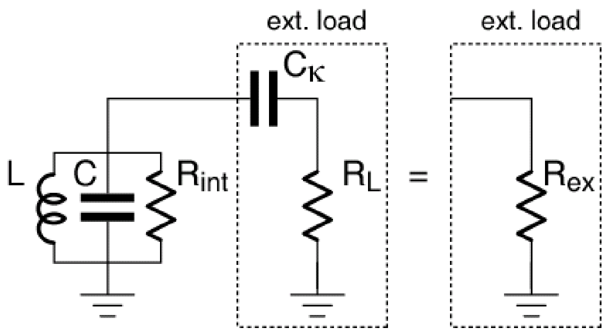
# Internal and External Dissipation in an LC Oscillator




internal losses: $R_{\mathrm{int}}$
conductor, dielectric

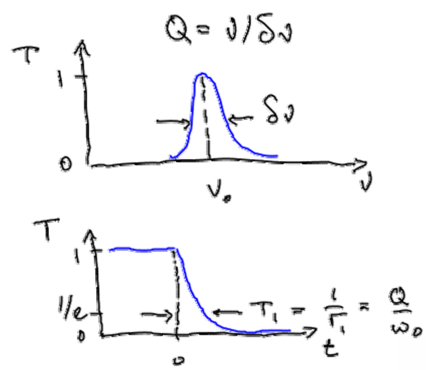
external losses: $R_{\mathrm{ext}}$
radiation, coupling

total losses $$\frac{1}{R} = \frac{1}{R_{\mathrm{int}}} + \frac{1}{R_{\mathrm{ext}}}$$

impedance $$Z = \sqrt{\frac{L}{C}}$$

quality factor $$Q = \frac{R}{Z} = \omega_0 RC$$

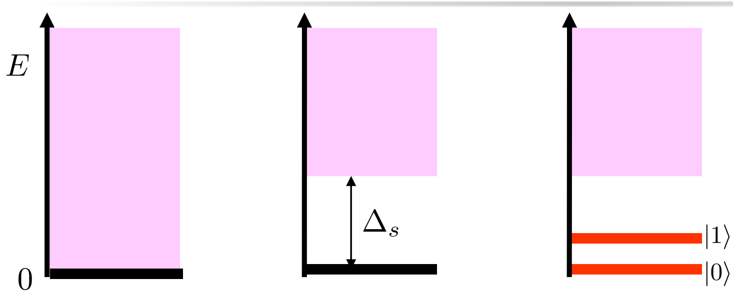
excited state decay rate $$\Gamma_1 = \frac{\omega_0}{Q} = \frac{1}{RC}$$




problem 2: internal and external dissipation



# Why Superconductors?



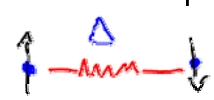

normal metal superconductor How to make qubit?

- single non-degenerate macroscopic ground state
- elimination of low-energy excitations

Superconducting materials (for electronics):

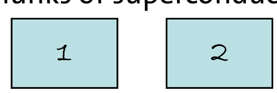
- Niobium (Nb): $2\Delta/h$ = 725 GHz, $T_c$ = 9.2 K
- Aluminum (Al): $2\Delta/h$ = 100 GHz, $T_c$ = 1.2 K

Cooper pairs:
bound electron pairs




Bosons (S=0, L=0)

2 chunks of superconductors

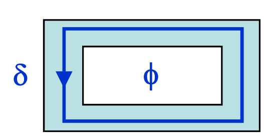



macroscopic wave function

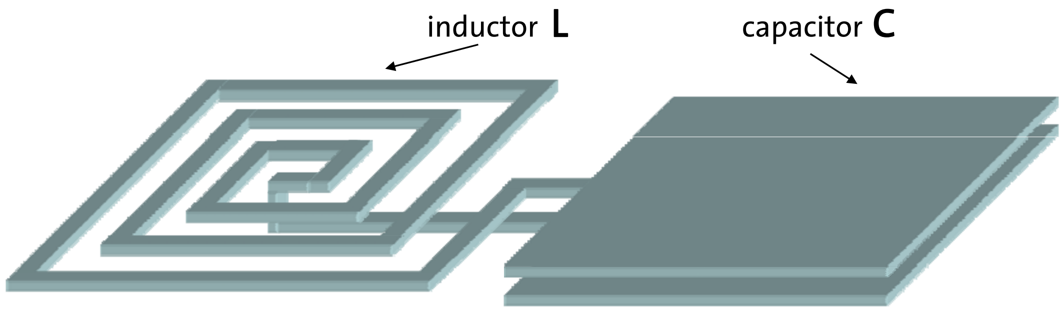
$$\Psi_i = \sqrt{n_i}\, e^{i\delta_i}$$

Cooper pair density $n_i$
and global phase $\delta_i$

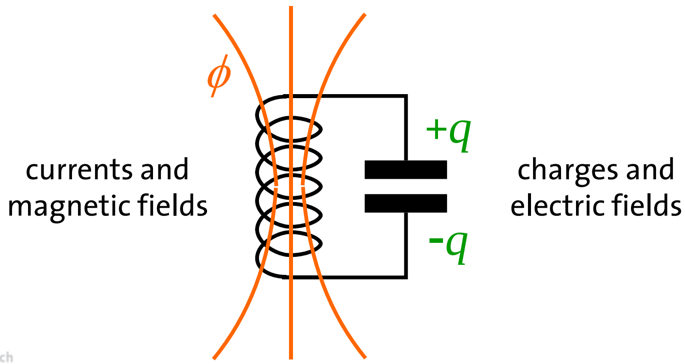
phase quantization: $\delta = n\, 2\pi$
flux quantization: $\phi = n\, \phi_0$



## Realization of H.O.: Lumped Element Resonator



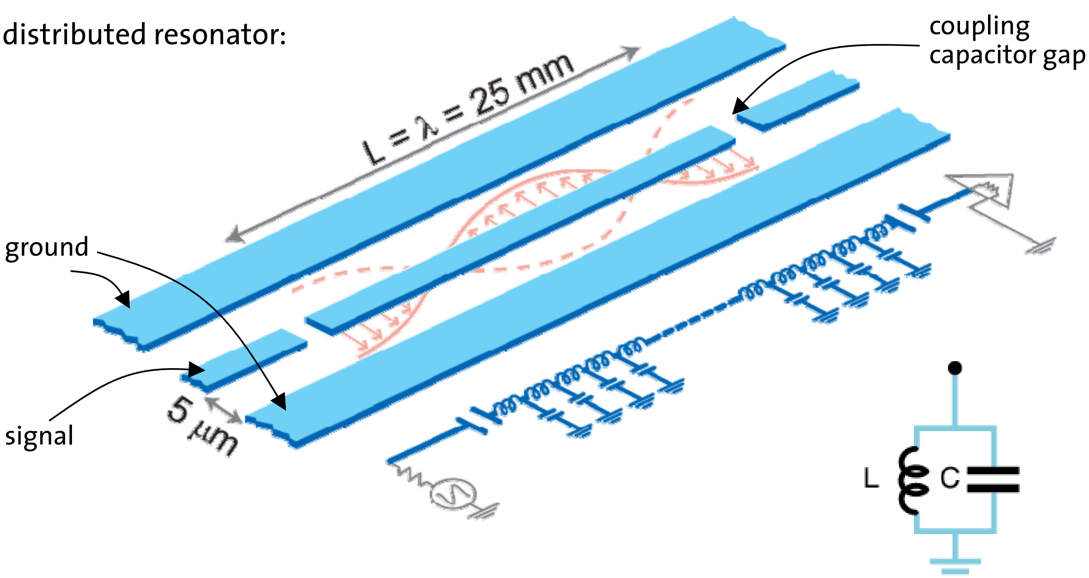

a harmonic oscillator






## Realization of H.O.: Transmission Line Resonator

distributed resonator:

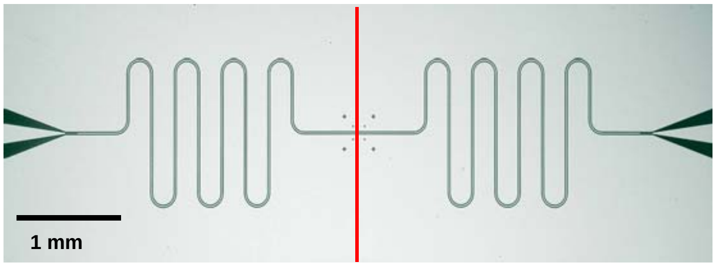



- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

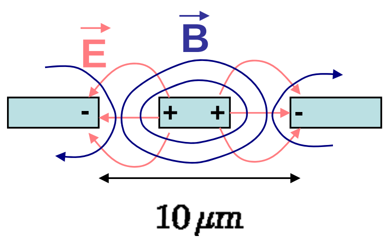
M. Goeppl *et al.*, Coplanar Waveguide Resonators for Circuit QED, *Journal of Applied Physics* **104**, 113904 (2008)

# Realization of Transmission Line Resonator

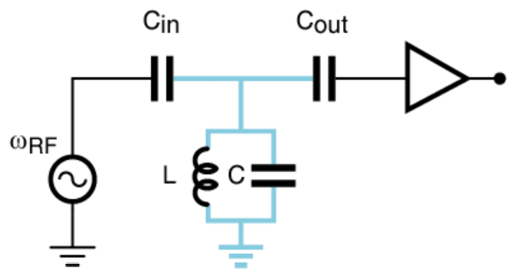
coplanar waveguide:




cross-section of transm. line (TEM mode):



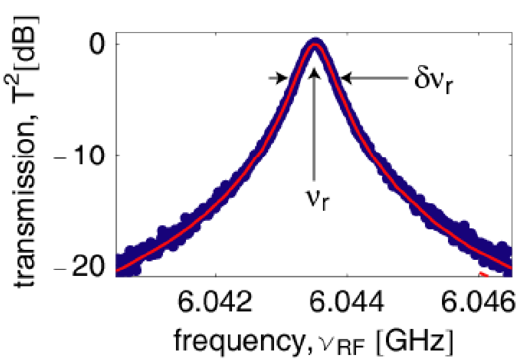

measuring the resonator:

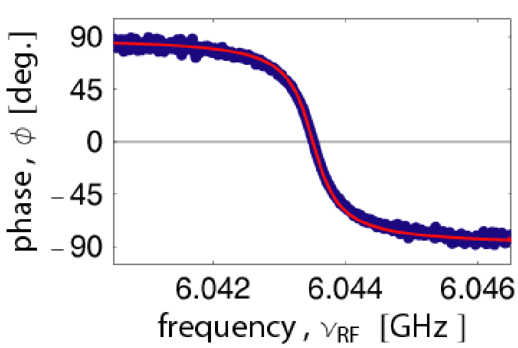



photon lifetime (quality factor) controlled by coupling capacitors $C_{in/out}$



# Resonator Quality Factor and Photon Lifetime




resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

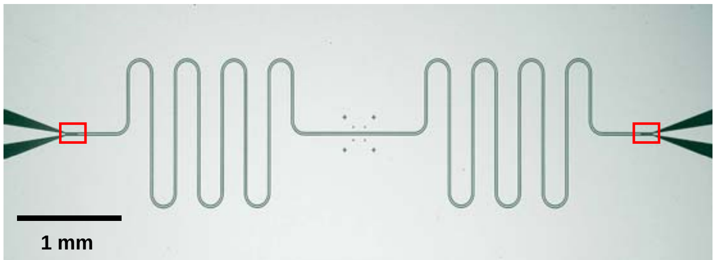
photon decay rate:

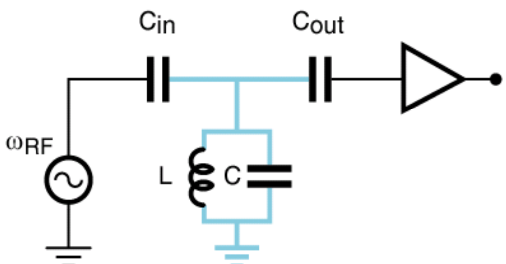
$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

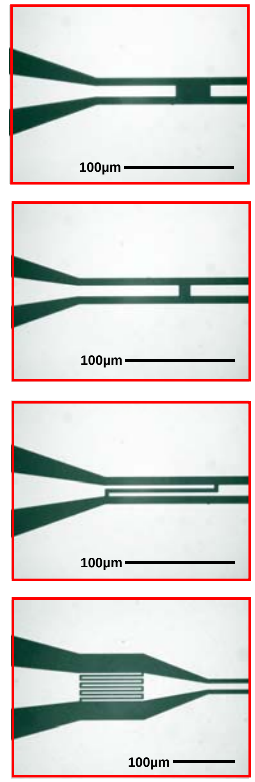
photon lifetime:

$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

# Controlling the Photon Life Time



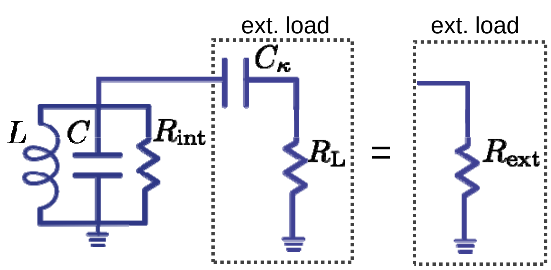



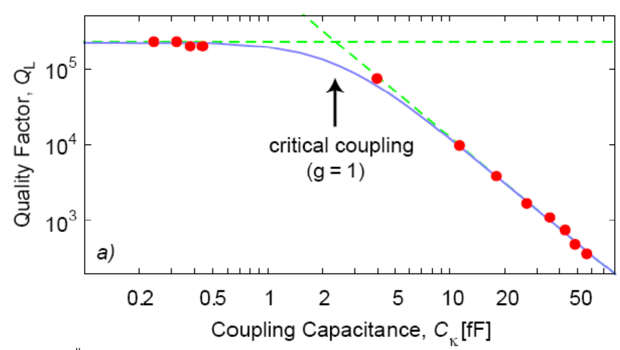



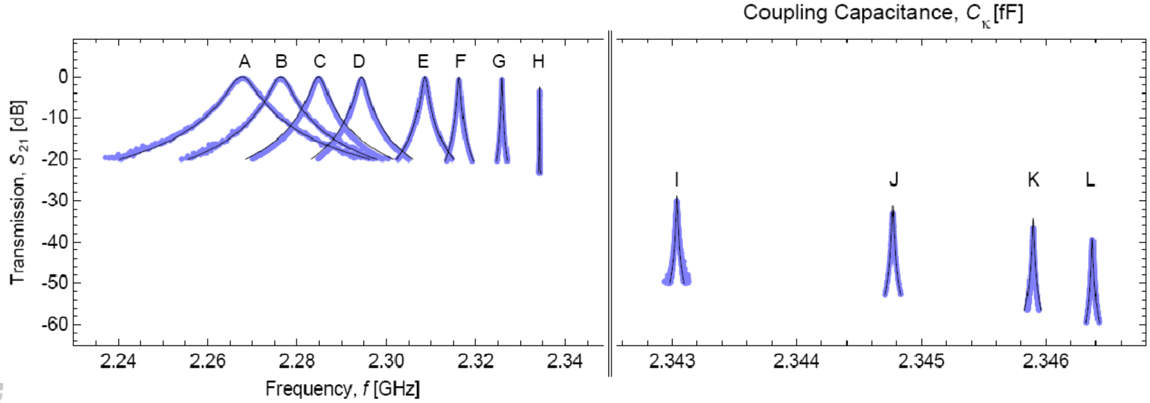

photon lifetime (quality factor)
controlled by coupling capacitor $C_{in/out}$



# Quality Factor Measurement

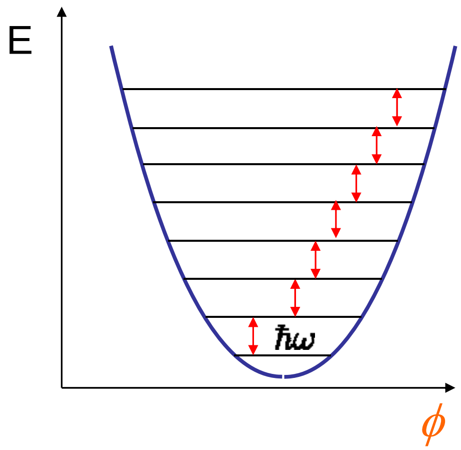



M. Goeppl *et al., J. Appl. Phys.* **104**, 113904 (2008)

# Quantum Harmonic Oscillator at Finite Temperature




thermal occupation:

$$\langle n_{\text{th}} \rangle = \frac{1}{\exp\left(h\nu / k_B T\right) - 1}$$

low temperature required:

$$\hbar\omega \gg k_B T$$

10 GHz ~ 500 mK ↗ ↖ 20 mK

$$\langle n_{\text{th}} \rangle \sim 10^{-11}$$



# How to Prove that a Harmonic Oscillator is Quantum?

measure:

- resonance frequency
- average charge (momentum)
- average flux (position)

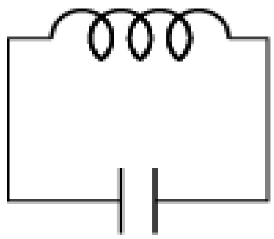
all averaged quantities are identical for a purely harmonic oscillator in the classical or quantum regime

solution:

- make oscillator non-linear in a controllable way

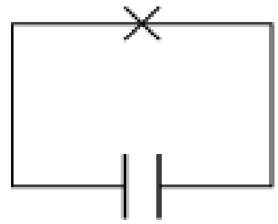
# Superconducting Nonlinear Oscillators

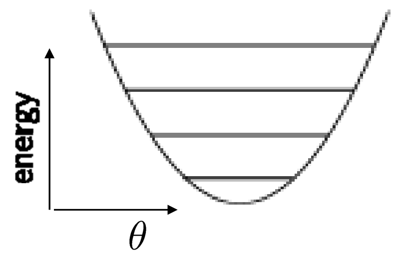
LC resonator



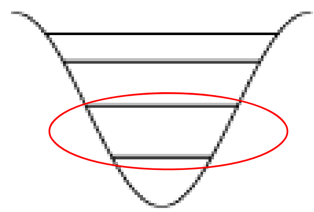
Josephson junction resonator

Josephson junction = nonlinear inductor





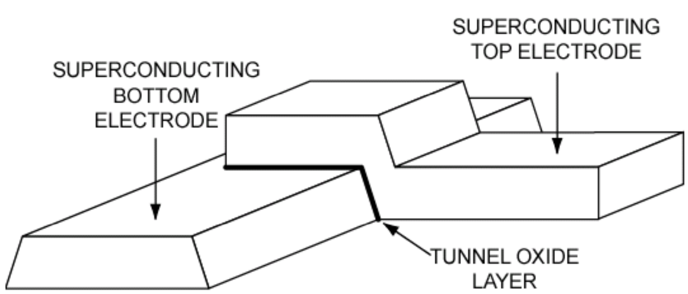

anharmonicity → effective two-level system





# A Low-Loss Nonlinear Element

a (superconducting) Josephson junction

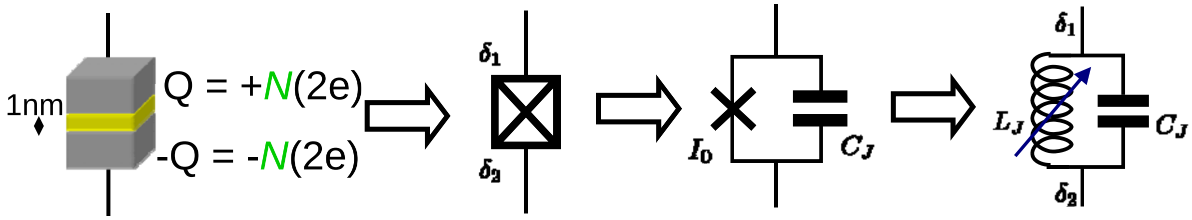



- superconductors: Nb, Al
- tunnel barrier: $AlO_x$

M. Tinkham, *Introduction to Superconductivity* (Krieger, Malabar, 1985).

# Josephson Tunnel Junction

the only non-linear LC resonator with no dissipation (BCS, $k_B T \ll \Delta$)




**tunnel junction parameters:**

- critical current $I_0$
- junction capacitance $C_J$
- high internal resistance $R_J$

**Josephson relations:**

$$I = I_0 \sin\delta$$

$$V = \phi_0 \frac{\partial\delta}{\partial t}$$

**flux quantum:** $\phi_0 = \frac{h}{2e}$

**phase difference:** $\delta = \delta_2 - \delta_1$

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derivation of Josephson effect, see e.g.: chap. 21 in R. A. Feynman: Quantum mechanics, The Feynman Lectures on Physics. Vol. 3 (Addison-Wesley, 1965)

The Josephson junction as a non-linear inductor

induction law: $V = -L \frac{\partial I}{\partial t}$

Josephson effect: dc-Josephson equation $I = I_c \sin\delta$

$$\frac{\partial I}{\partial t} = I_c \cos\delta \frac{\partial\delta}{\partial t}$$

ac-Josephson equation

$$V = \frac{\phi_0}{2\pi}\frac{\partial\delta}{\partial t} = \underbrace{\frac{\phi_0}{2\pi I_c}\frac{1}{\cos\delta}}_{L_J}\frac{\partial I}{\partial t}$$

Josephson inductance

$$L_J = \frac{\phi_0}{2\pi I_c}\frac{1}{\cos\delta}$$

specific Josephson inductance ($\frac{\phi_0}{2\pi I_c}$) — nonlinearity ($\frac{1}{\cos\delta}$)

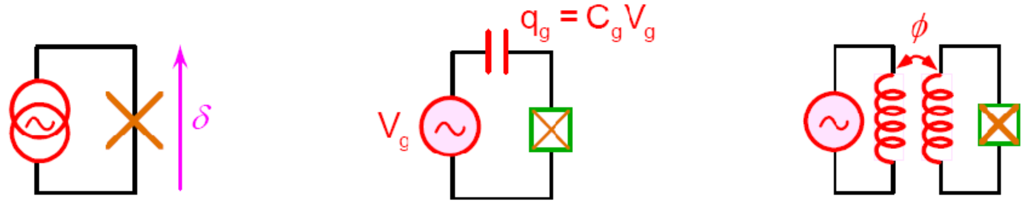
A typical characteristic Josephson inductance for a tunnel junction with $I_c = 100$ nA is $L_{J0} \sim 3$ nH.

review: M. H. Devoret *et al.*, Quantum tunneling in condensed media, *North-Holland*, (1992)

# How to Make Use of the Josephson Junction in Qubits?

different bias (control) circuits:

| phase qubit | charge qubit | flux qubit |
|---|---|---|




| current bias | charge bias | flux bias |
|---|---|---|

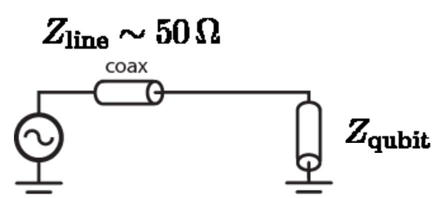
How is the control circuit important?



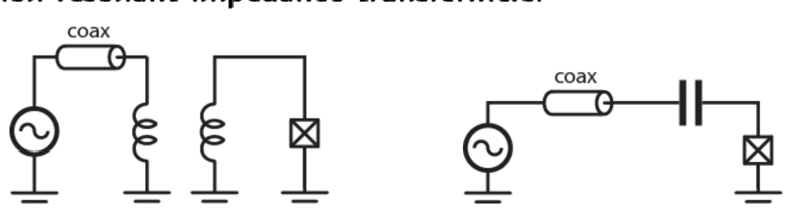
# Controlling Coupling to the E.M. Environment

coupling to environment (bias wires):

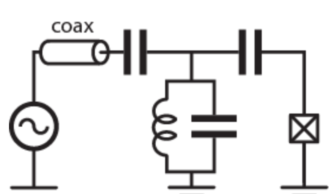
**decoherence**
**from energy relaxation**
**(spontaneous emission)**




decoupling using non-resonant impedance transformers:




using resonant impedance transformers




control spontaneous emission
by circuit design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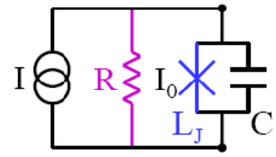
## Current Biased Phase Qubit




The bias current $I$ distributes into a Josephson current through an ideal Josephson junction with critical current $I_c$, through a resistor $R$ and into a displacement current over the capacitor $C$.

Kirchhoff's law:

$$I_b = I_s + I_R + I_C$$
$$= I_c \sin\delta + \frac{V}{R} + C\dot{V}$$

$$I_C = \dot{Q}_C = C\dot{V}$$
$$I_R = V/R$$
$$I_s = I_c \sin\delta$$

use Josephson equations:

$$I_b = I_c \sin\delta + \frac{\Phi_0}{2\pi R}\dot{\delta} + \frac{\Phi_0 C}{2\pi}\ddot{\delta}$$


W.C. Stewart, Appl. Phys. Lett. **2**, 277, (1968)
D.E. McCumber, J. Appl. Phys. **39**, 3 113 (1968)


looks like equation of motion for a particle with mass $m$ and coordinate $\delta$ in an external potential $U$:

$$m\ddot{\delta} + m\frac{1}{RC}\dot{\delta} + \frac{\partial U(\delta)}{\partial\delta} = 0$$

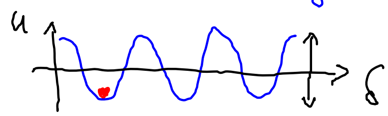
particle mass: $m = C\,(\Phi_0/2\pi)^2$

external potential: $U(\delta) = \frac{I_c\Phi_0}{2\pi}\left(-\frac{I_b}{I_c} - \cos\delta\right)$

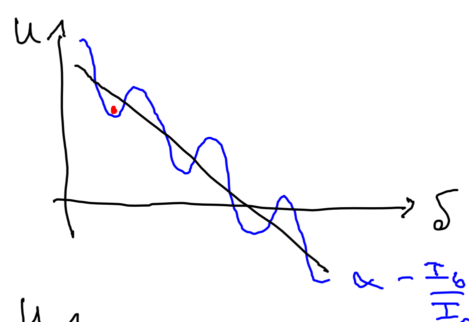
## Phase particle in a potential well

$$U(\delta) = \frac{I_c\Phi_0}{2\pi}\left(-\frac{I_b}{I_c}\delta - \cos\delta\right) \qquad E_J = \frac{I_c\Phi_0}{2\pi}$$

cosine potential for $I_b = 0$:




'tilted washboard' potential for $I_b \neq 0$:



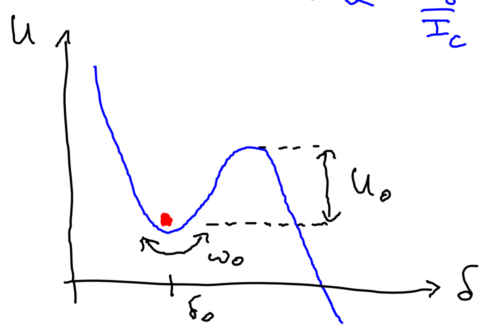

potential barrier:

$$U_0 = 2E_J\left[\sqrt{1-\gamma^2} - \gamma \arccos\gamma\right]$$

oscillation frequency:

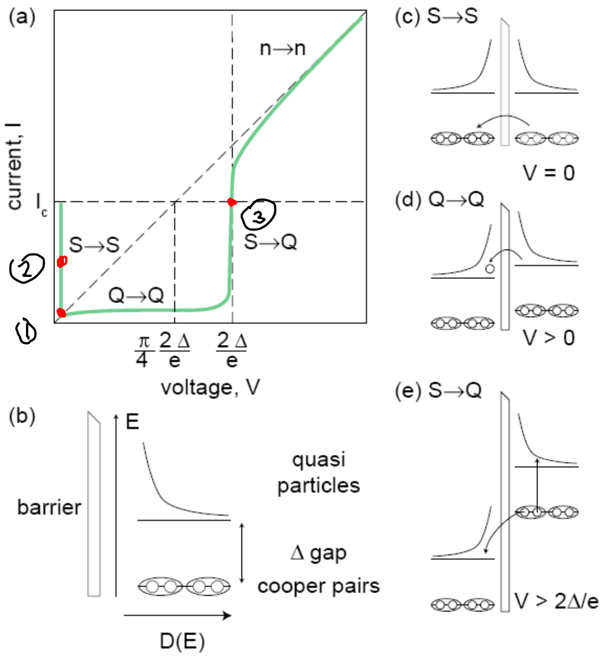
$$\omega_0 = \omega_p(1-\gamma^2)^{1/4} = \sqrt{\frac{U''(\delta_0)}{m}}$$

with: $\gamma = I_b/I_c$ ; $\omega_p = \sqrt{\frac{2\pi I_c}{\Phi_0 C}}$

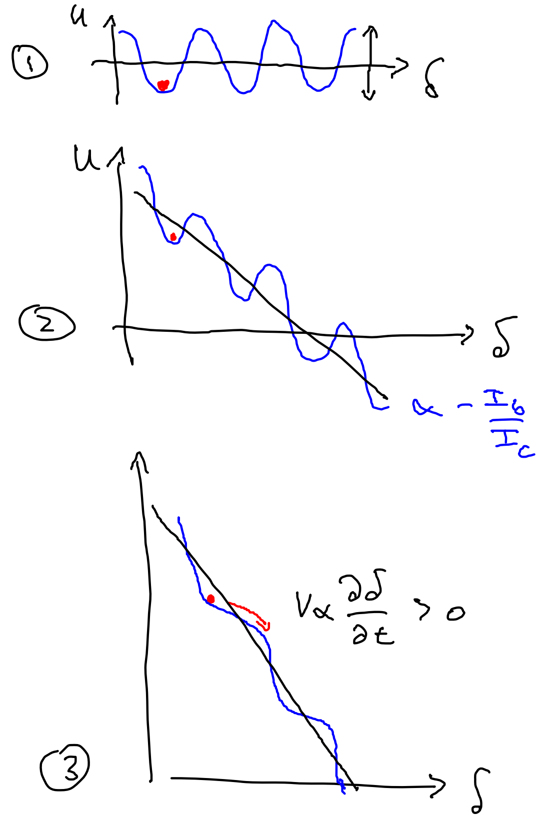


## Current-voltage characteristics

typical I-V curve of underdamped Josephson junctions:




band diagram




## Thermal Activation and Quantum Tunneling:

bias current dependence

$\omega_0(\gamma)\ ;\ U_0(\gamma)$

thermal activation rate:

$$\Gamma_{th} = a_t \frac{\omega_0}{2\pi} \exp\left(-\frac{U_0}{k_B T}\right)$$

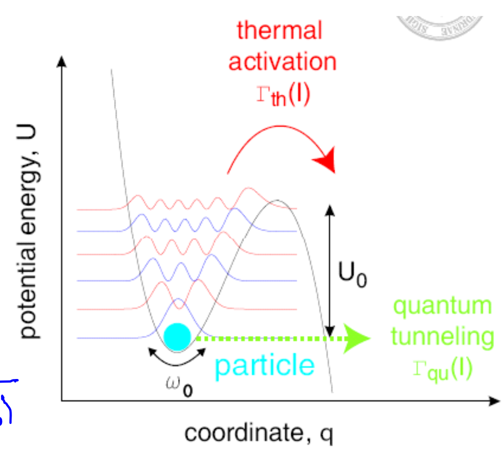
damping dependent prefactor

quantum tunneling rate:

$$\Gamma_{qu} = a_q \frac{\omega_0}{2\pi} \exp\left(-\frac{36}{5}\frac{U_0}{\hbar\omega_0}\right)$$

calculated using WKB method (exercise)

$$\Gamma_q = a_q\, \omega_0 \exp - \int_{\delta_1}^{\delta_2} \frac{1}{\hbar}\sqrt{2m\,(u(\delta) - E_0)}$$




energy level quantization:

$$E_n \approx \hbar\omega_0\left(n + \frac{1}{2}\right)$$

neglecting non-linearity


**Quantum Mechanics of a Macroscopic Variable: The Phase Difference of a Josephson Junction**
JOHN CLARKE, ANDREW N. CLELAND, MICHEL H. DEVORET, DANIEL ESTEVE, and JOHN M. MARTINIS
*Science* 26 February 1988 239: 992-997 [DOI: 10.1126/science.239.4843.992] (in Articles) Abstract » References » PDF »

**Macroscopic quantum effects in the current-biased Josephson junction**
M. H. Devoret, D. Esteve, C. Urbina, J. Martinis, A. Cleland, J. Clarke
*in* Quantum tunneling in condensed media, North-Holland (1992)

# Early Results (1980's)

search for macroscopic quantum effects in superconducting circuits

**theoretical predictions:**

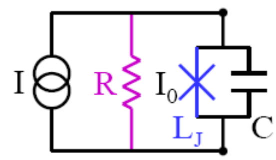
- tunneling ✓
- energy level quantization ✓
- coherence ✗

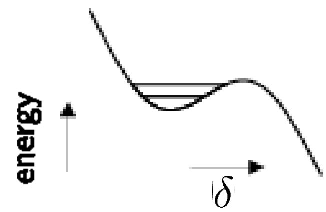
A.J. Leggett *et al.*,
*Prog. Theor. Phys. Suppl.* **69**, 80 (1980),
*Phys. Scr.* **T102**, 69 (2002).

short coherence times due to
strong coupling to em environment

**experimental verification:**

current biased JJ = phase qubit



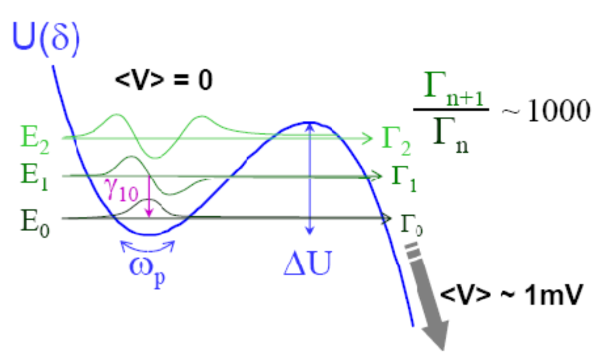



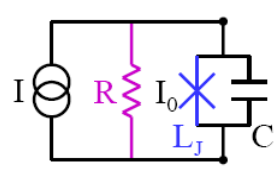

J. Clarke, J. Martinis, M. Devoret *et al.*, *Science* **239**, 992 (1988).



# The Current Biased Phase Qubit

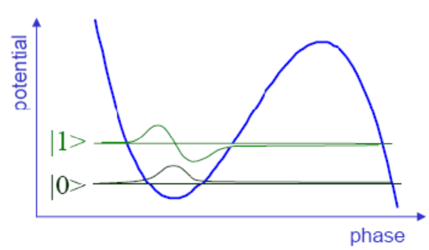
operating a current biased Josephson junction as a superconducting qubit:




<u>initialization:</u>

wait for |1> to decay to |0>, e.g. by
spontaneous emission at rate $\gamma_{10}$

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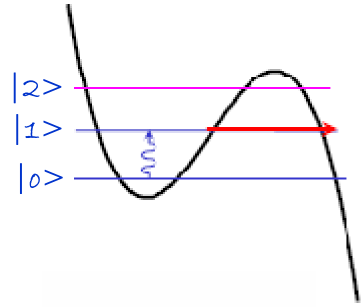
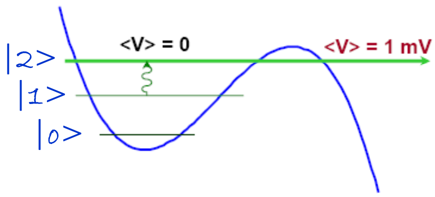
# Read-Out Ideas

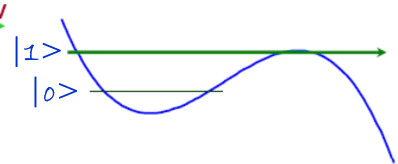
*measuring the state of a current biased phase qubit*




tunneling:

- prepare state |1> (pump)
- wait ($\Gamma_1 \sim 10^3\,\Gamma_0$)
- detect voltage
- |1> = voltage, |0> = no voltage




pump and probe pulses:

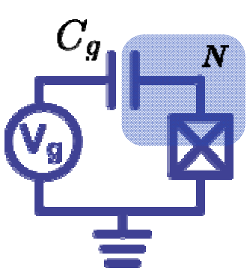
- prepare state |1> (pump)
- drive $\omega_{21}$ transition (probe)
- observe tunneling out of |2>

tipping pulse:

- prepare state |1>
- apply current pulse to suppress $U_0$
- observe tunneling out of |1>



# A Charge Qubit: The Cooper Pair Box

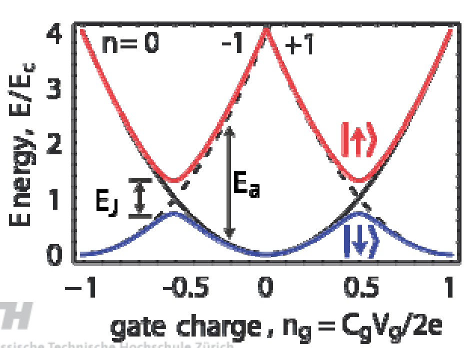



$$H_{el} = E_C N^2$$

$$H = E_C (N - N_g)^2 - E_J \cos\delta$$

$$[\delta, N] = i \quad \Rightarrow \quad e^{\pm i\delta}|n\rangle = |n \pm 1\rangle$$

$$H = \sum_N \left[ E_C (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} \left( |N\rangle\langle N+1| + |N+1\rangle\langle N| \right) \right]$$




Charging energy: $E_C = \dfrac{(2e)^2}{2C_\Sigma}$

Gate charge: $N_g = \dfrac{C_g V_g}{2e}$

Josephson energy: $E_J = \dfrac{I_0 \Phi_0}{2\pi} = \dfrac{h\Delta}{8e^2 R_J}$

Bouchiat et al. Physica Scripta 176, 165 (1998)

Cooper pair box Hamiltonian:

Hamiltonian: $\hat{H} = \underbrace{E_c(\hat{N}-N_g)^2}_{\text{electrostatic}} - \underbrace{E_J \cos\hat{\delta}}_{\text{magnetic energy}}$ $= \frac{E_J}{2}(e^{i\hat{\delta}} + e^{-i\hat{\delta}})$ gate charge $N_g = \frac{C_g V_g}{2e}$

charging energy Josephson coupling Energy

$E_c = \frac{(2e)^2}{2C_\Sigma}$ $E_J = \frac{\phi I_c}{2\pi}$

Hamiltonian in charge representation:

$$\hat{H} = E_c(N-N_g)^2 |N\rangle\langle N| - \frac{E_J}{2}\sum_N \left(|N+1\rangle\langle N| + |N\rangle\langle N+1|\right)$$

easy to diagonalize numerically

$$\hat{H} = \begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & E_c(-1-N_g)^2 & -E_J/2 & 0 & \cdots \\ \cdots & -E_J/2 & E_c(0-N_g)^2 & -E_J/2 & \cdots \\ \cdots & 0 & -E_J/2 & E_c(1-N_g)^2 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

relation between **phase** and **number basis**:

$$|\delta\rangle = \frac{1}{\sqrt{2\pi}}\sum_N e^{i\hat{N}\delta}|N\rangle \quad \text{with} \quad e^{i\hat{\delta}}|N\rangle = |N+1\rangle$$

**Phase representation** of Cooper pair box Hamiltonian:

$$\hat{H} = E_c(\hat{N}-N_g)^2 - E_J\cos\hat{\delta} = E_c\left(-i\frac{\partial}{\partial\delta} - N_g\right)^2 - E_J\cos\hat{\delta}$$

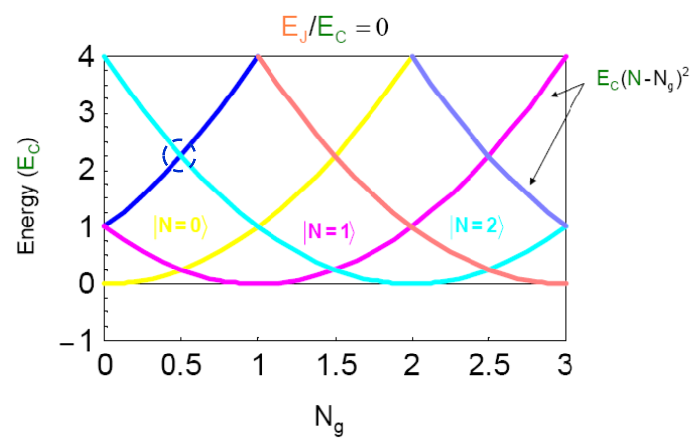
with $\hat{N} = \frac{\hat{Q}}{2e} = -i\hbar\frac{1}{2e}\frac{\partial}{\partial\phi} = -i\frac{\partial}{\partial\delta}$

Equivalent solution to the Hamiltonian can be found in both representations, e.g. by numerically solving the Schrödinger equation for the charge ($N$) representation or analytically solving the Schrödinger equation for the phase ($\delta$) representation.

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

solutions for $E_J = 0$:

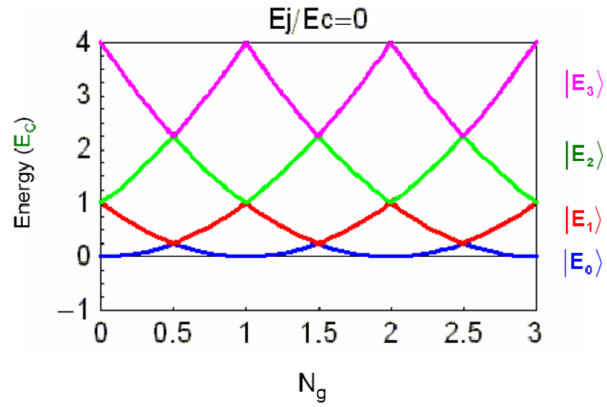
- crossing points are charge degeneracy points



# Energy Levels

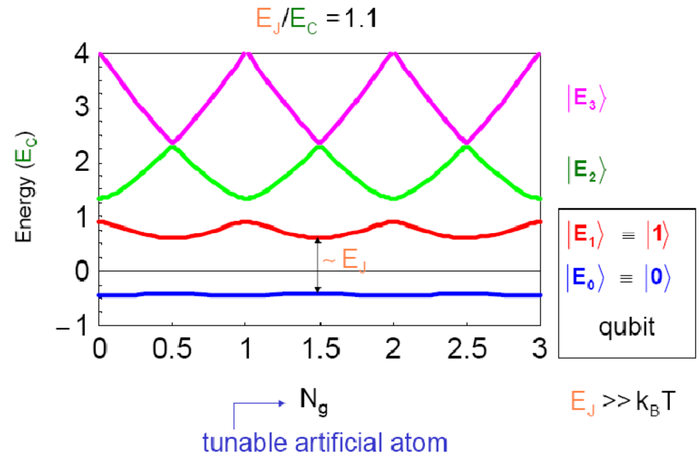
energy level diagram for $E_J$=0:

- energy bands are formed
- bands are periodic in $N_g$



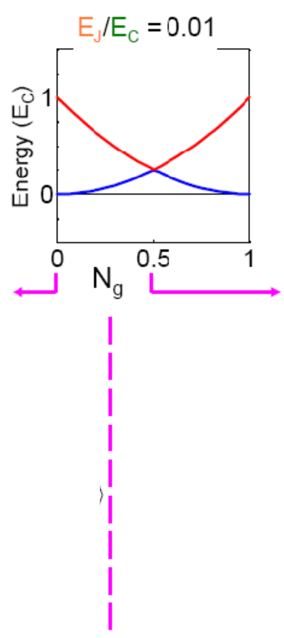

energy bands for finite $E_J$

- Josephson coupling lifts degeneracy
- $E_J$ scales level separation at charge degeneracy





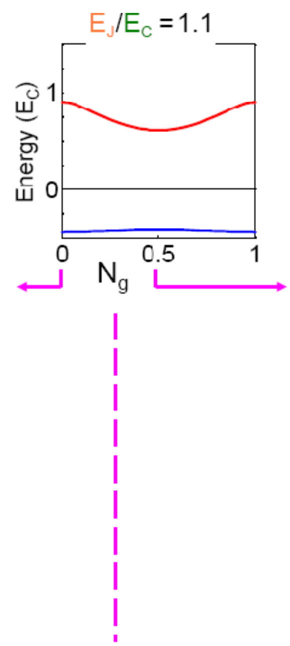

# Charge and Phase Wave Functions ($E_j$ << $E_c$)






courtesy Saclay

## Charge and Phase Wave Functions ($E_j \sim E_c$)

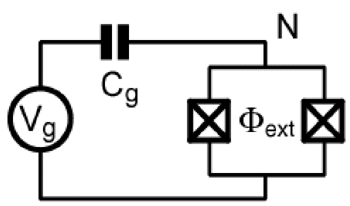





courtesy Saclay

## Tuning the Josephson Energy

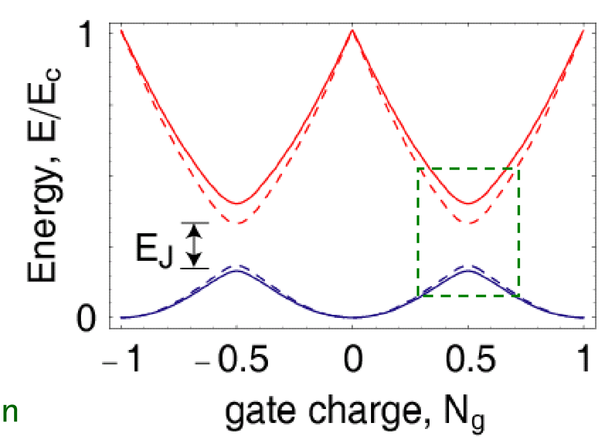
split Cooper pair box in perpendicular field




$$H = E_C\,(N - N_g)^2 - E_{J,\max}\cos\left(\pi\,\frac{\Phi_{\rm ext}}{\Phi_0}\right)$$

SQUID modulation of Josephson energy

$$E_J = E_{J,\max}\cos\left(\pi\,\frac{\Phi_{\rm ext}}{\Phi_0}\right)$$




consider two state approximation

J. Clarke, *Proc. IEEE* **77**, 1208 (1989)

# Two State Approximation

$$\mathbf{H}_{\text{CPB}} = \mathbf{H}_{\text{el}} + \mathbf{H}_{\text{J}} = E_C(N - N_g)^2 - E_J \cos\delta$$

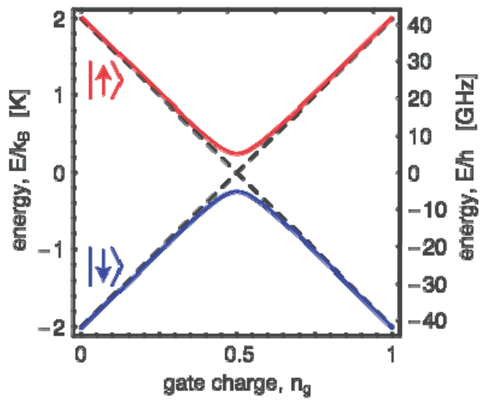
$$\mathbf{H}_{\text{CPB}} = \sum_N \left[ E_C(N - N_g)^2 \left|N\right\rangle \left\langle N\right. - \frac{E_{\text{J}}}{2} \left( \left|N\right\rangle \left\langle N+1\right. + \left|N-1\right\rangle \left\langle N\right| \right) \right]$$

Restricting to a two-charge Hilbert space:

$$N = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1 - \sigma_z}{2}$$

$$\cos\delta = \frac{\sigma_x}{2}$$

$$\begin{aligned} \mathbf{H}_{\text{CPB}} &= -\frac{E_C}{2}(1 - 2N_g)\sigma_z - \frac{E_J}{2}\sigma_x \\ &= -\frac{1}{2}(E_{\text{el}}\sigma_z + E_J\sigma_x) \end{aligned}$$




Shnirman *et al., Phys. Rev. Lett.* **79**, 2371 (1997)