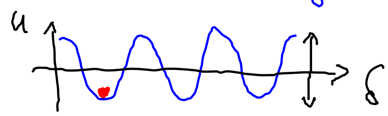
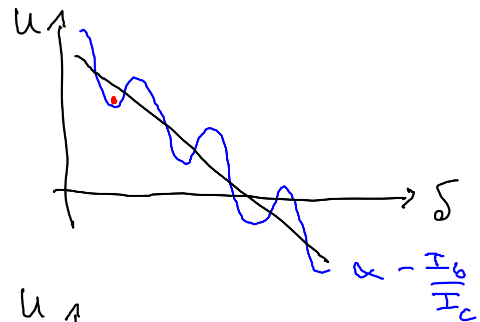
Phase particle in a potential well

$$U(\delta) = \frac{I_c \Phi_0}{2\pi}\left(-\frac{I_b}{I_c}\delta - \cos\delta\right) \qquad E_J = \frac{I_c \Phi_0}{2\pi}$$

cosine potential for $I_b = 0$:



'tilted washboard' potential for $I_b \neq 0$:



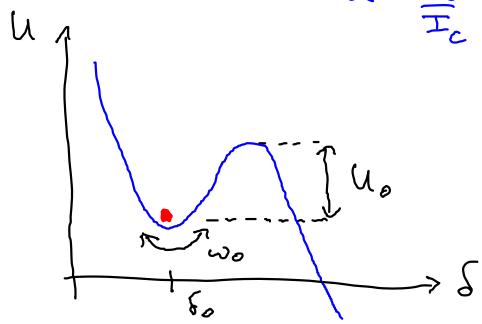
potential barrier:

$$U_0 = 2E_J\left[\sqrt{1-\gamma^2} - \gamma \arccos\gamma\right]$$

oscillation frequency:

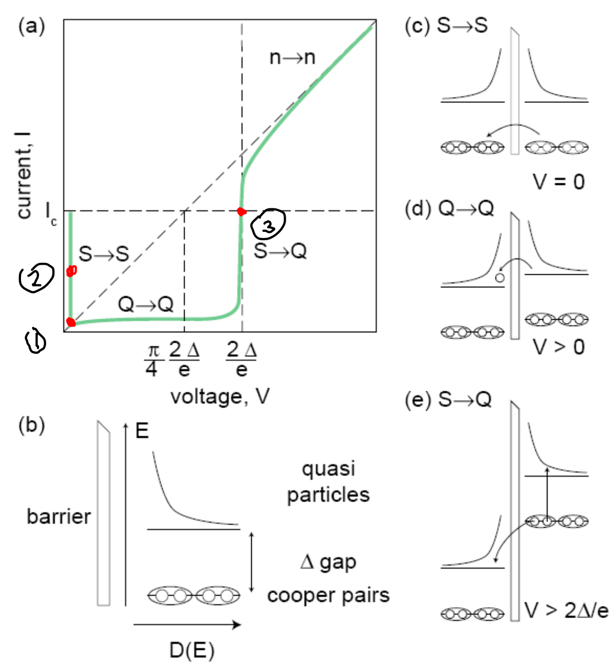
$$\omega_0 = \omega_p (1-\gamma^2)^{1/4} = \sqrt{\frac{U''(\delta_0)}{m}}$$

with: $\gamma = I_b/I_c$; $\omega_p = \sqrt{\frac{2\pi I_c}{\Phi_0 C}}$

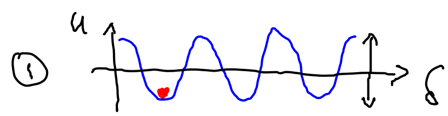


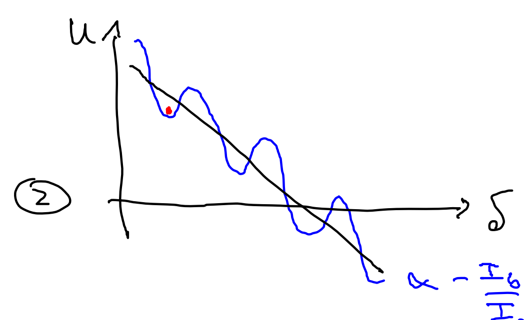
Current-voltage characteristics

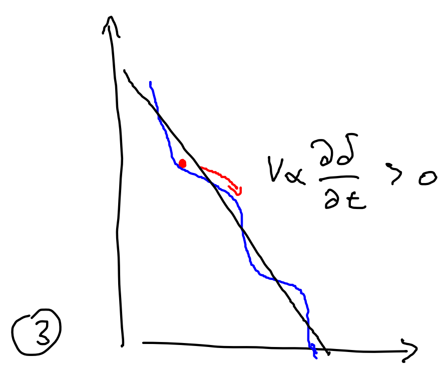
typical I-V curve of underdamped Josephson junctions:



band diagram







$V \propto \frac{\partial\delta}{\partial t} > 0$

Thermal Activation and Quantum Tunneling:

bias current dependence

$\omega_0(\gamma)$; $U_0(\gamma)$

thermal activation rate:

$$\Gamma_{th} \approx a_t \frac{\omega_0}{2\pi} \exp\left(-\frac{U_0}{k_B T}\right)$$

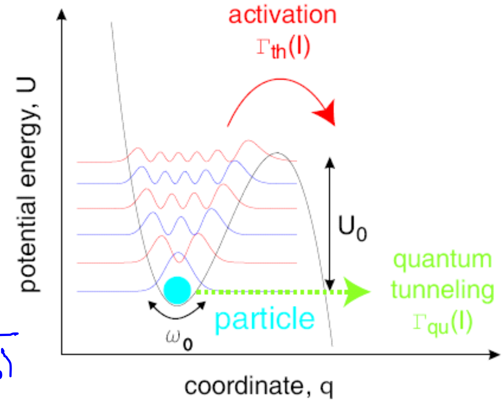
damping dependent prefactor

quantum tunneling rate:

$$\Gamma_{qu} = a_q \frac{\omega_0}{2\pi} \exp\left(-\frac{36}{5}\frac{U_0}{\hbar\omega_0}\right)$$

calculated using WKB method (exercise)

$$\Gamma_q = a_q\, \omega_0 \exp\left(-\int_{\delta_1}^{\delta_2} \frac{1}{\hbar}\sqrt{2m(U(\delta)-E_0)}\right)$$



energy level quantization:

$$E_n \approx \hbar\omega_0\left(n + \frac{1}{2}\right)$$

neglecting non-linearity

Quantum Mechanics of a Macroscopic Variable: The Phase Difference of a Josephson Junction
JOHN CLARKE, ANDREW N. CLELAND, MICHEL H. DEVORET, DANIEL ESTEVE, and JOHN M. MARTINIS
Science 26 February 1988 239: 992-997 [DOI: 10.1126/science.239.4843.992] (in Articles) Abstract » References » PDF »

Macroscopic quantum effects in the current-biased Josephson junction
M. H. Devoret, D. Esteve, C. Urbina, J. Martinis, A. Cleland, J. Clarke
in Quantum tunneling in condensed media, North-Holland (1992)

Early Results (1980's)

search for macroscopic quantum effects in superconducting circuits

theoretical predictions:

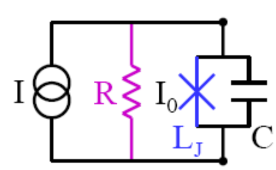
- **tunneling** ✓
- **energy level quantization** ✓
- **coherence** ✗

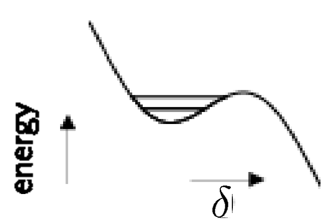
A.J. Leggett *et al.,*
Prog. Theor. Phys. Suppl. **69**, 80 (1980),
Phys. Scr. **T102**, 69 (2002).

short coherence times due to strong coupling to em environment

experimental verification:

current biased JJ = phase qubit

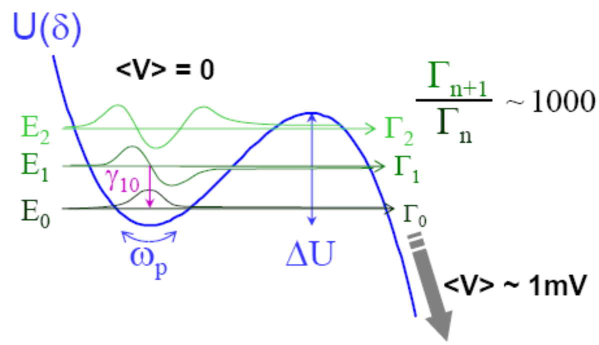


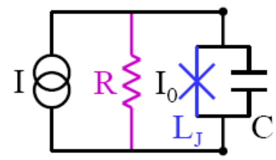


J. Clarke, J. Martinis, M. Devoret *et al.*, *Science* **239**, 992 (1988).

The Current Biased Phase Qubit

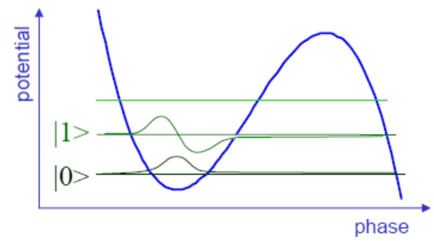
operating a current biased Josephson junction as a superconducting qubit:





<u>initialization:</u>

wait for $|1\rangle$ to decay to $|0\rangle$, e.g. by spontaneous emission at rate γ_{10}

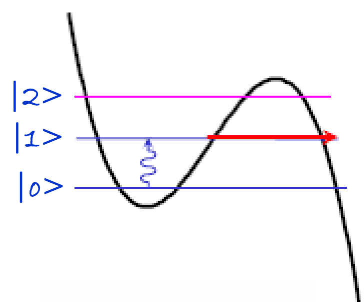


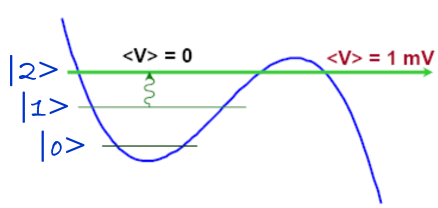
Read-Out Ideas

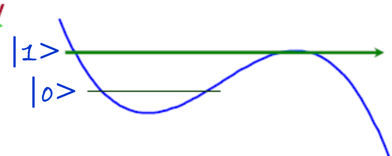
measuring the state of a current biased phase qubit

<u>tunneling:</u>

- prepare state $|1\rangle$ (pump)
- wait ($\Gamma_1 \sim 10^3\, \Gamma_0$)
- detect voltage
- $|1\rangle$ = voltage, $|0\rangle$ = no voltage







<u>pump and probe pulses:</u>

- prepare state $|1\rangle$ (pump)
- drive ω_{21} transition (probe)
- observe tunneling out of $|2\rangle$

<u>tipping pulse:</u>

- prepare state $|1\rangle$
- apply current pulse to suppress U_0
- observe tunneling out of $|1\rangle$

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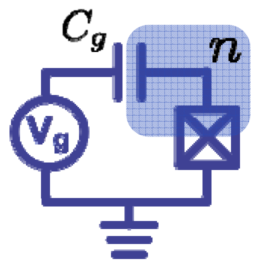
A Charge Qubit: The Cooper Pair Box

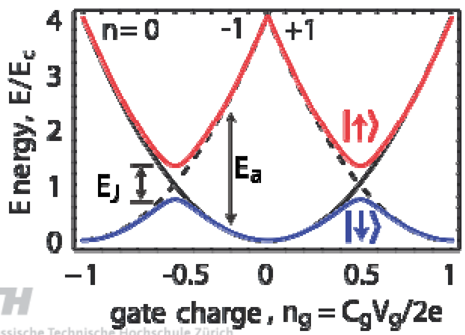


$$H = 4E_C n^2$$

$$H = 4E_C(n - n_g)^2 - E_J \cos\delta$$

$$[\delta, n] = i \;\Rightarrow\; e^{\pm i\delta}|n\rangle = |n \pm 1\rangle$$

$$H = \sum_n \left[4E_C(n-n_g)^2 |n\rangle\langle n| - \frac{E_J}{2}\left(|n\rangle\langle n+1| + |n+1\rangle\langle n|\right)\right]$$



Charging energy: $E_C = \dfrac{e^2}{2(C_g + C_J)}$

Gate charge: $n_g = \dfrac{C_g V_g}{2e}$

Josephson energy: $E_J = \dfrac{I_0 \Phi_0}{2\pi} = \dfrac{h\Delta}{8e^2 R_J}$

Bouchiat et al. Physica Scripta 176, 165 (1998)

Cooper pair box Hamiltonian:

Hamiltonian: $\hat{H} = E_c(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$, $E_J \cos\hat{\delta} = \frac{E_J}{2}\left(e^{i\hat{\delta}} + e^{-i\hat{\delta}}\right)$ gate charge $N_g = \dfrac{C_g V_g}{2e}$

electrostatic charging energy — magnetic energy Josephson coupling Energy

$E_c = \dfrac{(2e)^2}{2C_\Sigma}$ $E_J = \dfrac{\phi I_c}{2\pi}$

Hamiltonian in charge representation:

$$\hat{H} = E_c(N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2}\sum_N \left(|N+1\rangle\langle N| + |N\rangle\langle N+1|\right)$$

easy to diagonalize numerically

$$\hat{H} = \begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & E_c(-1-N_g)^2 & -E_J/2 & 0 & \cdots \\ \cdots & -E_J/2 & E_c(0-N_g)^2 & -E_J/2 & \cdots \\ \cdots & 0 & -E_J/2 & E_c(1-N_g)^2 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

relation between phase and number basis:

$$|\delta\rangle = \frac{1}{\sqrt{2\pi}} \sum_N e^{iN\delta} |N\rangle \quad \text{with} \quad e^{i\hat{\delta}}|N\rangle = |N+1\rangle$$

Phase representation of Cooper pair box Hamiltonian:

$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta} \qquad \text{with} \qquad \hat{N} = \frac{\hat{Q}}{2e} = -i\hbar\frac{1}{2e}\frac{\partial}{\partial\phi}$$

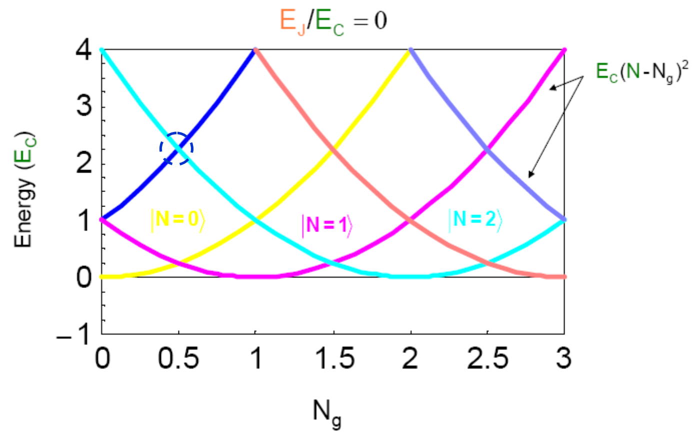
$$= E_C\left(-i\frac{\partial}{\partial\delta} - N_g\right)^2 - E_J\cos\hat{\delta} \qquad\qquad = -i\frac{\partial}{\partial\delta}$$

Equivalent solution to the Hamiltonian can be found in both representations, e.g. by numerically solving the Schrödinger equation for the charge (N) representation or analytically solving the Schrödinger equation for the phase (δ) representation.

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

solutions for $E_J = 0$:

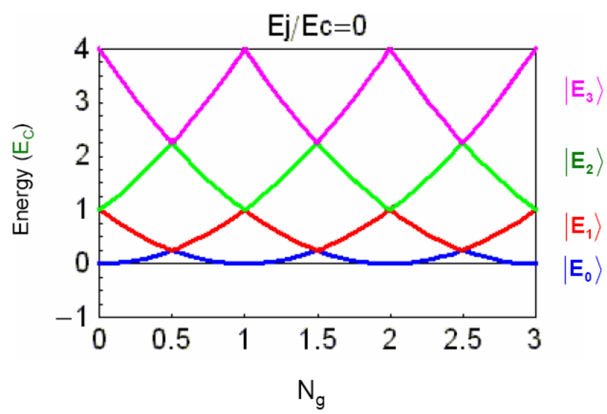
- crossing points are charge degeneracy points



Energy Levels

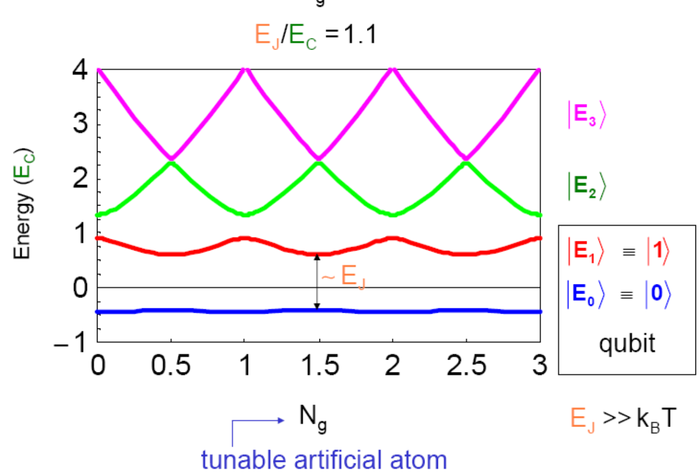
energy level diagram for $E_J=0$:

- energy bands are formed
- bands are periodic in N_g

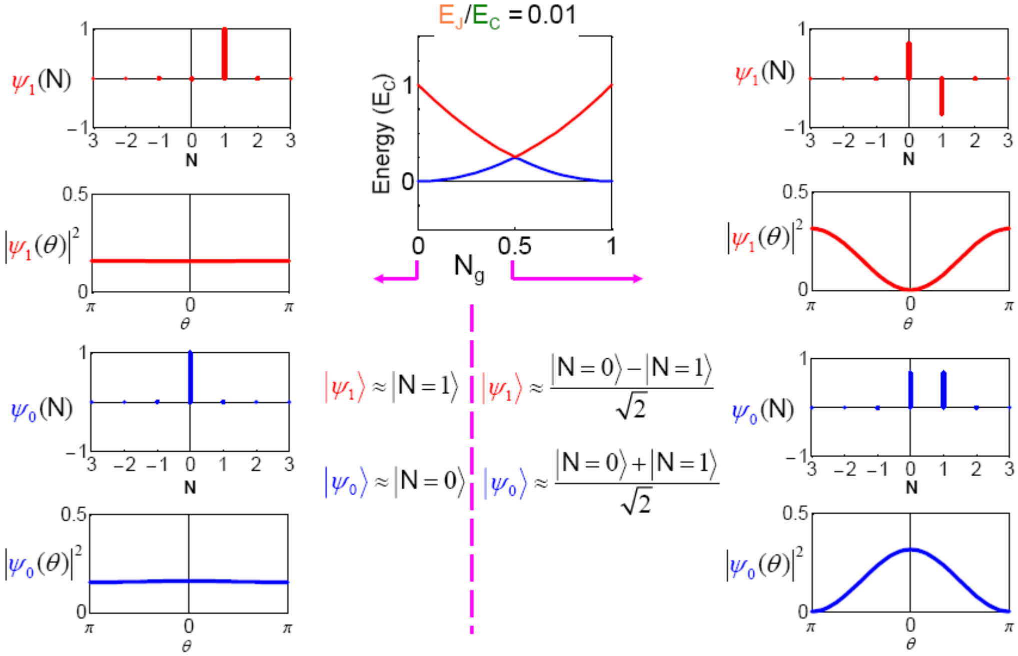


energy bands for finite E_J

- Josephson coupling lifts degeneracy
- E_J scales level separation at charge degeneracy



Charge and Phase Wave Functions ($E_j << E_c$)



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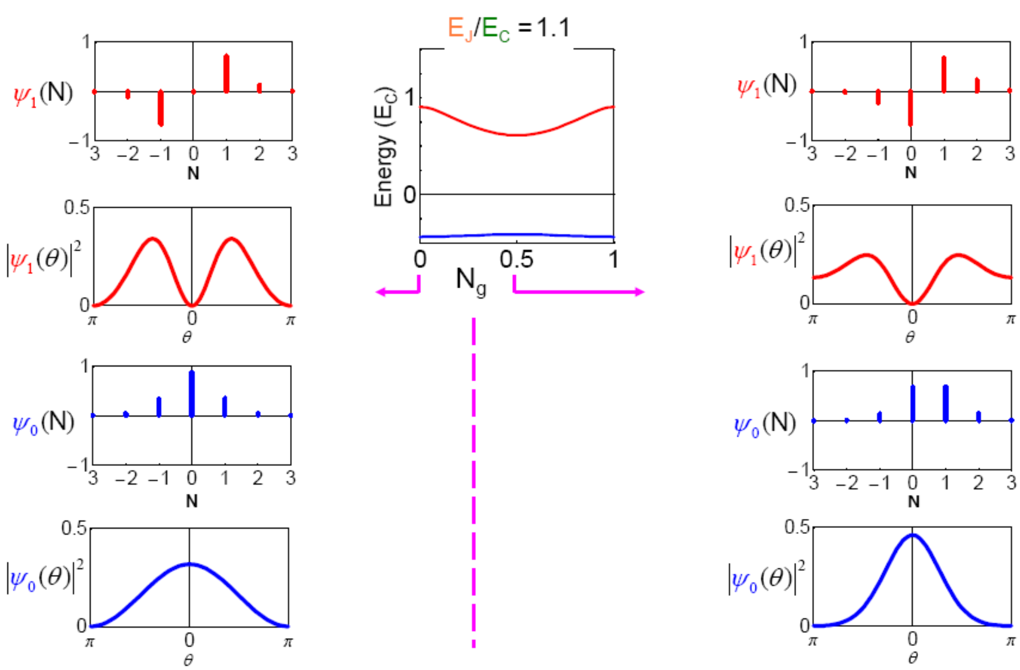
courtesy Saclay

Charge and Phase Wave Functions ($E_j \sim E_c$)



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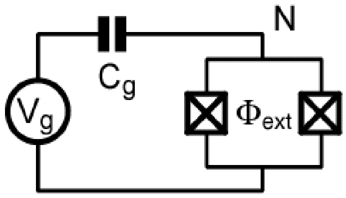
courtesy Saclay

Tuning the Josephson Energy

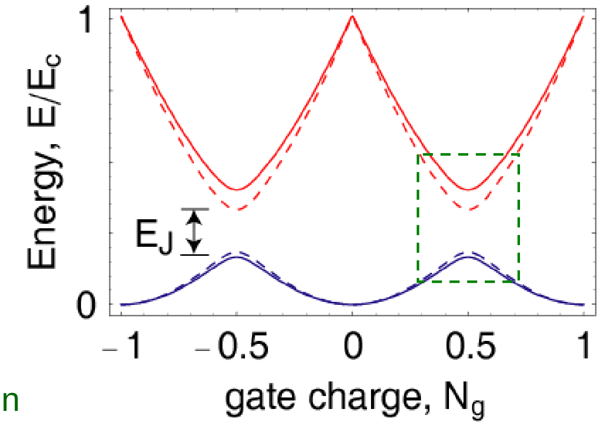
split Cooper pair box in perpendicular field



$$H = E_C\left(N - N_g\right)^2 - E_{J,\max}\cos\left(\pi\frac{\Phi_{\rm ext}}{\Phi_0}\right)$$

SQUID modulation of Josephson energy

$$E_J = E_{J,\max}\cos\left(\pi\frac{\Phi_{\rm ext}}{\Phi_0}\right)$$



consider two state approximation

J. Clarke, *Proc. IEEE* **77**, 1208 (1989)

Two State Approximation

$$\mathbf{H_{CPB}} = \mathbf{H_{el}} + \mathbf{H_J} = E_C(N - N_g)^2 - E_J\cos\delta$$

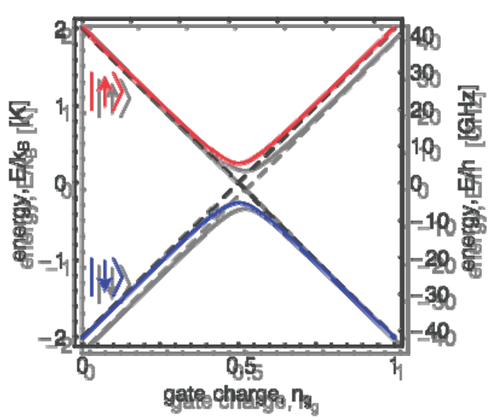
$$\mathbf{H_{CPB}} = \sum_N \left[E_C(N - N_g)^2\left|N\right\rangle\left\langle N\right| - \frac{E_J}{2}\left(\left|N\right\rangle\left\langle N+1\right| + \left|N+1\right\rangle\left\langle N\right|\right)\right]$$

Restricting to a two-charge Hilbert space:

$$N = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1-\sigma_z}{2}$$

$$\cos\delta = \frac{\sigma_x}{2}$$

$$\begin{aligned}\mathbf{H_{CPB}} &= -\frac{E_C}{2}(1 - 2N_g)\sigma_z - \frac{E_J}{2}\sigma_x \\ &= -\frac{1}{2}(E_{el}\sigma_z + E_J\sigma_x)\end{aligned}$$



Shnirman et al. Phys. Rev. Lett. 79, 2371 (1997)

Cavity QED with Electronic Circuits

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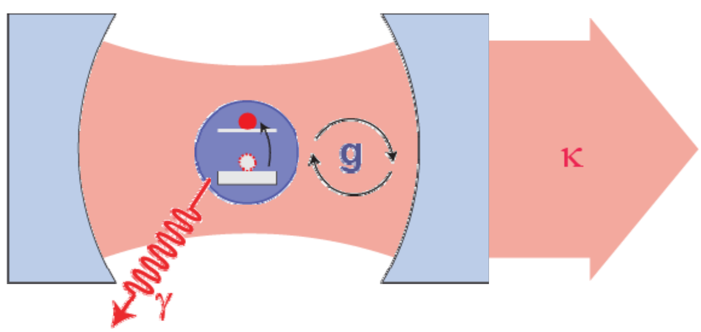
Cavity Quantum Electrodynamics

coupling photons to qubits:



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g (a^\dagger \sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit ($g = dE_0/\hbar > \gamma,\ \kappa\,, 1/t_{\mathrm{transit}}$)

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D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Dressed States Energy Level Diagram

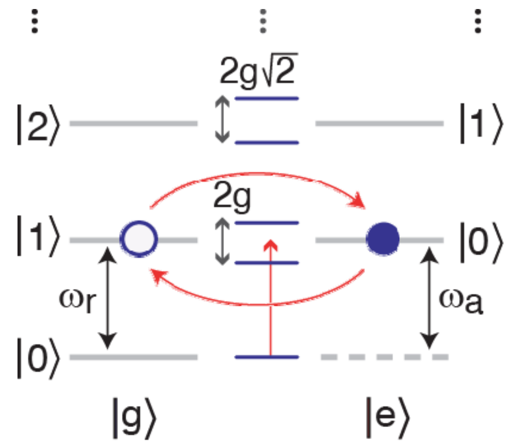
$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

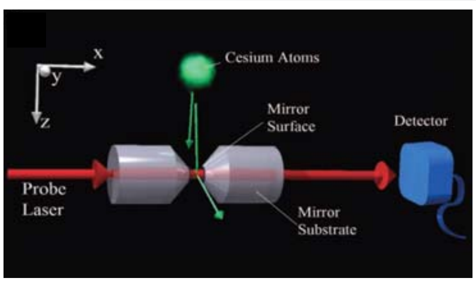
$$g = \frac{dE_0}{\hbar} > \gamma,\, \kappa$$



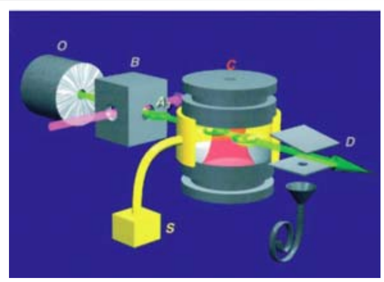
Jaynes-Cummings Ladder

Atomic cavity quantum electrodynamics reviews:
J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)
S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

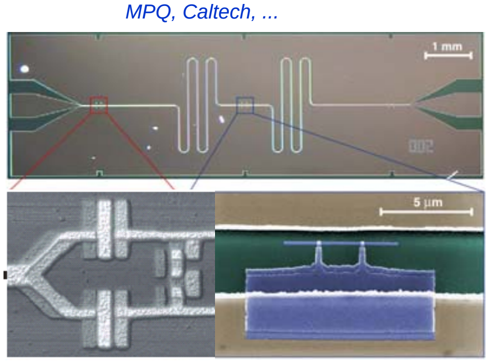
Cavity Quantum Electrodynamics (QED)



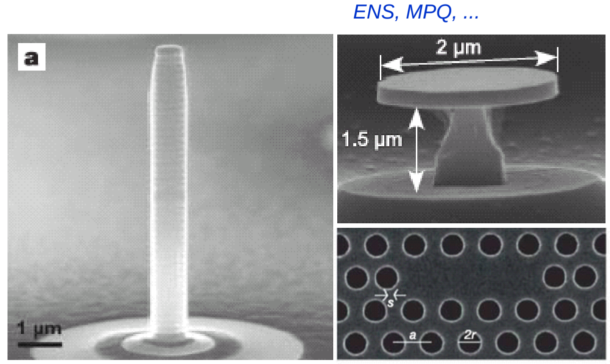
alkali atoms
MPQ, Caltech, ...



Rydberg atoms
ENS, MPQ, ...



superconductor circuits
Yale, Delft, NTT, ETHZ, NIST, …



semiconductor quantum dots
Wurzburg, ETHZ, Stanford …

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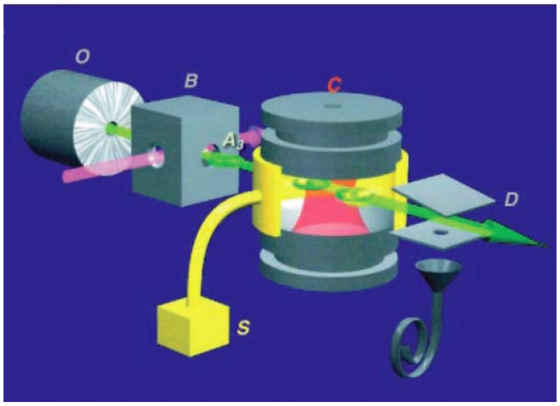
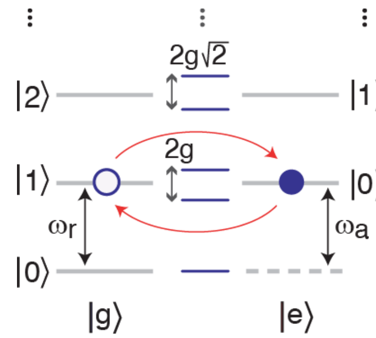
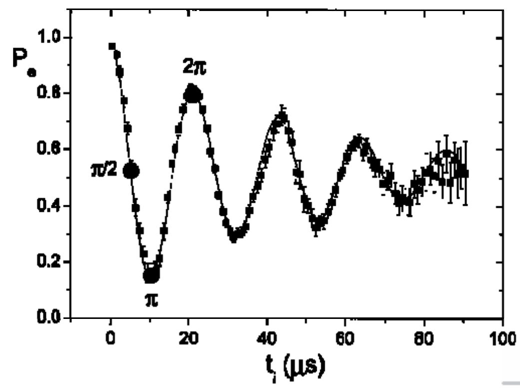
Vacuum Rabi Oscillations with Rydberg Atoms

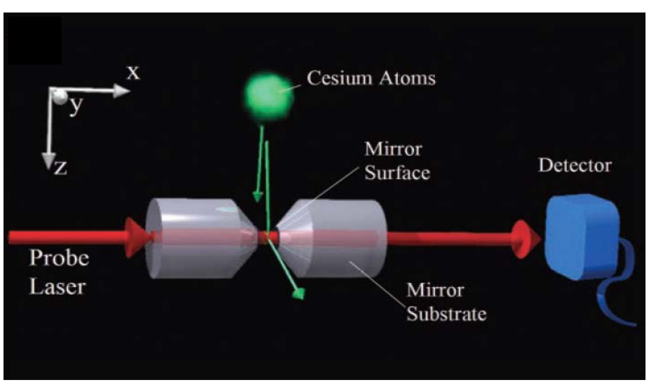


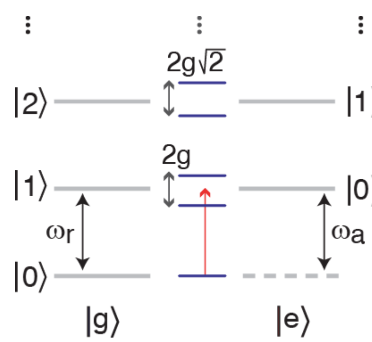


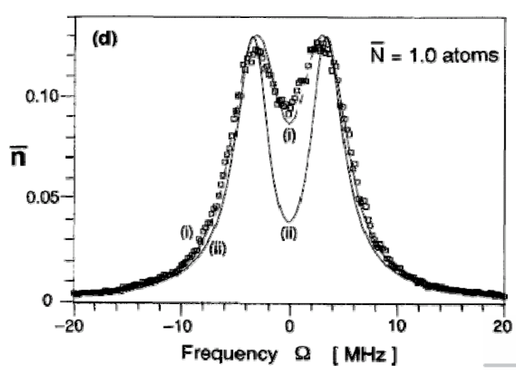


Review: J. M. Raimond, M. Brune, and S. Haroche
Rev. Mod. Phys. **73**, 565 (2001)
P. Hyafil, ..., J. M. Raimond, and S. Haroche,
Phys. Rev. Lett. **93**, 103001 (2004)

Vacuum Rabi Mode Splitting with Alkali Atoms

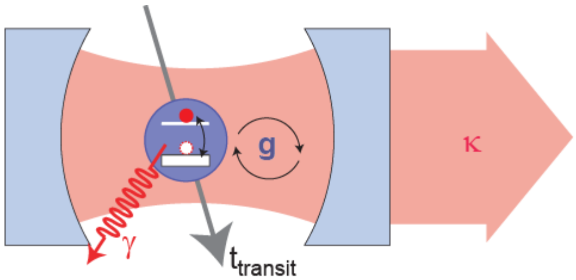






R. J. Thompson, G. Rempe, & H. J. Kimble,
Phys. Rev. Lett. **68** 1132 (1992)
A. Boca, ... , J. McKeever, & H. J. Kimble
Phys. Rev. Lett. **93**, 233603 (2004)

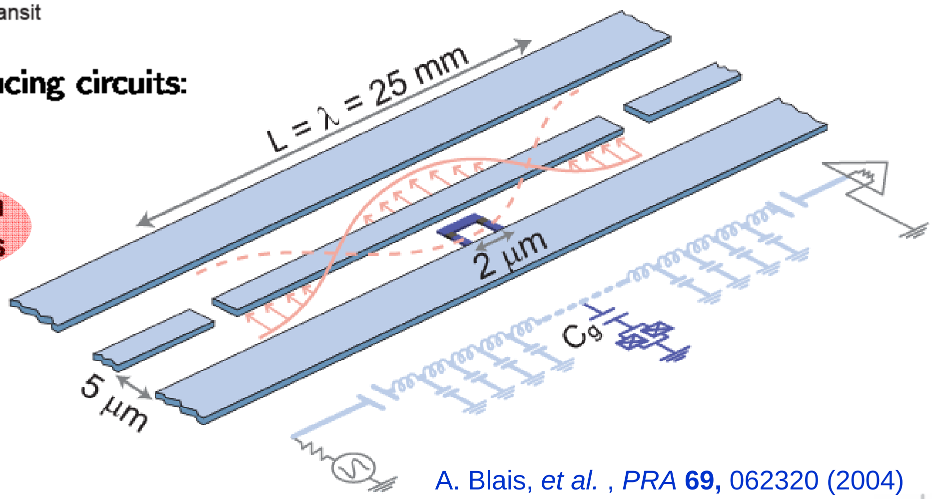
Cavity QED with Superconducting Circuits



coherent quantum mechanics
with individual photons and qubits ...

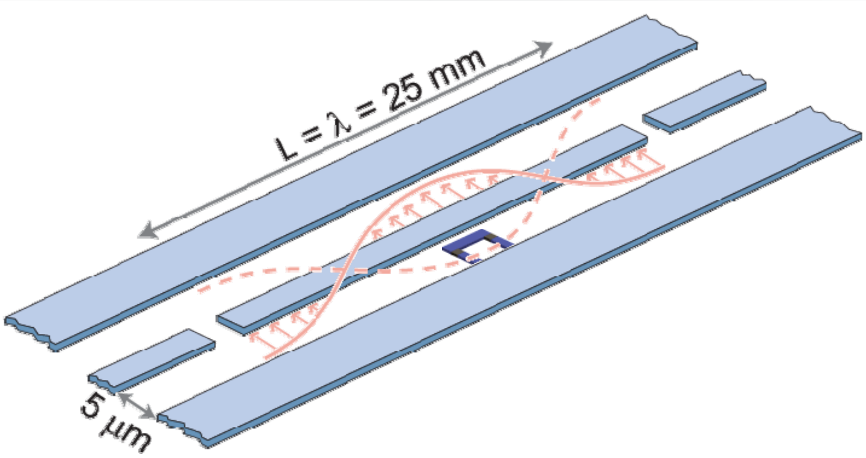
... in superconducing circuits:

circuit quantum electrodynamics



A. Blais, *et al.* , *PRA* **69,** 062320 (2004)
A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

Circuit Quantum Electrodynamics

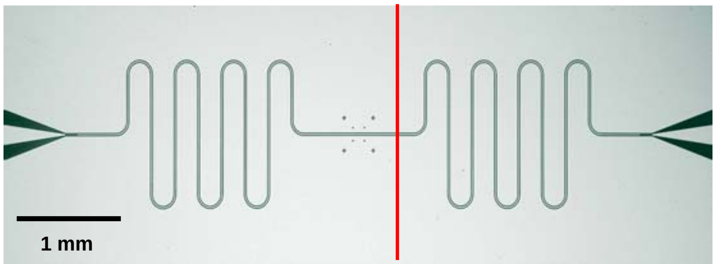


elements

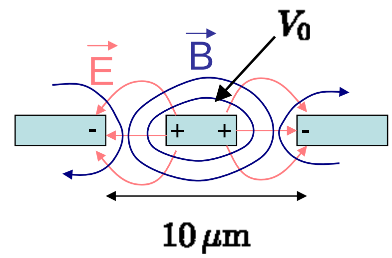
- the cavity: a superconducting 1D transmission line resonator with large vacuum field E_0 and long photon life time $1/\kappa$
- the artificial atom: a Cooper pair box with large E_J/E_C with large dipole moment d and long coherence time $1/\gamma$

A. Blais *et al.*, *PRA* **69,** 062320 (2004)

Vacuum Field in 1D Cavity



cross-section
of transm. line (TEM mode):



voltage across resonator in vacuum state ($n = 0$)

$$V_{0,\mathrm{rms}} = \sqrt{\frac{\hbar\omega_r}{2C}} \approx 1\,\mu\mathrm{V}$$

harmonic oscillator

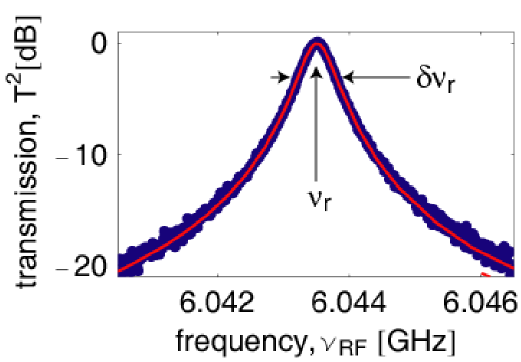
$$H_r = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right)$$

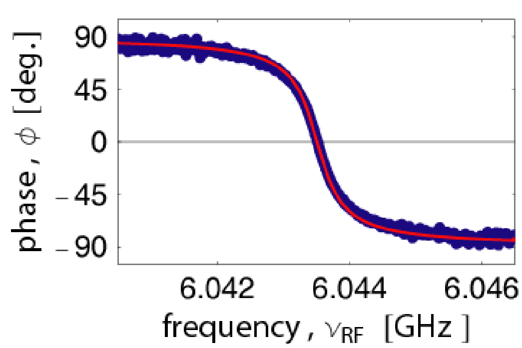
$$E_0 = \frac{V_{0,\mathrm{rms}}}{b} \approx 0.2\,\mathrm{V/m}$$

$\times 10^6$ larger than E_0
in 3D microwave cavity

for $\omega_r/2\pi \approx 6\,\mathrm{GHz}$ ($C \sim 1\,\mathrm{pF}$), $b \approx 5\,\mu\mathrm{m}$

Resonator Quality Factor and Photon Lifetime





resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

photon decay rate:

$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

photon lifetime:

$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

ETH
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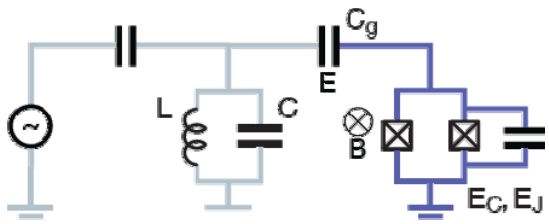
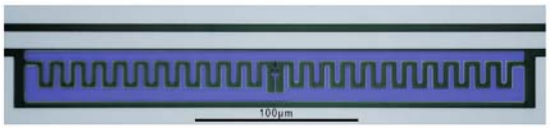
Swiss Federal Institute of Technology Zurich

Qubit/Photon Coupling in a Circuit



qubit coupled to resonator



coupling strength:

$$\hbar g = e V_{0,\mathrm{rms}} \frac{C_g}{C_\Sigma}$$

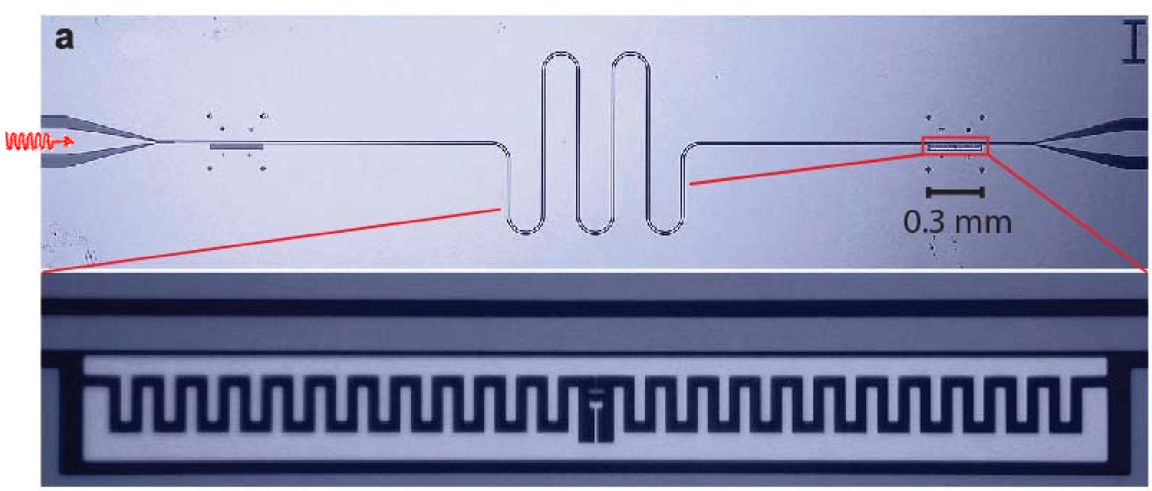
$$\Longrightarrow \nu_{\mathrm{vac}} = \frac{g}{\pi} \approx 1 \ldots 300\,\mathrm{MHz}$$

$g \gg [\kappa, \gamma]$ possible!

large effective dipole moment

$$d = \frac{\hbar g}{E_0} \sim 10^2 \ldots 10^4\, e a_0$$

Circuit QED with One Photon

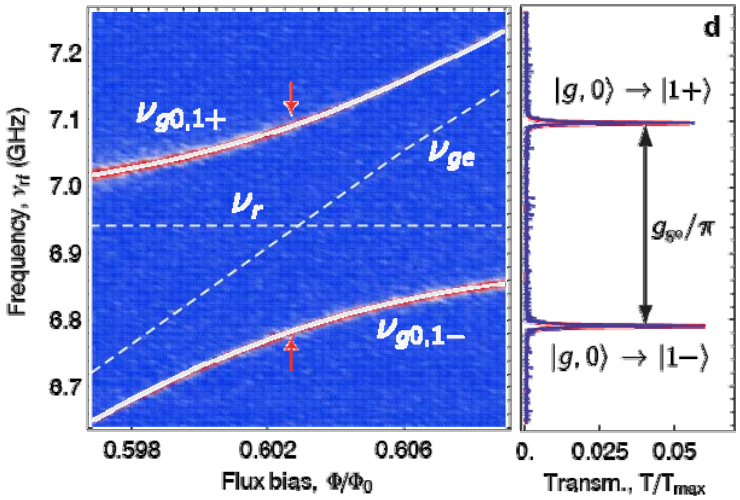


superconducting cavity QED circuit

A. Wallraff, …, R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004)

Resonant Vacuum Rabi Mode Splitting …

… with one photon ($n = 1$):

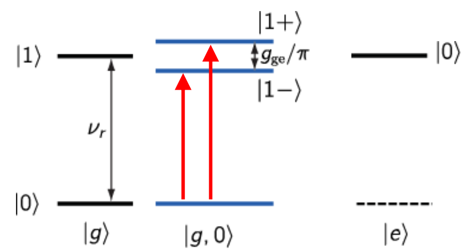


very strong coupling:

$g_{ge}/\pi = 308\,\mathrm{MHz}$

$\kappa, \gamma < 1\ \mathrm{MHz}$

$g_{ge} \gg \kappa, \gamma$



forming a 'molecule' of a qubit and a photon

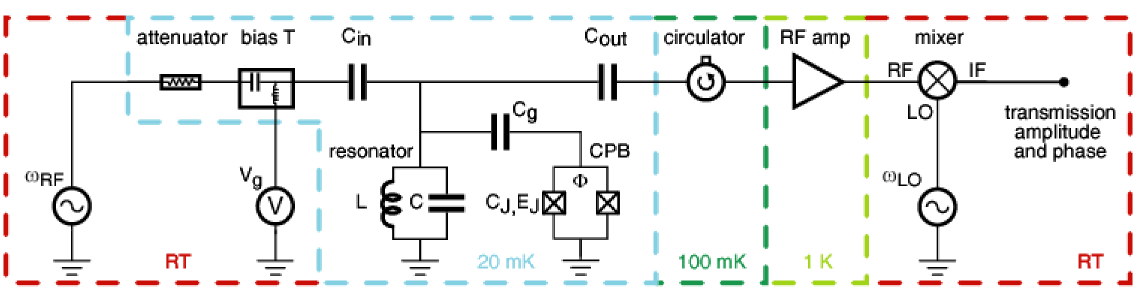
$$|1\pm\rangle = \left(|g,1\rangle \pm |e,0\rangle\right)/\sqrt{2}$$

first demonstration: A. Wallraff, … and R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004)
this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

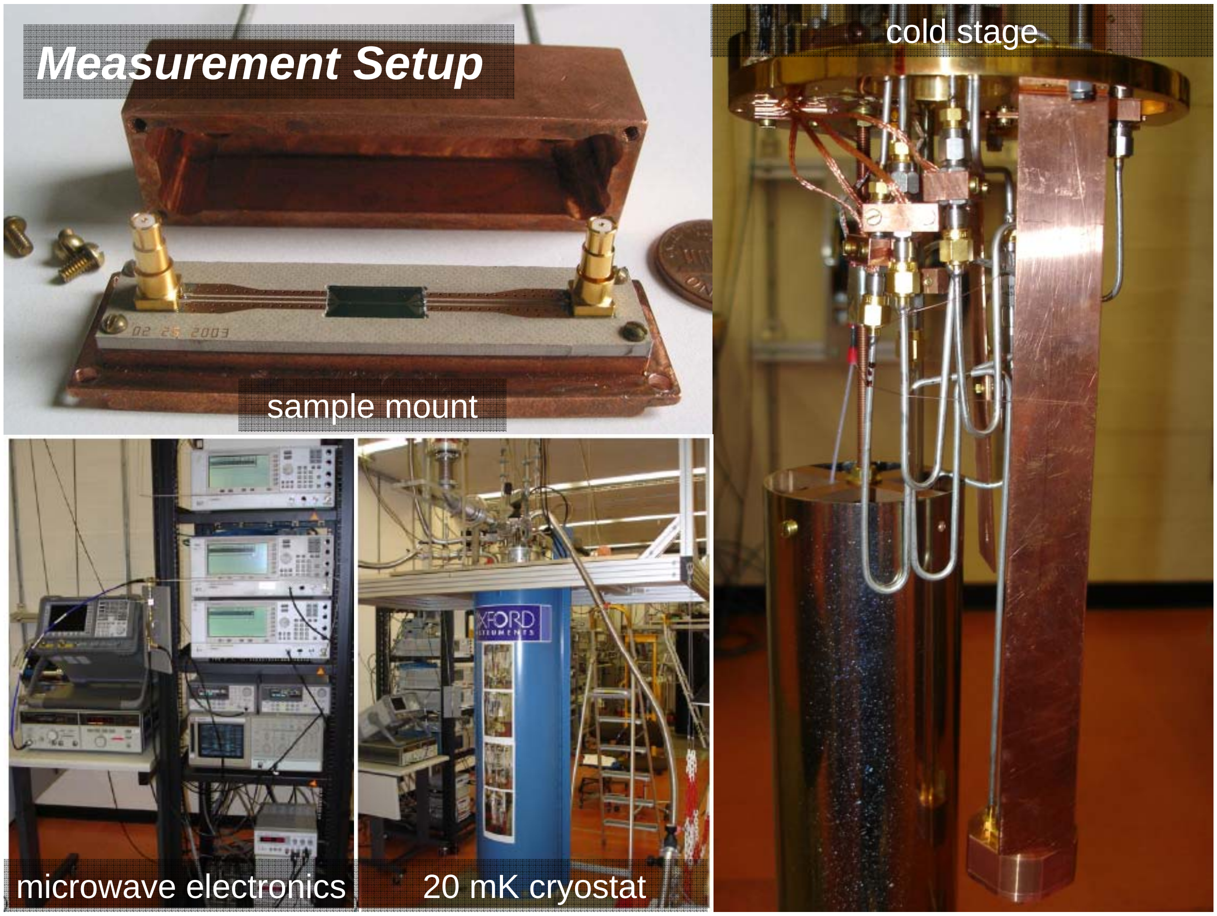
How to Measure Single Microwave Photons

- average power to be detected

$$\rightarrow \langle n = 1\rangle \hbar\omega_r\kappa/2 \approx P_{RF} = -140\,\mathrm{dBm} = 10^{-17}\,\mathrm{W}$$



- efficient with cryogenic low noise HEMT amplifier ($T_N = 6\,\mathrm{K}$)
- prevent leakage of thermal photons (cold attenuators and circulators)


Measurement Setup
cold stage
02 26 2003
sample mount
microwave electronics
20 mK cryostat