}^1S_0$
55	Cs	Cesium	132.90545	[Xe]$6s$	3.8939	${}^2S_{1/2}$
56	Ba	Barium	137.327	[Xe]$6s^2$	5.2117	${}^1S_0$
57	La	Lanthanum	138.9055	[Xe]$5d6s^2$	5.5769	${}^2D_{3/2}$
58	Ce	Cerium	140.116	[Xe]$4f5d6s^2$	5.5387	${}^1G^\circ_4$
59	Pr	Praseodymium	140.90765	[Xe]$4f^36s^2$	5.473	${}^4I^\circ_{9/2}$
60	Nd	Neodymium	144.24	[Xe]$4f^46s^2$	5.5250	${}^5I_4$
61	Pm	Promethium	(145)	[Xe]$4f^56s^2$	5.582	${}^6H^\circ_{5/2}$
62	Sm	Samarium	150.36	[Xe]$4f^66s^2$	5.6436	${}^7F_0$
63	Eu	Europium	151.964	[Xe]$4f^76s^2$	5.6704	${}^8S^\circ_{7/2}$
64	Gd	Gadolinium	157.25	[Xe]$4f^75d6s^2$	6.1501	${}^9D^\circ_2$
65	Tb	Terbium	158.92534	[Xe]$4f^96s^2$	5.8638	${}^6H^\circ_{15/2}$
66	Dy	Dysprosium	162.50	[Xe]$4f^{10}6s^2$	5.9389	${}^5I_8$
67	Ho	Holmium	164.93032	[Xe]$4f^{11}6s^2$	6.0215	${}^4I^\circ_{15/2}$
68	Er	Erbium	167.26	[Xe]$4f^{12}6s^2$	6.1077	${}^3H_6$
69	Tm	Thulium	168.93421	[Xe]$4f^{13}6s^2$	6.1843	${}^2F^\circ_{7/2}$
70	Yb	Ytterbium	173.04	[Xe]$4f^{14}6s^2$	6.2542	${}^1S_0$
71	Lu	Lutetium	174.967	[Xe]$4f^{14}5d6s^2$	5.4259	${}^2D_{3/2}$
72	Hf	Hafnium	178.49	[Xe]$4f^{14}5d^26s^2$	6.8251	${}^3F_2$
73	Ta	Tantalum	180.9479	[Xe]$4f^{14}5d^36s^2$	7.5496	${}^4F_{3/2}$
74	W	Tungsten	183.84	[Xe]$4f^{14}5d^46s^2$	7.8640	${}^5D_0$
75	Re	Rhenium	186.207	[Xe]$4f^{14}5d^56s^2$	7.8335	${}^6S_{5/2}$
76	Os	Osmium	190.23	[Xe]$4f^{14}5d^66s^2$	8.4382	${}^5D_4$
77	Ir	Iridium	192.217	[Xe]$4f^{14}5d^76s^2$	8.9670	${}^4F_{9/2}$
78	Pt	Platinum	195.078	[Xe]$4f^{14}5d^96s$	8.9587	${}^3D_3$
79	Au	Gold	196.96655	[Xe]$4f^{14}5d^{10}6s$	9.2255	${}^2S_{1/2}$
80	Hg	Mercury	200.59	[Xe]$4f^{14}5d^{10}6s^2$	10.4375	${}^1S_0$
81	Tl	Thallium	204.3833	[Hg]$6p$	6.1082	${}^2P^\circ_{1/2}$
82	Pb	Lead	207.2	[Hg]$6p^2$	7.4167	${}^3P_0$
83	Bi	Bismuth	208.98038	[Hg]$6p^3$	7.2856	${}^4S^\circ_{3/2}$
84	Po	Polonium	(209)	[Hg]$6p^4$	8.417 ?	${}^3P_2$
85	At	Astatine	(210)	[Hg]$6p^5$		${}^2P^\circ_{3/2}$
86	Rn	Radon	(222)	[Hg]$6p^6$	10.7485	${}^1S_0$
87	Fr	Francium	(223)	[Rn]$7s$	4.0727	${}^2S_{1/2}$
88	Ra	Radium	(226)	[Rn]$7s^2$	5.2784	${}^1S_0$
89	Ac	Actinium	(227)	[Rn]$6d7s^2$	5.17	${}^2D_{3/2}$
90	Th	Thorium	232.0381	[Rn]$6d^27s^2$	6.3067	${}^3F_2$
91	Pa	Protactinium	231.03588	[Rn]$5f^26d7s^2$	5.89	${}^4K_{11/2}$
92	U	Uranium	238.0289	[Rn]$5f^36d7s^2$	6.1941	${}^5L^\circ_6$
93	Np	Neptunium	(237)	[Rn]$5f^46d7s^2$	6.2657	${}^6L_{11/2}$
94	Pu	Plutonium	(244)	[Rn]$5f^67s^2$	6.0262	${}^7F_0$
95	Am	Americium	(243)	[Rn]$5f^77s^2$	5.9738	${}^8S^\circ_{7/2}$
96	Cm	Curium	(247)	[Rn]$5f^76d7s^2$	5.9915	${}^9D^\circ_2$
97	Bk	Berkelium	(247)	[Rn]$5f^97s^2$	6.1979	${}^6H^\circ_{15/2}$
98	Cf	Californium	(251)	[Rn]$5f^{10}7s^2$	6.2817	${}^5I_8$
99	Es	Einsteinium	(252)	[Rn]$5f^{11}7s^2$	6.42	${}^4I^\circ_{15/2}$
100	Fm	Fermium	(257)	[Rn]$5f^{12}7s^2$	6.50	${}^3H_6$
101	Md	Mendelevium	(258)	[Rn]$5f^{13}7s^2$	6.58	${}^2F^\circ_{7/2}$
102	No	Nobelium	(259)	[Rn]$5f^{14}7s^2$	6.65	${}^1S_0$
103	Lr	Lawrencium	(262)	[Rn]$5f^{14}7s^27p$?	4.9 ?	${}^2P^\circ_{1/2}$?
104	Rf	Rutherfordium	(261)	[Rn]$5f^{14}6d^27s^2$?	6.0 ?	${}^3F_2$?
105	Db	Dubnium	(262)			
106	Sg	Seaborgium	(263)			
107	Bh	Bohrium	(264)			
108	Hs	Hassium	(265)			
109	Mt	Meitnerium	(268)			
110	Uun	Ununnilium	(269)			
111	Uuu	Unununium	(272)			
112	Uub	Ununbium				

☐ Solids
▨ Liquids
▨ Gases
▨ Artificially Prepared

For a description of the atomic data, visit physics.nist.gov/atomic

Trapped ion quantum bits

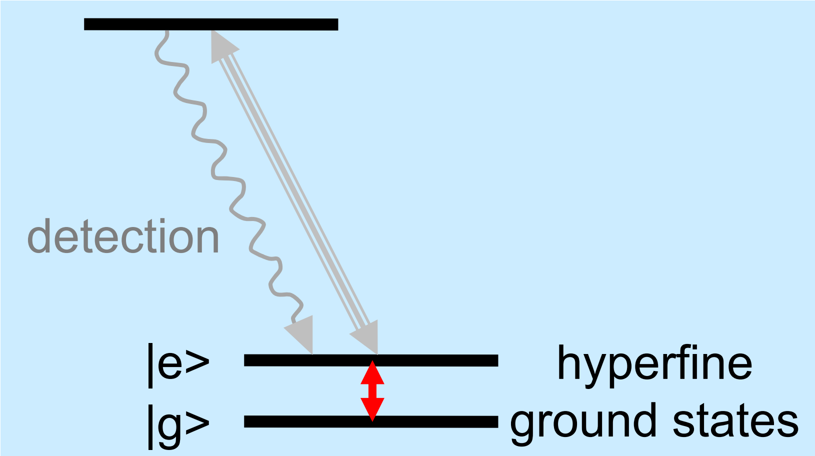
Ions with optical transition to metastable level: $^{40}Ca^+$, $^{88}Sr^+$, $^{172}Yb^+$



„optical qubit“

qubit manipulation requires ultrastable laser

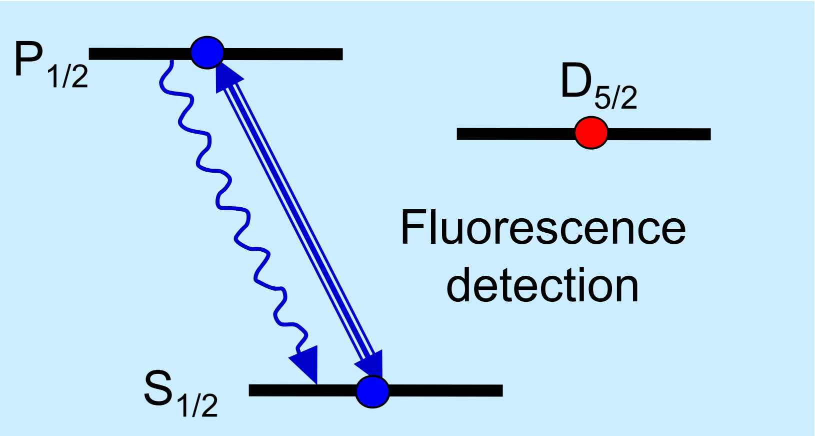
Ions with hyperfine structure: $^{9}Be^+$, $^{43}Ca^+$, $^{111}Cd^+$, $^{171}Yb^+$…



„hyperfine qubit“

qubit manipulation with microwaves or lasers

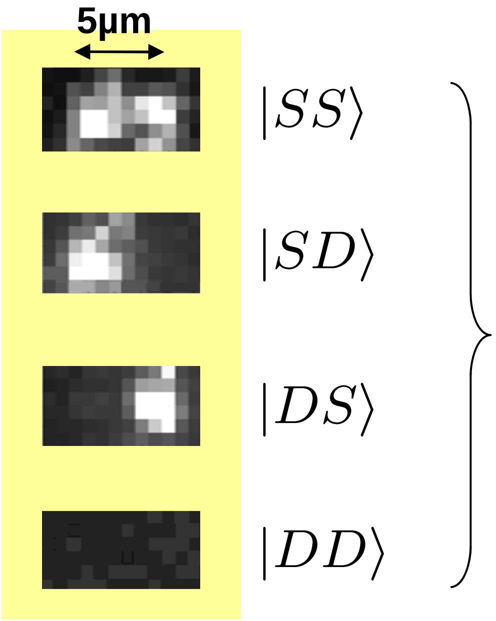
Experimental sequence



1. Initialization in a pure quantum state
2. Quantum state manipulation on $S_{1/2} – D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

Two ions:

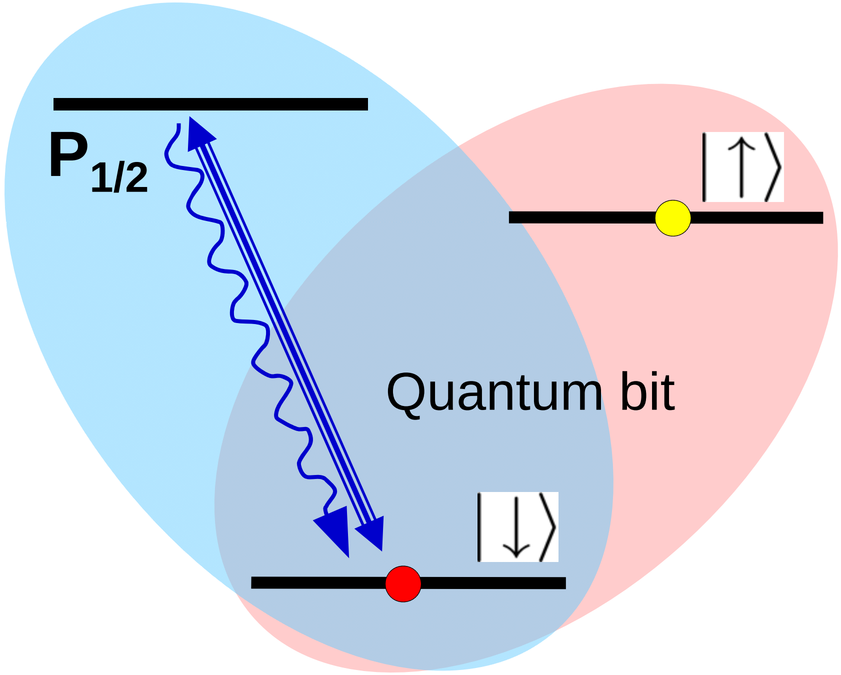
Spatially resolved detection with CCD camera:



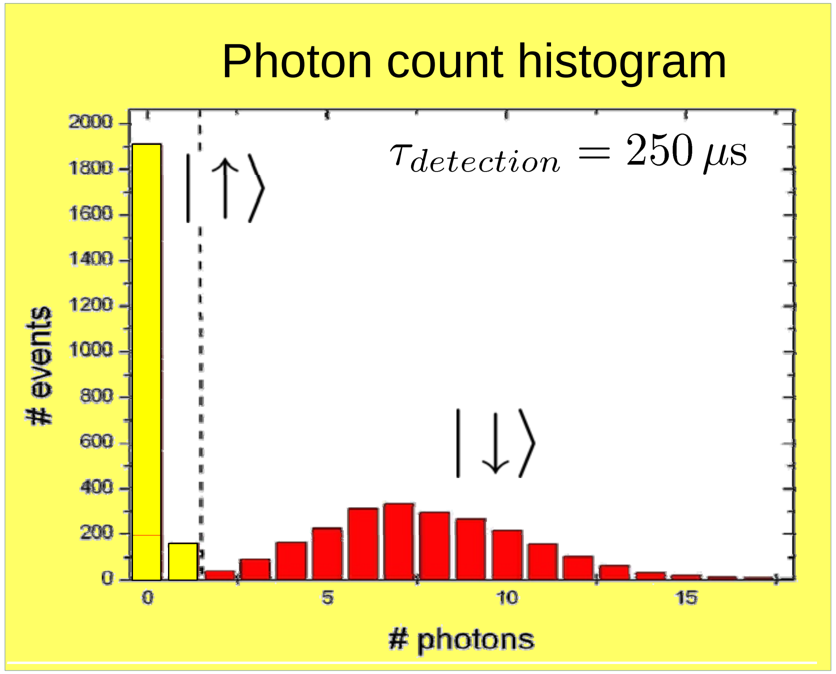
50 experiments / s

Repeat experiments 100-200 times

Quantum state detection



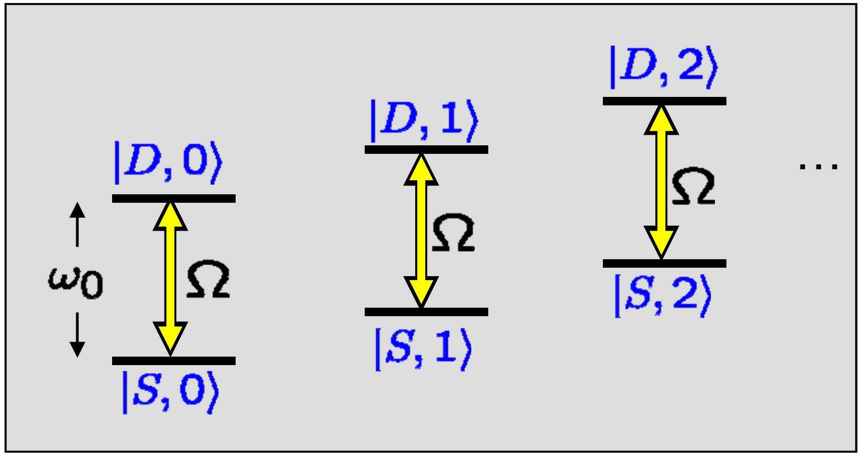
Measurement of σ_z



Trapped-ion laser interactions

$\omega_{laser} = \omega_0$

Carrier resonance



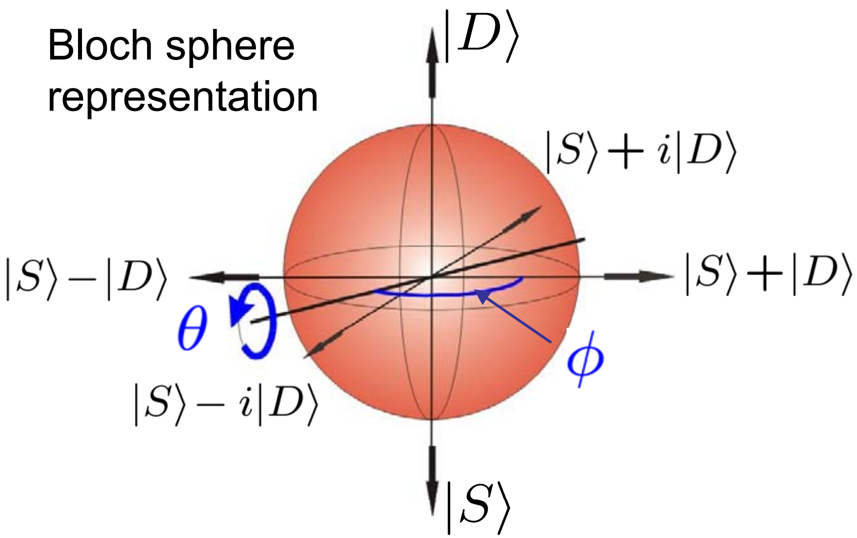
$$H \propto \sigma_+ + \sigma_-$$

Coherent excitation: Rabi oscillations

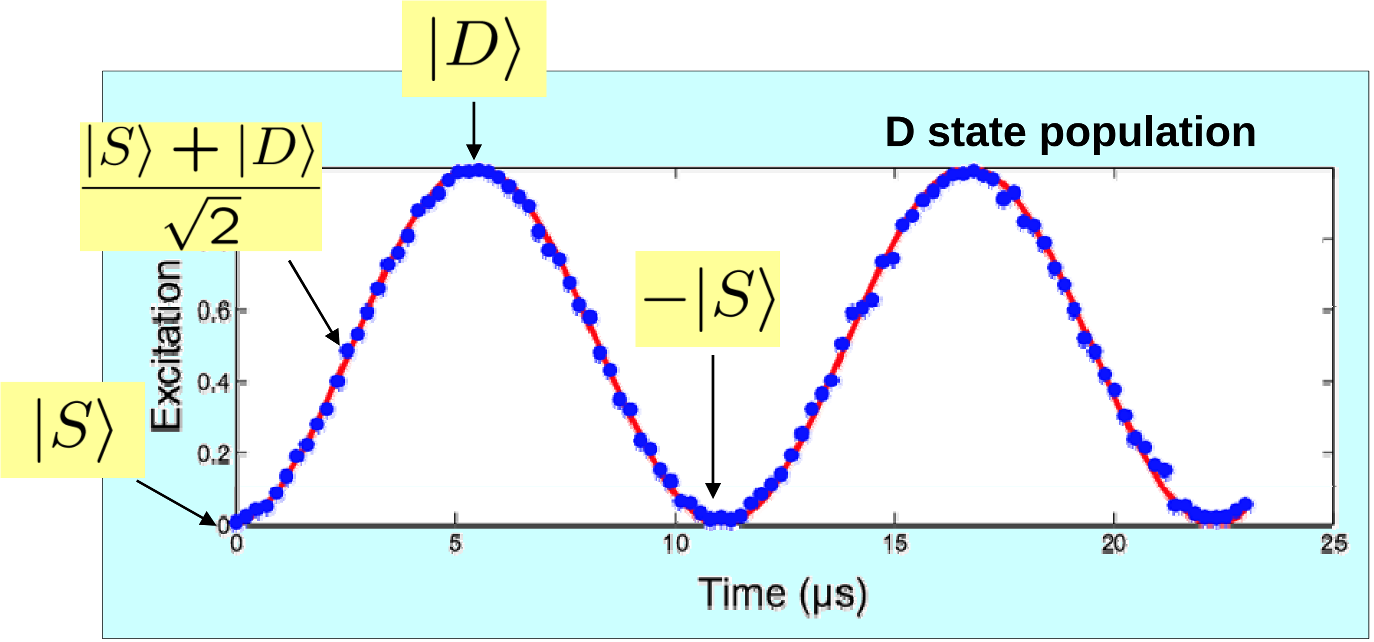
„Carrier“ pulses: $|S\rangle \longleftrightarrow |D\rangle$

$$H \propto \sigma_+ e^{i\phi} + \sigma_- e^{-i\phi}$$
$$\propto \sigma_x \cos\phi - \sigma_y \sin\phi$$

Bloch sphere representation



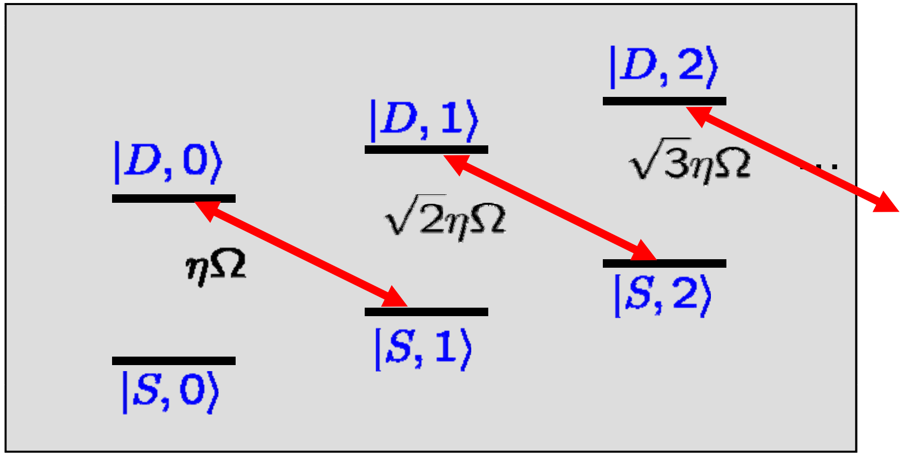
$\theta/2 = \Omega\tau$



Trapped-ion laser interactions

$$\omega_{laser} = \omega_0 - \nu$$

Red sideband



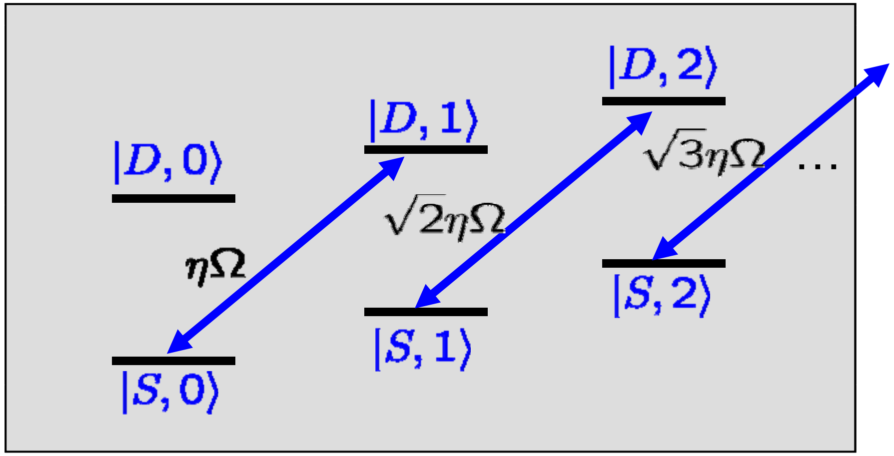
Jaynes-Cummings model

$$H \propto \sigma_+ a + \sigma_- a^\dagger$$

Trapped-ion laser interactions

$$\omega_{laser} = \omega_0 + \nu$$

Blue sideband

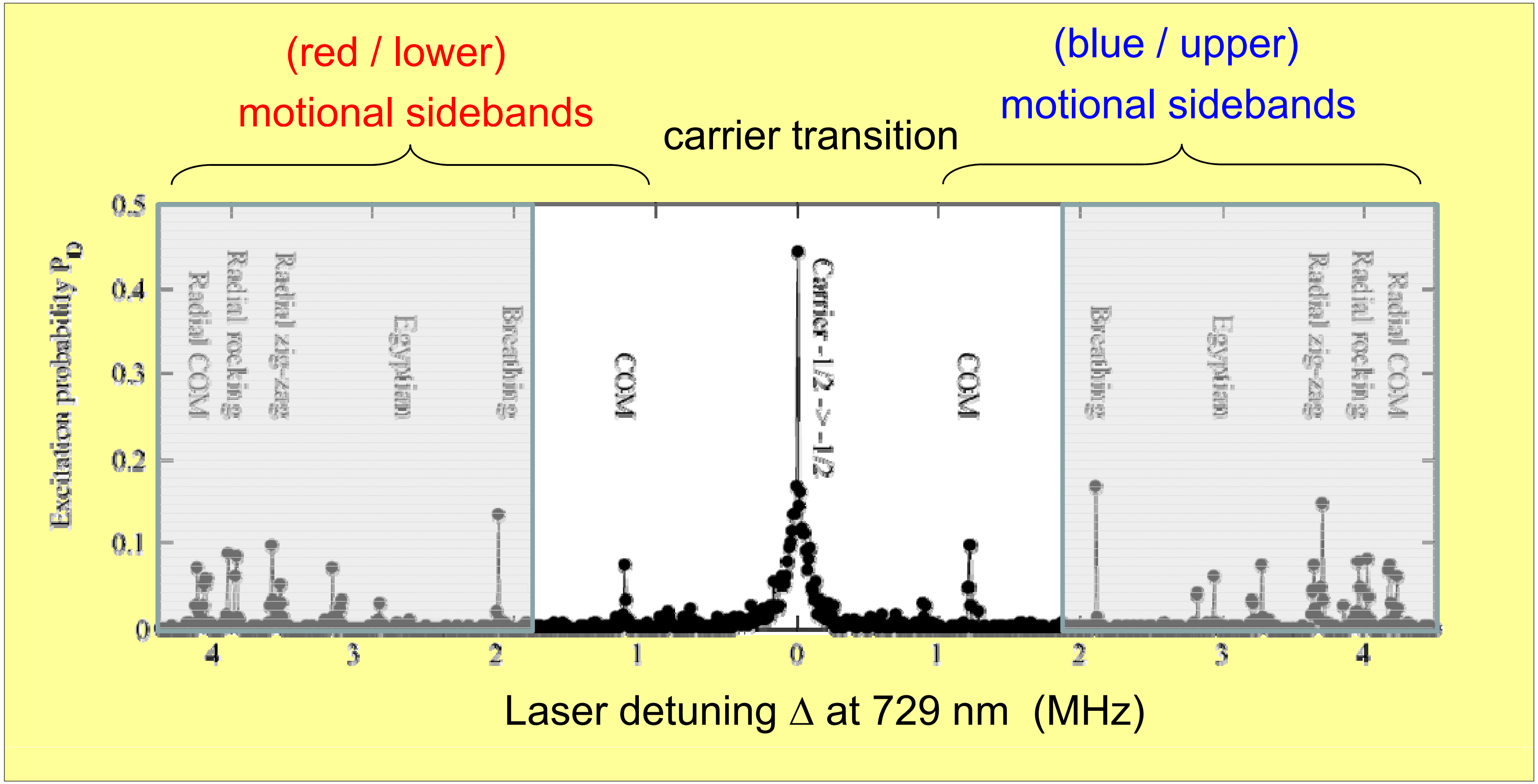


Anti-Jaynes Cummings model

$$H \propto \sigma_+ a^\dagger + \sigma_- a$$

Carrier and sidebands: excitation spectrum (3 ions)

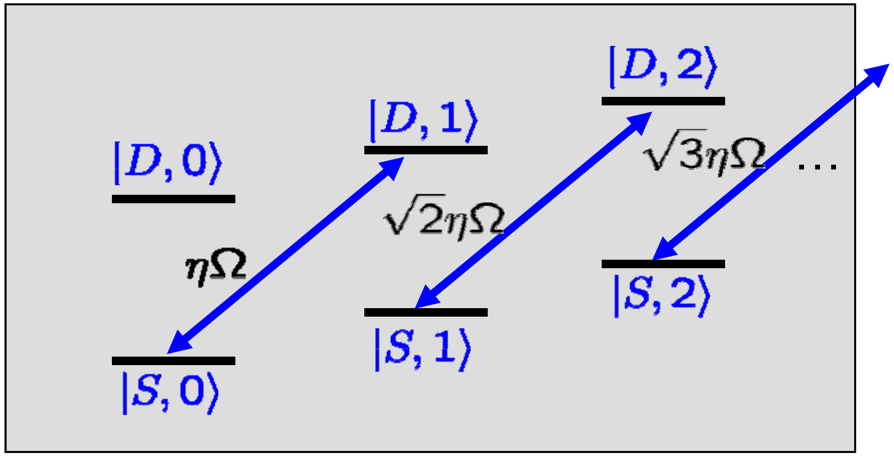
$$S_{1/2}, m = -1/2 \longleftrightarrow D_{5/2}, m = -1/2$$



Coherent excitation on the sideband

„Blue sideband“ pulses:

$$|S\rangle|n\rangle \longleftrightarrow |D\rangle|n+1\rangle$$

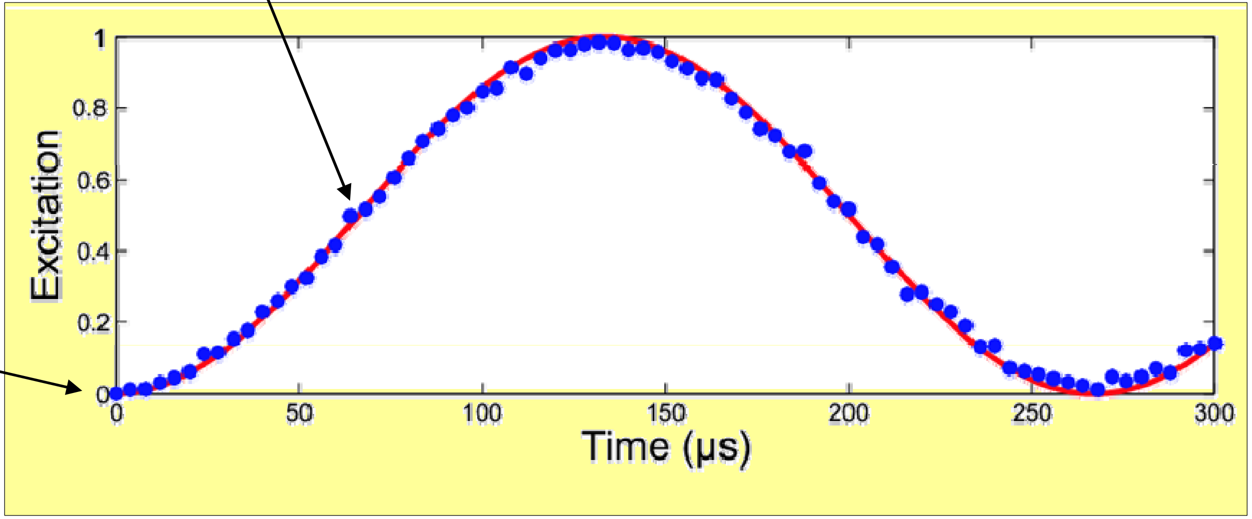


$\theta = \pi/2$: Entanglement between internal and motional state !

$$\frac{1}{\sqrt{2}}(|S,n=0\rangle + |D,n=1\rangle)$$

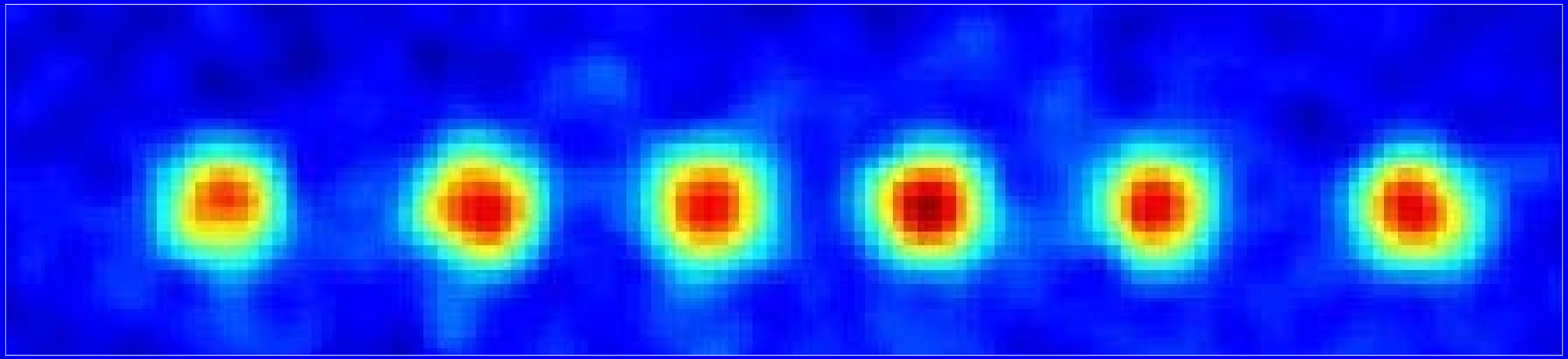
D state population

$$|S,n=0\rangle$$



Entangling quantum gate operations

- Cirac-Zoller CNOT gate
- Mølmer-Sørensen gate
- Conditional phase gate

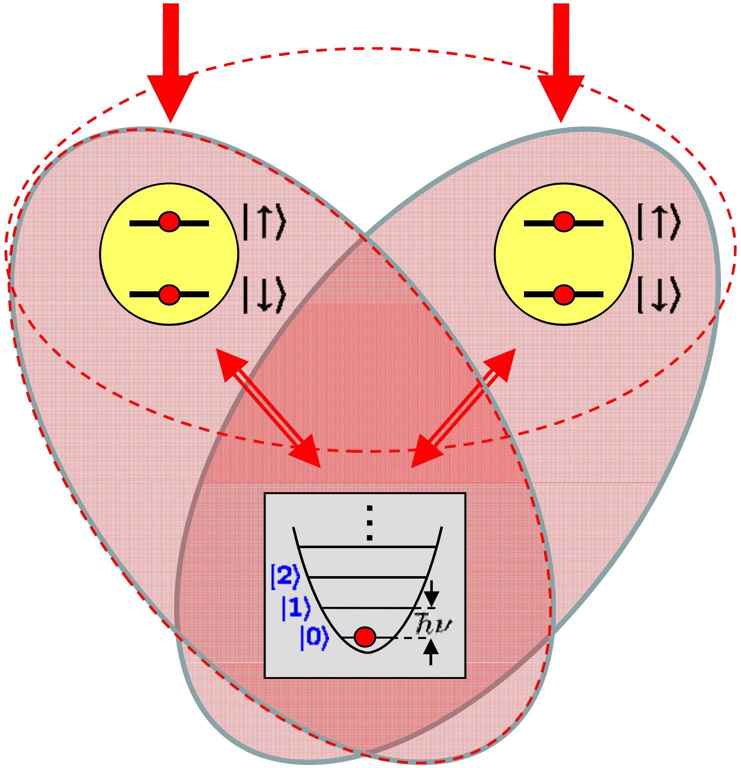


Entangling two qubits

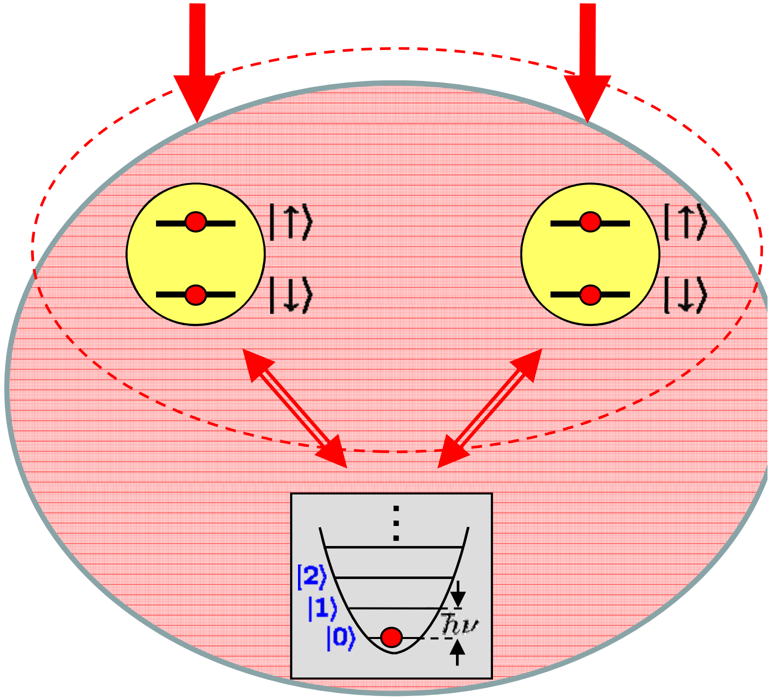
First strategy:

A focussed laser interacts with a single qubit at a time.

- Cirac-Zoller controlled-NOT gate



Entangling two qubits



First strategy:

A focussed laser interacts with a single qubit at a time.

- Cirac-Zoller controlled-NOT gate

Second strategy:

A laser interacts with a several qubits at the same time.

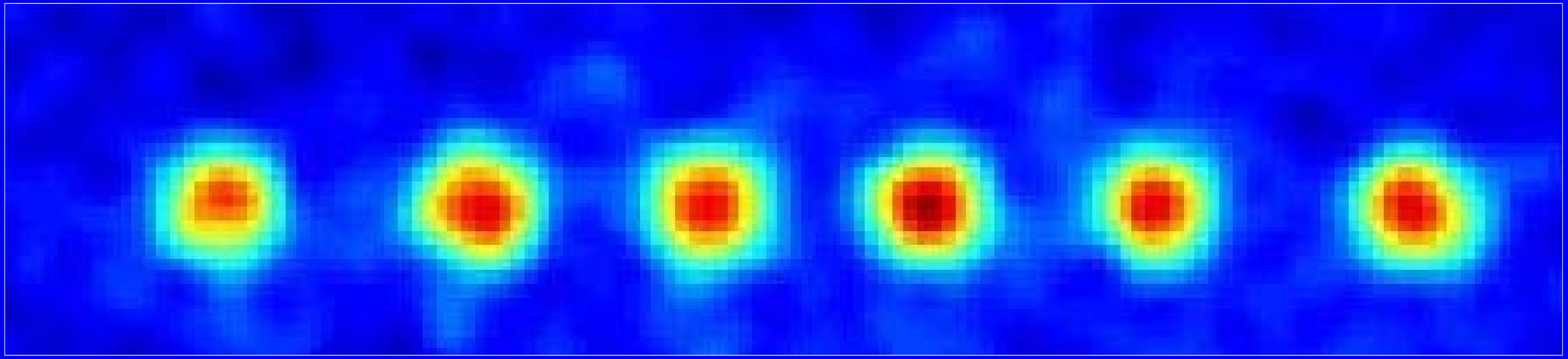
- Mølmer-Sørensen gate
- controlled-phase gate

Mølmer-Sørensen gates

How does it work ?

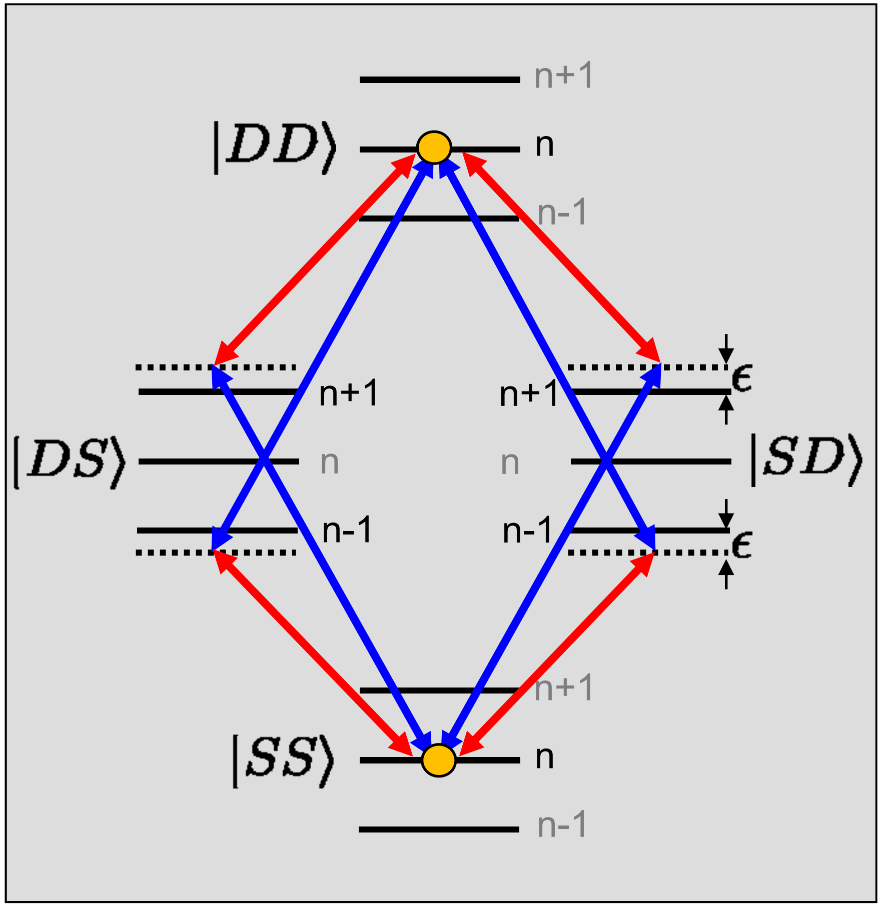
Bell states: creation and verification

GHZ states



Mølmer-Sørensen gate

Two ions



Bichromatic lasers:

$$\omega_r = \omega_0 - (\nu + \epsilon)$$

$$\omega_b = \omega_0 + (\nu + \epsilon)$$

$$\omega_b + \omega_r = 2\omega_0$$

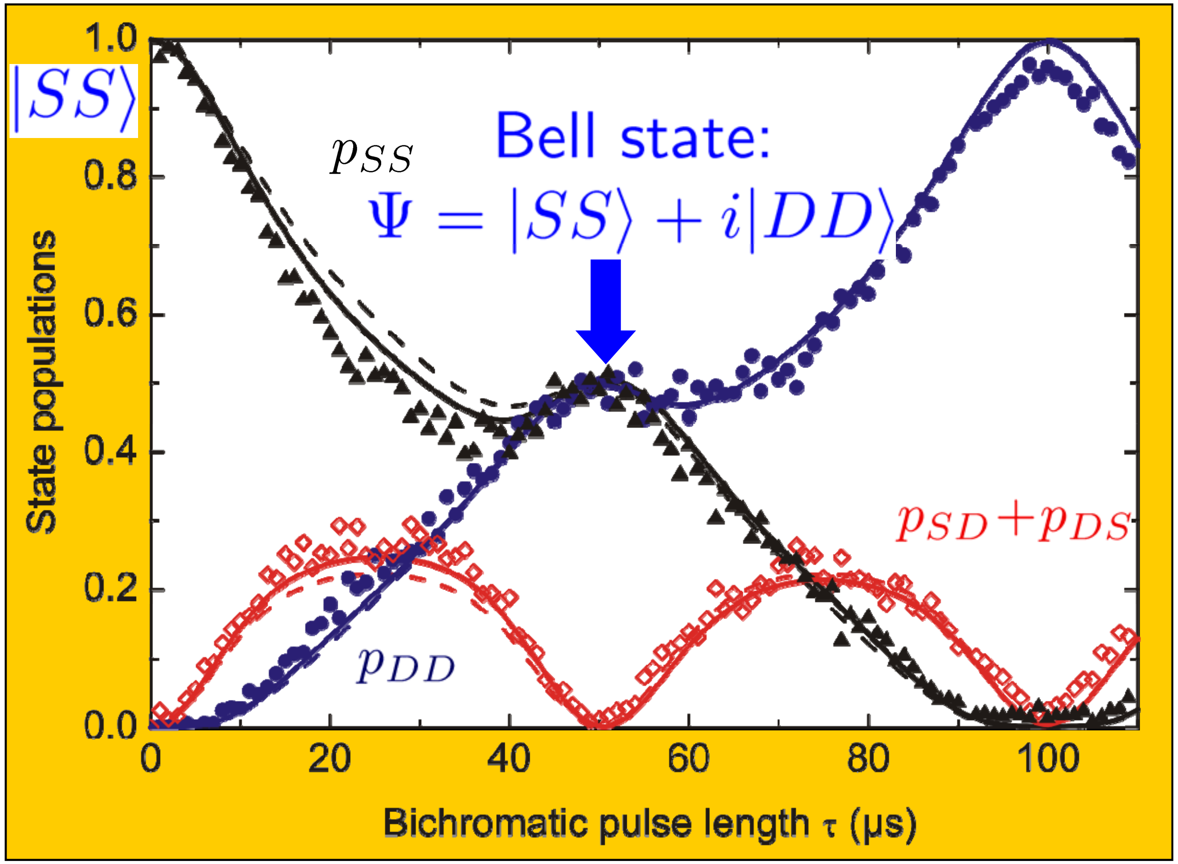
After time $\tau = \frac{2\pi}{\epsilon}$:

$$\boxed{U = e^{-i\Phi\sigma_x^{(1)}\otimes\sigma_x^{(2)}}}$$

Maximally entangling gate

A. Sørensen, K. Mølmer, Phys. Rev. A **62**, 022311 (2000)

Mølmer-Sørensen gate: time evolution

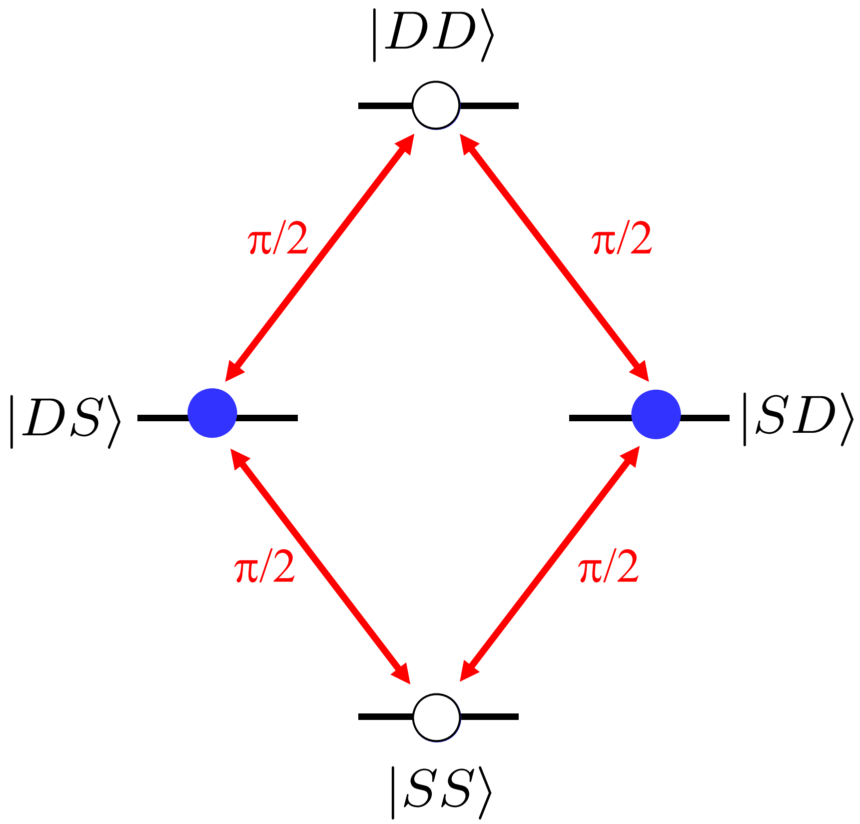


$$\tau_{gate} = 50\,\mu\text{s}$$

$p_{SS}+p_{DD}$ = 0.9965(4) 13,000 measurements

Entanglement check : interference

$$|SS\rangle + |DD\rangle$$

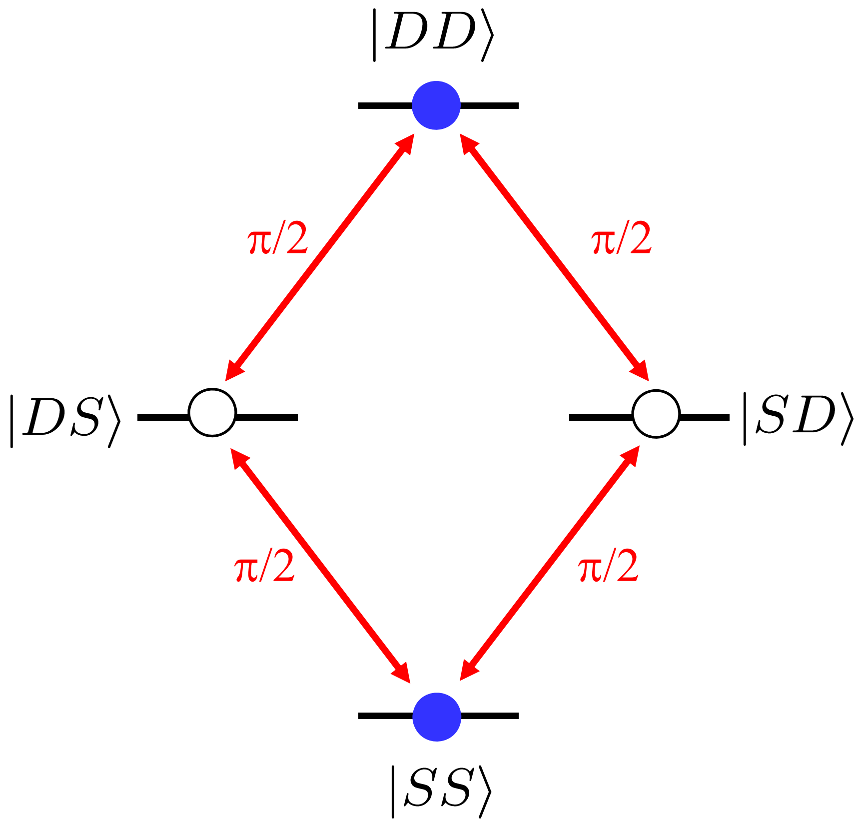


constructive
interference

$$|SS\rangle + |DD\rangle \longrightarrow |SD\rangle + |DS\rangle$$

Entanglement check : interference

$|SS\rangle + |DD\rangle$



constructive interference

Parity

$|SS\rangle + |DD\rangle \longrightarrow |SD\rangle + |DS\rangle$ - 1

$\searrow |SS\rangle + |DD\rangle$ + 1

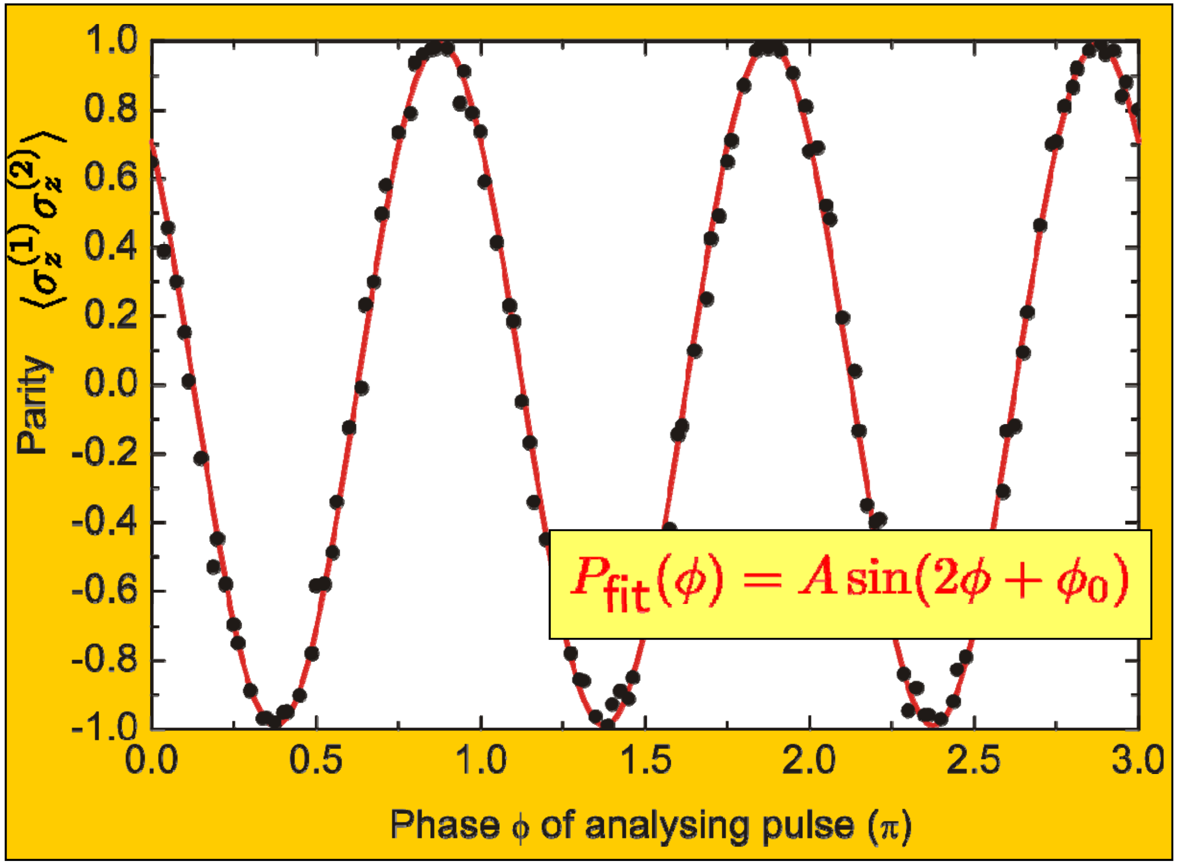
destructive interference

Final state depends on phase of $\pi/2$ pulse

Coherence measurement:
Scan ϕ and measure parity

Mølmer-Sørensen gate: parity oscillations

Parity oscillation contrast: $|\langle SS|\rho_\Psi|DD\rangle|$



Bell state:

$\Psi = |SS\rangle + i|DD\rangle$

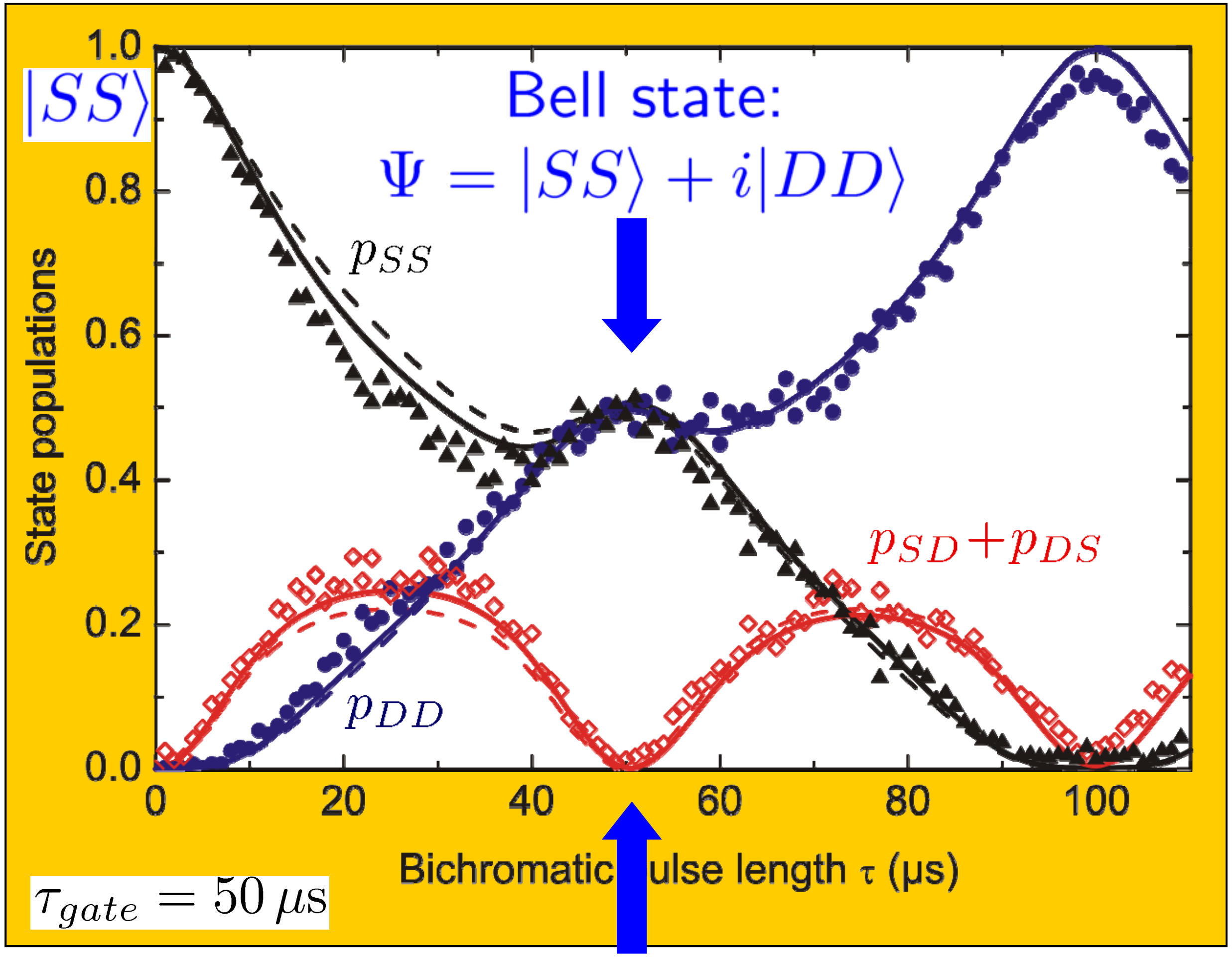
A = 0.990(1) 29,400 measurements

$p_{SS}+p_{DD}$ = 0.9965(4) 13,000 measurements

Bell state fidelity

F=99.3(1)%

Creating Bell states



Bell state:
$\Psi = |SS\rangle + i|DD\rangle$

$\tau_{gate} = 50\,\mu s$

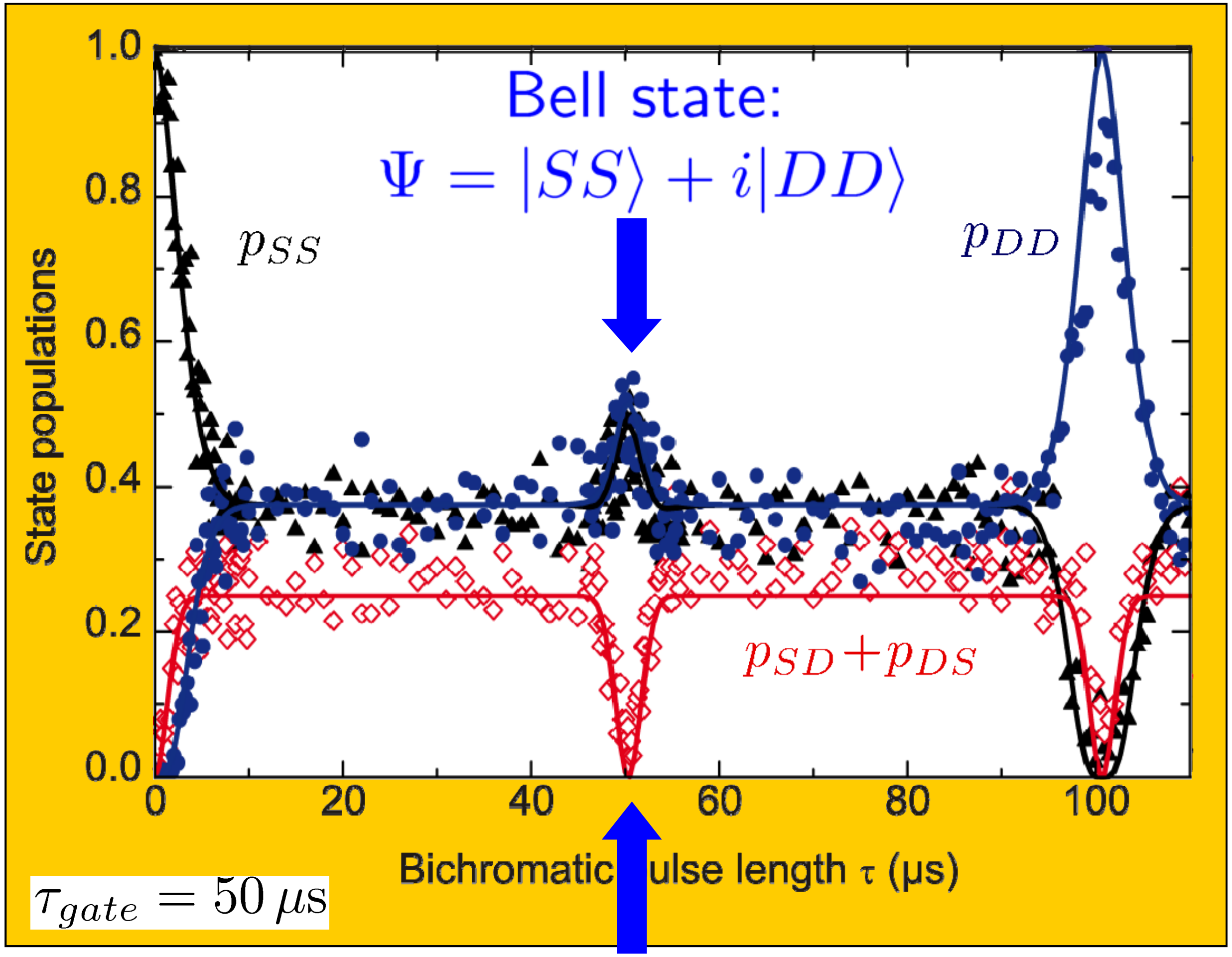
Fidelity

F=99.3(1)%

$\langle \bar{n} \approx 0 \rangle$

J. Benhelm *et al.*, Nat. Phys. **4**, 463 (2008)

‘Hot’ Bell states



Fidelity

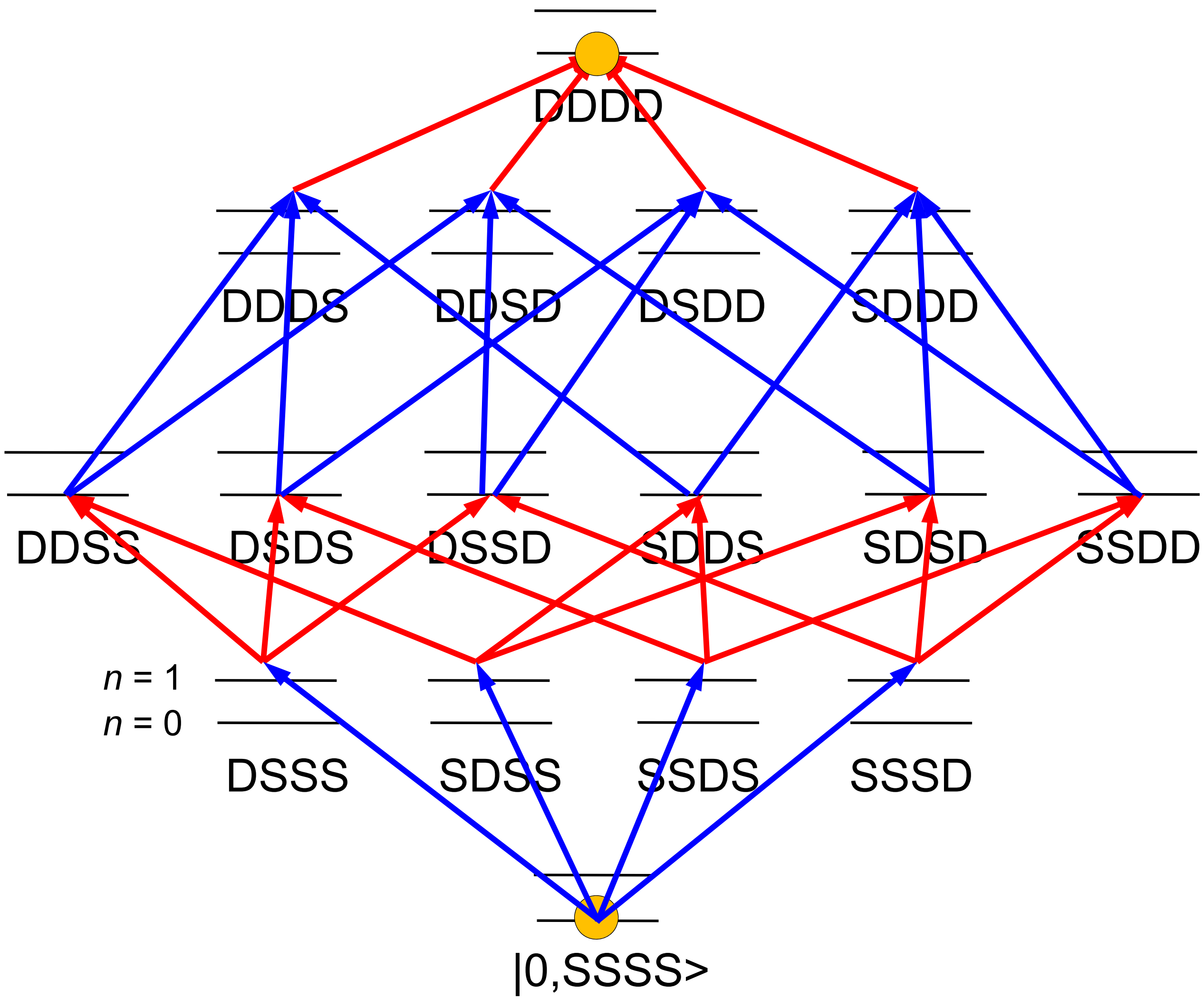
F ≈ 98%

Doppler-cooled ions !

$\langle \bar{n} \approx 18 \rangle$

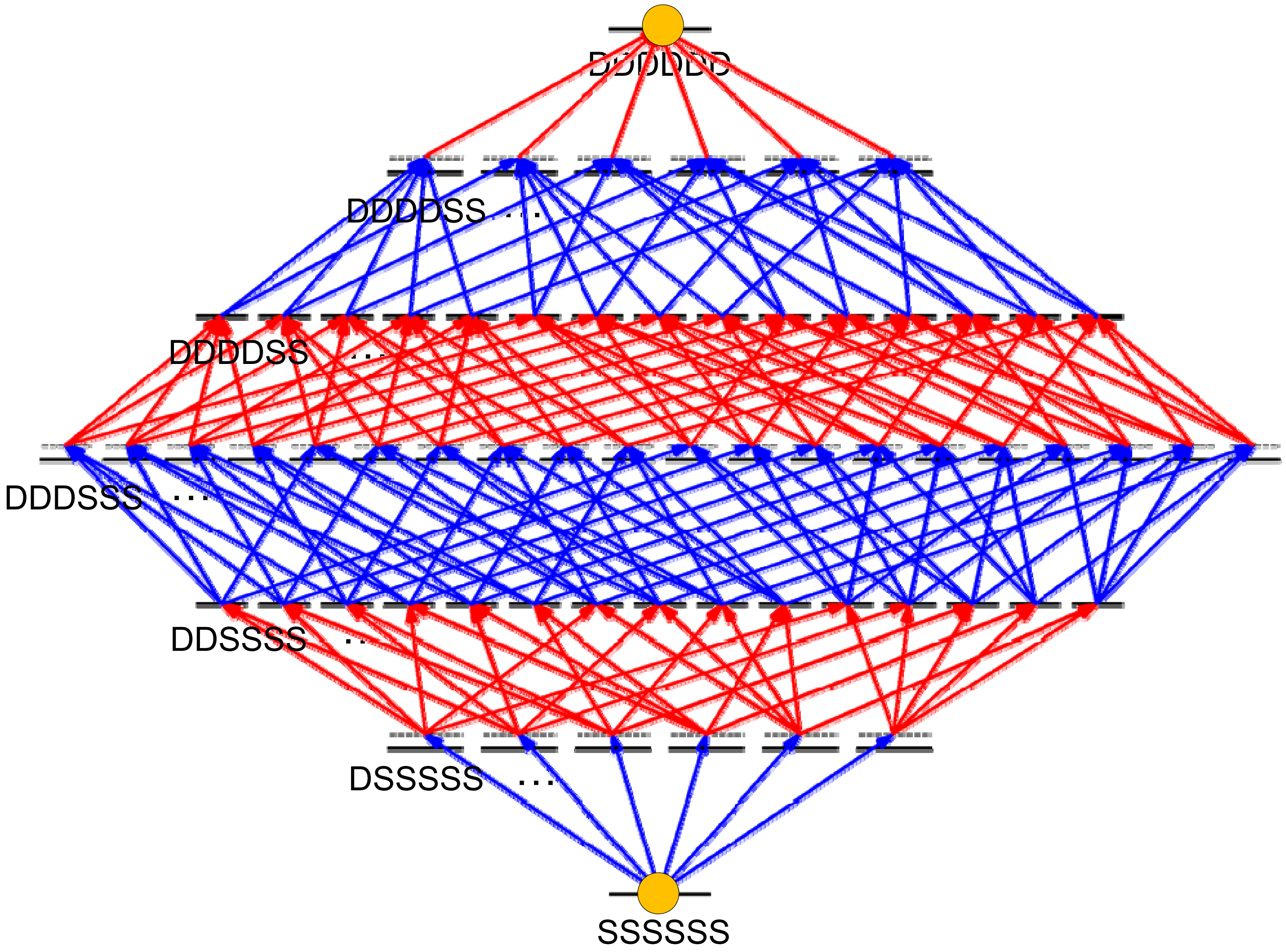
G. Kirchmair *et al.*, New J. Phys **11**, 023002 (2009)

Creating GHZ-states with 4 ions



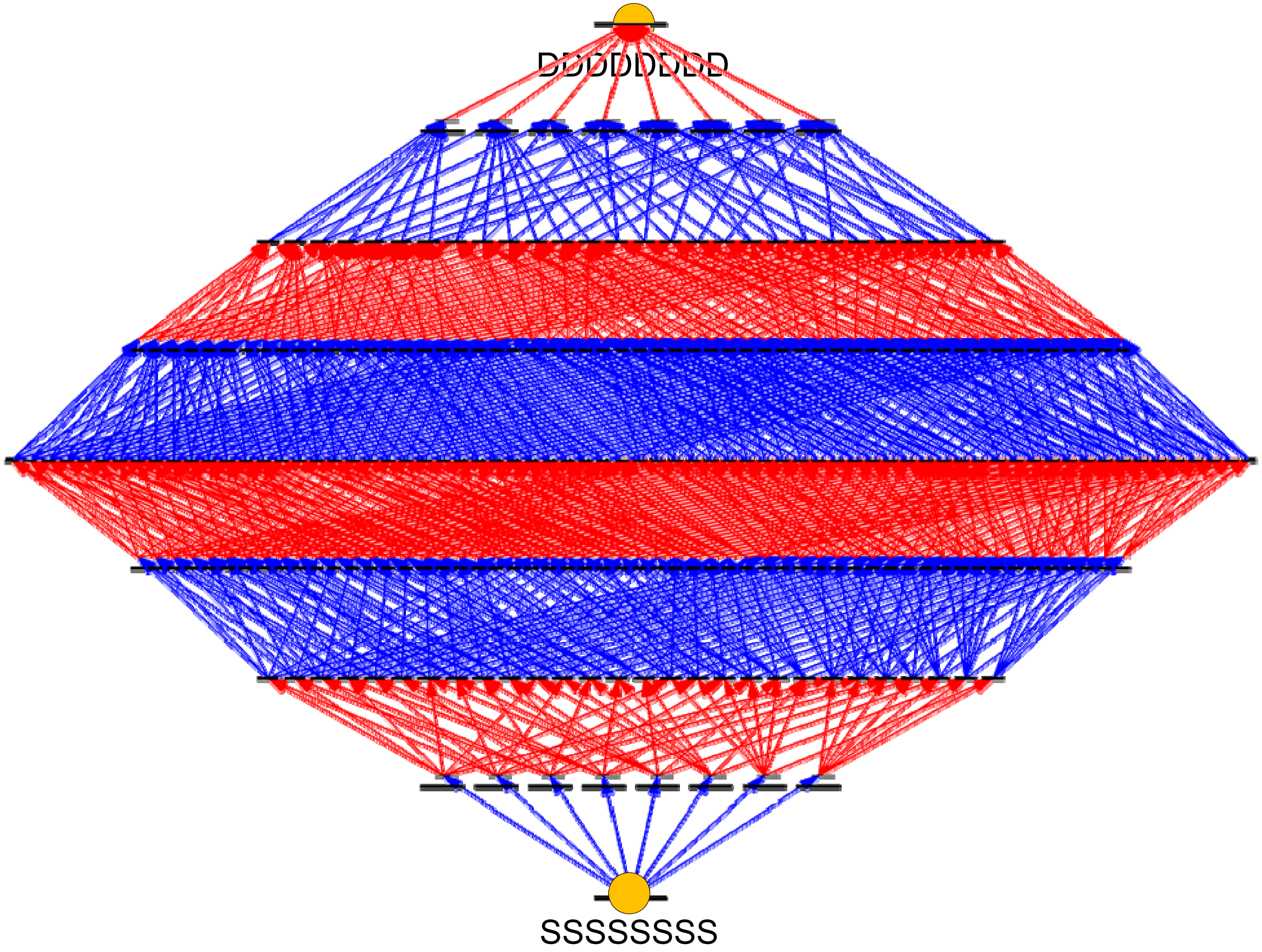
$$|SSSS\rangle \longrightarrow (|SSSS\rangle + |DDDD\rangle)/\sqrt{2}$$

Creating GHZ-states with 6 ions



$$|SSSSSS\rangle \longrightarrow (|SSSSSS\rangle + |DDDDDD\rangle)/\sqrt{2}$$

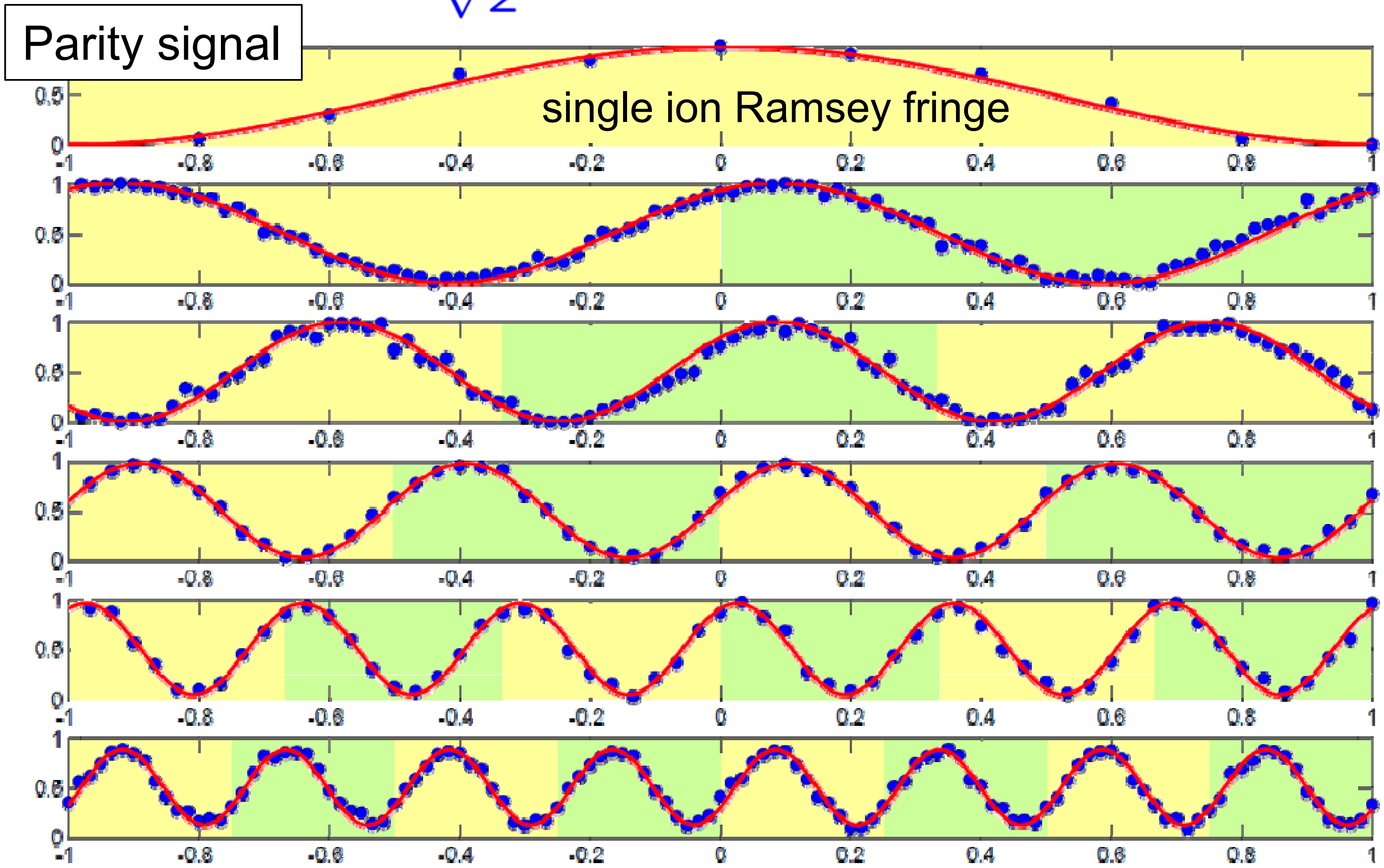
Creating GHZ-states with 8 ions



$$|SSSSSSSS\rangle \longrightarrow (|SSSSSSSS\rangle + |DDDDDDDD\rangle)/\sqrt{2}$$

N - qubit GHZ state generation

$$\Psi = \frac{1}{\sqrt{2}}(|SS\ldots S\rangle + |DD\ldots D\rangle)$$



$N =$	Fidelity (%)
1	99.5(7)
2	99.6(1.6)
3	98.7(2.0)
4	95.8(1.5)
6	91.9(3.0)
8	82.1(2.8)

T. Monz, P. Schindler, J. Barreiro, M. Hennrich, Innsbruck (2009)

Further literature

Review articles:

D. Leibfried et al.,*Quantum dynamics of single trapped ions*‘,
Rev. Mod. Phys. **75**, 281 (2003)

H. Häffner et al.,*Quantum computing with trapped ions*‘,
Phys. Rep. **469**, 155 (2008)

R. Blatt, D. Wineland, ‚*Entangled states of trapped atomic ions*‘,
Nature **453**, 1008 (2008)