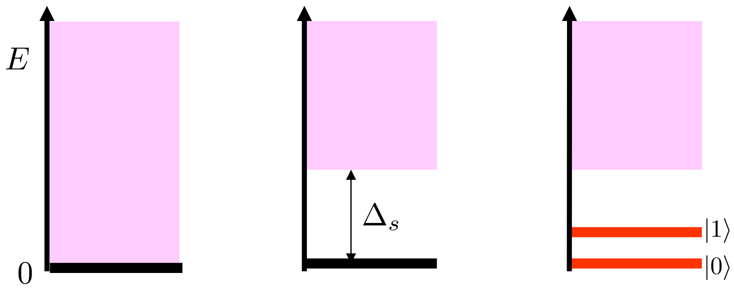
Why Superconductors?



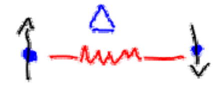
normal metal — superconductor — How to make qubit?

- single non-degenerate macroscopic ground state
- elimination of low-energy excitations

Superconducting materials (for electronics):

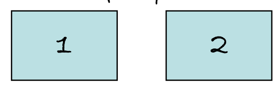
- Niobium (Nb): $2\Delta/h = 725$ GHz, $T_c = 9.2$ K
- Aluminum (Al): $2\Delta/h = 100$ GHz, $T_c = 1.2$ K

Cooper pairs:
bound electron pairs



are Bosons (S=0, L=0)

2 chunks of superconductors

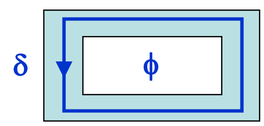


macroscopic wave function

$$\Psi_i = \sqrt{n_i^{CP}}\, e^{i\delta_i}$$

Cooper pair density n_i
and global phase δ_i

phase quantization: $\delta = n\, 2\pi$
flux quantization: $\phi = n\, \phi_0$

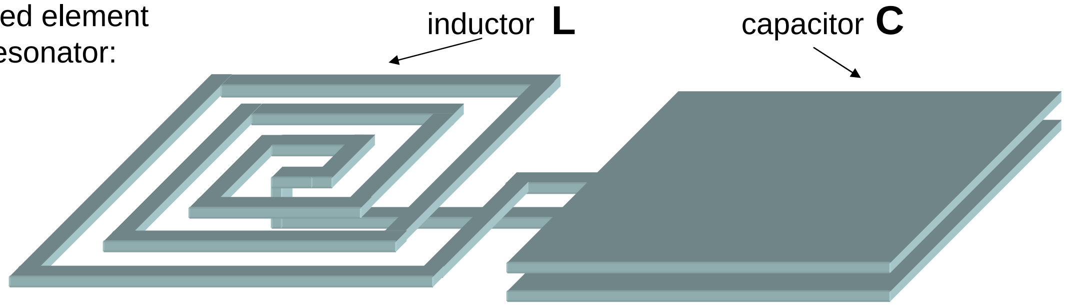


Can it be done?

lumped element
LC resonator:

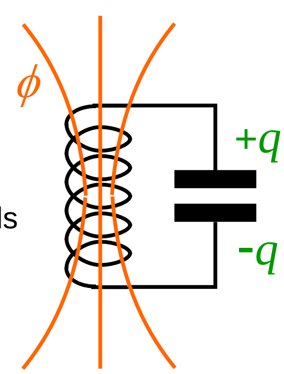
inductor **L**

capacitor **C**



a harmonic oscillator

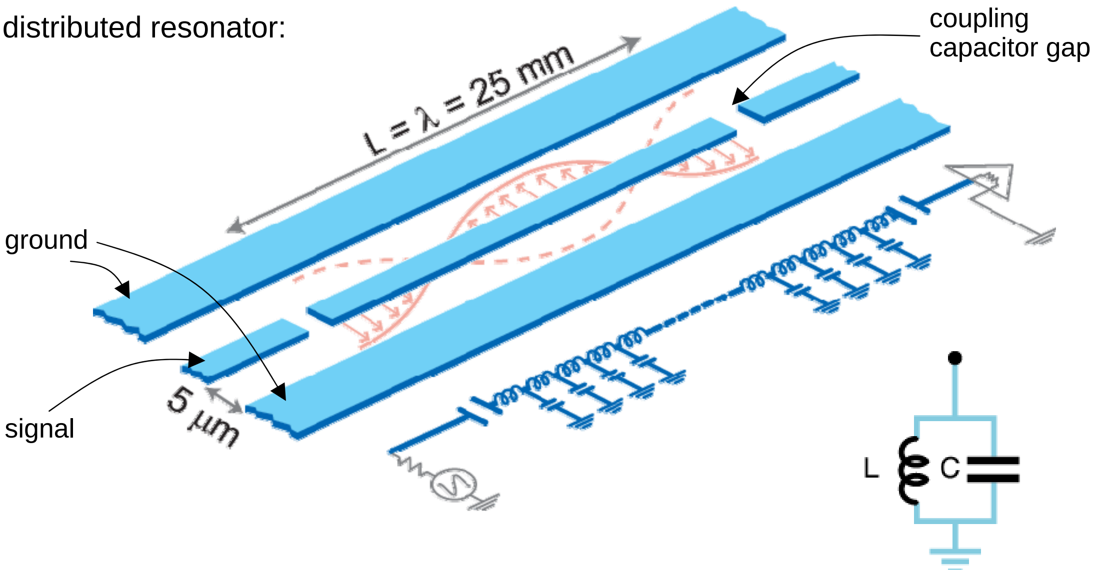
currents and
magnetic fields



charges and
electric fields

Transmission Line Resonator

distributed resonator:



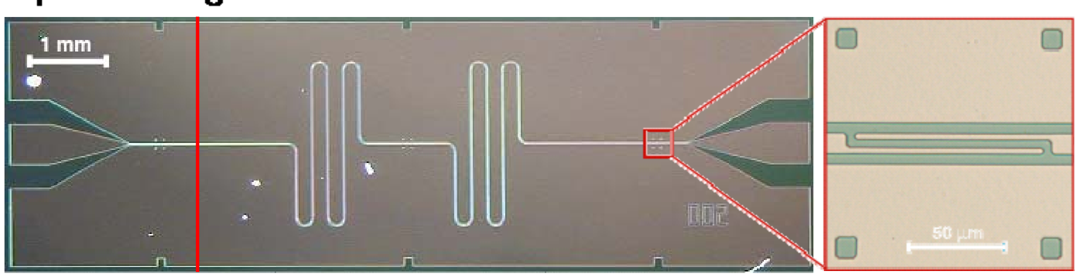
- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

M. Goppl *et al.*, Coplanar Waveguide Resonators for Circuit Quantum Electrodynamics, *arXiv:0807.4094v1* (2008)

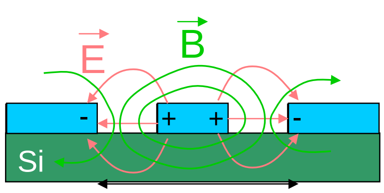
Transmission Line Resonator

$$H_r = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right)$$

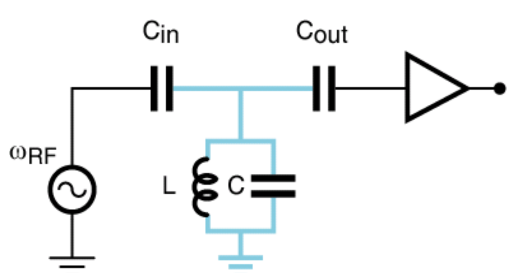
coplanar waveguide:



cross section:

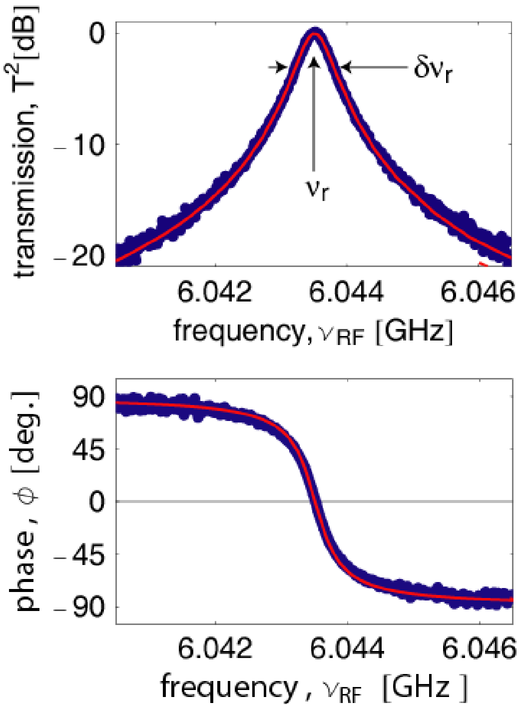


measuring the resonator:



photon lifetime (quality factor) controlled by coupling $C_{\mathrm{in/out}}$

Resonator Quality Factor and Photon Lifetime



resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

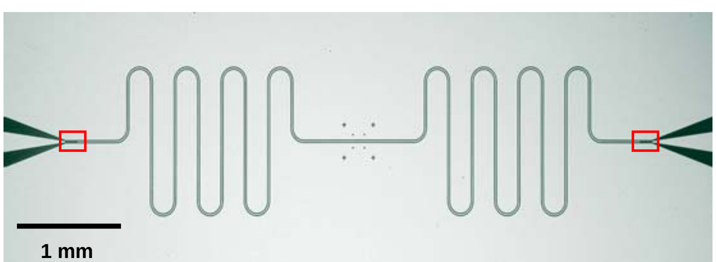
photon decay rate:

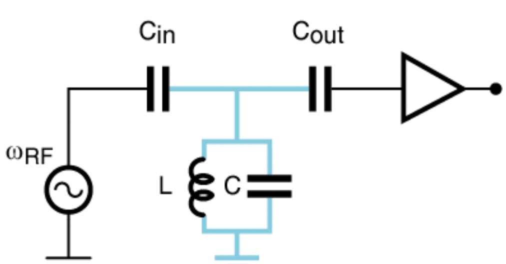
$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

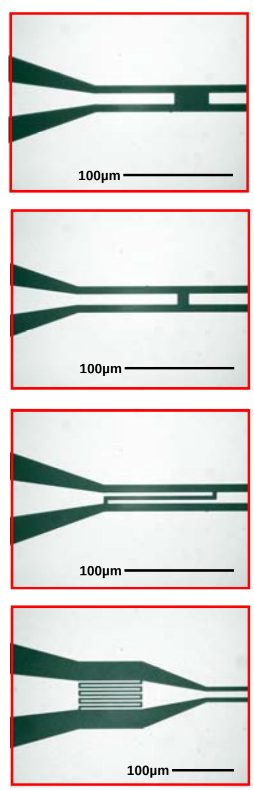
photon lifetime:

$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

Controlling the Photon Life Time



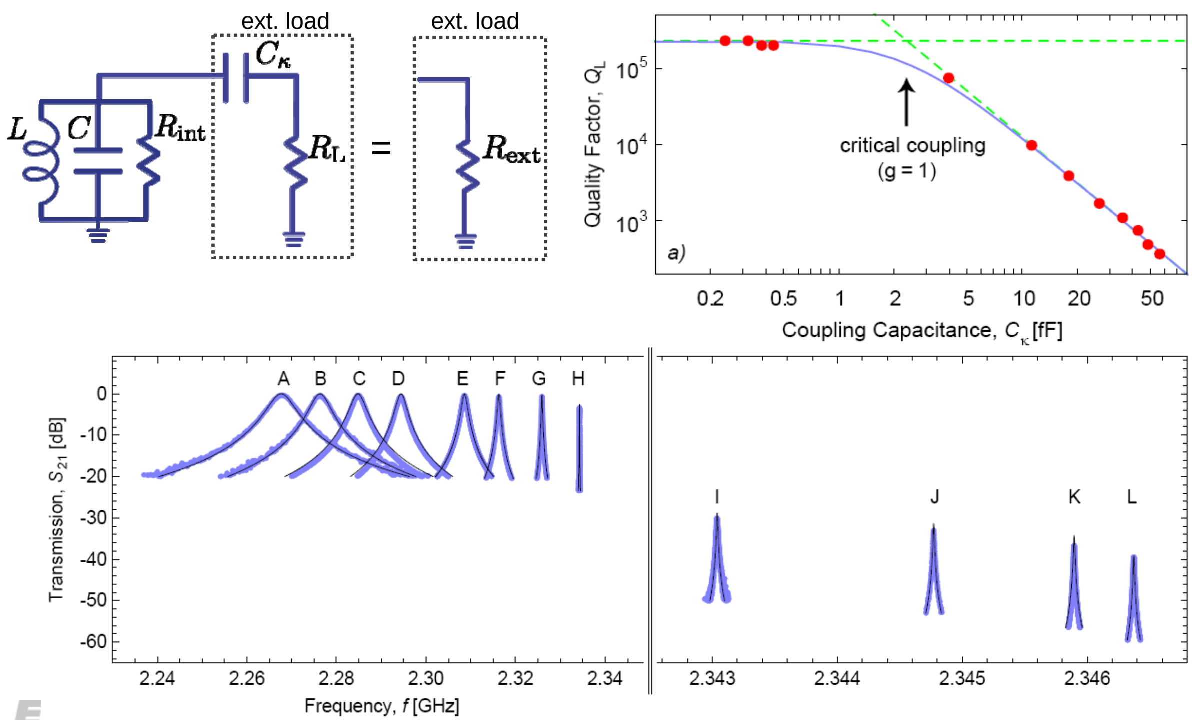




photon lifetime (quality factor)
controlled by coupling capacitor $C_{in/out}$

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Coupling Dependent Quality Factor



M. Goppl, A. Fragner *et al. arXiv:0807.4094* (2008)

How to prove that the h.o. is quantum?

measure**:**

- resonance frequency
- average charge (momentum)
- average flux (position)

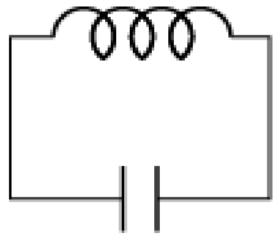
all averaged quantities are identical for a purely harmonic oscillator in the classical or quantum regime

solution:

- make oscillator non linear in a controllable way

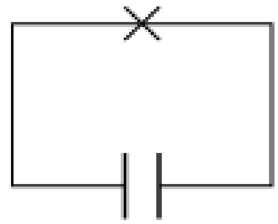
Superconducting Nonlinear Oscillators

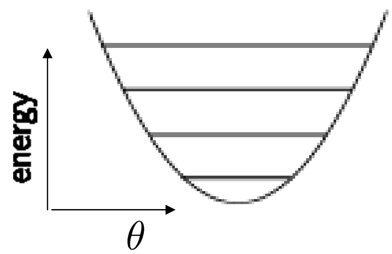
LC resonator



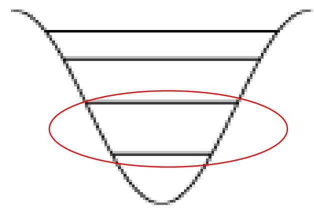
Josephson junction resonator

Josephson junction = nonlinear inductor





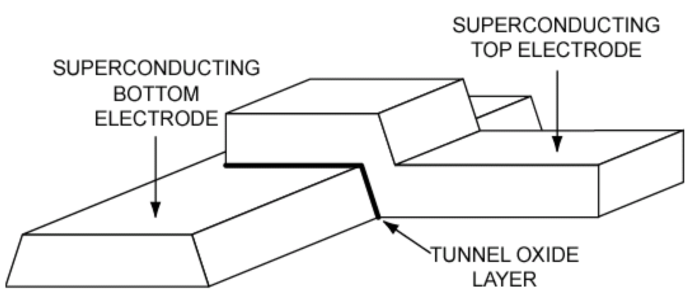
anharmonicity → effective two-level system



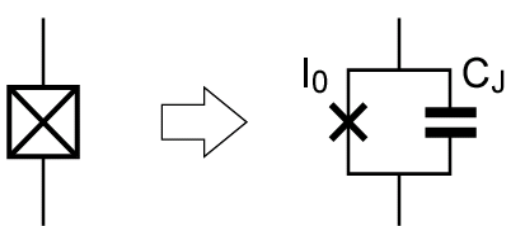
solution to problem 1

A Low-Loss Nonlinear Element

a (superconducting) Josephson junction



- superconductors: Nb, Al
- tunnel barrier: AlO_x

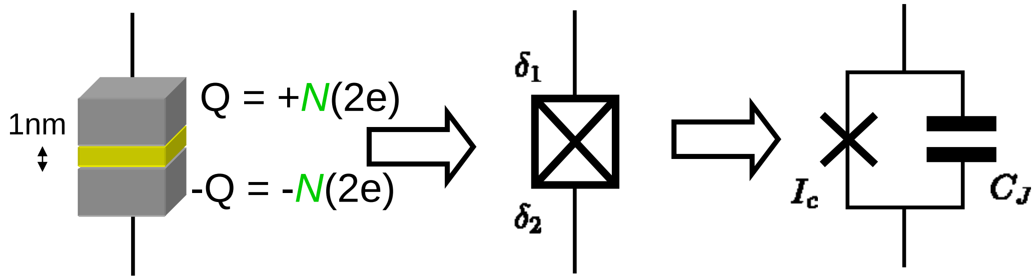


- critical current I_c
- junction capacitance C_J
- high internal resistance R_J

M. Tinkham, *Introduction to Superconductivity* (Krieger, Malabar, 1985).

Josephson Tunnel Junction

the only non-linear LC resonator with no dissipation (BCS, $k_B T \ll \Delta$)



tunnel junction parameters:

- critical current I_c
- junction capacitance C_J
- high internal resistance R_J

Josephson relations:

$$I_0 = I_c \sin\delta$$

$$V = \phi_0 \frac{\partial \delta}{\partial t}$$

flux quantum: $\phi_0 = \frac{h}{2e}$

phase difference: $\delta = \delta_2 - \delta_1$

derivation of Josephson effect, see e.g.: chap. 21 in R. A. Feynman: Quantum mechanics, The Feynman Lectures on Physics. Vol. 3 (Addison-Wesley, 1965)

The Josephson junction as a non-linear inductor

induction law: $V = -L \frac{\partial I}{\partial t}$

Josephson effect: dc-Josephson equation $I = I_c \sin\delta$

$$\frac{\partial I}{\partial t} = I_c \cos\delta \frac{\partial \delta}{\partial t}$$

ac-Josephson equation

$$V = \frac{\phi_0}{2\pi} \frac{\partial \delta}{\partial t} = \underbrace{\frac{\phi_0}{2\pi I_c} \frac{1}{\cos\delta}}_{L_J} \frac{\partial I}{\partial t}$$

Josephson inductance

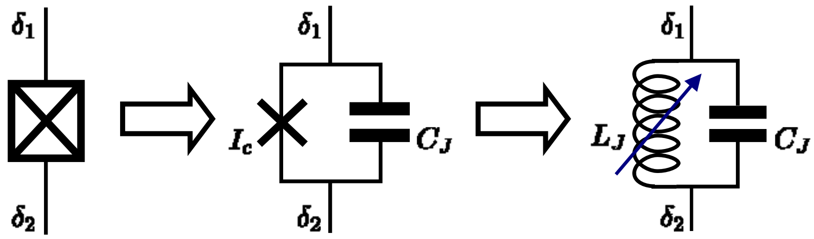
$$L_J = \underbrace{\frac{\phi_0}{2\pi I_c}}_{\text{specific Josephson inductance}} \underbrace{\frac{1}{\cos\delta}}_{\text{nonlinearity}}$$

A typical characteristic Josephson inductance for a tunnel junction with $I_c = 100$ nA is $L_{J0} \sim 3$ nH.

review: M. H. Devoret *et al.*, Quantum tunneling in condensed media, *North-Holland,* (1992)

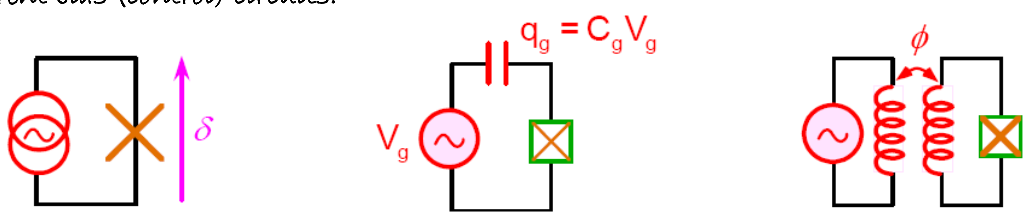
A Non-Linear Tunable Inductor w/o Dissipation

the Josephson junction as a circuit element:



How to Make Use of the Josephson Junction in Qubits?

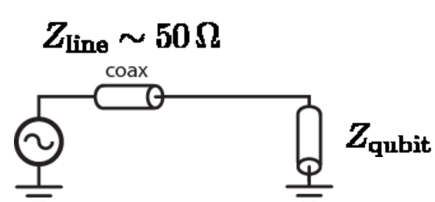
different bias (control) circuits:



Coupling to the Electromagnetic Environment

strong coupling to environment (bias wires):

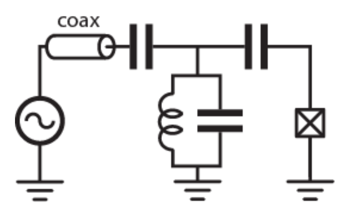
decoherence
from energy relaxation

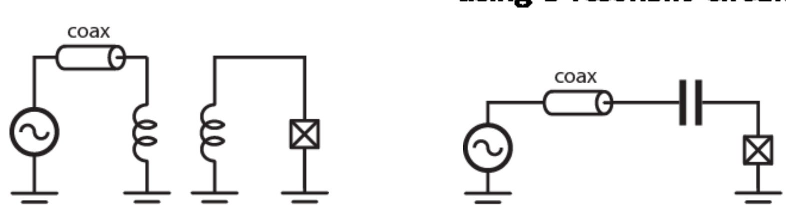


decoupling using impedance transformers:

control decoherence
by circuit design

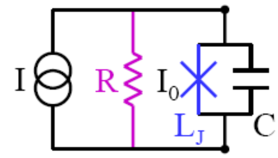
solution to problem 2





using non-resonant impedance transformers

Current Biased Phase Qubit



The bias current I distributes into a Josephson current through an ideal Josephson junction with critical current I_c, through a resistor R and into a displacement current over the capacitor C.

Kirchhoff's law:

$$I_b = I_s + I_R + I_C$$
$$= I_c \sin\delta + \frac{V}{R} + C\dot{V}$$

$$I_C = \dot{Q}_C = C\dot{V}$$
$$I_R = V/R$$
$$I_s = I_c \sin\delta$$

use Josephson equations:

$$I_b = I_c \sin\delta + \frac{\phi_0}{2\pi R}\dot{\delta} + \frac{\phi_0 C}{2\pi}\ddot{\delta}$$

W.C. Stewart, Appl. Phys. Lett. **2**, 277, (1968)
D.E. McCumber, J. Appl. Phys. **39**, 3 113 (1968)

looks like equation of motion for a particle with mass m and coordinate δ in an external potential U:

$$m\ddot{\delta} + m\frac{1}{RC}\dot{\delta} + \frac{\partial U(\delta)}{\partial\delta} = 0$$

particle mass: $m = C\,(\phi_0/2\pi)^2$

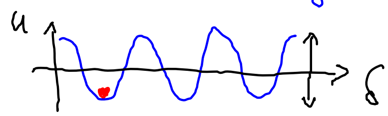
external potential: $U(\delta) = \frac{I_c\phi_0}{2\pi}\left(-\frac{I_b}{I_c} - \cos\delta\right)$

Phase particle in a potential well

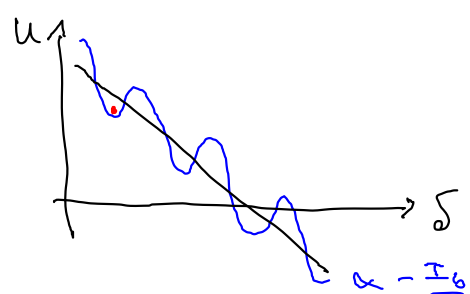
$$U(\delta) = \frac{I_c\phi_0}{2\pi}\left(-\frac{I_b}{I_c}\delta - \cos\delta\right)$$

$$E_J = \frac{I_c\phi_0}{2\pi}$$

cosine potential for $I_b = 0$:



'tilted washboard' potential for $I_b \neq 0$:



potential barrier:

$$U_0 = 2E_J\left[\sqrt{1-\gamma^2} - \gamma \arccos\gamma\right]$$

oscillation frequency:

$$\omega_0 = \omega_p(1-\gamma^2)^{1/4} = \sqrt{\frac{U''(\delta_0)}{m}}$$

with: $\gamma = I_b/I_c$; $\omega_p = \sqrt{\frac{2\pi I_c}{\phi_0 C}}$