ION TRAPS – STATE OF THE ART QUANTUM GATES

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ION TRAPS – STATE OF THE ART

QUANTUM GATES

I. Fault-tolerant computing & the Mølmer-Sørensen gate with ion traps

II. Quantum Toffoli gate

I. FAULT-TOLERANT COMPUTING & THE MØLMER-SØRENSEN GATE

Towards fault-tolerant quantum computing with ion traps,

J.Benhelm et.al, Nature 2008, doi:10.1038/nphys961

MØLMER-SØRENSEN GATE

- Motivation
    - Ion traps are a promising candidate for universal quantum computation
    - Fault tolerant computing only if the errors are small
    - High fidelity single & multi qubit gates are needed
    - Single qubit gates have low error rates
    - Multi qubit gates are more difficult to perform
    - Error range ~10^{-2} - 10^{-4}
    - Recently shown: 2 qubit entangling gate with high fidelity "Mølmer-Sørensen gate"

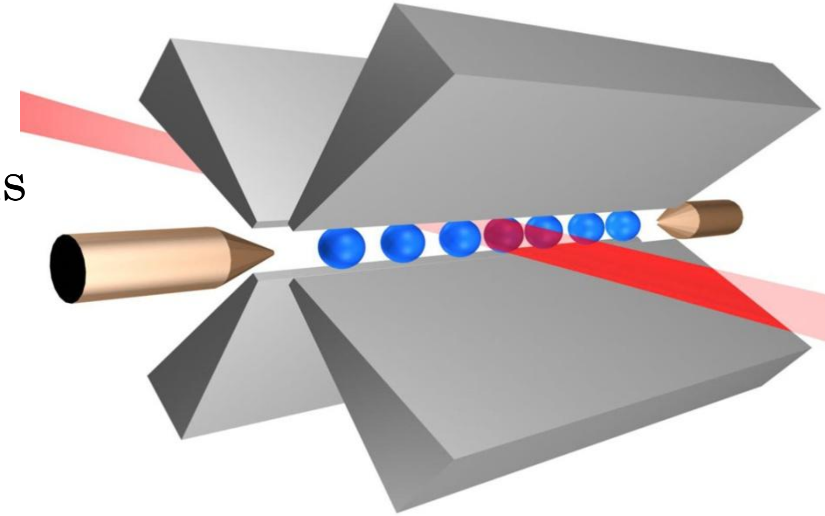
MØLMER-SØRENSEN GATE

- Properties of the gate
  - Scalable multi qubit entangling gate
  - Performs collective “spin“ flips

- Experimental setup
  - Paul trap w/ two $^{40}Ca^+$ ions
  - Bichromatic laser field

$$\omega_{+/-} = \omega_0 +/- \delta,\ \delta > \upsilon$$

υ the phonon frequency



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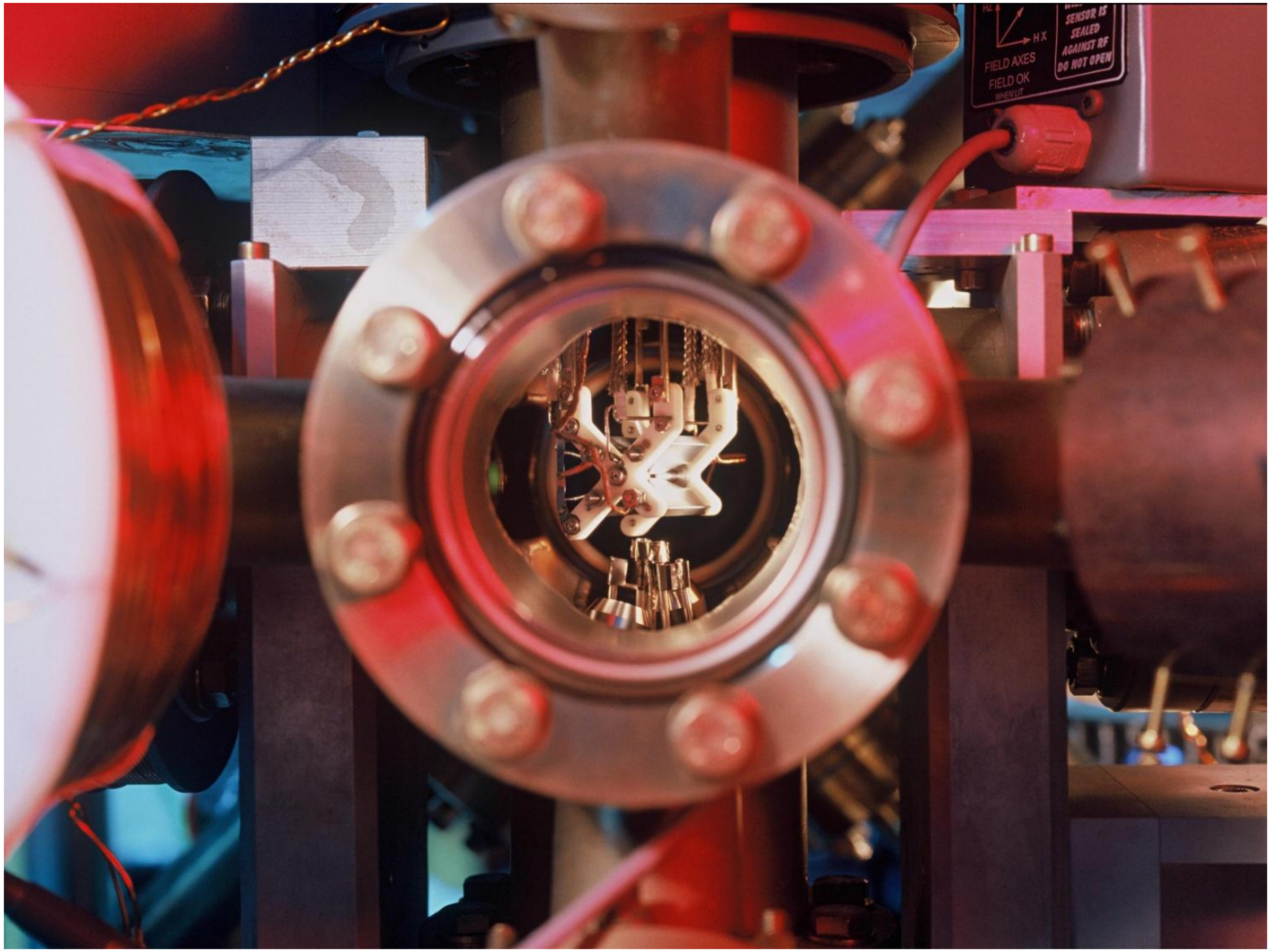
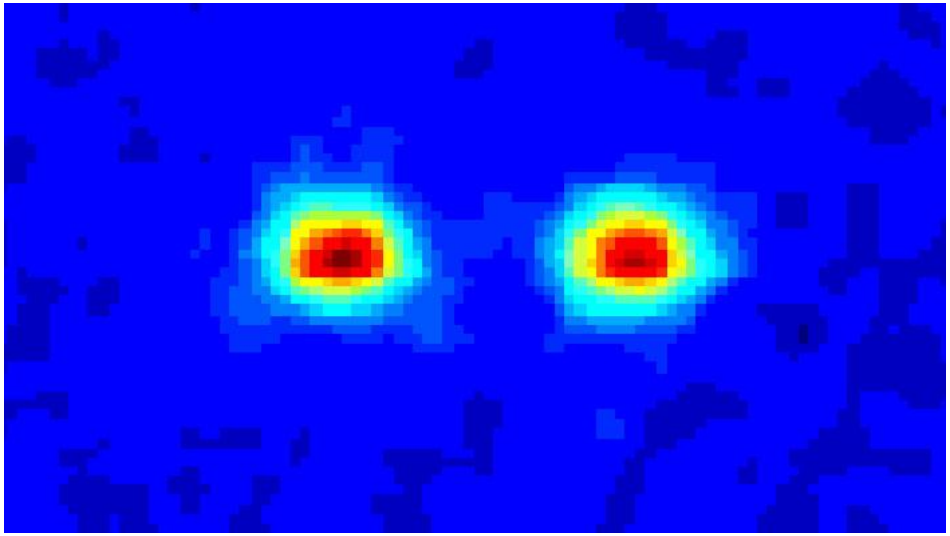
- Paul trap at the University of Innsbruck



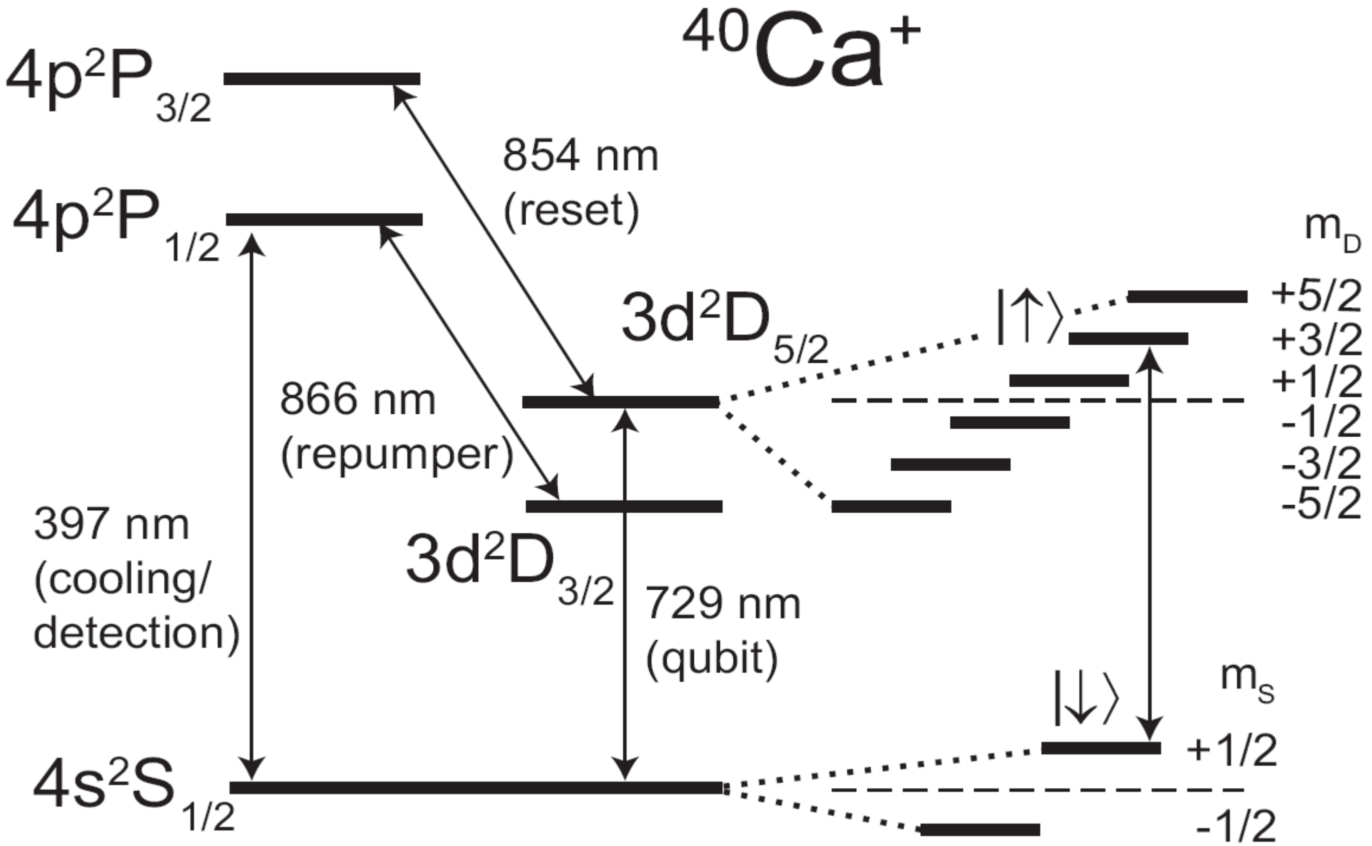
MØLMER-SØRENSEN GATE

- Procedure
  - Doppler cooling & optical pumping with a laser for initialization to the ground state |SS>
  - Applying the bichromatic laser field (gate)
  - Readout with the CCD camera



MØLMER-SØRENSEN GATE

- Energy scheme for the $^{40}Ca^+$ ions



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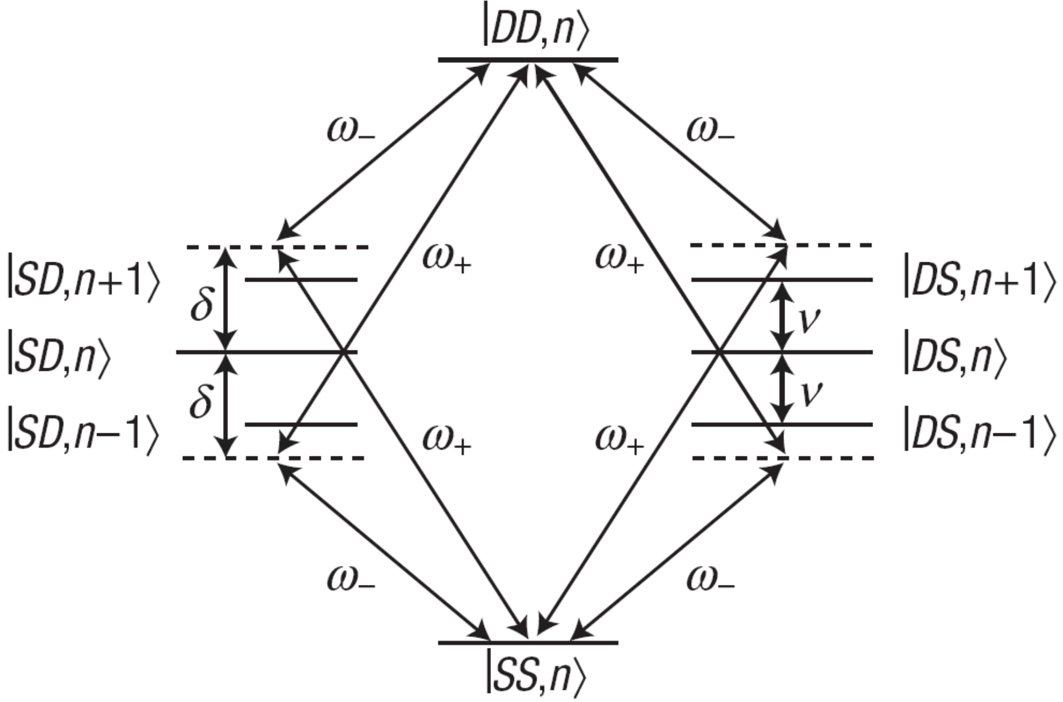
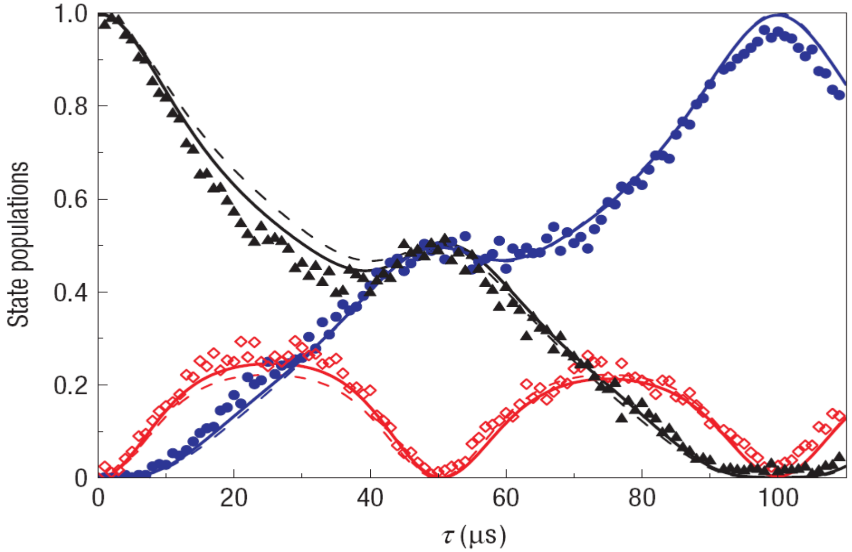
- Gate mechanism



  - $\omega_{+/-} = \omega_0 +/- \delta$, $\delta > \upsilon$, ω_0: single ion excitation frequency
  - Gate operation: interference of 4 2-photon-processes

MØLMER-SØRENSEN GATE

- Final measurements

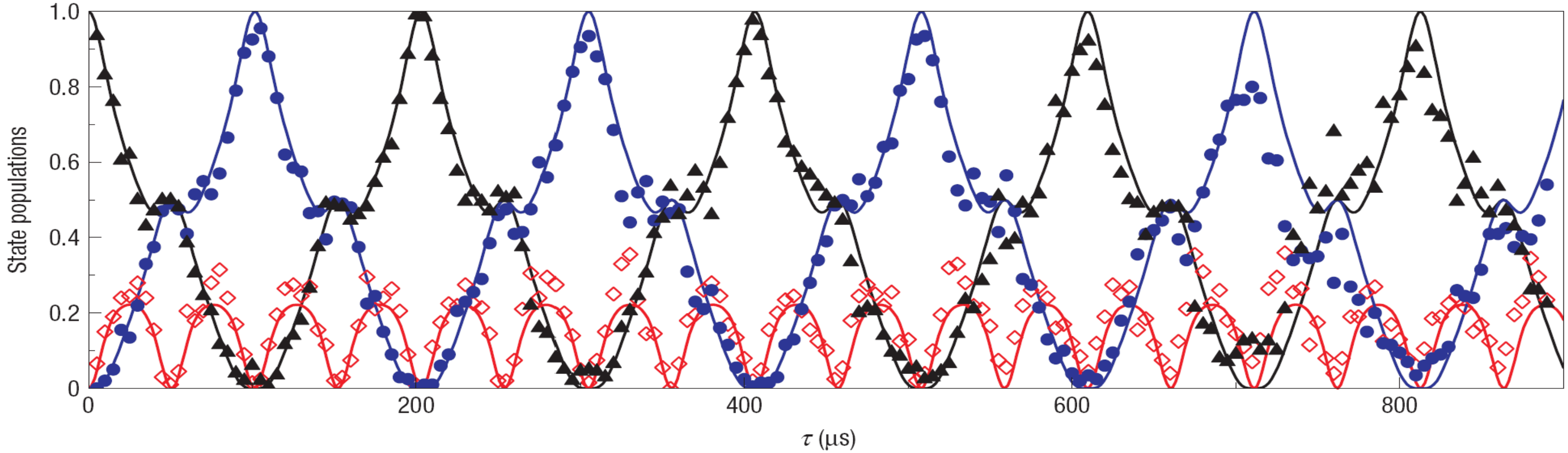


- $|SS\rangle$ --τ_{gate}-- $|SS\rangle+e^{i\varphi}|DD\rangle$ --τ_{gate}-- $|DD\rangle$
  - Black: probability p_2 of finding 2 ions in state $|S\rangle$
  - Red: probability p_1 of finding 1 ion in state $|S\rangle$
  - Blue: probability p_0 of finding 0 ions in state $|S\rangle$

MØLMER-SØRENSEN GATE

- Max. entanglement for $\tau = m^{*}\tau_{\text{gate, m=1,3,...}}$
- "Spin" flip for m=2,4,...

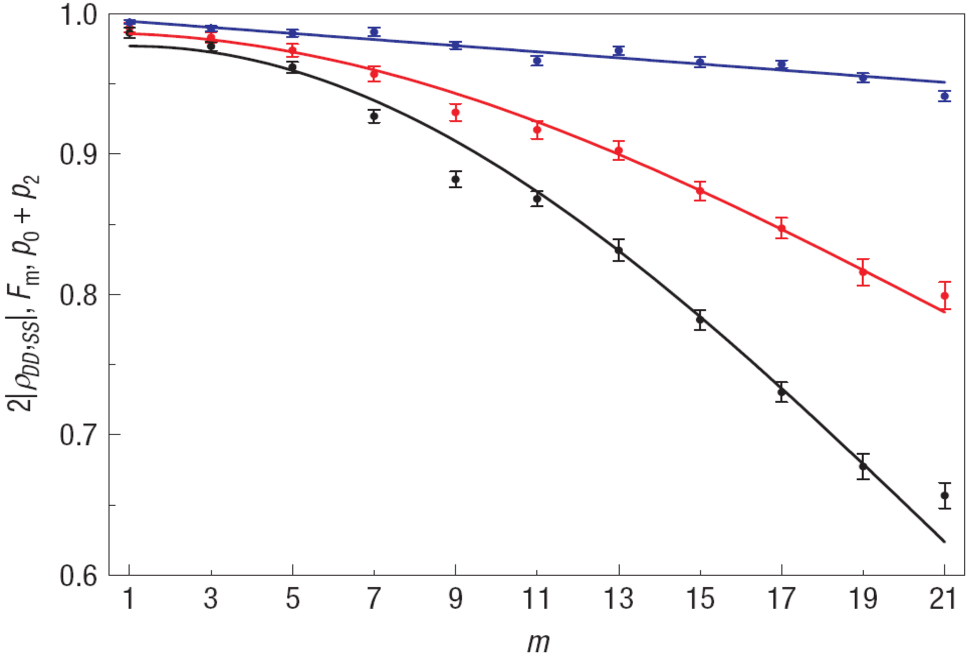
- Test the fidality of the gate after multiple operations



- 21 successive gate operations shown, $\tau_{\text{gate = 50µs}}$

MØLMER-SØRENSEN GATE

- Gate imperfections as function of pulse length



  - Blue: state populations of p_0+p_2
  - Red: resulting Bell state fidelity
  - Black: magnitude of coherence of the system

MØLMER-SØRENSEN GATE

- Conclusions
  - High Bell state fidelity of F=99,3(1)% achieved
  - Infidelity is less than 10^{-2} threshold
  - Further advances needed
  - Good candidate for multi qubit entangling gates with single laser interaction for more than 2 qubits

MØLMER-SØRENSEN GATE

- References
  - Towards fault-tolerant quantum computing with ion traps, J.Benhelm et.al, Nature 2008, doi:10.1038/nphys961
  - Entangled states of trapped atomic ions, R. Blatt & D. Wineland, Nature 2008, doi:10.1038/nature07125
  - Deterministic entanglement of ions in thermal states of motion, G.Kirchmair et. al., arXiv:0810.0670v1
  - Scalable Entanglement of Trapped Ions, C. Monroe et. al., http://www.boulder.nist.gov/timefreq/general/pdf/1397.pdf
  - Optimierung verschränkender Quantengatter für Experimente mit Ionenfallen, Volckmar Nebendahl, Diplomarbeit 2008, Universität Hamburg

II. TOFFOLI GATE

Realization of the quantum Toffoli gate with trapped ions,

Monz, T; Kim, K; Haensel, W; et al. (not yet published)

PLAN

1. What is a Toffoli gate ?
2. Why use a single gate ?
3. General Principle
4. Results
5. Conclusion

1. WHAT IS A TOFFOLI GATE ?

- Performs a NOT operation on a target qubit depending on the state of two control qubits

$$\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\\0&1&1\\1&0&0\\1&0&1\\1&1&1\\1&1&0\end{pmatrix} = \left(\begin{array}{c|c} \begin{matrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{matrix} & 0 \\ \hline 0 & \begin{array}{c|c} \begin{matrix}1&0\\0&1\end{matrix} & 0 \\ \hline 0 & \begin{matrix}\mathbf{0}&\mathbf{1}\\\mathbf{1}&\mathbf{0}\end{matrix} \end{array} \end{array}\right) \begin{pmatrix}0&0&0\\0&0&1\\0&1&0\\0&1&1\\1&0&0\\1&0&1\\1&1&0\\1&1&1\end{pmatrix}$$

- Application in quantum error correction

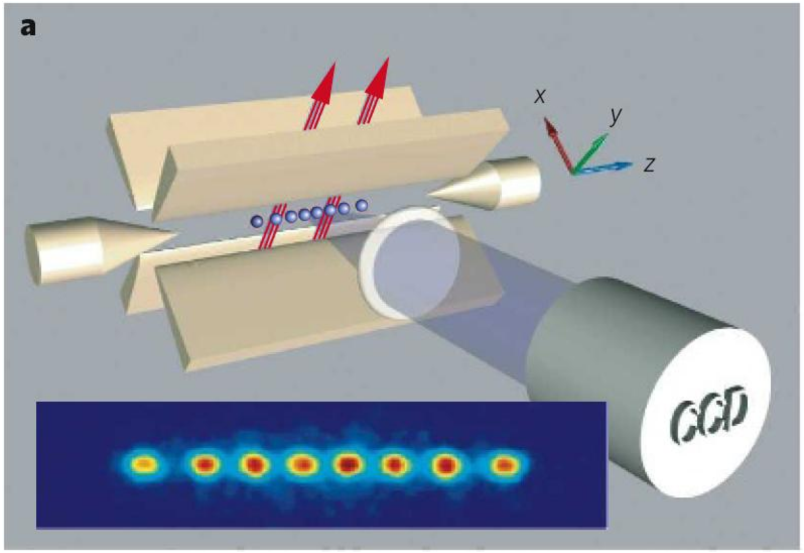
2. WHY USE A SINGLE GATE ?

- Could be done with concatenated two-qubit gates

- Advantages of a single gate:
  - Simplify complex quantum operations
  - Higher fidelity
  - Faster

3. GENERAL PRINCIPLE

- System = string of $^{40}\mathrm{Ca}^+$ ions confined in a linear Paul trap



- Ground state: $\mathrm{S}_{1/2}(\mathrm{m}{=}-1/2) = |S\rangle \equiv |1\rangle$
- Excited state: $\mathrm{D}_{5/2}(\mathrm{m}{=}-1/2) = |D\rangle \equiv |0\rangle$

- Use of the centre-of-mass (COM) vibrational mode of the ion string as intermediate

3. GENERAL PRINCIPLE (II)

- 3 major steps:

1. Encoding of the joint quantum information of the control qubits $|c_1\rangle$ and $|c_2\rangle$ in the vibrational COM mode
2. NOT operation on the target qubit controlled by the vibrational mode
3. Decoding of the qubits (reversal of the encoding step)

3. GENERAL PRINCIPLE (III)

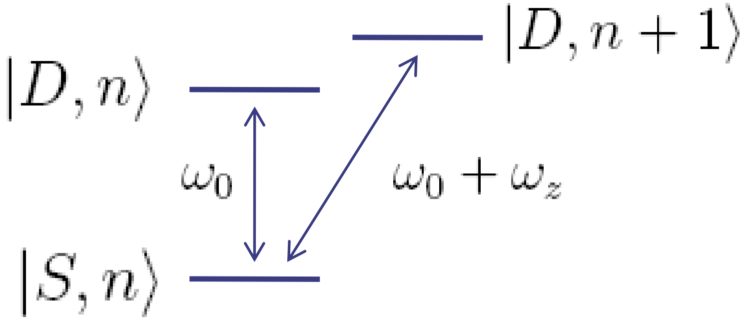
- Ideal unitary map implemented:

$$U_{\mathrm{T}} = \exp(-i\pi \frac{1}{2\sqrt{2}}\sigma_{Z,t}) \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \end{pmatrix}$$

In the basis

$$\{|c_1, c_2, t\rangle\} = \{|DDD\rangle, |DDS\rangle, |DSD\rangle, |DSS\rangle, |SDD\rangle, |SDS\rangle, |SSD\rangle, |SSS\rangle\}$$

3. ENCODING SECTION



- State of the quantum qubits at the end:

$$|SS,0\rangle \rightarrow |DD,2\rangle$$

$$|DS,0\rangle \rightarrow \sin\frac{\pi}{2\sqrt{2}}|DD,1\rangle + \cos\frac{\pi}{2\sqrt{2}}|DS,0\rangle$$

$$|SD,0\rangle \rightarrow \cos\frac{\pi}{2\sqrt{2}}|DD,1\rangle - \sin\frac{\pi}{2\sqrt{2}}|DS,0\rangle$$

$$|DD,0\rangle \rightarrow |DD,0\rangle$$

3. INFORMATION IN THE COM MODE

- Initially it contains no phonons

$$|vib\rangle = |n = 0\rangle$$

- Encoding → 2 phonons ($|c_1 c_2\rangle = |SS\rangle$)
  or 1 phonon (other cases)

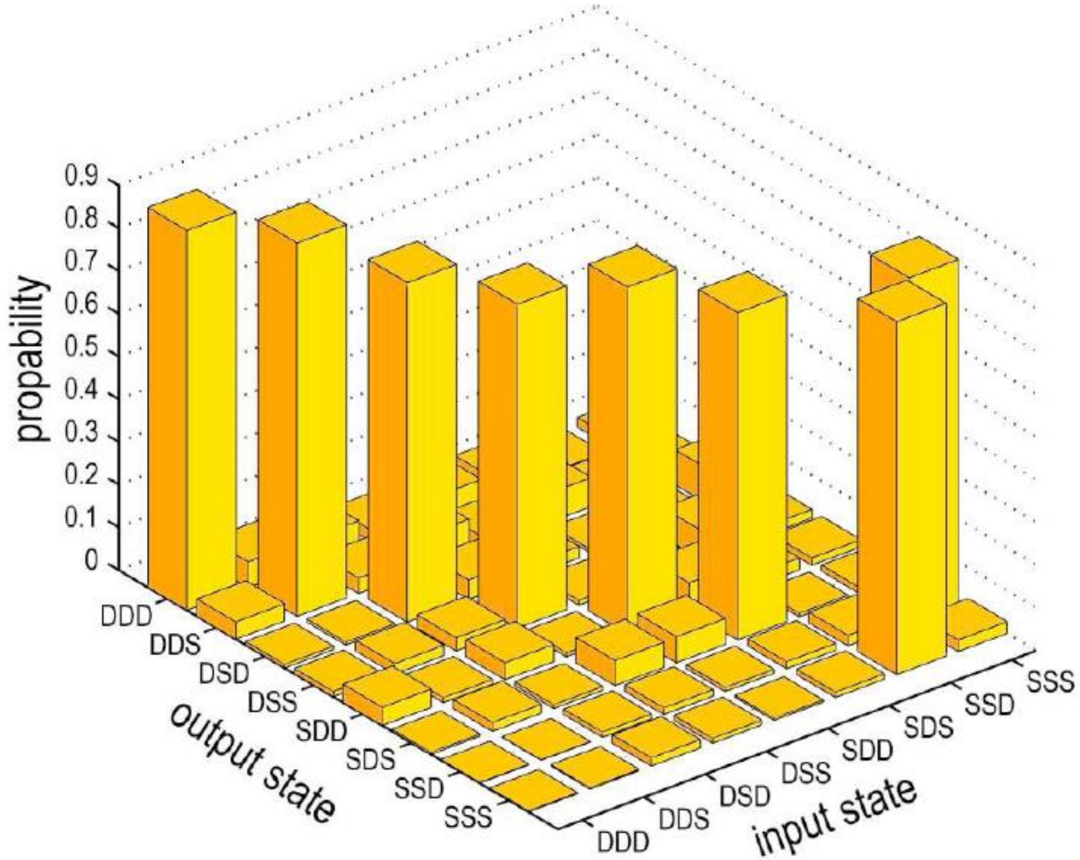
- Removal of one phonon

$$(c_1 \text{ AND } c_2) = 1 \Rightarrow |n = 1\rangle$$

$$(c_1 \text{ AND } c_2) = 0 \Rightarrow |n = 0\rangle$$

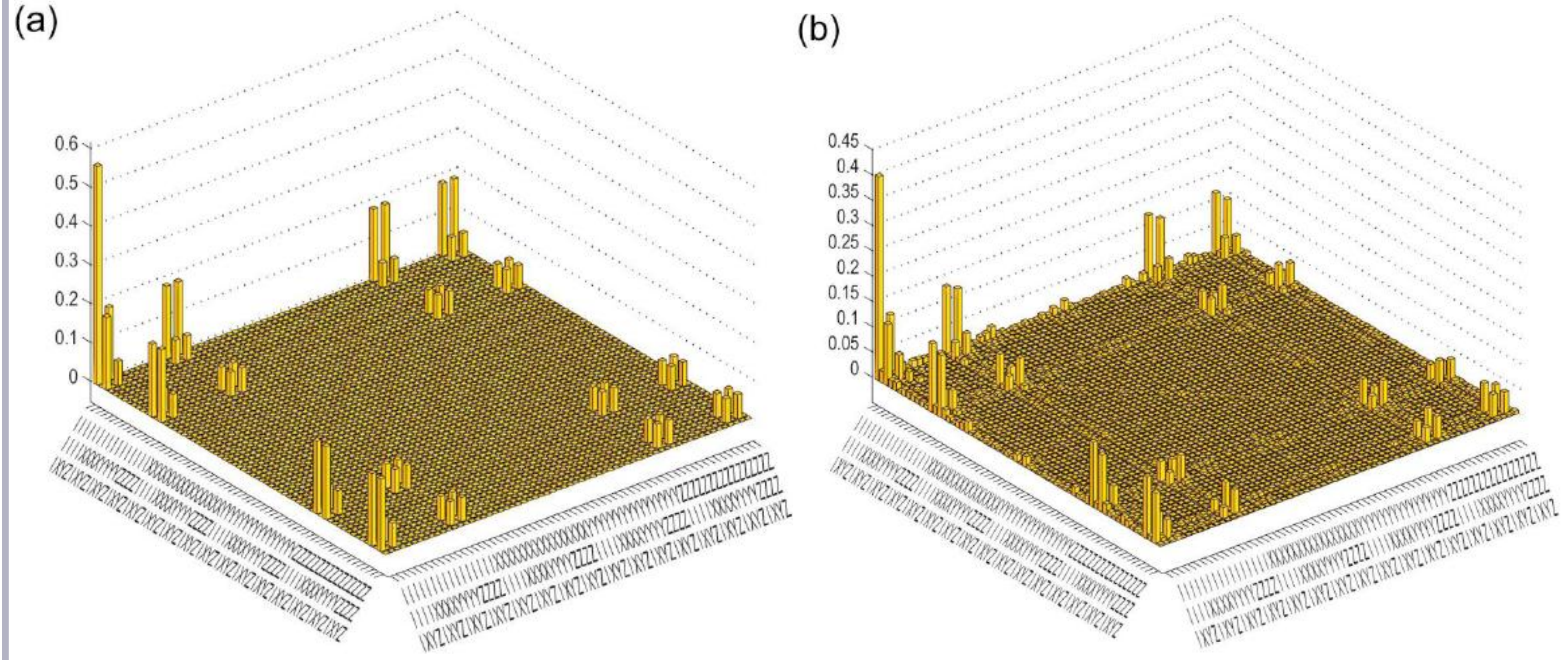
4. RESULTS

- Probabilities of 81 (± 5)% that the ion ends up in the correct output state.



4. COMPARAISON OF X-MATRIX

- Obtained by quantum process tomography



Basis: $\sigma_{c1} \otimes \sigma_{c2} \otimes \sigma_t \in \{I \otimes I \otimes I, I \otimes I \otimes X, \ldots, Z \otimes Z \otimes Z\}$

- Mean fidelity of approximately 71%

4. SOURCES OF INFIDELITY

- Mainly due to technical imperfections:
  - Rabi frequency imprecisions (12%)
  - Temperature changes and voltage fluctuations (7%)
  - Initialization of the COM mode in the ground state (1%)
  - Laser linewidth and magnetic field fluctuations (1%)
  - Ion state initialization (0.5% per ion)

  - Statistical uncertainties in the tomographic measurements

- Decoherence time of quantum information stored in the vibrational mode: 85ms

5. COMPARAISON WITH CNOT GATES BASED REALIZATION

- Need of 6 CNOT gates to make a Toffoli gate

- Fidelity: $F = (92.6\%)^6 \sim 63\%$

- Duration of the gate = 3 times longer than the 3-qubits gate (1.5ms)

QUESTIONS ?

PULSE SEQUENCE

	Pulse	Comment	Logical part
1	$R_1^+\left(\pi, \frac{3\pi}{2}\right)$	Encode first target qubit onto motion	Encoding
2	$R_2^+\left(\frac{\pi}{\sqrt{2}}, \frac{3\pi}{2}\right)$	Encode second target qubit onto motion	
3	$R_1^+\left(\frac{\pi}{2\sqrt{2}}, \frac{\pi}{2}\right)$	Composite pulse to remove one phonon	
4	$R_1^+\left(\pi, 0\right)$		
5	$R_1^+\left(\frac{\pi}{2\sqrt{2}}, \frac{\pi}{2}\right)$		
6	$R_3\left(\frac{\pi}{2}, 0\right)$	Prepare target qubit for motion controlled NOT	controlled NOT
7	$R_3^+\left(\frac{\pi}{2}, 1\right)$	Composite phase gate	
8	$R_3^+\left(\sqrt{2}\pi, \frac{\pi}{2}\right)$		
9	$R_3^+\left(\frac{\pi}{2}, 0\right)$		
10	$R_3\left(\frac{\pi}{2}, (\frac{1}{\sqrt{2}} - 1)\pi\right)$	Complete motion controlled NOT on target qubit	
11	$R_1^+\left(\frac{\pi}{2\sqrt{2}}, (-\frac{1}{2} + \frac{1}{\sqrt{2}})\pi\right)$	Undo encoding algorithm	Decoding
12	$R_1^+\left(\pi, (-1 + \frac{1}{\sqrt{2}})\pi\right)$		
13	$R_1^+\left(\frac{\pi}{2\sqrt{2}}, (-\frac{1}{2} + \frac{1}{\sqrt{2}})\pi\right)$		
14	$R_2^+\left(\frac{\pi}{\sqrt{2}}, (\frac{1}{2} + \frac{1}{\sqrt{2}})\pi\right)$	Decoding finished for second control qubit	
15	$R_1^+\left(\pi, (\frac{1}{2} + \frac{1}{\sqrt{2}})\pi\right)$	Decoding finished for first control qubit, Toffoli complete	