Lecture

Quantum Systems for Information Technology

fall term (HS) 2008

Lecturer:
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Basic Structure of Course

- Part I: Introduction to Quantum Information Processing (QIP)
  - basic concepts
  - qubits, qubit control, measurement, gate operations
  - circuit model of quantum computation
  - examples of quantum algorithms
- Part II: Superconducting Quantum Electronic Circuits for QIP
  - qubit realizations, characterization, decoherence
  - qubit/photon interface: cavity quantum electrodynamics
  - physical realization of qubit control, tomography and qubit/qubit interactions
- Part III: Implementations
  - electrons and spins in semiconductor quantum dots
  - ions and neutral cold atoms
  - photons and linear optics
  - spins in nuclear magnetic resonance

Guest Lectures

- Ion Trap Quantum Computing (1.12.2008),
  Hartmut Haeffner (University of Innsbruck, Austria)

- Quantum Communication (date to be confirmed)
  Mikael Afzelius (University of Geneva)

- Error Correction (to be confirmed)
  Guido Burkhard (University of Konstanz, Germany)
  or Sasha Shnirman (University of Karlsruhe, Germany)

Exercise Classes

- part I & II (week 2 - 8)
  - discuss and practice topics of lecture

- part III (week 9 - 13)
  - student presentations

- teaching assistants:
  - Stefan Filipp (filipp@phys.ethz.ch)
  - Peter Leek (peterleek@phys.ethz.ch)

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Reading

- Quantum computation and quantum information
  Michael A. Nielsen & Isaac L. Chuang
  Cambridge : Cambridge University Press, 2000
  676 S.
  ISBN 0-521-63235-8

- additional reading material will be provided throughout the lecture and on the web page:
  qudev.ethz.ch/content/courses/coursesmain.html

Credit (Testat) Requirements

- active contribution to lectures and discussions

- successfully prepare and present a talk on one of the physical implementations of quantum information processing

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Student Presentations

- Topics: implementations of quantum information processing
- Goal: present key features of implementation and judge its prospects
- Material: research papers and review articles will be provided
- Preparation: teams of two students, 10 slots for teams available, advice and support by TAs
- Duration: presentation + discussion (30+15 minutes)
- Presentation: blackboard, transparencies, powerpoint …

Exam & Credits

- aural exam (20 mins) during summer or winter exam session
- exam dates as required by your program of study
- 8 credit points (KP) can be earned successfully completing this class

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Time and Place

- lecture: Monday (15-17), 14:45 – 16:30, HCI H 2.1
- exercises: Monday (11-13), 10:45 – 12:30, HCI H 8.1

- are there timing conflicts with other lectures?
  - TBD

- potential alternative time slots:
  - TBD

Registration & Contact Information

your registration and contact information

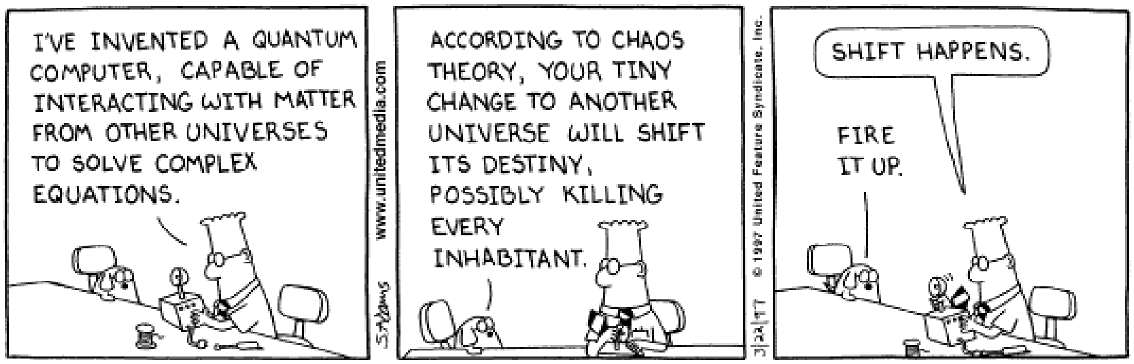
- please register online for the class

- in this way we can contact you

our contact information

- qsit-lecture@phys.etzh.ch
- www.qudev.ethz.ch/content/courses/coursesmain.html (will be updated)

Let's get started!



What is this lecture about?

Quantum Mechanics and its Applications in Information Processing

Questions:

- What are the fundamental concepts of quantum computation and quantum information?
- How did these concepts develop?
- How can one make use of these concepts?
- How does one go about actually building a quantum information processor?

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Is it really interesting?

Even fashion models talk about it!

You do not believe it?

Watch this!

Why one should care about Quantum Mechanics

And quantum physics is featured in popular talk shows!

Watch Conan O'Brien and Jim Carrey on the 'Late Night' show.

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1.0 Introduction to Quantum Systems for Information Technology

1.1 Motivation

What is quantum mechanics good for?

traditional historical perspective:

- beginning of 20th century:
  classical physics fails to explain phenomena observed in nature
  - stability of atoms
  - discrete spectra of light emitted by atoms
  - spectrum of black body radiation

- use quantum mechanics to explain phenomena occurring in nature
  - properties of microscopic systems (atoms, nuclei, electrons, elementary particles)
    - energy level quantization
    - tunneling
    - entanglement
    - ...
  - properties of macroscopic systems
    - superconductivity
    - electronic band structure of semiconductors
    - …

- quantum mechanics is a hugely successful theory ...

- … but its concepts are difficult to grasp
  - EPR paradox
  - entanglement
  - quantum measurement

... Motivation

- early on study of quantum information and quantum computation is motivated by desire to better understand quantum mechanics
  - relation between information and physics
    Rolf Landauer: information is physical
  - 80's: Can quantum mechanics be used to transmit information faster than light?
    No: shown in the context of the *no-cloning theorem.*

Efforts to try to make use of quantum mechanics:

- Quantum computation and quantum information is the study of information processing that can be accomplished with quantum mechanical systems.
  - it took a long time after the development of QM to invent this new field

quantum information processing is enabled by new technologies:

- 70's: develop complete control over single quantum systems
  - single atoms/ions/molecules
  - single photons
  - 90's: single electrons/spins/flux quanta in solid state
  - …
- explore new regimes of nature that only occur in single isolated quantum systems
- different from prior experiments in quantum phenomena in ensembles
  - superconductivity, collective quantum effect of 10^{23} electrons
    - no information over individual electrons
  - particle physics: analysis of constituents of matter
    - no control over individual particles

… Motivation

- now: control collections of individual quantum systems and their interactions
  - arrays of ions interacting electrically
  - arrays of atoms interacting in collisions
  - …
- demonstrate information processing with quantum systems
  - small systems have been realized (up to ten quantum objetcs)
  - larger systems remain a major physics and engineering challenge

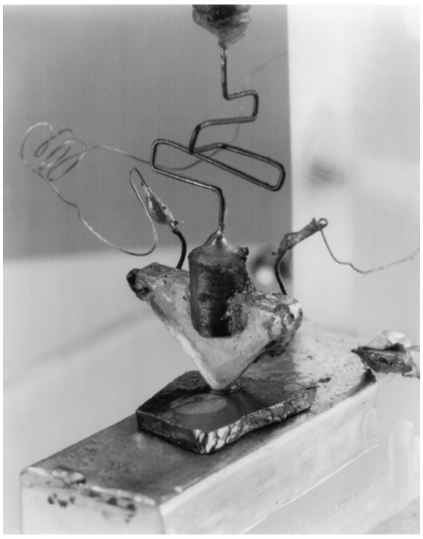
Up to now we have discussed the physics perspective.

What about the **computer science** perspective?

- (1936) Turing machine
  - model for any realizable classical computer
  - But are there alternative computing schemes?
- realization of first electronic computers
  - 1947: the transistor is invented
  - great success up to now: Moore's Law (1965)

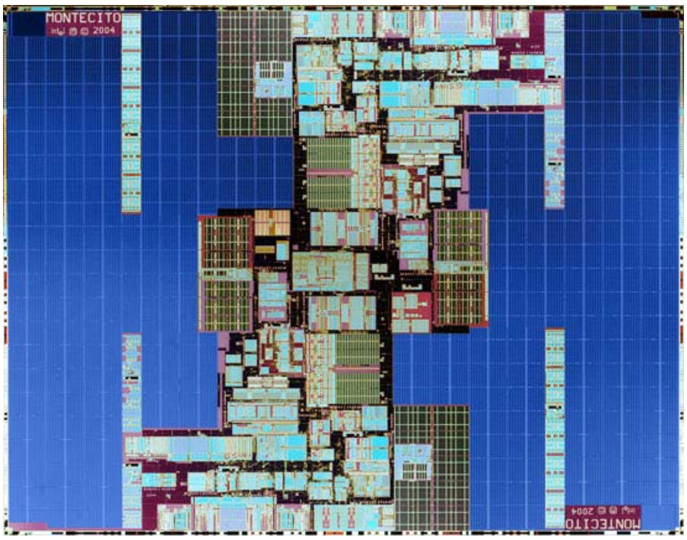
Classical information processing with electronic circuits

- first transistor at Bell Labs (1947) invented by John Bardeen, Walter Brittain, and Will Shockley

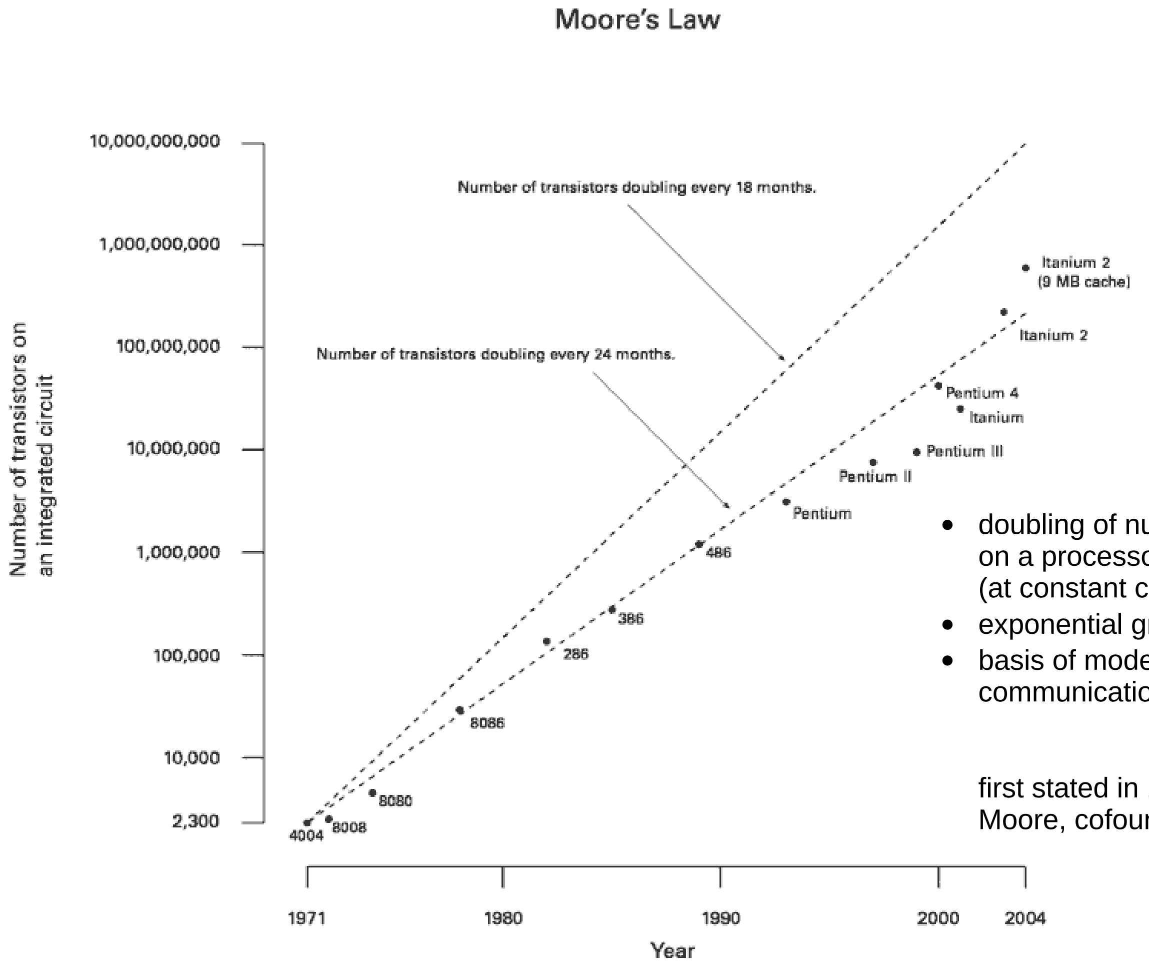


- 1 transistor
- size a few cm

- intel dual core processor (2006)



- 2.000.000.000 transistors
- smallest feature size 65 nm
- clock speed ~ 2 GHz
- power consumption 10 W
- 5 nW per transistor
- 2.5 10^{-18}J per transistor per cycle



- doubling of number of transistors on a processor every 24 months (at constant cost)
- exponential growth
- basis of modern information and communication based society

first stated in 1965 by Gordon E. Moore, cofounder of Intel

… Motivation

- What will happen when electronic circuit components reach atomic sizes?
  - Will quantum mechanics be a problem?
  - Or will it be an opportunity?

- Make use of quantum mechanics as an opportunity for novel approaches to computing.

- Quantum computing is a new paradigm in computer science.

quantum information processing (QIP):

- Deutsch (1985)
  - finds a simple algorithm that is more efficient on a quantum computer
  - searches for computation device that could efficiently simulate any physical system (incl. quantum systems)
    - a device based on quantum mechanics in itself

- Shor (1994)
  - develops an efficient algorithm to find prime factors of an integer
  - exponential speed-up in comparison to classical algorithm
  - important because encryptions schemes (RSA) are based on difficulty of problem

- Grover (1995)
  - searching in unstructured data bases (quadratic speed up)

- Feynman (1982)
  - simulate complex quantum systems
  - potentially the most interesting application

… Motivation

state of the art:

- difficult to realize and control even a small quantum computer
- BUT the concepts do work and have been demonstrated
  - prime factors of 15 = 3 * 5 have been calculated on a nuclear magnetic resonance (NMR) quantum computer
- ongoing research into realizing scalable hardware for a quantum computer
  - solid state systems
  - ions
- ongoing quest for quantum algorithms
  - difficult to find efficient quantum algorithms that are better than classical ones
  - any classical algorithm can be run on a quantum computer
  - develop of novel approaches to information processing that are enabled by quantum mechanics

quantum communication (QC):

- efficient encoding of information in photons
  - super dense coding (Bennett '92)
- unconditionally secure communication using individual photons
  - quantum cryptography (Bennett, Brassard '84)

state of the art:

- quantum cryptography is used in commercial applications for distributing keys in optical fiber networks [http://www.idquantique.com/]
- limited by loss of photons in optical fibers
- ongoing research into quantum repeaters to extend range

1.2 Goals of Lecture: Quantum Systems for Information Technology

- Introduction to Quantum Information Processing (QIP)
  - understand basic concepts
    - What are qubits?
    - What are their properties?
    - How to process information with quantum systems?
    - Which algorithms can a quantum computer execute efficiently?
  - get to know physical realizations
    - How to realize a quantum information processor?
    - Example: Superconducting Electronic Circuits
      - characterization of qubits
      - initialization, control and read-out of qubits
      - realization of quantum logic
    - gain general understanding of methods used to characterize physical realizations of quantum systems
  - learn how to evaluate the physical properties and prospects of different qubit implementations
    - atomic qubits
    - photonic qubits
    - spin qubits
    - semiconductor qubits
    - ...

1.3 Structure of Course: Quantum Systems for Information Technology

- Introduction to Quantum Information Processing (QIP)
  - basic concepts
    - qubits and their properties
    - single qubit control and measurement
    - multiple qubits
    - qubit/qubit interactions and logical operations
  - basic quantum algorithms
    - Deutsch-Josza
    - Teleportation
    - later: basic principles of factorization (Shor) and search algorithms (Grover)

- Quantum Systems for Information Processing
  - qubits based on superconducting quantum electronic circuits
    - realizations of qubits in electronic circuits
      - harmonic oscillators
      - types of superconducting qubits
      - qubit initialization
    - measurement of the qubit state
      - dispersive read-out
      - other types of state measurements
      - spectroscopy
    - qubit state control and basic time-resolved measurements
      - Rabi oscillations
      - Ramsey fringes
      - spin echo

Structure: Quantum Systems for Information Technology

- Quantum Systems for Information Processing
  - qubits based on superconducting quantum electronic circuits (continued)
    - decoherence
      - sources of decoherence
      - reducing decoherence
    - quantum state tomography
      - single and two-qubit read-out
    - two-qubit interactions
      - realization of logic gates
    - summary

  - physical systems for QIP
    - atomic qubits
      - ions
      - neutral atoms
    - spin qubits
      - nuclear spins
      - electron spins
    - semiconductor quantum dots
      - electrostatic quantum dots
      - self-assembled systems
    - qubit/photon interactions
      - cavity quantum electrodynamics

2.0 Basic Introduction to Quantum Information Processing

2.1 Classical information processing

2.1.1 The carrier of information

- binary representation of information as **bits** (Binary digITs).

- classical bits can take values **either** 0 **or** 1

- information is represented (and stored) in a physical system
  - for example, as a voltage level at the input of a transistor in a digital circuit

- in Transistor-Transistor-Logic (TTL)
  - "low" = logical 0 = 0 - 0.8 V
  - "high" = logical 1 = 2.2 - 5 V
- similar in other approaches
  - CMOS: complementary metal oxide semiconductor
  - ECL: emitter coupled logic

- information is processed by operating on bits using physical processes
  - e.g. realizing logical gates with transistors

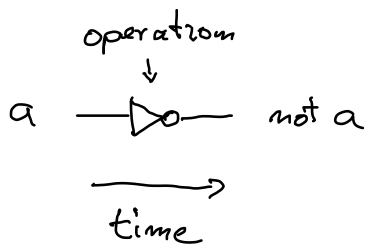
2.1.2 Processing information with classical logic

- decomposition of logical operations in **single bit and two-bit operations**

truth table of operation

		IN	OUT
- trivial single bit logic gate:	**Identity**	1	1
		0	0
- non-trivial single bit logic gate:	**NOT**	0	1
		1	0

- circuit representation

input bit  output bit

- representation of time evolution of information
- each wire represents a bit and transports information in time
- each gate operation represented by a symbol changes the state of the bit

2.1.3 The universal two-bit logic gate

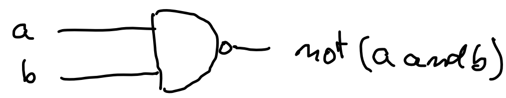
- logical operations between two bits: AND, OR, XOR, NOR ...
  - can all be implemented using NAND gates

- Negation of AND : **NAND**
  AND followed by NOT

  truth table

IN	OUT
00	1
01	1
10	1
11	0

- circuit representation of the NAND gate:



Universality of the NAND gate:

- Any function operating on bits can be computed using NAND gates.
- Therefore NAND is called **a universal logic gate**.

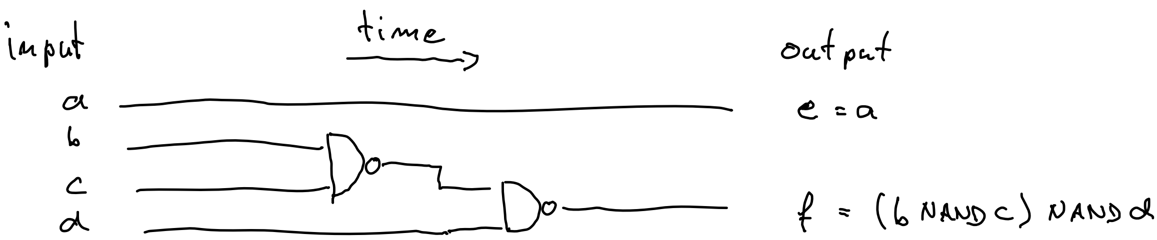
read: Nielsen, M. A. & Chuang, I. L., QC and QI, chapter 3, *Cambridge University Press,* (2000)

For quantum computation a set of universal gates has been identified

- single qubit operations and the CNOT gate form a universal set of gates for operation of a quantum computer

2.1.4 Circuit representation

- Any computable function can be represented as a circuit composed of universal gates acting on a set of input bits generating a set of output bits.



logical circuit computing a function

- properties of classical circuits representing a function
  - wires preserve states of bits
  - FANOUT: single input bit can be copied
  - additional working bits (ancillas) are allowed
  - CROSSOVER: interchange of the value of two bits
  - AND, XOR or NOT gates operate on bits
    - can be replaced by NAND gates using ancillas and FANOUT

Note:
- number of output bits can be smaller than number of input bits
  - information is lost, the process is not reversible
- no loops are allowed
  - the process has to be acyclic

- A similar circuit approach is useful to describe the operation of a quantum computer.
  - But how to make good quantum wires?
  - Can quantum information be copied?
  - How to make two-bit logic reversible?
  - What is a set of universal gates?

2.1.5 Conventional classical logic versus quantum logic

Conventional electronic circuits for information processing

- work according to the laws of **classical physics**
- quantum mechanics does not play a role in information processing

However:
- some devices used for information processing (LASERs, tunnel diodes, semiconductor heterostructures) operate using quantum mechanical effects on a microscopic level
- but macroscopic degrees of freedom (currents, voltages, charges) do usually not display quantum properties

Quantum mechanics for information processing

Questions:

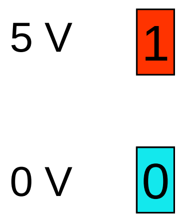
- How can we make use of **quantum mechanics** for information processing?
- Is there something to be gained?
- How can a quantum information processor be realized?
- Which physical systems are promising candidates to realize a quantum information processor?
- Macroscopic solid state systems
  - What happens when circuits are miniaturized to near atomic scales?
  - Do they continue working the same way?
  - Does quantum mechanics get in the way or can it be used?
- Microscopic atomic systems
  - How to realize and control a fixed number of microscopic degrees of freedom individually?
  - Can systems be scaled up to large enough size to be interesting for information processing?

2.2 Quantum Bits

2.2.1 Classical Bits versus Quantum Bits

classical bit (**b**inary dig**it**)

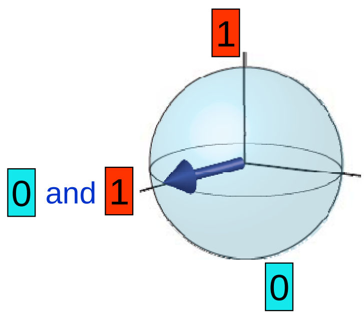
- can take values 0 or 1



- realized e.g. as a voltage level 0 V or 5 V in a circuit

qubit (**qu**antum **bit**) [Schumacher '95]

- can take values 0 and 1 'simultaneously'



- realized as the quantum states of a physical system
- we will explore algorithms where the possibility to generate such states of the information carrying bit are essential

Schumacher, B., Quantum coding, *Phys. Rev. A* **51**, 2738-2747 (1995)

2.2.2 Definition of a Quantum Bit

Quantum **bit**s (qubits) are quantum mechanical systems with two distinct quantum mechanical states.

Qubits can be realized in a wide variety of physical systems displaying quantum mechanical properties.

- atoms, ions, molecules
- electronic and nuclear magnetic moments
- charges in semiconductor quantum dots
- charges and fluxes in superconducting circuits
- and many more ...

A suitable realization of a qubit should fulfill the so called **DiVincenzo criteria**.

Quantum Mechanical Description of a Qubit
A qubit has internal states that are represented as vectors in a 2-dimensional Hilbert space. A set of possible qubit (computational) basis states is:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; ; \; |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{(Dirac notation)}$$

Quantum Mechanics Reminder:
QM postulate I: The quantum state of an isolated physical system is completely described by its state vector in a complex vector space with a inner product (a **Hilbert Space** that is). The state vector is a unit vector in that space.

Note:
This mathematical representation of a qubit allows us to consider its abstract properties independent of its actual physical realization.

2.2.3 Superposition States of a Qubit

A **qu**antum **bit** can take values (quantum mechanical states) **|ψ>**

$$|0\rangle , \; |1\rangle$$

or both of them at the same time in which case the qubit is in a **superposition of states**

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad \text{where} \quad \alpha, \beta \in \mathbb{C}$$

- when the state of a qubit is measured one will find

$$|0\rangle \text{ with probability } |\alpha|^2 = \alpha \alpha^*$$
$$|1\rangle \quad \text{"} \quad |\beta|^2 = \beta \beta^*$$

- where the normalization condition is $\langle\psi|\psi\rangle = |\alpha|^2 + |\beta|^2 = 1$

$$\text{with } \langle\psi| = |\psi\rangle^\dagger = \alpha^* \langle 0| + \beta^* \langle 1| = (\alpha^*, \beta^*)$$

This just means that the sum over the probabilities of finding the qubit in any state must be unity.

Example: $$|\psi\rangle = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$ equal superposition state

2.2.4 Bloch sphere representation of qubit state space

alternative representation of qubit state vector useful for interpretation of qubit dynamics

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$= e^{i\gamma}\left[\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right]$$

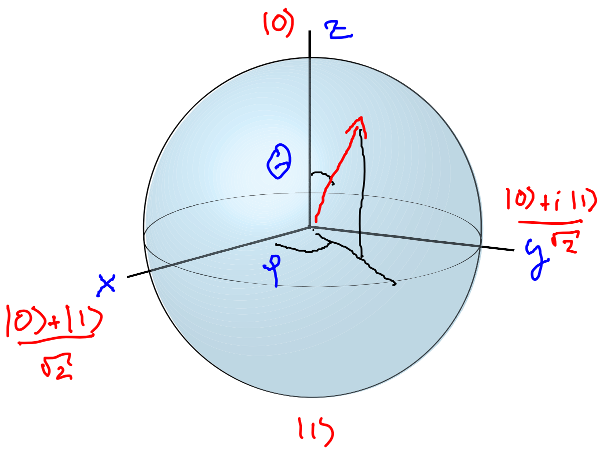
γ global phase factor

θ polar angle

φ azimuth angle

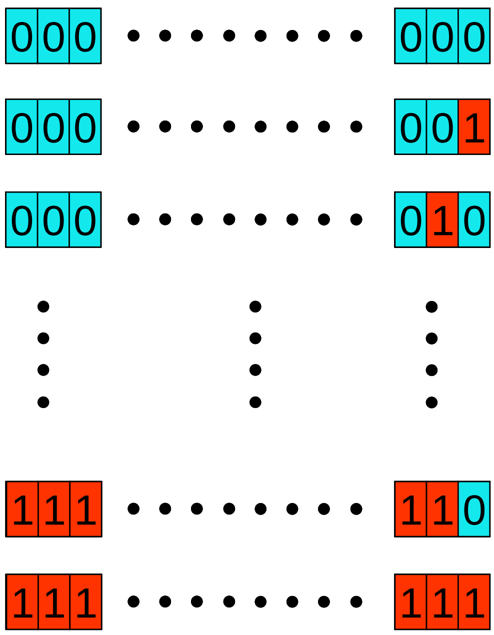
unit vector pointing at the surface of a sphere:

$$\vec{v} = (\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta)$$



- ground state **|0>** corresponds to a vector pointing to the north pole
- excited state **|1>** corresponds to a vector pointing to the south pole
- equal superposition state **$(|0\rangle + e^{i\phi}|1\rangle)/2^{1/2}$** is a vector pointing to the equator

2.2.5 A register of N quantum bits



classical register:

- has 2^N possible configurations
- but can store only 1 number

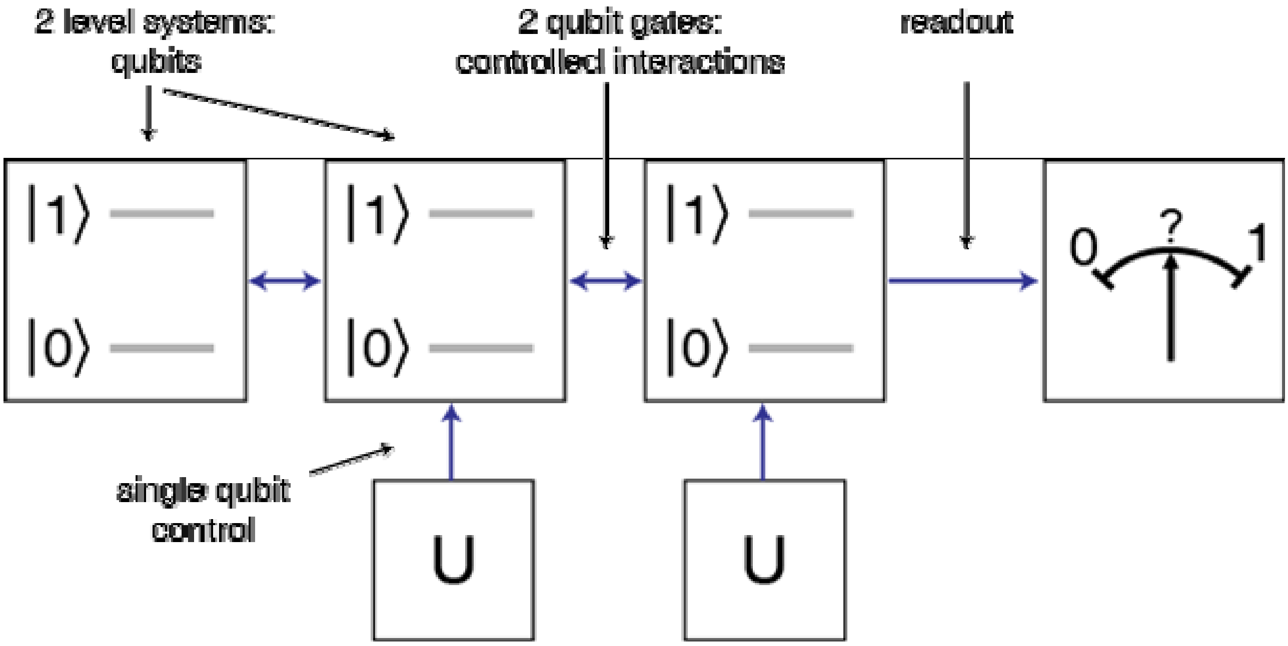
quantum register:

- has 2^N possible basis states
- can store superpositions of all numbers simultaneously

Goal: Try to process superposition of numbers simultaneously in a quantum computer.

- But what is needed to construct a quantum computer and how would it be operated?

2.3 Basic Components of a Generic Quantum Processor



2.3.1 The 5 DiVincenzo Criteria for Implementation of a Quantum Computer:

#1. A scalable physical system with well-characterized qubits.

#2. The ability to initialize the state of the qubits to a simple fiducial state.

#3. Long (relative) decoherence times, much longer than the gate-operation time.

#4. A universal set of quantum gates.

#5. A qubit-specific measurement capability.

in the standard (circuit approach) to **quantum information processing** (QIP)

plus two criteria requiring the possibility to transmit information:

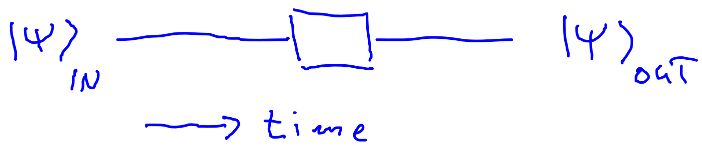
#6. The ability to interconvert stationary and mobile (or flying) qubits.

#7. The ability to faithfully transmit flying qubits between specified locations.

DiVincenzo, D., Quantum Computation, *Science* **270**, 255 (1995)

2.4 Single Qubit Logic Gates

2.4.1 Quantum circuits for single qubit gate operations



operations on single qubits:

X	bit flip	$|0\rangle \to |1\rangle$; $|1\rangle \to |0\rangle$
Y	bit flip*	$|0\rangle \to -i|1\rangle$; $|1\rangle \to i|0\rangle$
Z	phase flip	$|0\rangle \to |0\rangle$; $|1\rangle \to -|1\rangle$
I	identity	$|0\rangle \to |0\rangle$; $|1\rangle \to |1\rangle$

any single qubit operation can be represented as a rotation on a Bloch sphere

2.4.2 Pauli matrices

The action of the single qubit gates discussed before can be represented by Pauli matrices acting on the computational basis states:

bit flip (NOT gate) $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; $X|0\rangle = |1\rangle$; $X|1\rangle = |0\rangle$

bit flip*(with extra phase) $Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$; $Y|0\rangle = i|1\rangle$; $Y|1\rangle = -i|0\rangle$

phase flip $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$; $Z|0\rangle = |0\rangle$; $Z|1\rangle = -|1\rangle$

identity $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; $I|0\rangle = |0\rangle$; $I|1\rangle = |1\rangle$

all are unitary: $U = X, Y, Z, I$: $U^\dagger U = I$

exercise: calculate eigenvalues and eigenvectors of all Pauli matrices and represent them on the Bloch sphere

2.4.3 The Hadamard gate

a single qubit operation generating superposition states from the qubit computational basis states

$$|0\rangle \;—\boxed{H}—\; \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|1\rangle \;—\boxed{H}—\; \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

matrix representation of Hadamard gate:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}}(X + Z) \quad ; \quad H^{\dagger}H = I$$

exercise: write down the action of the Hadamard gate on the computational basis states of a qubit.

2.5 Dynamics of Quantum Systems

2.5.1 The Schrödinger equation

QM postulate: The time evolution of a state $|\psi\rangle$ of a closed quantum system is described by the **Schrödinger equation**

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$

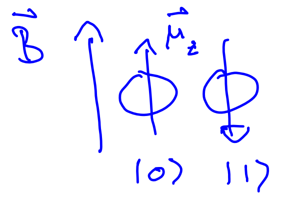
where H is the hermitian operator known as the **Hamiltonian** describing the closed system.

Reminder: A **closed quantum system** is one which does not interact with any other system.

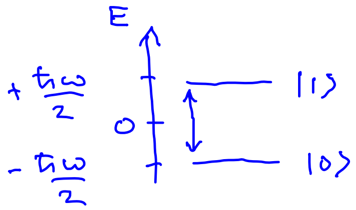
general solution for a time independent Hamiltonian H:

$$|\psi(t)\rangle = \exp\left[\frac{-iHt}{\hbar}\right]|\psi(0)\rangle$$

example: e.g. electron spin in a field



energy level diagram:



Hamiltonian for spin 1/2 in a magnetic field: $H = -\frac{\hbar\omega}{2} Z$

$$H = -\frac{\hbar\omega}{2}\left(|0\rangle\langle 0| - |1\rangle\langle 1|\right)$$

$$|\psi(0)\rangle = |0\rangle \longrightarrow |\psi(t)\rangle = e^{\frac{i\omega}{2}t}|0\rangle$$

$$|\psi(0)\rangle = |1\rangle \longrightarrow |\psi(t)\rangle = e^{-\frac{i\omega}{2}t}|1\rangle$$

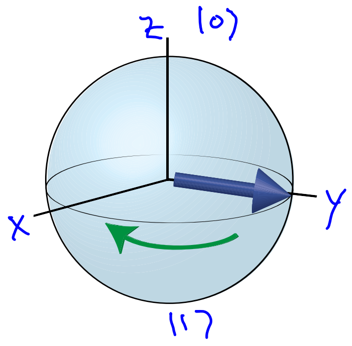
$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$= \frac{1}{\sqrt{2}} e^{i\frac{\omega}{2}t}\left(|0\rangle + e^{-i\omega t}|1\rangle\right)$$

$$|\psi\rangle = e^{i\gamma}\left(\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right)$$

$$\Longrightarrow \quad \theta = \frac{\pi}{2}, \quad \varphi = -\omega t$$

interpretation of dynamics on the Bloch sphere:



this is a rotation around the equator of the Bloch sphere with **Larmor precession frequency** ω

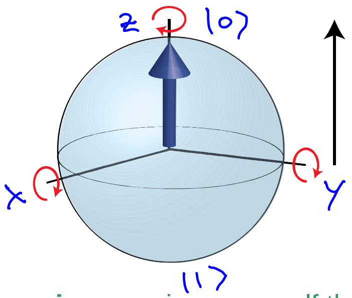
2.5.2 Rotation of qubit state vectors and rotation operators

when exponentiated the Pauli matrices give rise to rotation matrices around the three orthogonal axis in 3-dimensional space.

$$R_x(\theta) = e^{-i\theta X/2} = \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2} X = \begin{pmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

$$R_y(\theta) = e^{-i\theta Y/2} = \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2} Y = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

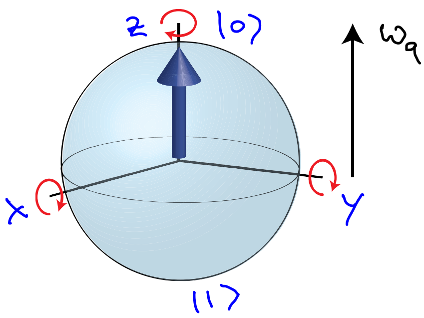
$$R_z(\theta) = e^{-i\theta Z/2} = \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2} Z = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$



If the Pauli matrices **X**, **Y** or **Z** are present in the Hamiltonian of a system they will give rise to rotations of the qubit state vector around the respective axis.

exercise: convince yourself that the operators $\mathbf{R}_{x,y,z}$ do perform rotations on the qubit state written in the Bloch sphere representation.

2.5.3 Preparation of specific qubit states



initial state **|0>:**

prepare excited state by rotating around **x** or **y** axis:

$\mathbf{X}_\pi$ pulse: $\Omega_x t = \pi$; $|0\rangle$ —$[X_\pi]$— $|1\rangle$

$\mathbf{Y}_\pi$ pulse: $\Omega_y t = \pi$; $|0\rangle$ —$[Y_\pi]$— $-i|1\rangle$

preparation of a superposition state:

$\mathbf{X}_{\pi/2}$ pulse: $\Omega_x t = \frac{\pi}{2}$ $\quad |0\rangle$ —$[X_{\pi/2}]$— $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$

$\mathbf{Y}_{\pi/2}$ pulse: $\Omega_y t = \frac{\pi}{2}$ $\quad |0\rangle$ —$[Y_{\pi/2}]$— $\frac{|0\rangle + i|1\rangle}{\sqrt{2}}$

in fact such a pulse of chosen length and phase can prepare any single qubit state, i.e. any point on the Bloch sphere can be reached

2.6 Quantum Measurement

Quantum measurement is done by having a closed quantum system interact in a controlled way with an external system from which the state of the quantum system under measurement can be recovered.

- example to be discussed: dispersive measurement in cavity QED

2.6.1 The quantum measurement postulate

QM postulate: **quantum measurement** is described by a set of operators **$\{M_m\}$** acting on the state space of the system. The **probability p of a measurement result m** occurring when the state ψ is measured is

$$p(m) = \langle \psi | M_m^\dagger M_m | \psi \rangle$$

the **state of the system after the measurement** is

$$|\psi'\rangle = \frac{M_m |\psi\rangle}{\sqrt{p(m)}}$$

completeness: the sum over all measurement outcomes has to be unity

$$1 = \sum_m p(m) = \sum_m \langle \psi | M_m^\dagger M_m | \psi \rangle$$

2.6.2 Example: projective measurement of a qubit in state ψ in its computational basis

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

measurement operators:

$$M_0 = |0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} ; \quad M_1 = |1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

measurement probabilities:

$$p(0) = \langle \psi | M_0^\dagger M_0 | \psi \rangle = \alpha^* \alpha \langle 0 | 0 \rangle = |\alpha|^2$$

$$p(1) = \langle \psi | M_1^\dagger M_1 | \psi \rangle = \beta^* \beta \langle 1 | 1 \rangle = |\beta|^2$$

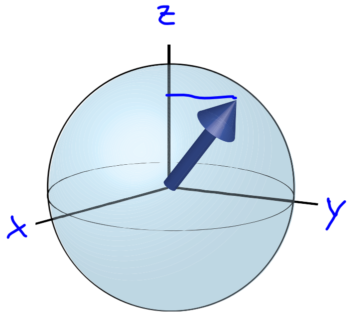
state after measurement:

$$\frac{M_0 |\psi\rangle}{\sqrt{p(0)}} = \frac{\alpha |0\rangle}{\sqrt{|\alpha|^2}} = \frac{\alpha}{|\alpha|} |0\rangle$$

$$\frac{M_1 |\psi\rangle}{\sqrt{p(1)}} = \frac{\beta |1\rangle}{\sqrt{|\beta|^2}} = \frac{\alpha}{|\alpha|} |1\rangle$$

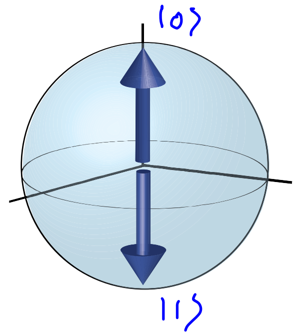
measuring the state again after a first measurement yields the same state as the initial measurement with unit probability

2.6.3 Interpretation of the Action of a Projective Measurement



One way to determine the state of a qubit is to measure the projection of its state vector along a given axis, say the z-axis.

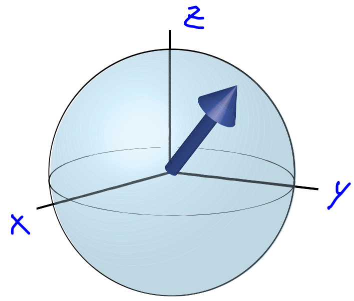
On the Bloch sphere this corresponds to the following operation:



After a projective measurement is completed the qubit will be in either one of its computational basis states.

In a repeated measurement the projected state will be measured with certainty.

Information content in a single qubit state



- infinite number of qubit states
- but single measurement reveals only 0 or 1 with probabilities $|\alpha|^2$ or $|\beta|^2$
- measurement will collapse state vector on basis state
- to determine α and β an infinite number of measurements has to be made

But if not measured the qubit contains 'hidden' information about α and β.

2.7 Multiple Qubits

2.7.1 Two Qubits

2 classical bits with states:

bit 1	bit 2
0	0
0	1
1	0
1	1

- 2^n different states (here n=2)
- but only one is realized at any given time

2 qubits with quantum states:

qubit 1 qubit 2

$|00\rangle$
$|01\rangle$
$|10\rangle$
$|11\rangle$

- 2^n basis states (n=2)
- can be realized simultaneously
- quantum parallelism

2^n complex coefficients describe quantum state

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

normalization condition

$$\sum_{ij} |\alpha_{ij}|^2 = 1$$

2.7.2 Composite quantum systems

QM postulate: The state space of a composite systems is the tensor product of the state spaces of the component physical systems. If the component systems have states ψ_i the composite system state is

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \dots \otimes |\psi_n\rangle$$

This is a product state of the individual systems.

example:

$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$
$$|\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

$$\rightarrow \quad |\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle = |\psi_1 \psi_2\rangle$$
$$= \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

exercise: Write down the state vector (matrix representation) of two qubits, i.e. the tensor product, in the computational basis. Write down the basis vectors of the composite system.

there is no generalization of Bloch sphere picture to many qubits

2.7.3 Information content in multiple qubits

- 2^n complex coefficients describe the state of a composite quantum system with n qubits
- Imagine to have 500 qubits, then 2^{500} complex coefficients describe their state.
- How to store this state?
    - 2^{500} is larger than the number of atoms in the universe.
    - It is impossible in classical bits.
    - This is also why it is hard to simulate quantum systems on classical computers.
- A quantum computer would be much more efficient than a classical computer at simulating quantum systems.
- Make use of the information that can be stored in qubits for quantum information processing!

2.7.4 Entanglement

Definition: An **entangled state** of a composite system is a state that cannot be written as a product state of the component systems.

example: an entangled 2-qubit state (one of the Bell states)

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

What is special about this state? Try to write it as a product state!

$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle \;;\; |\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

$$|\psi\rangle \overset{!}{=} |\psi_1\psi_2\rangle \Rightarrow \alpha_1\alpha_2 = \frac{1}{\sqrt{2}} \wedge \beta_1\beta_2 = \frac{1}{\sqrt{2}} \Rightarrow \alpha_1\beta_2 \neq 0 \wedge \alpha_2\beta_1 \neq 0 \,!$$

It is not possible! This state is special, it is entangled!

Use this property as a resource in quantum information processing:
- super dense coding
- teleportation
- error correction

2.7.5 Measurement of a single qubit in an entangled state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

measurement of first qubit:

$$p_1(0) = \langle\psi|(M_0\otimes I)^\dagger(M_0\otimes I)|\psi\rangle = \frac{1}{\sqrt{2}}\langle 00|\frac{1}{\sqrt{2}}|00\rangle = \frac{1}{2}$$

post measurement state:

$$|\psi'\rangle = \frac{(M_0\otimes I)|\psi\rangle}{\sqrt{p_1(0)}} = \frac{\frac{1}{\sqrt{2}}|00\rangle}{\frac{1}{\sqrt{2}}} = |00\rangle$$

measurement of qubit two will then result with certainty in the same result:

$$p_2(0) = \langle\psi'|(I\otimes M_0)^\dagger(I\otimes M_0)|\psi'\rangle = 1$$

The two measurement results are **correlated**!

- Correlations in quantum systems can be stronger than correlations in classical systems.
- This can be generally proven using the **Bell inequalities** which will be discussed later.
- Make use of such correlations as a **resource** for information processing
  - super dense coding, teleportation, error corrections

2.7.6 Super Dense Coding

task: Try to transmit two bits of classical information between Alice (A) and Bob (B) using only one qubit.

- As Alice and Bob are living in a quantum world they are allowed to use one pair of entangled qubits that they have prepared ahead of time.

protocol:
A) Alice and Bob each have one qubit of an entangled pair in their possession

$$|\psi\rangle = 1/\sqrt{2}\,(|00\rangle + |11\rangle)$$

B) Alice does a quantum operation on her qubit depending on which 2 classical bits she wants to communicate
C) Alice sends her qubit to Bob
D) Bob does one measurement on the entangled pair

shared entanglement $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

local operations $X_1,\ iY_1,\ Z_1,\ I_1$

send Alices qubit to Bob

Bob measures

bits to be transferred:	Alice's operation	resulting 2-qubit state	Bob's measurement
00	I_1	$I_1\|\psi\rangle = \frac{1}{\sqrt{2}}(\|00\rangle + \|11\rangle)$	measure in Bell basis
01	Z_1	$Z_1\|\psi\rangle = \frac{1}{\sqrt{2}}(\|00\rangle - \|11\rangle)$	
10	X_1	$X_1\|\psi\rangle = \frac{1}{\sqrt{2}}(\|10\rangle + \|01\rangle)$	
11	iY_1	$iY_1\|\psi\rangle = \frac{1}{\sqrt{2}}(\|10\rangle - \|01\rangle)$	

- all these states are entangled (try!)
- they are called the Bell states

comments:
- two qubits are involved in protocol BUT Alice only interacts with one and sends only one along her quantum communications channel
- two bits cannot be communicated sending a single classical bit along a classical communications channel

original proposal of super dense coding: Charles H. Bennett and Stephen J. Wiesner, Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states, **Phys. Rev. Lett. 69, 2881(1992)**

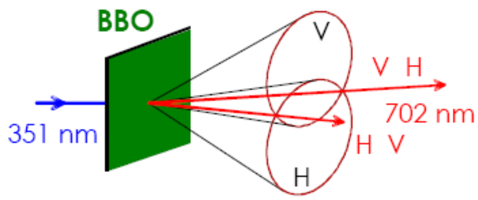
2.7.7 Experimental demonstration of super dense coding using photons

Generating polarization entangled photon pairs using **Parametric Down Conversion**:

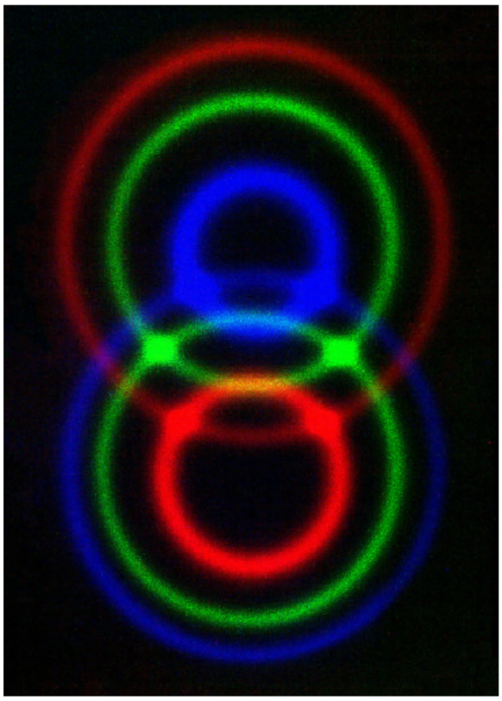
parametric down-conversion

- 1 UV-photon ➔ 2 "red" photons
- conservation of
  - energy $\omega_p = \omega_s + \omega_i$
  - momentum $\vec{k}_p = \vec{k}_s + \vec{k}_i$
- Polarisationskorrelationen (typ II)

optically nonlinear medium: BBO (BaB_2O_4)
beta barium borate

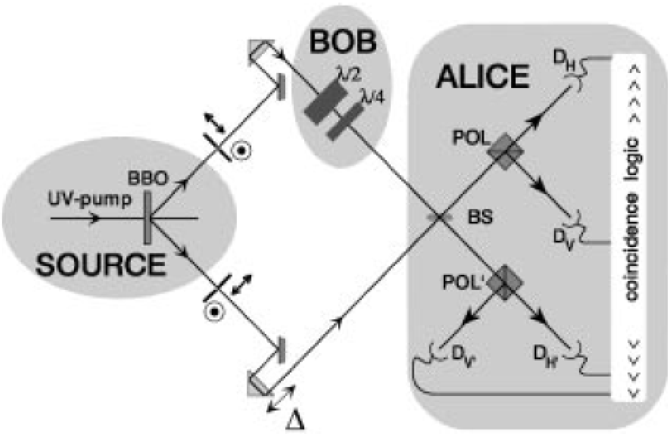


$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle|V\rangle - |V\rangle|H\rangle\right)$$



state manipulation

Bell state measurement



$$\Psi^- = \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right)$$ asym.

$$\Psi^+ = \frac{1}{\sqrt{2}}\left(|HV\rangle + |VH\rangle\right)$$

$$\phi^+ = \frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right)$$ sym.

$$\phi^- = \frac{1}{\sqrt{2}}\left(|HH\rangle - |VV\rangle\right)$$

H = horizontal polarization
V = vertical polarization

Klaus Mattle, Harald Weinfurter, Paul G. Kwiat, and Anton Zeilinger, Dense coding in experimental quantum communication, Phys. Rev. Lett.76, 4656 (1996)

2.8 Two Qubit Quantum Logic Gates

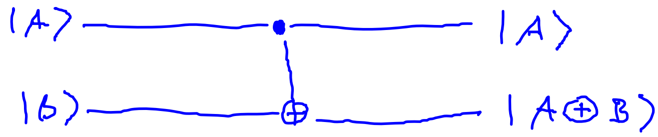
2.8.1 The controlled NOT gate (CNOT)

function:

$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |01\rangle$$
$$|10\rangle \rightarrow |11\rangle$$
$$|11\rangle \rightarrow |10\rangle$$

$$|A,B\rangle \rightarrow |A, A \oplus B\rangle$$ addition mod 2 of basis states

CNOT circuit:



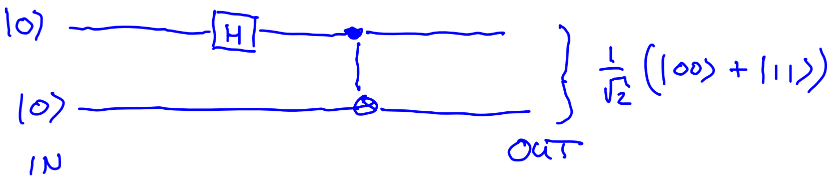
control qubit

target qubit

comparison with classical gates:
- XOR is not reversible
- CNOT is reversible (unitary)

Universality of controlled NOT:
Any multi qubit logic gate can be composed of CNOT gates and single qubit gates X,Y,Z.

2.8.2 **Application of CNOT: generation of entangled states (Bell states)**



$$|00\rangle \xrightarrow{H_1} \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|01\rangle \xrightarrow{H_1} \frac{1}{\sqrt{2}}(|01\rangle + |11\rangle) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|10\rangle \xrightarrow{H_1} \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|11\rangle \xrightarrow{H_1} \frac{1}{\sqrt{2}}(|01\rangle - |11\rangle) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

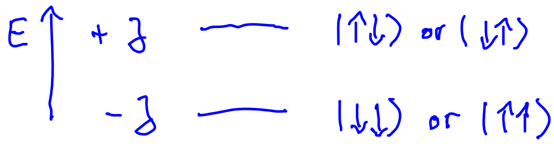
exercise: Write down the unitary matrix representations of the CNOT in the computational basis with qubit 1 being the control qubit. Write down the matrix in the same basis with qubit 2 being the control bit.

2.8.3 Implementation of CNOT using the Ising interaction

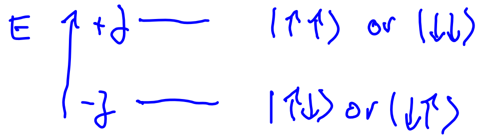
Ising interaction: $H = -\sum_{ij} J_{ij} \hat{z}_i \hat{z}_j$ pair wise spin interaction

generic two-qubit interaction: $H = -J \hat{z}_1 \hat{z}_2$

J > 0: ferromagnetic coupling



J < 0: anti-ferrom. coupling



2-qubit unitary evolution: $C(\gamma) = e^{-i \frac{\gamma}{2} \hat{z}_1 \hat{z}_2}$

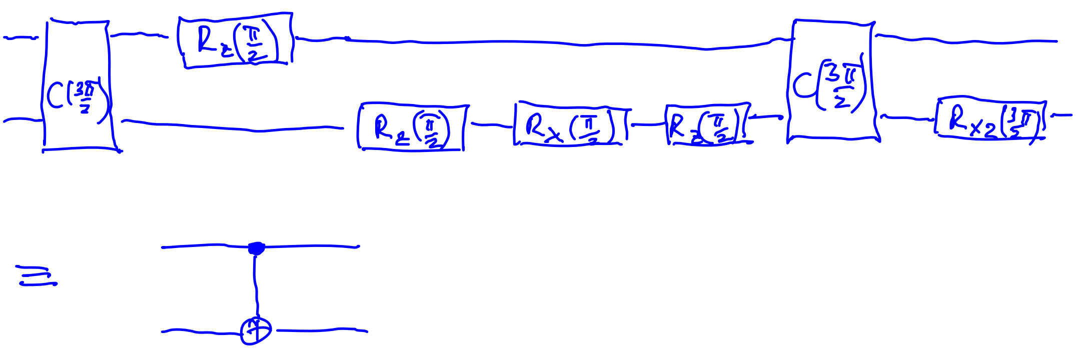
BUT this does not realize a CNOT gate yet. Additionally, single qubit operations on each of the qubits are required to realize a CNOT gate.

CNOT realization with the Ising-type interaction

CNOT - unitary:

$$C_{NOT} = e^{-i\frac{3\pi}{4}} R_{x2}\left(\frac{3\pi}{2}\right) C\left(\frac{3\pi}{2}\right) R_{z2}\left(\frac{\pi}{2}\right) R_{x2}\left(\frac{\pi}{2}\right) R_{z2}\left(\frac{\pi}{2}\right) R_{z1}\left(\frac{\pi}{2}\right) C\left(\frac{3\pi}{2}\right)$$

circuit representation:



Any physical two-qubit interaction that can produce entanglement can be turned into a universal two-qubit gate (such as the CNOT gate) when it is augmented by arbitrary single qubit operations.

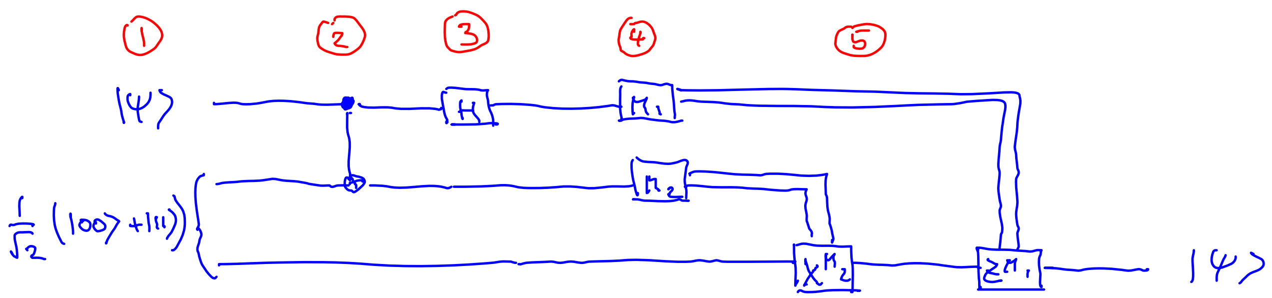
Bremner et al., *Phys. Rev. Lett.* **89**, 247902 (2002)

2.9 Quantum Teleportation

Task: Alice wants to transfer an unknown quantum state ψ to Bob only using **one entangled pair** of qubits and **classical information** as a resource.

note:
- Alice does not know the state to be transmitted
- Even if she knew it the classical amount of information that she would need to send would be infinite.

The **teleportation circuit**:



original article: Bennett, C. H. et al., Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels, *Phys. Rev. Lett.* **70**, 1895-1899 (1993)

2.9.1 How does it work?

① $$|\psi\rangle \otimes \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\right)$$

CNOT between qubit to be teleported and one bit of the entangled pair:

② $$\xrightarrow{CNOT_{12}} \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle\right)$$

Hadamard on qubit to be teleported:

③ $$\xrightarrow{H_1} \frac{1}{2}\Big[\left(|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle)\right.$$
$$\left. + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)\right)\Big]$$

measurement of qubit 1 and 2, classical information transfer and single bit manipulation on target qubit 3:

④ $\xrightarrow{M_1 \otimes M_2}$ ⑤

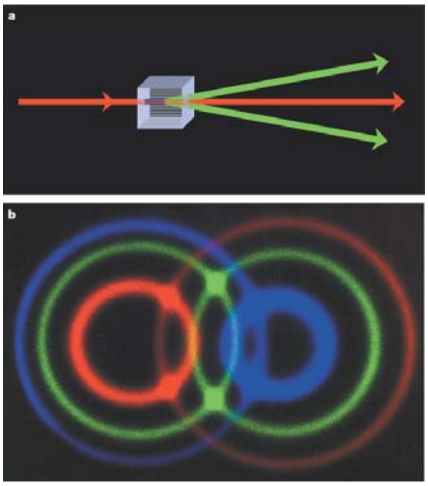
$$P_{00} = \frac{1}{4}\ ; \quad |\psi_3\rangle = \alpha|0\rangle + \beta|1\rangle \xrightarrow{I} |\psi\rangle$$

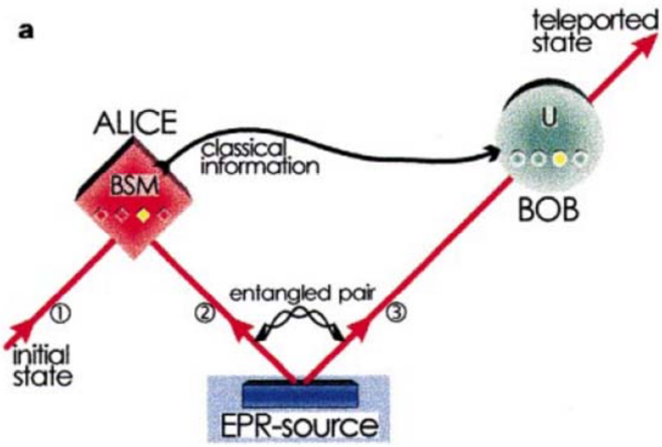
$$P_{10} = \frac{1}{4}\ ; \quad |\psi_3\rangle = \alpha|0\rangle - \beta|1\rangle \xrightarrow{Z} |\psi\rangle$$

$$P_{01} = \frac{1}{4}\ ; \quad |\psi_3\rangle = \alpha|1\rangle + \beta|0\rangle \xrightarrow{X} |\psi\rangle$$

$$P_{11} = \frac{1}{4}\ ; \quad |\psi_3\rangle = \alpha|1\rangle - \beta|0\rangle \xrightarrow{XZ} |\psi\rangle$$

2.9.2 (One) Experimental Realization of Teleportation using Photon Polarization:





- parametric down conversion (PDC) source of entangled photons
- qubits are polarization encoded

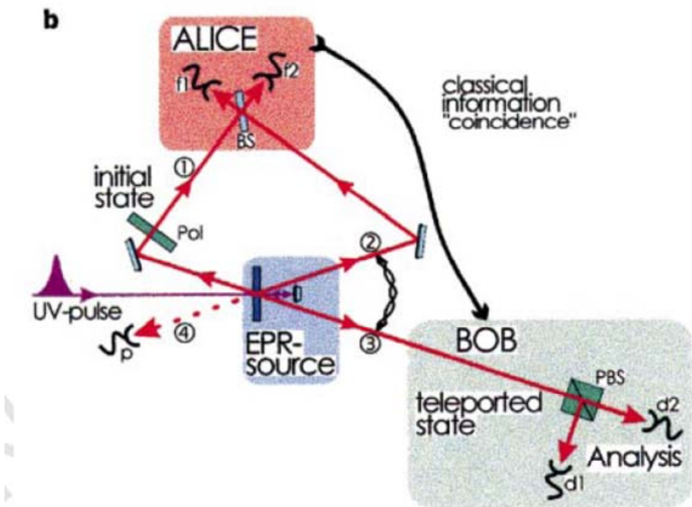
Dik Bouwmeester, Jian-Wei Pan, Klaus Mattle, Manfred Eibl, Harald Weinfurter, Anton Zeilinger, Experimental quantum teleportation *Nature* **390**, 575 (1997)

Experimental Implementation

start with states

$$|\psi_1\rangle = \alpha\,|H\rangle + \beta\,|V\rangle$$

$$|\psi_{23}\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right)$$



combine photon to be teleported (1) and one photon of entangled pair (2) on a 50/50 beam splitter (BS) and measure (at Alice) resulting state in Bell basis.

analyze resulting teleported state of photon (3) using polarizing beam splitters (PBS) single photon detectors

- polarizing beam splitters (PBS) as detectors of teleported states teleportation papers for you to present:

Experimental Realization of Teleporting an Unknown Pure Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels
D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu
Phys. Rev. Lett. **80**, 1121 (1998) [PROLA Link]

Unconditional Quantum Teleportation
A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik
Science 23 October 1998 282: 706-709 [DOI: 10.1126/science.282.5389.706] (in Research Articles)
Abstract » Full Text » PDF »

Complete quantum teleportation using nuclear magnetic resonance
M. A. Nielsen, E. Knill, R. Laflamme
Nature 396, 52 - 55 (05 Nov 1998) Letters to Editor
Abstract | Full Text | PDF | Rights and permissions | Save this link

Deterministic quantum teleportation of atomic qubits
M. D. Barrett, J. Chiaverini, T. Schaetz, J. Britton, W. M. Itano, J. D. Jost, E. Knill, C. Langer, D. Leibfried, R. Ozeri, D. J. Wineland
Nature 429, 737 - 739 (17 Jun 2004) Letters to Editor
Abstract | Full Text | PDF | Rights and permissions | Save this link

Deterministic quantum teleportation with atoms
M. Riebe, H. Haeffner, C. F. Roos, W. Haensel, J. Benhelm, G. P. T. Lancaster, T. W. Koerber, C. Becher, F. Schmidt-Kaler, D. F. V. James, R. Blatt
Nature 429, 734 - 737 (17 Jun 2004) Letters to Editor
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Quantum teleportation between light and matter
Jacob F. Sherson, Hanna Krauter, Rasmus K. Olsson, Brian Julsgaard, Klemens Hammerer, Ignacio Cirac, Eugene S. Polzik
Nature 443, 557 - 560 (05 Oct 2006) Letters to Editor
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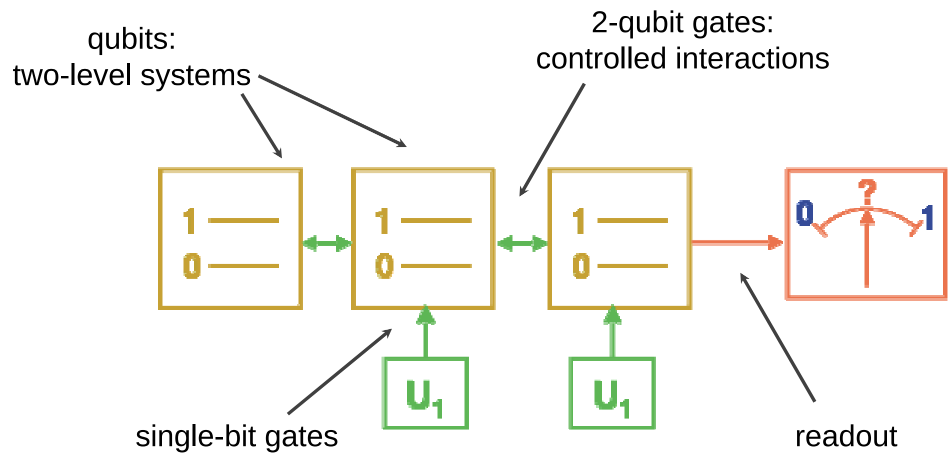
Generic Quantum Information Processor

The challenge:



- Quantum information processing requires excellent qubits, gates, ...
- Conflicting requirements: good isolation from environment while maintaining good addressability

M. Nielsen and I. Chuang,
Quantum Computation and Quantum Information (Cambridge, 2000)

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The 5 (+2) DiVincenzo Criteria for Implementation of a Quantum Computer:

in the standard (circuit approach) to **quantum information processing** (QIP)

#1. A scalable physical system with well-characterized qubits.

#2. The ability to initialize the state of the qubits to a simple fiducial state.

#3. Long (relative) decoherence times, much longer than the gate-operation time.

#4. A universal set of quantum gates.

#5. A qubit-specific measurement capability.

#6. The ability to interconvert stationary and mobile (or flying) qubits.

#7. The ability to faithfully transmit flying qubits between specified locations.

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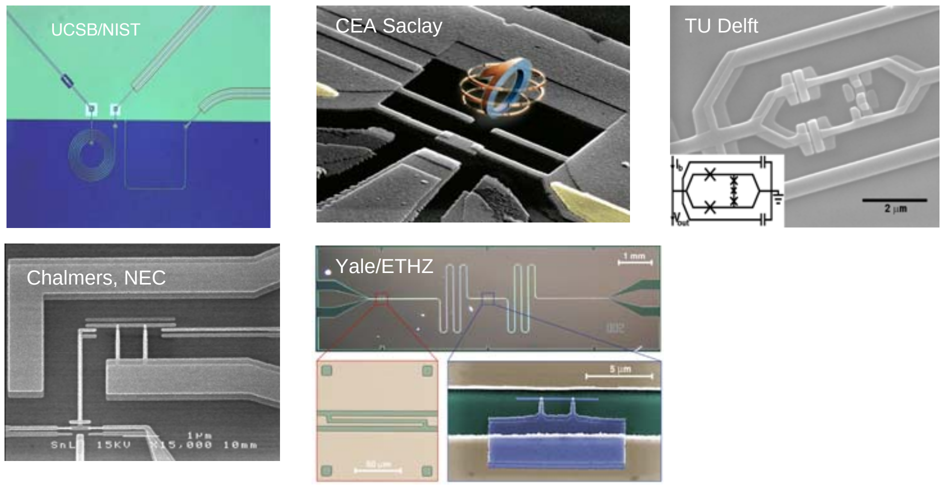
Quantum Information Processing with Superconducting Circuits



with material from
NIST, UCSB, Berkeley, NEC, NTT, CEA Saclay, Yale and ETHZ

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Outline

- realization of superconducting qubits
- harmonic oscillators
- the current biased phase qubit
- the charge qubit
- qubit read-out
- single qubit control
- decoherence
- two-qubit gates

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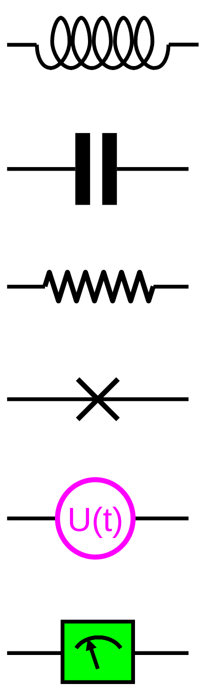
Some Basics ...

… on how to construct qubits
using superconducting circuit elements.

Building Quantum Electrical Circuits



- inductor
- capacitor
- resistor
- nonlinear element
- voltage source
- voltmeters

requirements for quantum circuits:

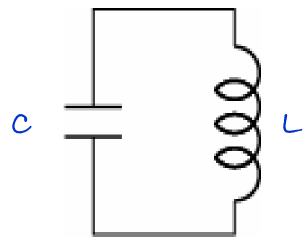
- low dissipation
- non-linear (non-dissipative elements)
- low (thermal) noise

a solution:

- use superconductors
- use Josephson tunnel junctions
- operate at low temperatures

Superconducting Harmonic Oscillator

a simple electronic circuit:

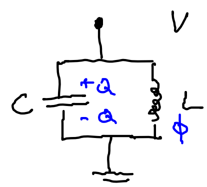


- typical inductor: $L = 1$ nH
- a wire in vacuum has inductance ~ 1 nH/mm
- typical capacitor: $C = 1$ pF
- a capacitor with plate size 10 μm x 10 μm and dielectric AlOx ($\varepsilon = 10$) of thickness 10 nm has a capacitance $C \sim 1$ pF
- resonance frequency

$$\frac{1}{2\pi\sqrt{LC}} \sim 5\,\mathrm{GHz}$$

Quantization of the electrical LC harmonic oscillator:

parallel LC oscillator circuit:



voltage across the oscillator:

$$V = \frac{Q}{C} = -L\frac{\partial I}{\partial t}$$

total energy (Hamiltonian):

$$H = \frac{1}{2} C V^2 + \frac{1}{2} L I^2 = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}\frac{\phi^2}{L}$$

with the charge Q stored on the capacitor $\quad Q = VC$

a flux ϕ stored in the inductor $\quad \phi = LI$

properties of Hamiltonian written in variables Q and ϕ:

$$\frac{\partial H}{\partial Q} = \frac{Q}{C} = -L\frac{\partial I}{\partial t} = -\dot{\phi}$$

$$\frac{\partial H}{\partial \phi} = \frac{\phi}{L} = I = \dot{Q}$$

Q and ϕ are canonical variables

see e.g.: Goldstein, Classical Mechanics, Chapter 8, Hamilton Equations of Motion

Quantum version of Hamiltonian

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

with commutation relation $\quad [\hat{\phi}, \hat{Q}] = i\hbar$

compare with particle in a harmonic potential:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega^2 \hat{x}^2$$

analogy with electrical oscillator:
- charge Q corresponds to momentum p
- flux ϕ corresponds to position x

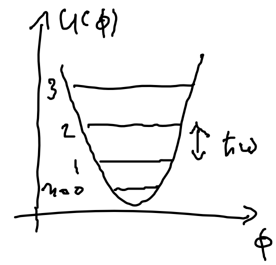
$$[\hat{x}, \hat{p}] = [\hat{x}, -i\hbar \frac{\partial}{\partial x}] = i\hbar$$

$$[\hat{\phi}, \hat{Q}] = [\hat{\phi}, -i\hbar \frac{\partial}{\partial \phi}] = i\hbar$$

Hamiltonian in terms of raising and lowering operators:

$$\hat{H} = \hbar\omega \left(a^\dagger a + \frac{1}{2}\right)$$

with oscillator resonance frequency: $\quad \omega = \frac{1}{\sqrt{LC}}$



Raising and lowering operators:

$$a^\dagger |n\rangle = \sqrt{n+1}\, |n+1\rangle \; ; \; \hat{a} |n\rangle = \sqrt{n}\, |n-1\rangle$$

$$a^\dagger a |n\rangle = n |n\rangle \qquad \text{number operator}$$

in terms of Q and ϕ:

$$\hat{a} = \frac{1}{\sqrt{2\hbar Z_c}} \left(Z_c \hat{Q} + i \hat{\phi}\right)$$

with Z_c being the characteristic impedance of the oscillator $\quad Z_c = \sqrt{\frac{L}{C}}$

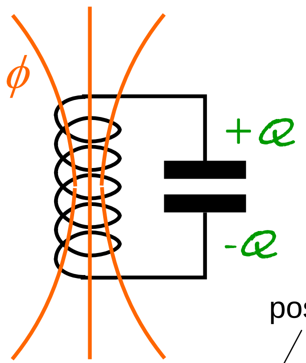
charge Q and flux ϕ operators can be expressed in terms of raising and lowering operators:

$$\hat{Q} = \sqrt{\frac{\hbar}{2 Z_c}} \left(a + a^\dagger\right)$$

$$\hat{\phi} = \sqrt{\frac{2 Z_c}{\hbar}} \left(a - a^\dagger\right)$$

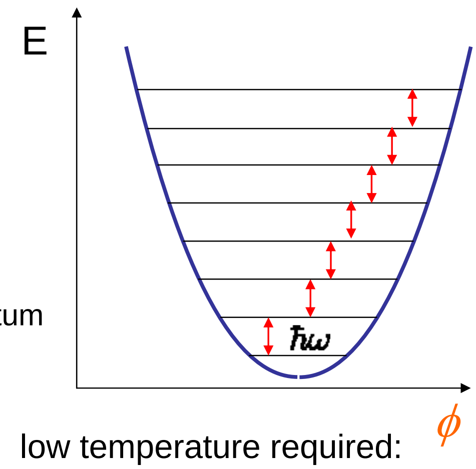
Exercise: Making use of the commutation relations for the charge and flux operators, show that the harmonic oscillator Hamiltonian in terms of the raising and lowering operators is identical to the one in terms of charge and flux operators.

Quantum LC Oscillator



$[\phi, q] = i\hbar$

position momentum



Hamiltonian $H = \frac{\phi^2}{2L} + \frac{q^2}{2C}$

$\omega = 1/\sqrt{LC}$

$H = \hbar\omega \left(a^\dagger a + \frac{1}{2} \right)$

low temperature required:

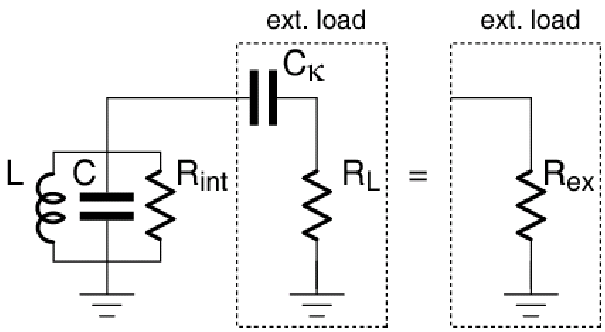
$\hbar\omega \gg k_B T$

10 GHz ~ 500 mK 20 mK

$\langle n_{th} \rangle = \frac{1}{\exp(h\nu / k_B T) - 1} \sim 10^{-11}$

problem 1: equally spaced energy levels (linearity)

Dissipation in an LC Oscillator



internal losses: R_{int}
conductor, dielectric

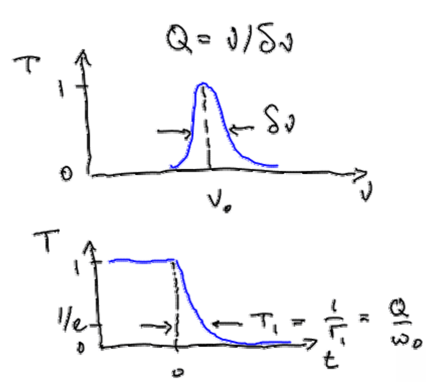
external losses: R_{ext}
radiation, coupling

total losses $\frac{1}{R} = \frac{1}{R_{\text{int}}} + \frac{1}{R_{\text{ext}}}$

impedance $Z = \sqrt{\frac{L}{C}}$

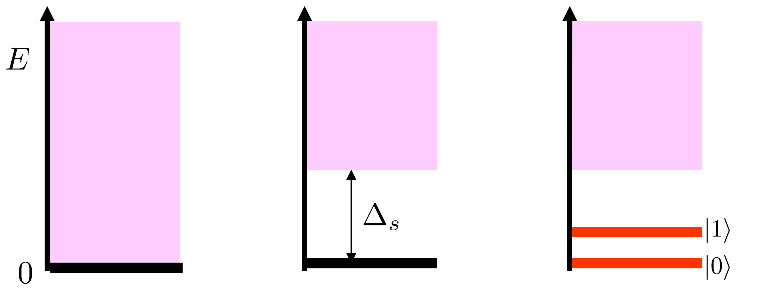
quality factor $Q = \frac{R}{Z} = \omega_0 RC$

excited state decay rate $\Gamma_1 = \frac{\omega_0}{Q} = \frac{1}{RC}$



problem 2: internal and external dissipation

Why Superconductors?



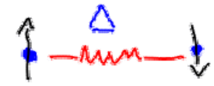
normal metal | superconductor | How to make qubit?

- single non-degenerate macroscopic ground state
- elimination of low-energy excitations

Superconducting materials (for electronics):

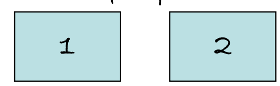
- Niobium (Nb): $2\Delta/h = 725$ GHz, $T_c = 9.2$ K
- Aluminum (Al): $2\Delta/h = 100$ GHz, $T_c = 1.2$ K

Cooper pairs:
bound electron pairs



are Bosons (S=0, L=0)

2 chunks of superconductors

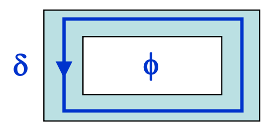


macroscopic wave function

$$\Psi_i = \sqrt{n_i^{CP}}\, e^{i\delta_i}$$

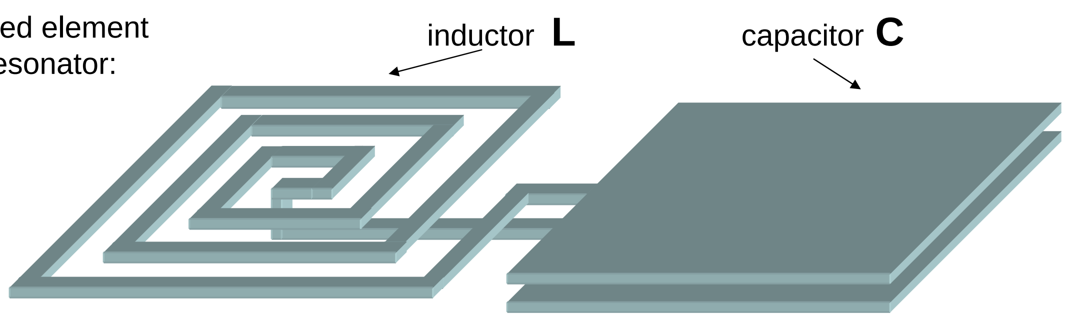
Cooper pair density n_i
and global phase δ_i

phase quantization: $\delta = n\, 2\pi$
flux quantization: $\phi = n\, \phi_0$

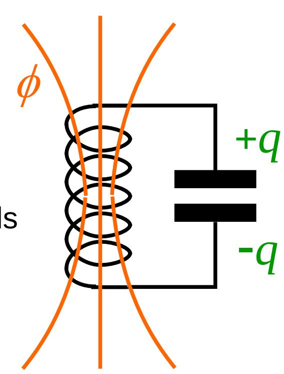


Can it be done?

lumped element
LC resonator:



a harmonic oscillator

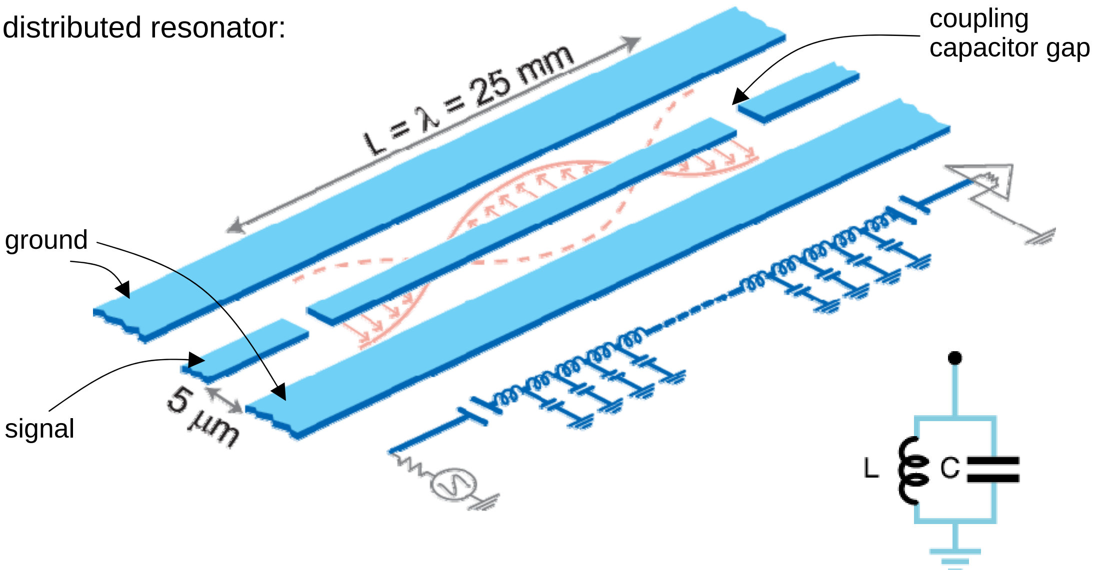


currents and magnetic fields

charges and electric fields

Transmission Line Resonator

distributed resonator:



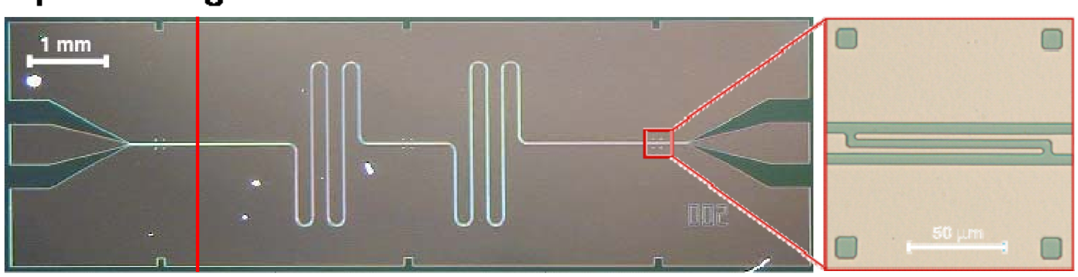
- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

M. Goppl *et al.*, Coplanar Waveguide Resonators for Circuit Quantum Electrodynamics, *arXiv:0807.4094v1* (2008)

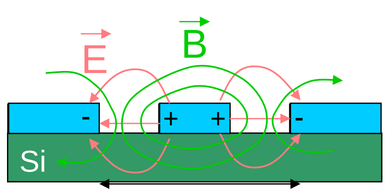
Transmission Line Resonator

$$H_r = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right)$$

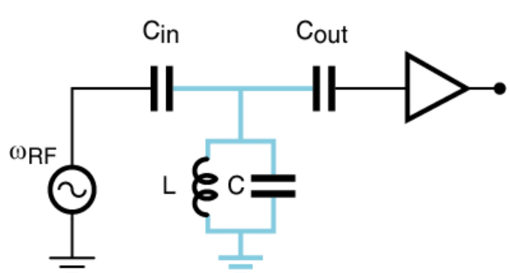
coplanar waveguide:



cross section:

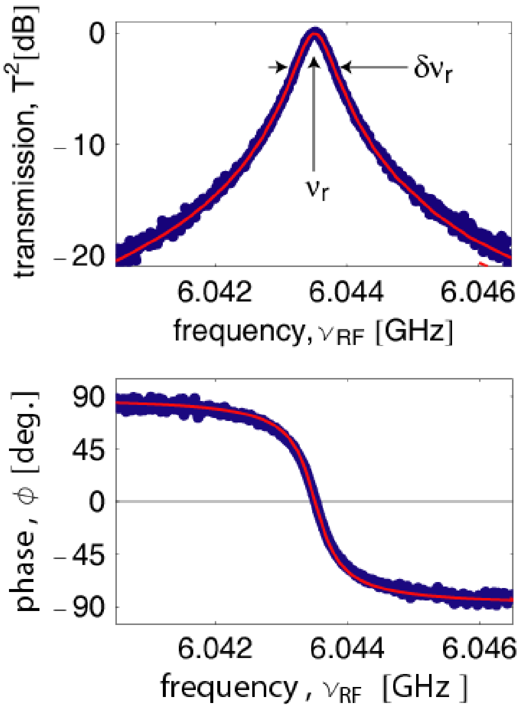


measuring the resonator:



photon lifetime (quality factor) controlled by coupling $C_{in/out}$

Resonator Quality Factor and Photon Lifetime



resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

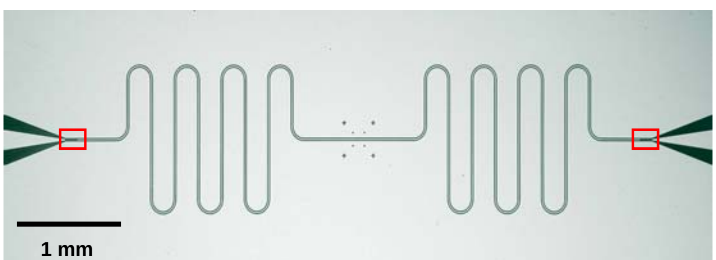
photon decay rate:

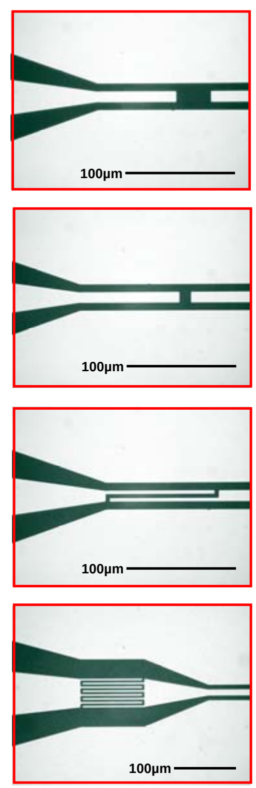
$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

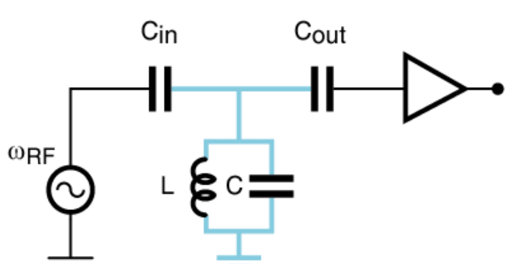
photon lifetime:

$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

Controlling the Photon Life Time

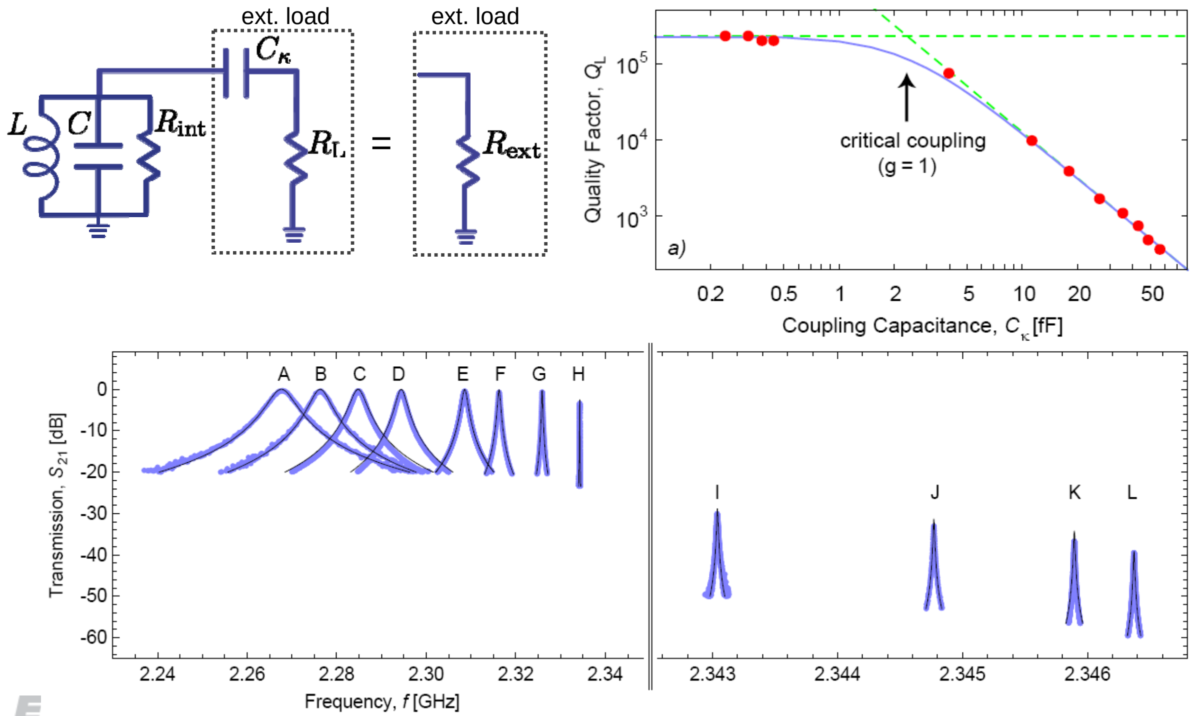






photon lifetime (quality factor)
controlled by coupling capacitor $C_{in/out}$

Coupling Dependent Quality Factor



M. Goppl, A. Fragner *et al. arXiv:0807.4094* (2008)

How to prove that the h.o. is quantum?

measure**:**

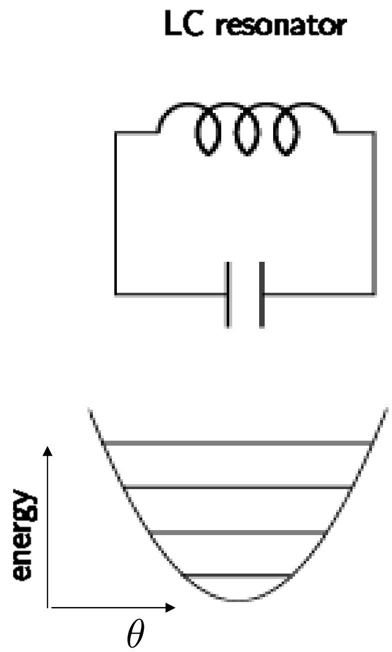
- resonance frequency
- average charge (momentum)
- average flux (position)

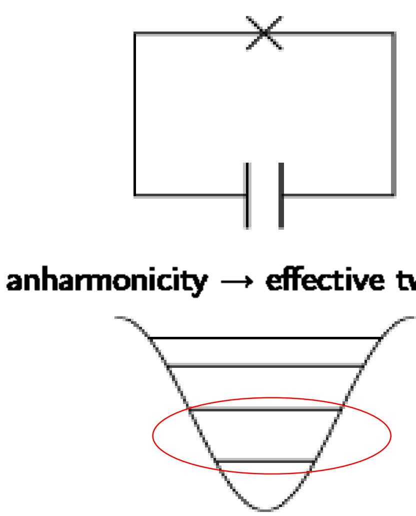
all averaged quantities are identical for a purely harmonic oscillator in the classical or quantum regime

solution:

- make oscillator non linear in a controllable way

Superconducting Nonlinear Oscillators

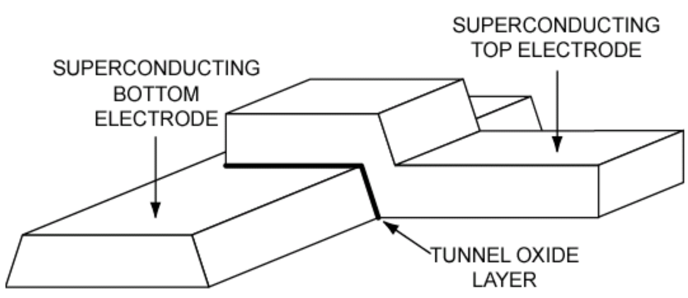




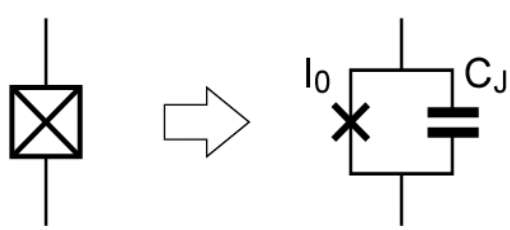
solution to problem 1

A Low-Loss Nonlinear Element

a (superconducting) Josephson junction



- superconductors: Nb, Al
- tunnel barrier: AlO_x

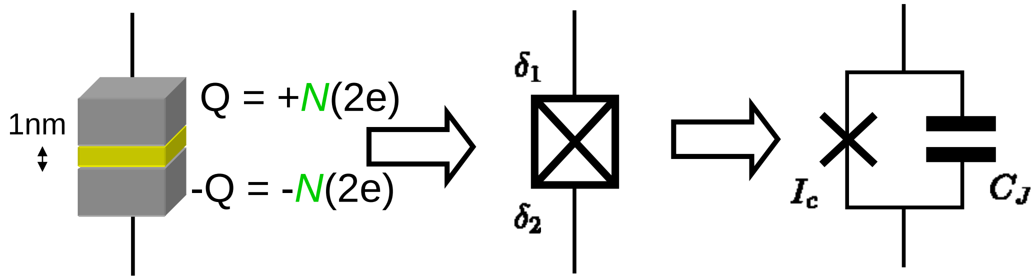


- critical current I_c
- junction capacitance C_J
- high internal resistance R_J

M. Tinkham, *Introduction to Superconductivity* (Krieger, Malabar, 1985).

Josephson Tunnel Junction

the only non-linear LC resonator with no dissipation (BCS, $k_B T \ll \Delta$)



tunnel junction parameters:

- critical current I_c
- junction capacitance C_J
- high internal resistance R_J

Josephson relations:

$$I_0 = I_c \sin\delta$$

$$V = \phi_0 \frac{\partial \delta}{\partial t}$$

flux quantum: $\phi_0 = \frac{h}{2e}$

phase difference: $\delta = \delta_2 - \delta_1$

derivation of Josephson effect, see e.g.: chap. 21 in R. A. Feynman: Quantum mechanics, The Feynman Lectures on Physics. Vol. 3 (Addison-Wesley, 1965)

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The Josephson junction as a non-linear inductor

induction law: $V = -L \frac{\partial I}{\partial t}$

Josephson effect: dc-Josephson equation $I = I_c \sin\delta$

$$\frac{\partial I}{\partial t} = I_c \cos\delta \frac{\partial \delta}{\partial t}$$

ac-Josephson equation $V = \frac{\phi_0}{2\pi}\frac{\partial\delta}{\partial t} = \underbrace{\frac{\phi_0}{2\pi I_c}\frac{1}{\cos\delta}}_{L_J}\frac{\partial I}{\partial t}$

Josephson inductance $L_J = \underbrace{\frac{\phi_0}{2\pi I_c}}\ \frac{1}{\cos\delta}$

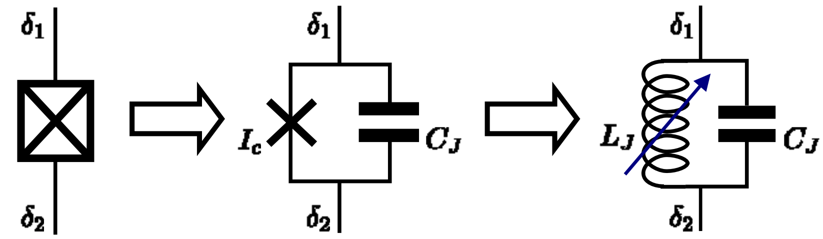
specific Josephson inductance; nonlinearity

A typical characteristic Josephson inductance for a tunnel junction with $I_c = 100$ nA is $L_{J0} \sim 3$ nH.

review: M. H. Devoret *et al.*, Quantum tunneling in condensed media, *North-Holland,* (1992)

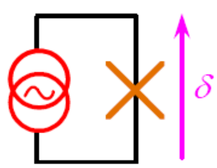
A Non-Linear Tunable Inductor w/o Dissipation

the Josephson junction as a circuit element:

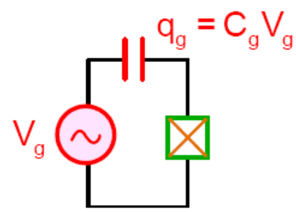


How to Make Use of the Josephson Junction in Qubits?

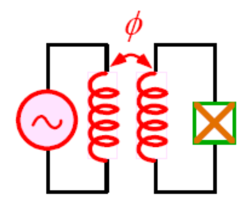
different bias (control) circuits:



current bias



charge bias

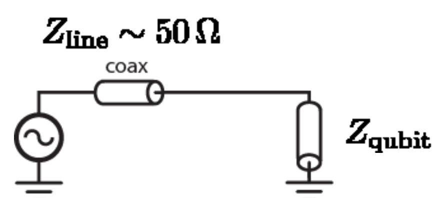


flux bias

Coupling to the Electromagnetic Environment

strong coupling to environment (bias wires):

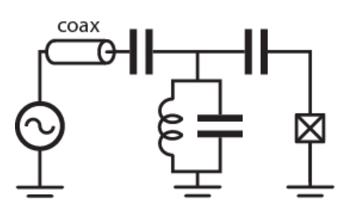
decoherence
from energy relaxation



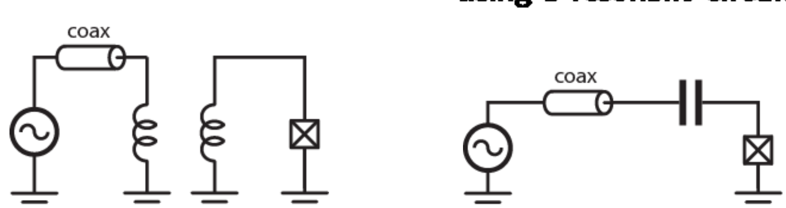
decoupling using impedance transformers:

control decoherence
by circuit design

solution to problem 2

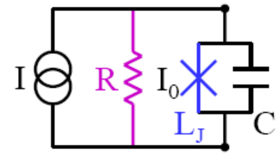


using a resonant circuit



using non-resonant impedance transformers

Current Biased Phase Qubit



The bias current I distributes into a Josephson current through an ideal Josephson junction with critical current I_c, through a resistor R and into a displacement current over the capacitor C.

Kirchhoff's law:

$$I_b = I_s + I_R + I_C$$
$$= I_c \sin\delta + \frac{V}{R} + C\dot{V}$$

$$I_C = \dot{Q}_C = C\dot{V}$$
$$I_R = V/R$$
$$I_s = I_c \sin\delta$$

use Josephson equations:

$$I_b = I_c \sin\delta + \frac{\phi_0}{2\pi R}\dot{\delta} + \frac{\phi_0 C}{2\pi}\ddot{\delta}$$

W.C. Stewart, Appl. Phys. Lett. **2**, 277, (1968)
D.E. McCumber, J. Appl. Phys. **39**, 3 113 (1968)

looks like equation of motion for a particle with mass m and coordinate δ in an external potential U:

$$m\ddot{\delta} + m\frac{1}{RC}\dot{\delta} + \frac{\partial U(\delta)}{\partial \delta} = 0$$

particle mass: $m = C\,(\phi_0/2\pi)^2$

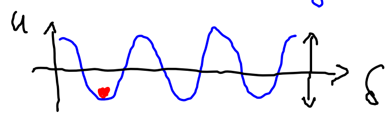
external potential: $U(\delta) = \frac{I_c \phi_0}{2\pi}\left(-\frac{I_b}{I_c} - \cos\delta\right)$

Phase particle in a potential well

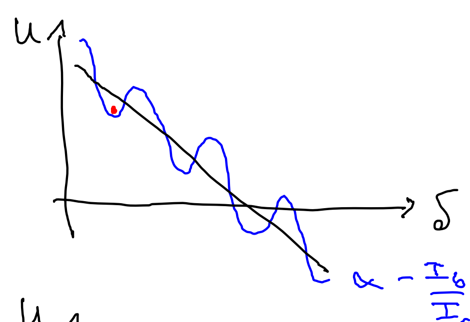
$$U(\delta) = \frac{I_c \phi_0}{2\pi}\left(-\frac{I_b}{I_c}\delta - \cos\delta\right)$$

$$E_J = \frac{I_c \phi_0}{2\pi}$$

cosine potential for $I_b = 0$:



'tilted washboard' potential for $I_b \neq 0$:



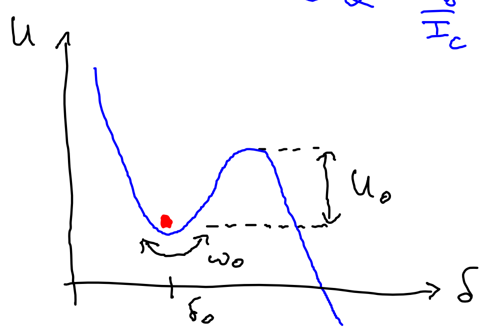
potential barrier:

$$U_0 = 2E_J\left[\sqrt{1-\gamma^2} - \gamma \arccos\gamma\right]$$

oscillation frequency:

$$\omega_0 = \omega_p (1-\gamma^2)^{1/4} = \sqrt{\frac{U''(\delta_0)}{m}}$$

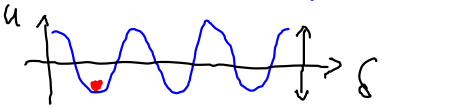
with: $\gamma = I_b/I_c$; $\omega_p = \sqrt{\frac{2\pi I_c}{\phi_0 C}}$



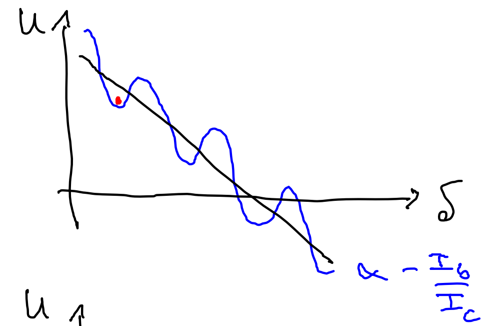
Phase particle in a potential well

$$U(\delta) = \frac{I_c \phi_0}{2\pi}\left(-\frac{I_b}{I_c}\delta - \cos\delta\right) \qquad E_J = \frac{I_c \phi_0}{2\pi}$$

cosine potential for $I_b = 0$:



'tilted washboard' potential for $I_b \neq 0$:



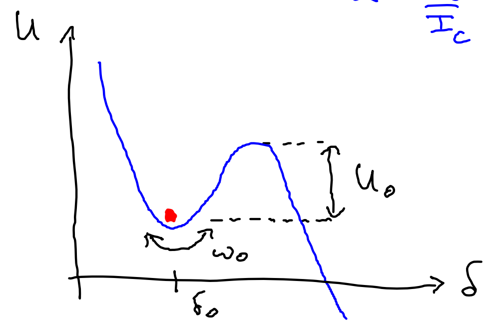
potential barrier:

$$U_0 = 2E_J\left[\sqrt{1-\gamma^2} - \gamma \arccos\gamma\right]$$

oscillation frequency:

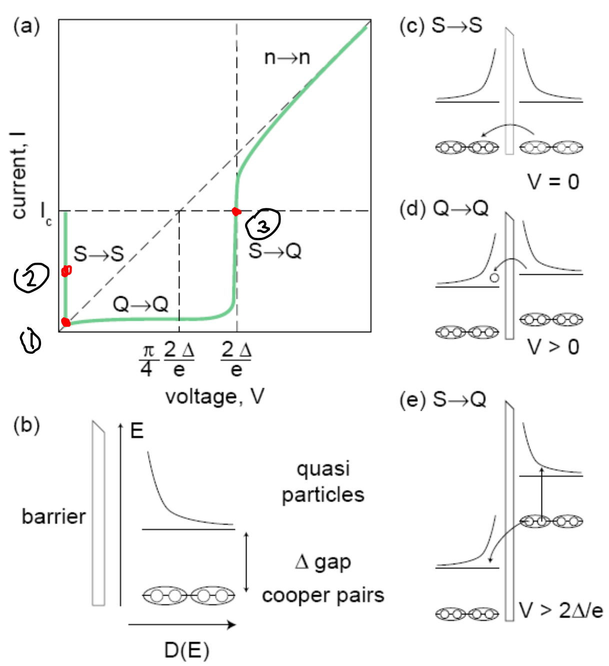
$$\omega_0 = \omega_p (1-\gamma^2)^{1/4} = \sqrt{\frac{U''(\delta_0)}{m}}$$

with: $\gamma = I_b/I_c$; $\omega_p = \sqrt{\frac{2\pi I_c}{\phi_0 C}}$

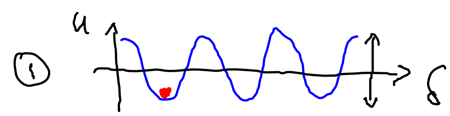


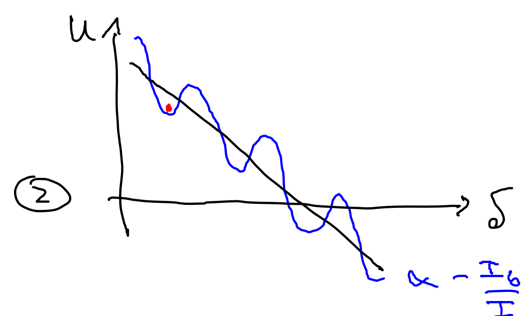
Current-voltage characteristics

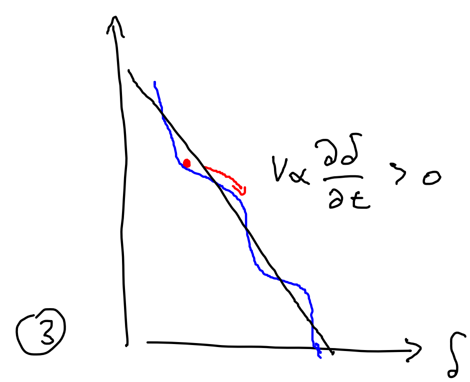
typical I-V curve of underdamped Josephson junctions:



band diagram







$$V \propto \frac{\partial\delta}{\partial t} > 0$$

Thermal Activation and Quantum Tunneling:

bias current dependence

$\omega_0(\gamma)\ ;\ U_0(\gamma)$

thermal activation rate:

$$\Gamma_{th} \approx a_t \frac{\omega_0}{2\pi} \exp\left(-\frac{U_0}{k_B T}\right)$$

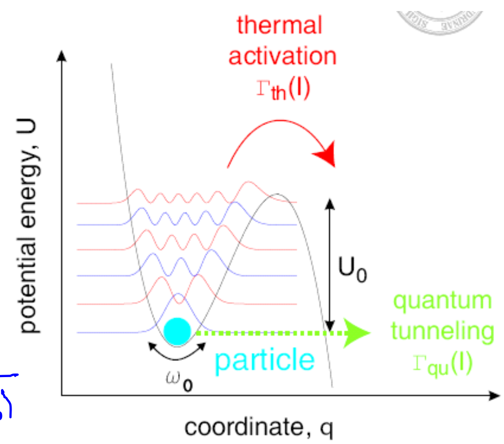
damping dependent prefactor

quantum tunneling rate:

$$\Gamma_{qu} = a_q \frac{\omega_0}{2\pi} \exp\left(-\frac{36}{5}\frac{U_0}{\hbar\omega_0}\right)$$

calculated using WKB method (exercise)

$$\Gamma_q = a_q\, \omega_0 \exp\left(-\int_{\delta_1}^{\delta_2} \frac{1}{\hbar}\sqrt{2m(U(\delta)-E_0)}\right)$$



energy level quantization:

$$E_n \approx \hbar\omega_0\left(n+\frac{1}{2}\right)$$

neglecting non-linearity

Quantum Mechanics of a Macroscopic Variable: The Phase Difference of a Josephson Junction
JOHN CLARKE, ANDREW N. CLELAND, MICHEL H. DEVORET, DANIEL ESTEVE, and JOHN M. MARTINIS
Science 26 February 1988 239: 992-997 [DOI: 10.1126/science.239.4843.992] (in Articles) Abstract » References » PDF »

Macroscopic quantum effects in the current-biased Josephson junction
M. H. Devoret, D. Esteve, C. Urbina, J. Martinis, A. Cleland, J. Clarke
in Quantum tunneling in condensed media, North-Holland (1992)

Early Results (1980's)

search for macroscopic quantum effects in superconducting circuits

theoretical predictions:

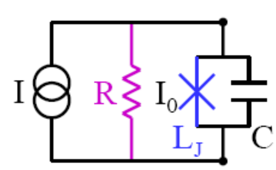
- tunneling ✓
- energy level quantization ✓
- coherence ✗

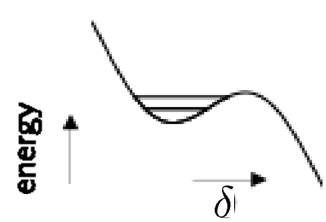
A.J. Leggett *et al.*,
Prog. Theor. Phys. Suppl. **69**, 80 (1980),
Phys. Scr. **T102**, 69 (2002).

short coherence times due to strong coupling to em environment

experimental verification:

current biased JJ = phase qubit

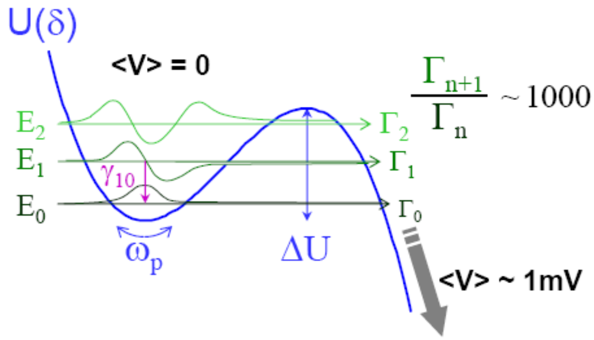


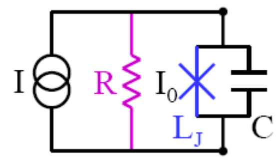


J. Clarke, J. Martinis, M. Devoret *et al.*, *Science* **239**, 992 (1988).

The Current Biased Phase Qubit

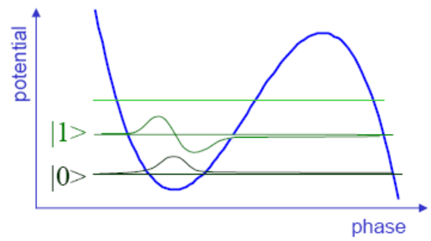
operating a current biased Josephson junction as a superconducting qubit:





initialization:

wait for |1> to decay to |0>, e.g. by spontaneous emission at rate γ_{10}

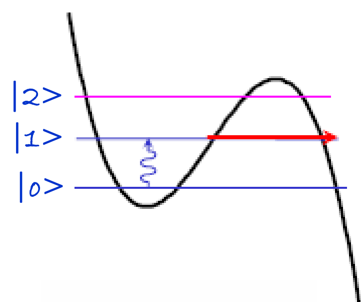


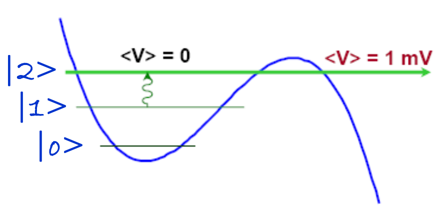
Read-Out Ideas

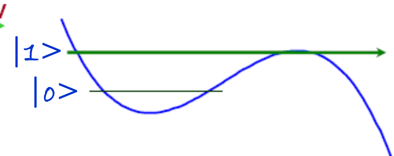
measuring the state of a current biased phase qubit

tunneling:

- prepare state |1> (pump)
- wait ($\Gamma_1 \sim 10^3\, \Gamma_0$)
- detect voltage
- |1> = voltage, |0> = no voltage







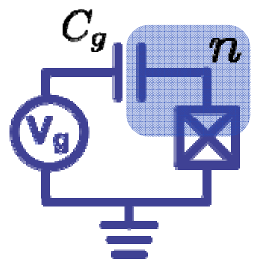
pump and probe pulses:

- prepare state |1> (pump)
- drive ω_{21} transition (probe)
- observe tunneling out of |2>

tipping pulse:

- prepare state |1>
- apply current pulse to suppress U_0
- observe tunneling out of |1>

A Charge Qubit: The Cooper Pair Box

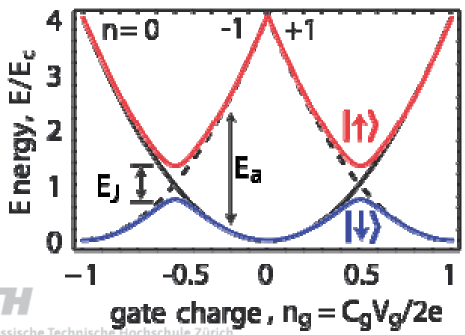


$$H = 4E_C n^2$$

$$H = 4E_C(n - n_g)^2 - E_J \cos\delta$$

$$[\delta, n] = i \rightarrow e^{\pm i\delta}|n\rangle = |n \pm 1\rangle$$

$$H = \sum_n \left[4E_C(n-n_g)^2 |n\rangle\langle n| - \frac{E_J}{2}\left(|n\rangle\langle n+1| + |n+1\rangle\langle n|\right)\right]$$



Charging energy: $E_C = \dfrac{e^2}{2(C_g + C_J)}$

Gate charge: $n_g = \dfrac{C_g V_g}{2e}$

Josephson energy: $E_J = \dfrac{I_0 \Phi_0}{2\pi} = \dfrac{h\Delta}{8e^2 R_J}$

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Bouchiat et al. Physica Scripta 176, 165 (1998)

Cooper pair box Hamiltonian:

Hamiltonian: $\hat{H} = E_c(\hat{N} - N_g)^2 - E_J \cos\hat{\delta} = \frac{E_J}{2}(e^{i\hat{\delta}} + e^{-i\hat{\delta}})$ gate charge $N_g = \dfrac{C_g V_g}{2e}$

electrostatic charging energy — magnetic energy Josephson coupling Energy

$E_c = \dfrac{(2e)^2}{2C_\Sigma}$ $E_J = \dfrac{\phi I_c}{2\pi}$

Hamiltonian in charge representation:

$$\hat{H} = E_c(N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2}\sum_N (|N+1\rangle\langle N| + |N\rangle\langle N+1|)$$

easy to diagonalize numerically

$$\hat{H} = \begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & E_c(-1-N_g)^2 & -E_J/2 & 0 & \cdots \\ \cdots & -E_J/2 & E_c(0-N_g)^2 & -E_J/2 & \cdots \\ \cdots & 0 & -E_J/2 & E_c(1-N_g)^2 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

relation between phase and number basis:

$$|\delta\rangle = \frac{1}{\sqrt{2\pi}}\sum_N e^{iN\delta}|N\rangle \quad \text{with} \quad e^{i\hat{\delta}}|N\rangle = |N+1\rangle$$

Phase representation of Cooper pair box Hamiltonian:

$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta} \qquad \text{with} \qquad \hat{N} = \frac{\hat{Q}}{2e} = -i\hbar\frac{1}{2e}\frac{\partial}{\partial\phi}$$

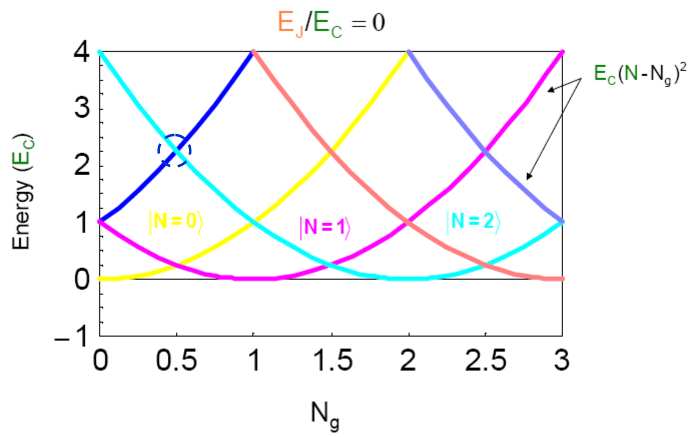
$$= E_C\left(-i\frac{\partial}{\partial\delta} - N_g\right)^2 - E_J\cos\hat{\delta} \qquad\qquad = -i\frac{\partial}{\partial\delta}$$

Equivalent solution to the Hamiltonian can be found in both representations, e.g. by numerically solving the Schrödinger equation for the charge (N) representation or analytically solving the Schrödinger equation for the phase (δ) representation.

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

solutions for $E_J = 0$:

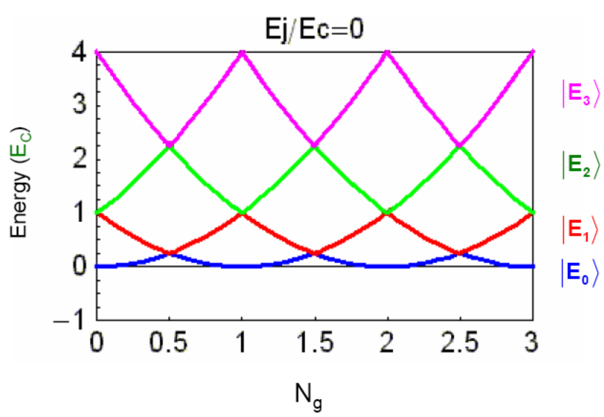
- crossing points are charge degeneracy points



Energy Levels

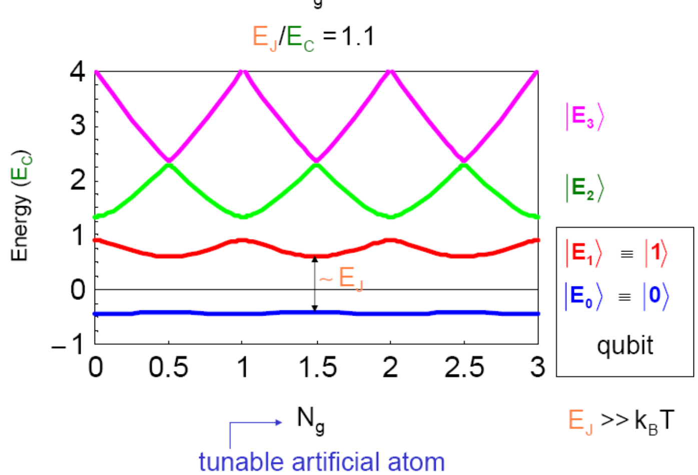
energy level diagram for $E_J=0$:

- energy bands are formed
- bands are periodic in N_g



energy bands for finite E_J

- Josephson coupling lifts degeneracy
- E_J scales level separation at charge degeneracy



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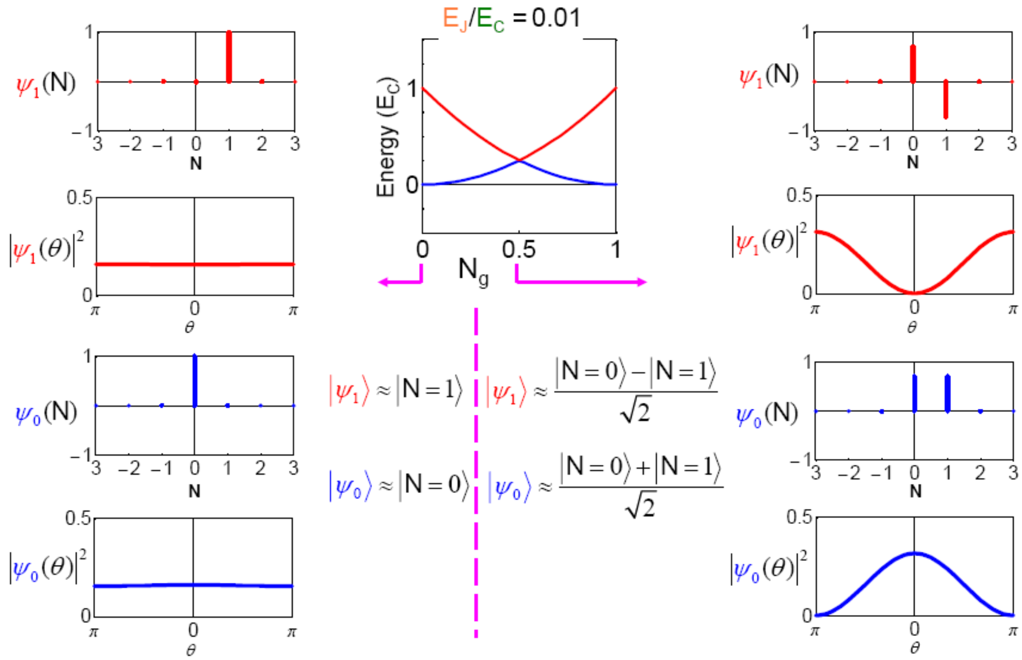
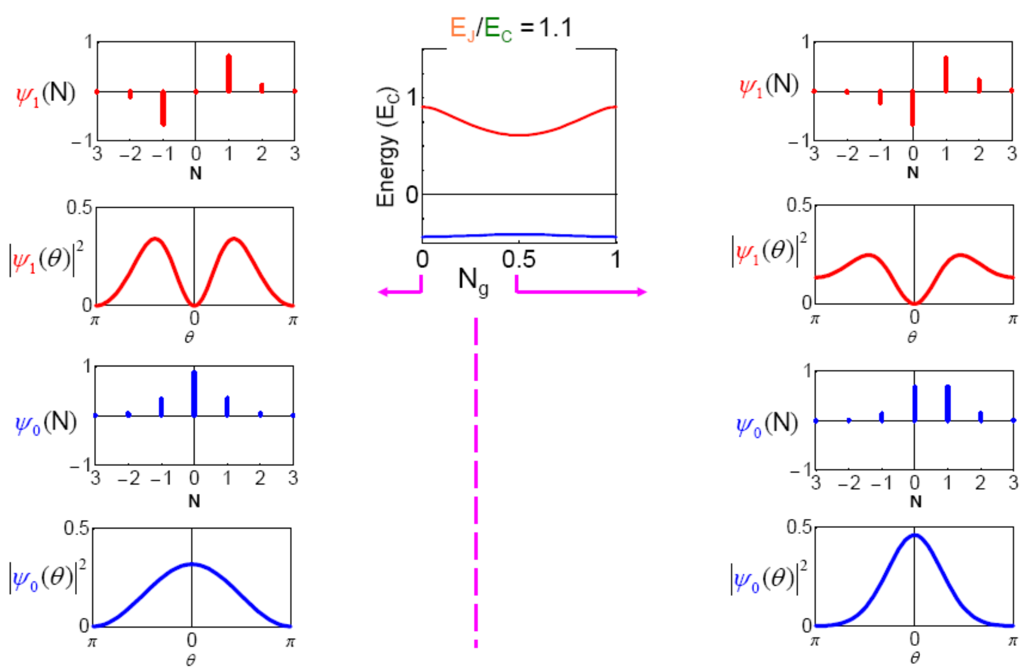
Charge and Phase Wave Functions ($E_j << E_c$)



courtesy Saclay

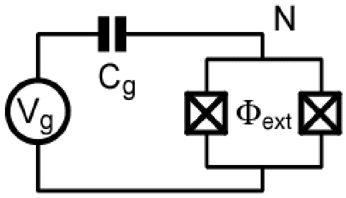
Charge and Phase Wave Functions ($E_j \sim E_c$)



courtesy Saclay

Tuning the Josephson Energy

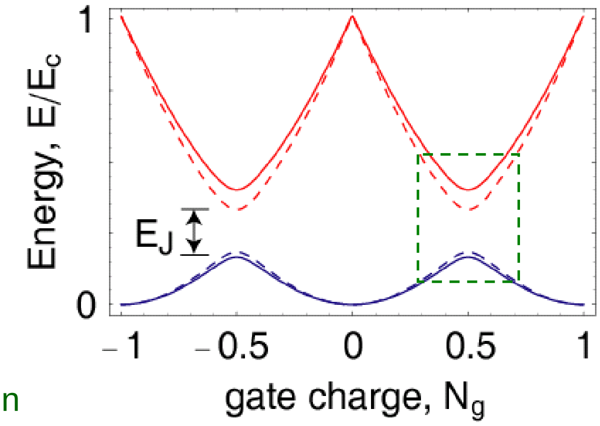
split Cooper pair box in perpendicular field



$$H = E_C\,(N - N_g)^2 - E_{J,\max}\cos\left(\pi\frac{\Phi_{\rm ext}}{\Phi_0}\right)$$

SQUID modulation of Josephson energy

$$E_J = E_{J,\max}\cos\left(\pi\frac{\Phi_{\rm ext}}{\Phi_0}\right)$$



consider two state approximation

J. Clarke, *Proc. IEEE* **77**, 1208 (1989)

Two State Approximation

$$\mathbf{H_{CPB}} = \mathbf{H_{el}} + \mathbf{H_J} = E_C(N - N_g)^2 - E_J\cos\delta$$

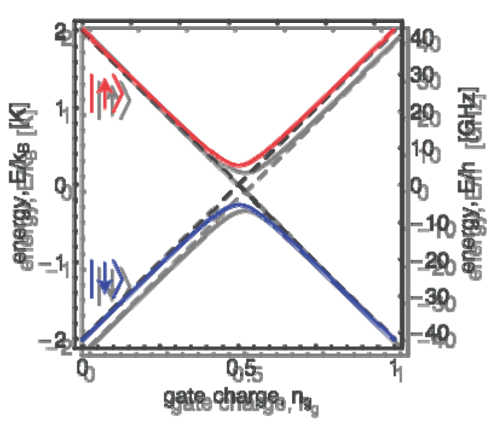
$$\mathbf{H_{CPB}} = \sum_N \left[E_C(N-N_g)^2\,|N\rangle\langle N| - \frac{E_J}{2}\left(|N\rangle\langle N+1| + |N+1\rangle\langle N|\right)\right]$$

Restricting to a two-charge Hilbert space:

$$N = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1-\sigma_z}{2}$$

$$\cos\delta = \frac{\sigma_x}{2}$$

$$\begin{aligned}\mathbf{H_{CPB}} &= -\frac{E_C}{2}(1-2N_g)\sigma_z - \frac{E_J}{2}\sigma_x \\ &= -\frac{1}{2}(E_{el}\sigma_z + E_J\sigma_x)\end{aligned}$$



Shnirman et al. Phys. Rev. Lett. 79, 2371 (1997)

Cavity QED with Electronic Circuits

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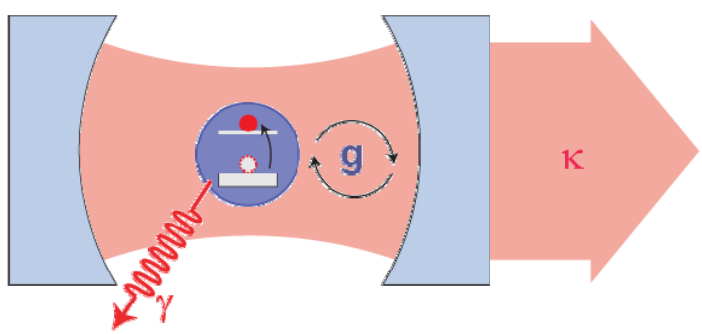
Swiss Federal Institute of Technology Zurich

Cavity Quantum Electrodynamics

coupling photons to qubits:



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit ($g = dE_0/\hbar > \gamma,\ \kappa\,, 1/t_{\mathrm{transit}}$)

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D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Dressed States Energy Level Diagram

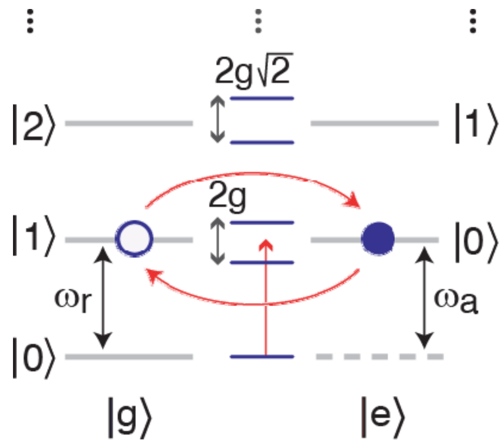
$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

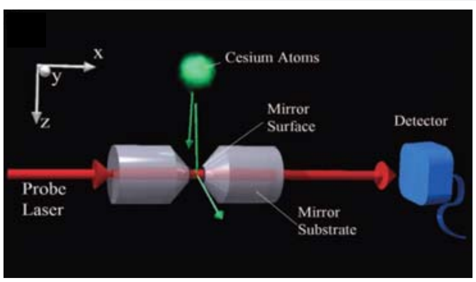
$$g = \frac{dE_0}{\hbar} > \gamma, \kappa$$



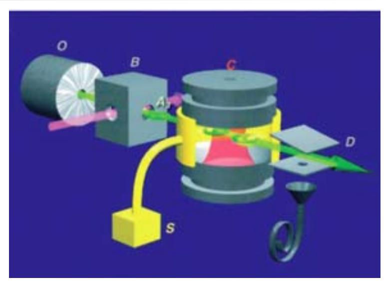
Jaynes-Cummings Ladder

Atomic cavity quantum electrodynamics reviews:
J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)
S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

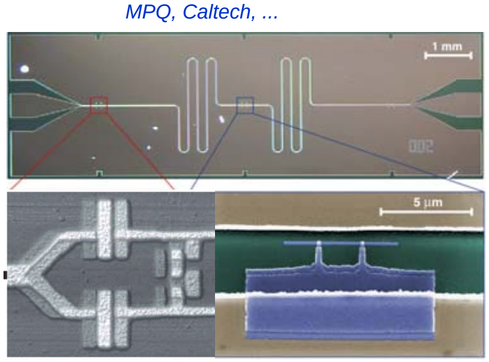
Cavity Quantum Electrodynamics (QED)



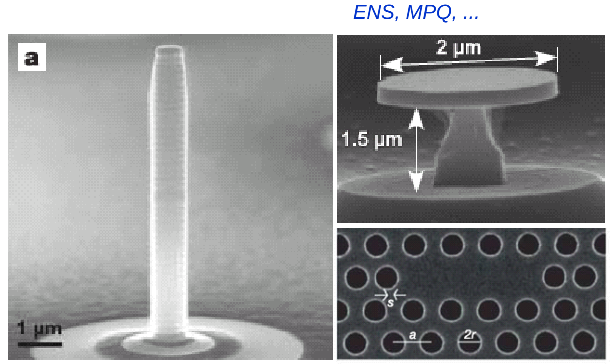
alkali atoms
MPQ, Caltech, ...



Rydberg atoms
ENS, MPQ, ...



superconductor circuits
Yale, Delft, NTT, ETHZ, NIST, ...



semiconductor quantum dots
Wurzburg, ETHZ, Stanford ...

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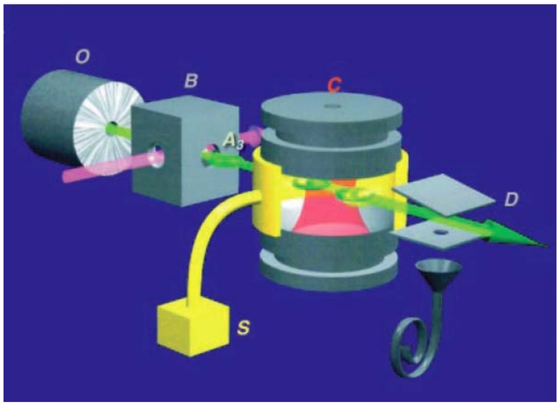
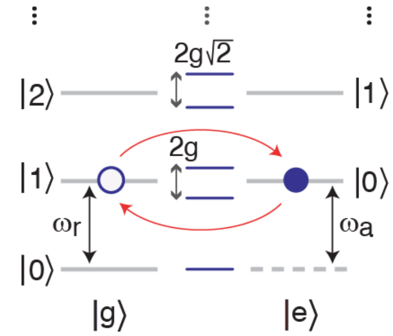
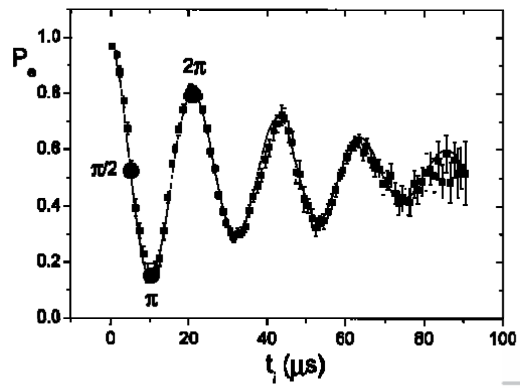
Vacuum Rabi Oscillations with Rydberg Atoms

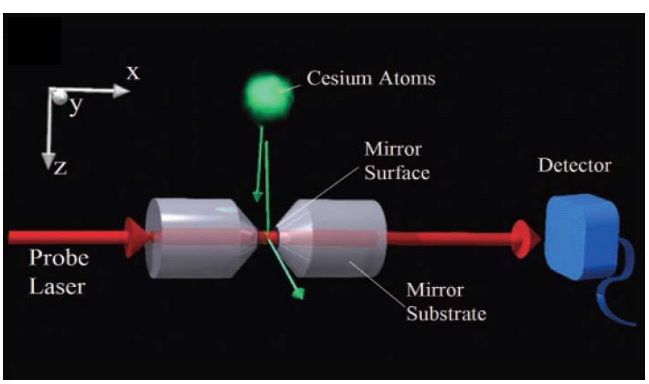


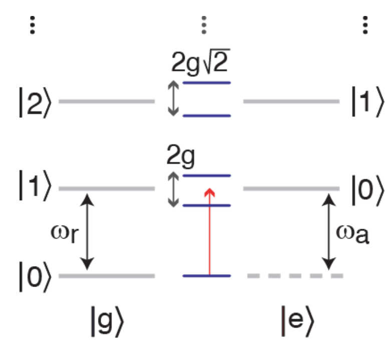


Review: J. M. Raimond, M. Brune, and S. Haroche
Rev. Mod. Phys. **73**, 565 (2001)
P. Hyafil, ..., J. M. Raimond, and S. Haroche,
Phys. Rev. Lett. **93**, 103001 (2004)

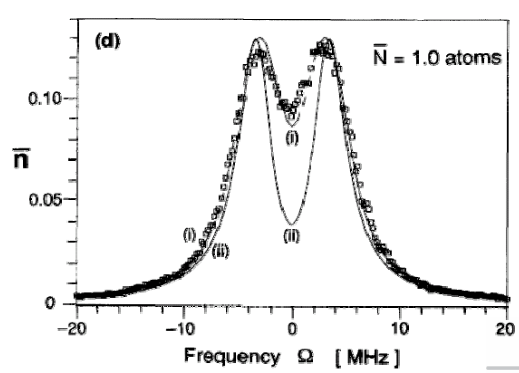


Vacuum Rabi Mode Splitting with Alkali Atoms





R. J. Thompson, G. Rempe, & H. J. Kimble,
Phys. Rev. Lett. **68** 1132 (1992)
A. Boca, ... , J. McKeever, & H. J. Kimble
Phys. Rev. Lett. **93**, 233603 (2004)



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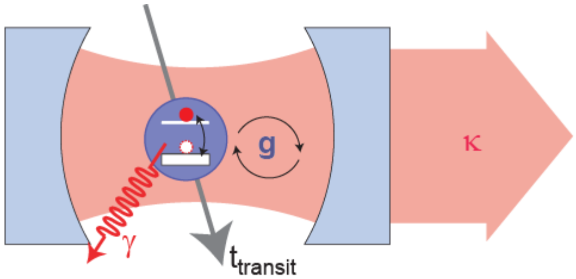
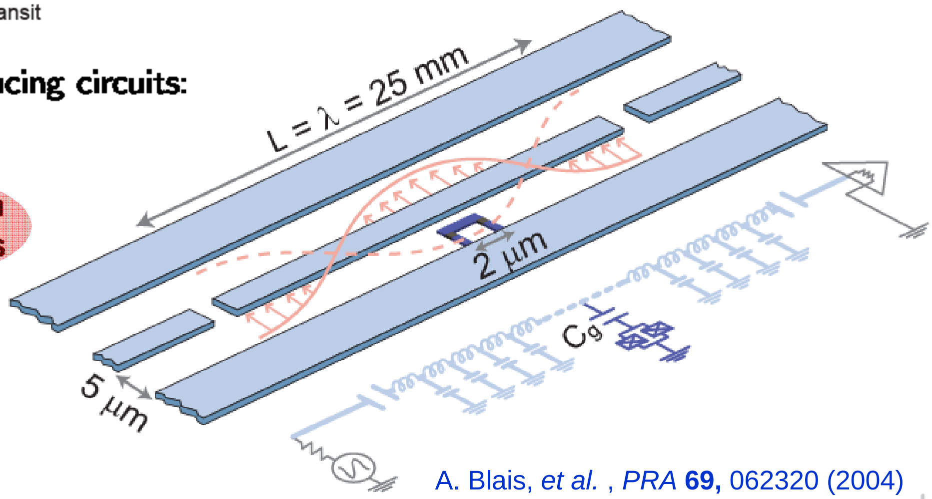
Cavity QED with Superconducting Circuits



coherent quantum mechanics with individual photons and qubits ...

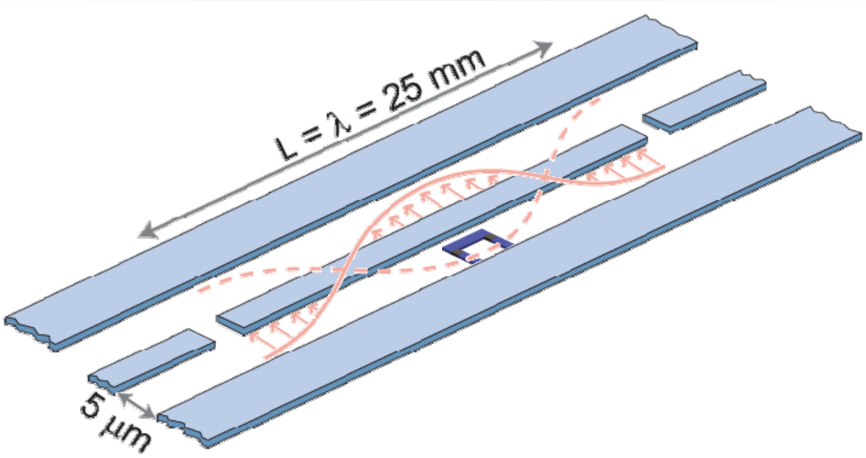
... in superconducing circuits:

circuit quantum electrodynamics



A. Blais, *et al.* , *PRA* **69,** 062320 (2004)
A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

Circuit Quantum Electrodynamics

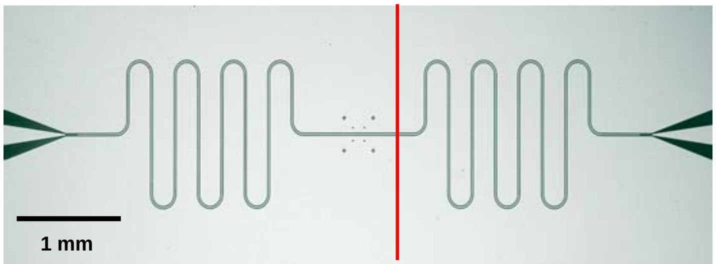


elements

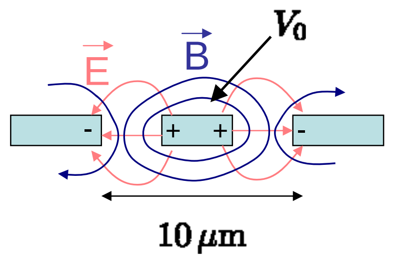
- the cavity: a superconducting 1D transmission line resonator with large vacuum field E_0 and long photon life time $1/\kappa$
- the artificial atom: a Cooper pair box with large E_J/E_C with large dipole moment d and long coherence time $1/\gamma$

A. Blais *et al.*, *PRA* **69,** 062320 (2004)

Vacuum Field in 1D Cavity



cross-section of transm. line (TEM mode):



voltage across resonator in vacuum state ($n = 0$)

$$V_{0,\mathrm{rms}} = \sqrt{\frac{\hbar\omega_r}{2C}} \approx 1\,\mu\mathrm{V}$$

harmonic oscillator

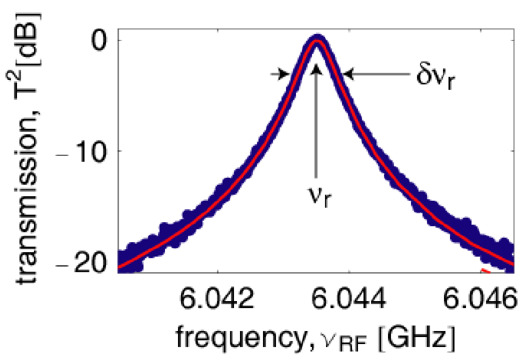
$$H_r = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right)$$

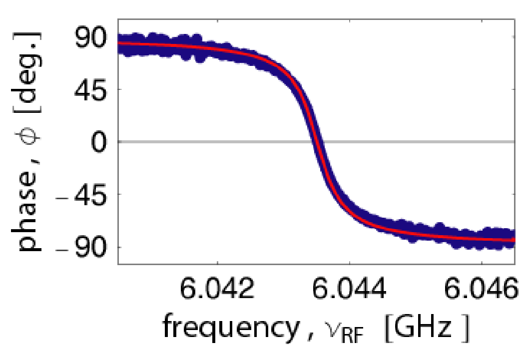
$$E_0 = \frac{V_{0,\mathrm{rms}}}{b} \approx 0.2\,\mathrm{V/m}$$

$\times 10^6$ larger than E_0 in 3D microwave cavity

for $\omega_r/2\pi \approx 6\,\mathrm{GHz}$ ($C \sim 1\,\mathrm{pF}$), $b \approx 5\,\mu\mathrm{m}$

Resonator Quality Factor and Photon Lifetime





resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

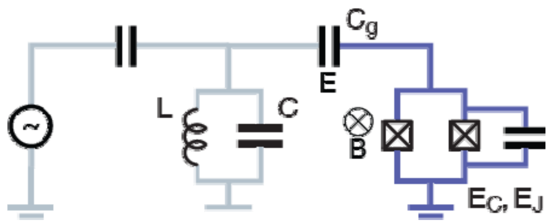
photon decay rate:

$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

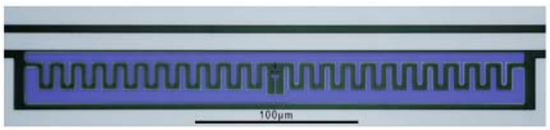
photon lifetime:

$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

Qubit/Photon Coupling in a Circuit



qubit coupled to resonator



coupling strength:

$$\hbar g = e V_{0,\mathrm{rms}} \frac{C_g}{C_\Sigma}$$

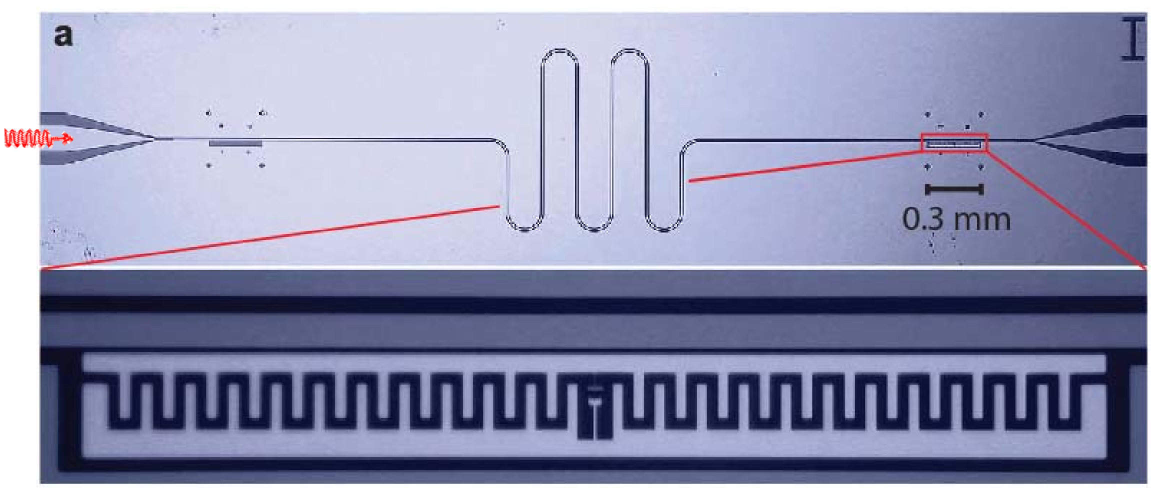
$$\Longrightarrow \nu_{\mathrm{vac}} = \frac{g}{\pi} \approx 1 \ldots 300\,\mathrm{MHz}$$

$g \gg [\kappa, \gamma]$ possible!

large effective dipole moment

$$d = \frac{\hbar g}{E_0} \sim 10^2 \ldots 10^4\, e a_0$$

Circuit QED with One Photon



superconducting cavity QED circuit

A. Wallraff, …, R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004)

Resonant Vacuum Rabi Mode Splitting …

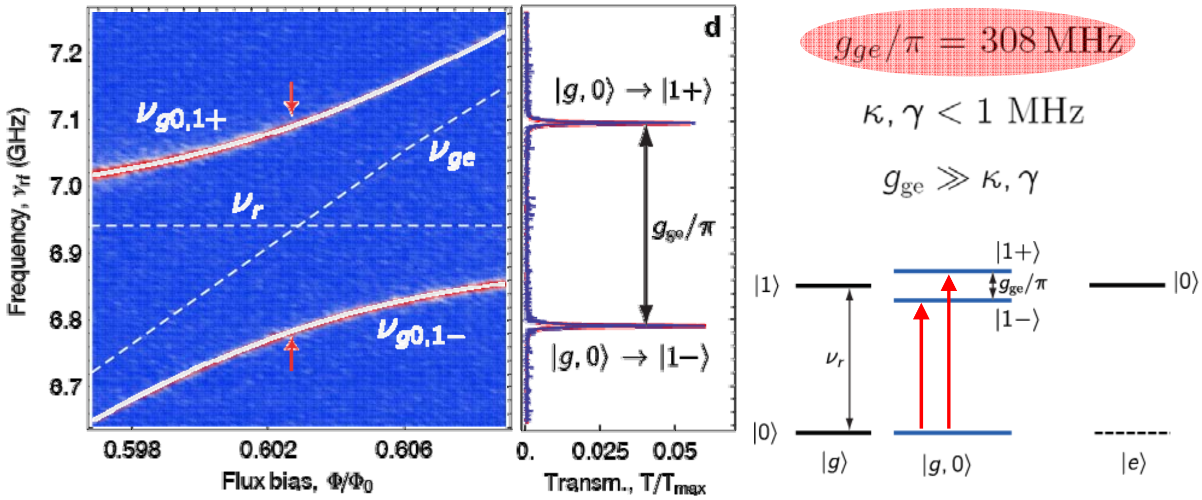
… with one photon ($n = 1$):

very strong coupling:

$g_{ge}/\pi = 308\,\mathrm{MHz}$

$\kappa, \gamma < 1\ \mathrm{MHz}$

$g_{ge} \gg \kappa, \gamma$



forming a 'molecule' of a qubit and a photon

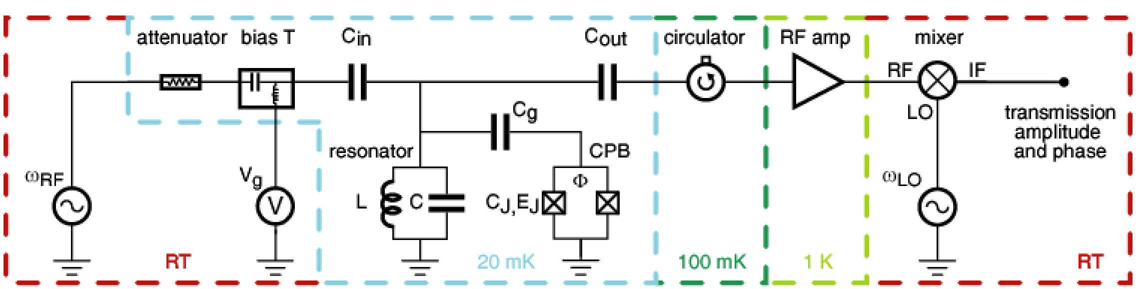
$$|1\pm\rangle = \left(|g,1\rangle \pm |e,0\rangle\right)/\sqrt{2}$$

first demonstration: A. Wallraff, … and R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004)
this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

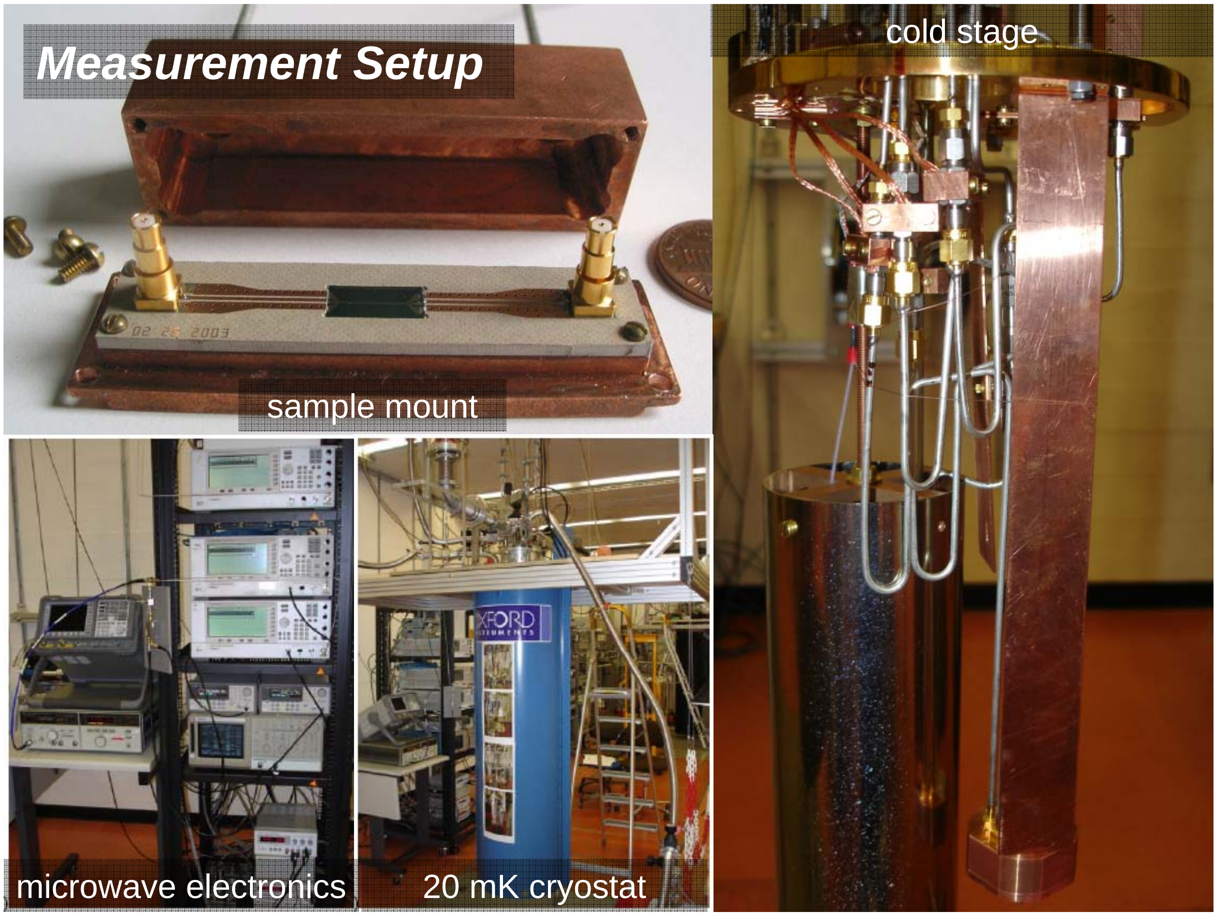
How to Measure Single Microwave Photons

- average power to be detected

$$\rightarrow \langle n = 1\rangle \hbar\omega_r\kappa/2 \approx P_{RF} = -140\,\mathrm{dBm} = 10^{-17}\,\mathrm{W}$$

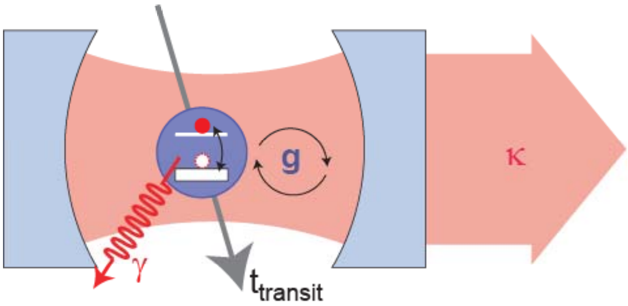


- efficient with cryogenic low noise HEMT amplifier ($T_N = 6\,\mathrm{K}$)
- prevent leakage of thermal photons (cold attenuators and circulators)



Measurement Setup
sample mount
02 25 2003
microwave electronics
20 mK cryostat
cold stage

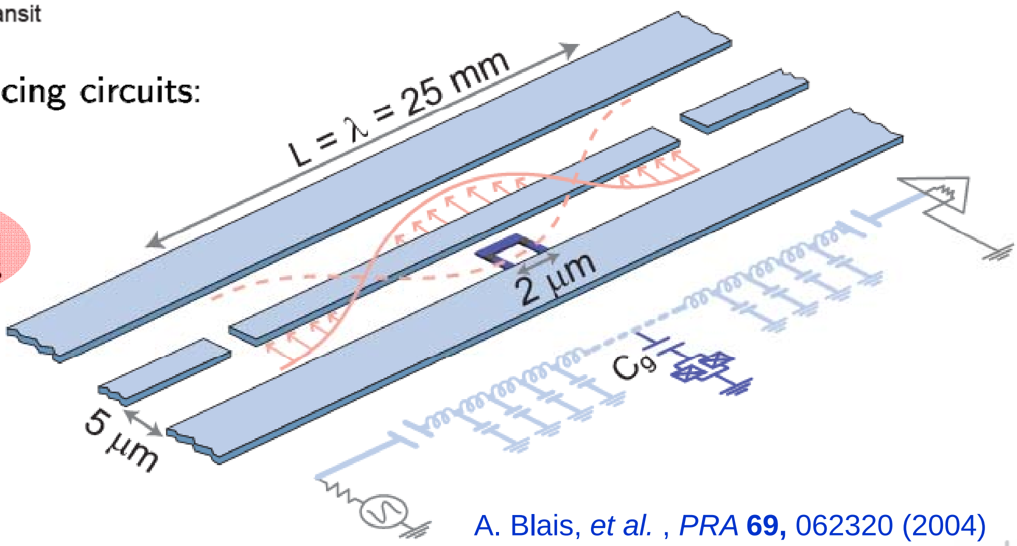
Cavity QED with Superconducting Circuits



coherent quantum mechanics
with individual photons and qubits ...

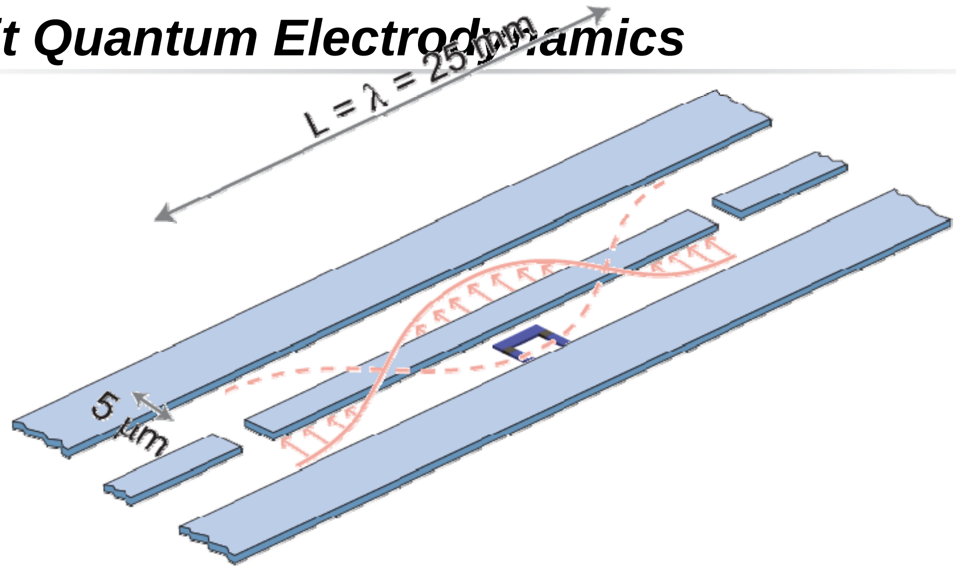
... in superconducing circuits:

circuit quantum electrodynamics



A. Blais, *et al.* , *PRA* **69,** 062320 (2004)
A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

Circuit Quantum Electrodynamics



elements

- the cavity: a superconducting 1D transmission line resonator with large vacuum field E_0 and long photon life time $1/\kappa$
- the artificial atom: a Cooper pair box with large dipole moment *d* and long coherence time $1/\gamma$

A. Blais *et al.*, *PRA* **69,** 062320 (2004)

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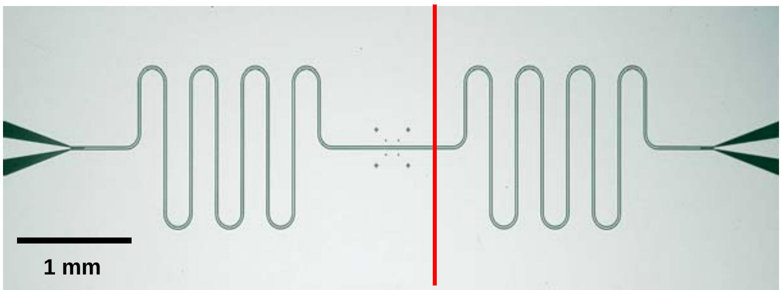
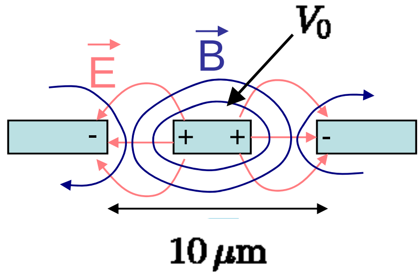
Vacuum Field in 1D Cavity



cross-section of transm. line (TEM mode):



voltage across resonator in vacuum state ($n = 0$)

$$V_{0,\mathrm{rms}} = \sqrt{\frac{\hbar\omega_r}{2C}} \approx 1\,\mu\mathrm{V}$$

harmonic oscillator

$$H_r = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right)$$

$$E_0 = \frac{V_{0,\mathrm{rms}}}{b} \approx 0.2\,\mathrm{V/m}$$

10^3 larger than in 3D cavity

for $\omega_r/2\pi \approx 6\,\mathrm{GHz}$ ($C \sim 1\,\mathrm{pF}$), $b \approx 5\,\mu m$

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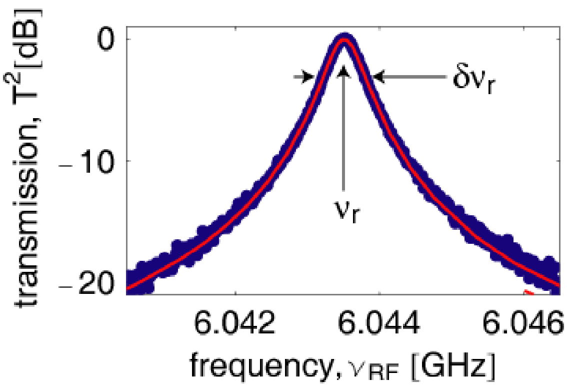
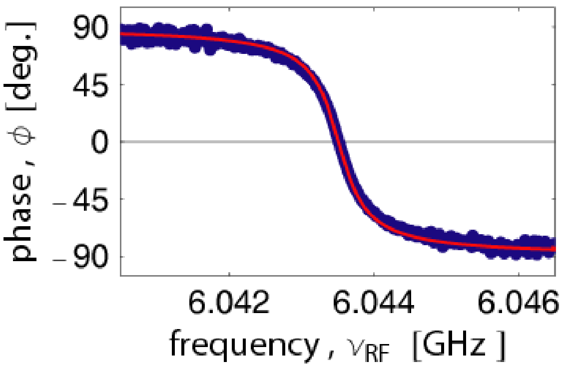
Resonator Quality Factor and Photon Lifetime





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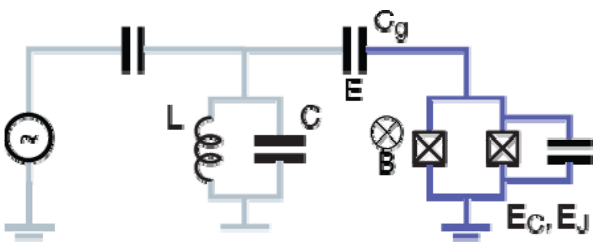

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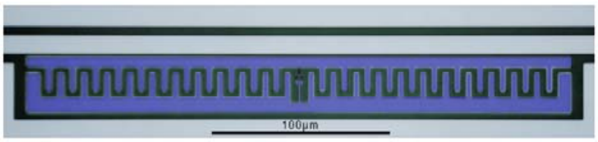

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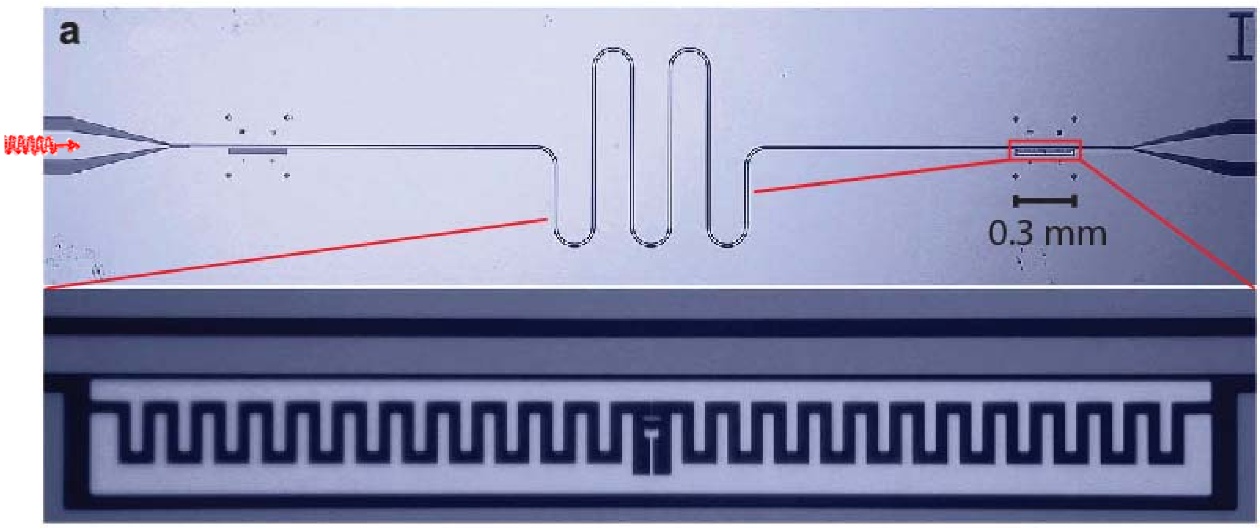

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A. Wallraff, …, R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004)

Resonant Vacuum Rabi Mode Splitting …

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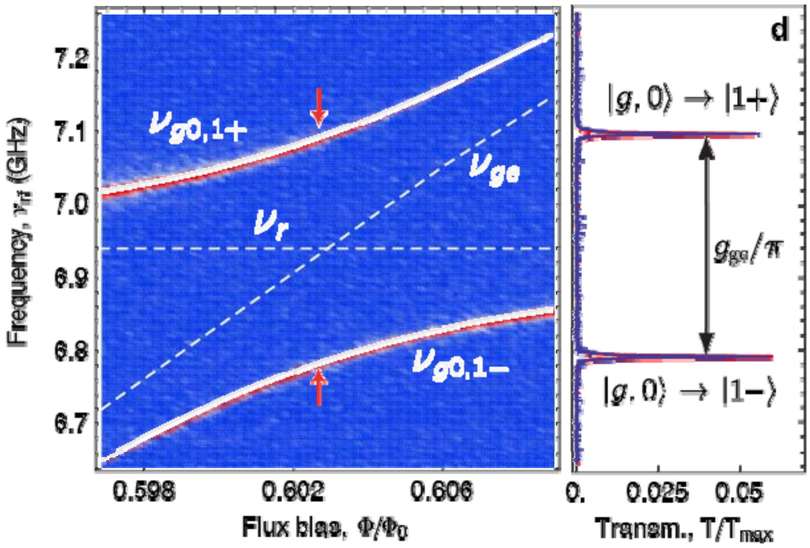

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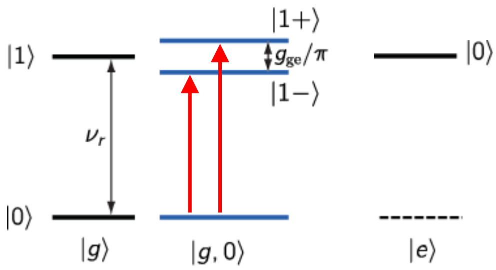

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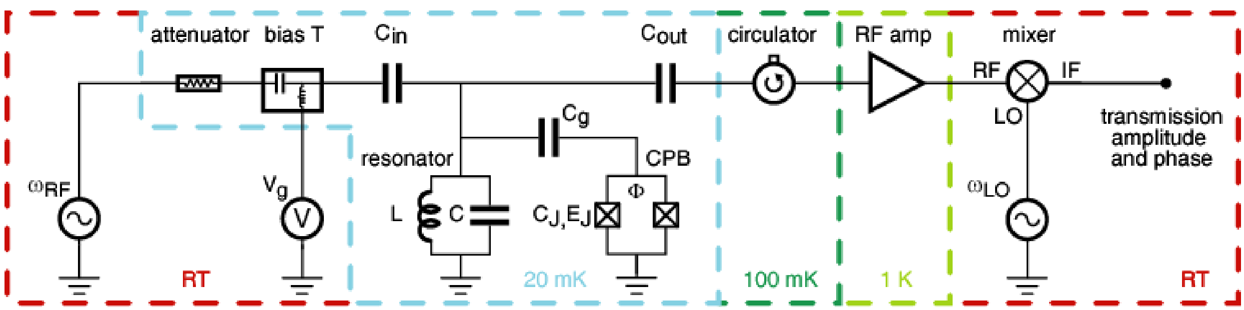

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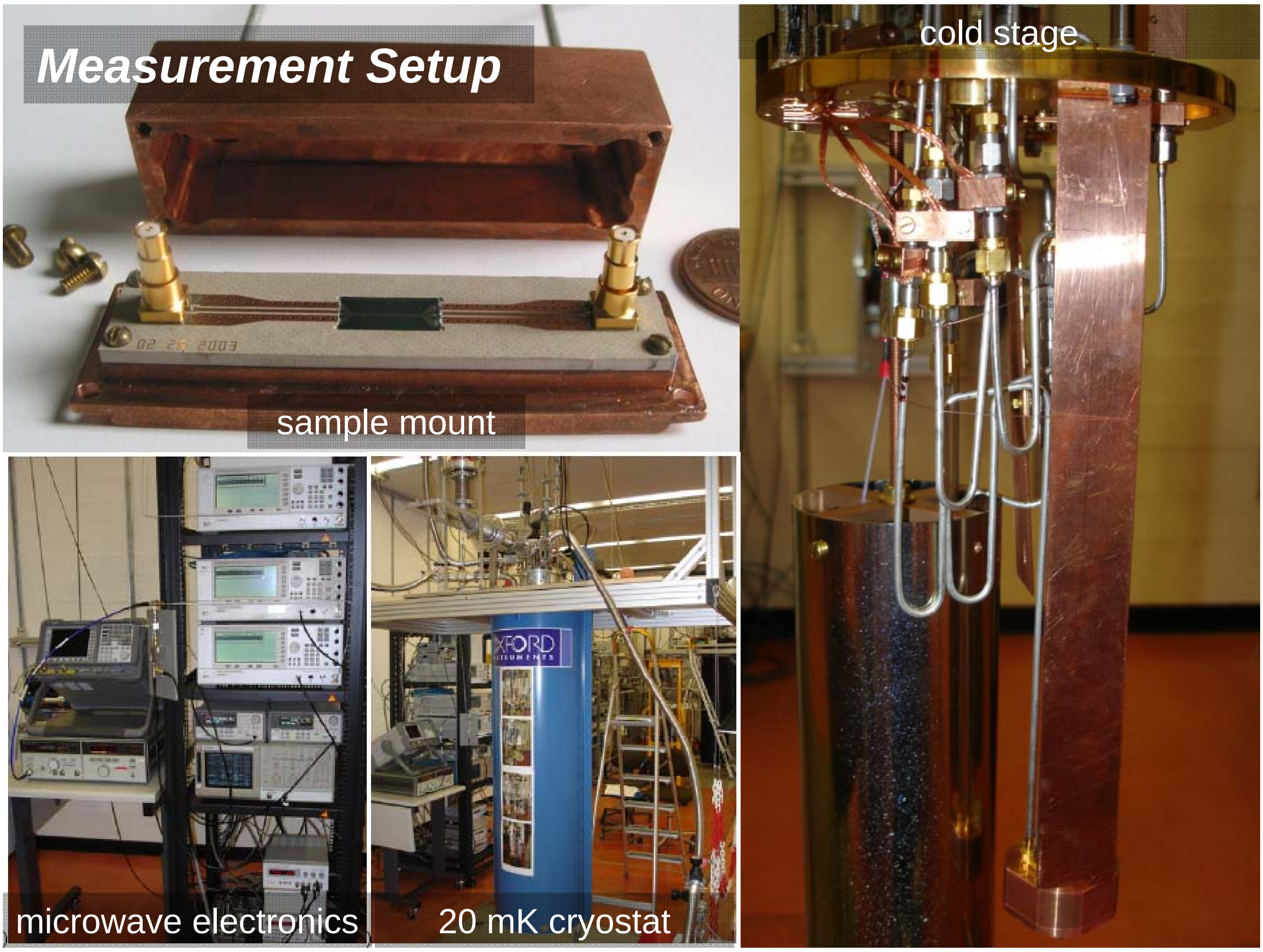
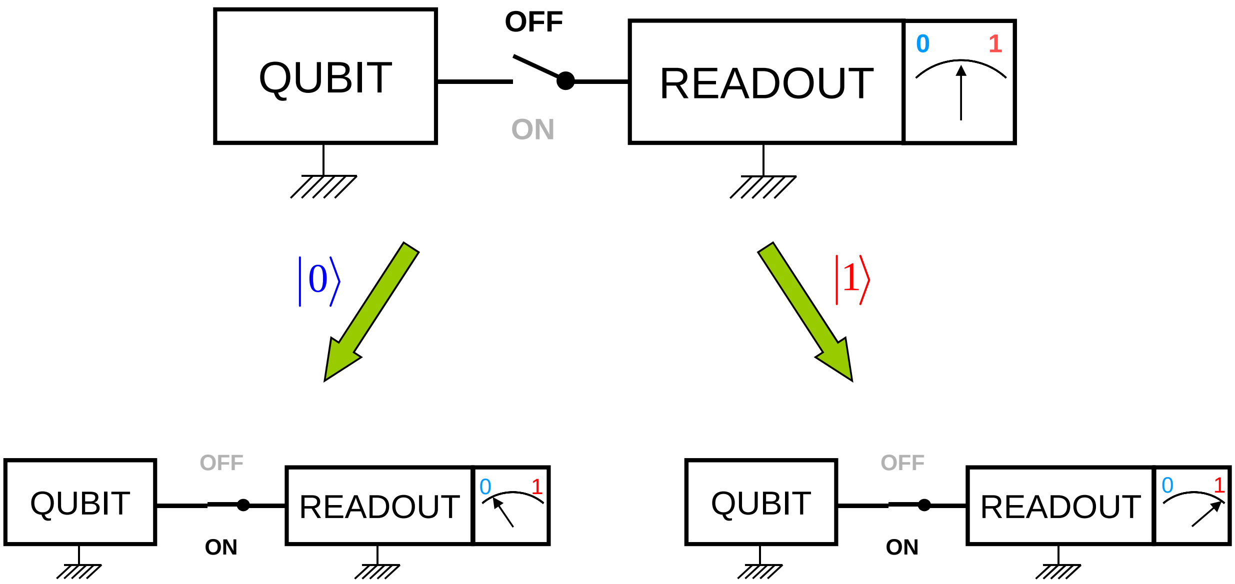
- prevent leakage of thermal photons (cold attenuators and circulators)

Measurement Setup



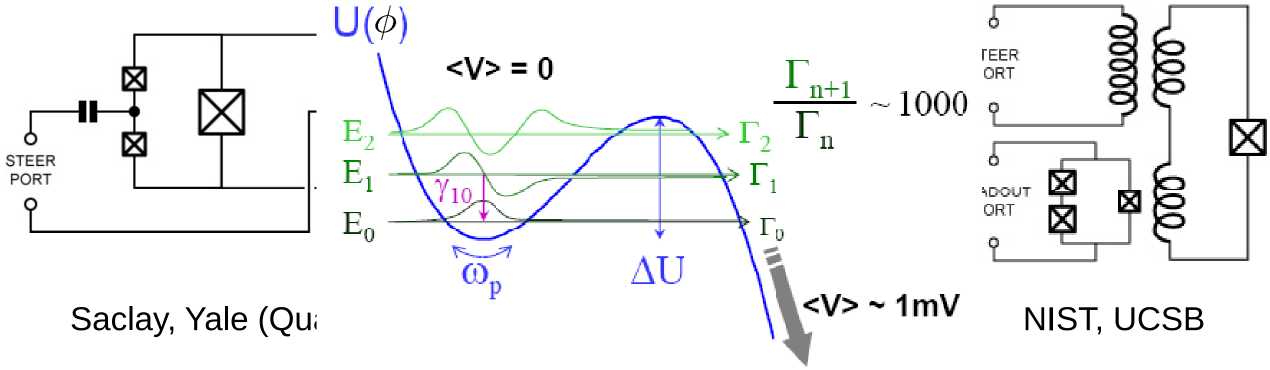
Qubit Read Out



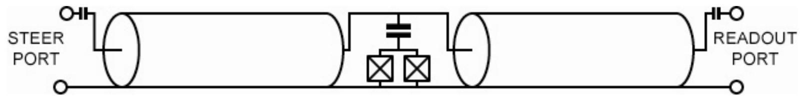
desired: good on/off ratio
no relaxation in on state (QND)

Read Out Strategies

demolition measurements (switching/latching measurements)



quantum non-demolition (QND) measurements



Yale (circuit QED)

now also: Chalmers, Delft, Yale (JBA)

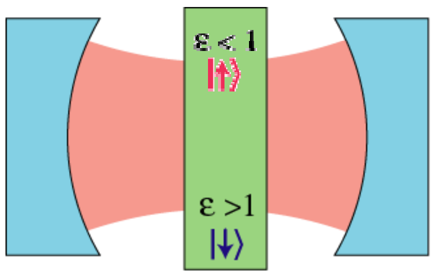
Non-Resonant Interaction: Qubit Readout

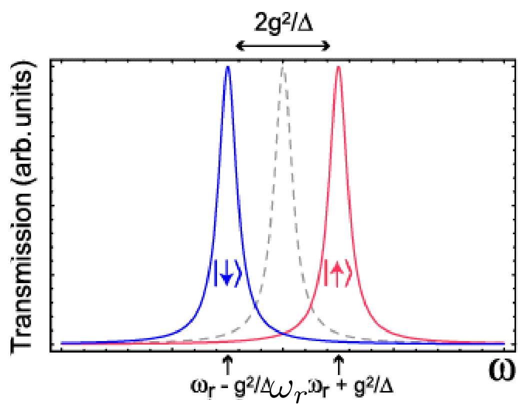
approximate diagonalization in the dispersive limit $|\Delta| = |\omega_a - \omega_r| \gg g$

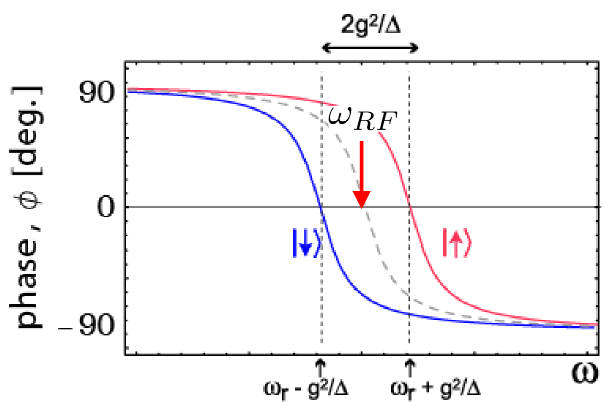
$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta} \sigma_z \right) a^\dagger a + \frac{1}{2} \hbar \left(\omega_a + \frac{g^2}{\Delta} \right) \sigma_z$$

cavity frequency shift *and* qubit ac-Stark shift

Lamb shift







A. Blais *et al.*, *PRA* **69,** 062320 (2004)

Non-Resonant Coupling for Qubit Readout

approximate diagonalization for $|\Delta| = |\omega_a - \omega_r| \gg g$

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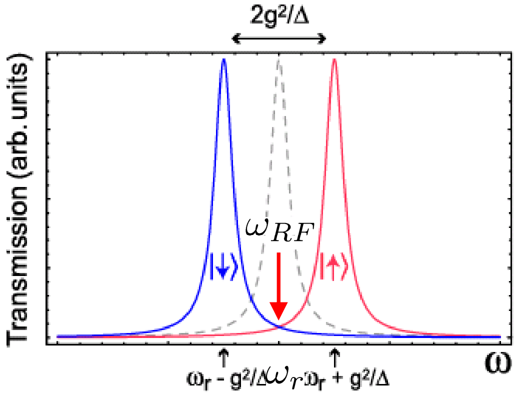
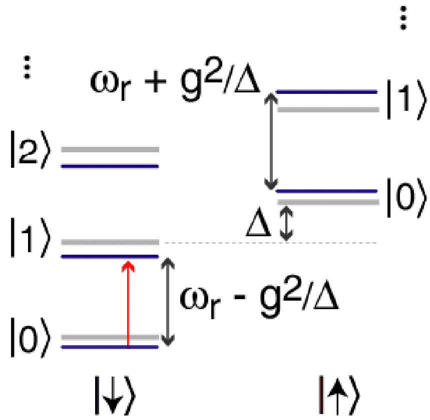
cavity frequency shift *and* qubit ac-Stark shift

Lamb shift

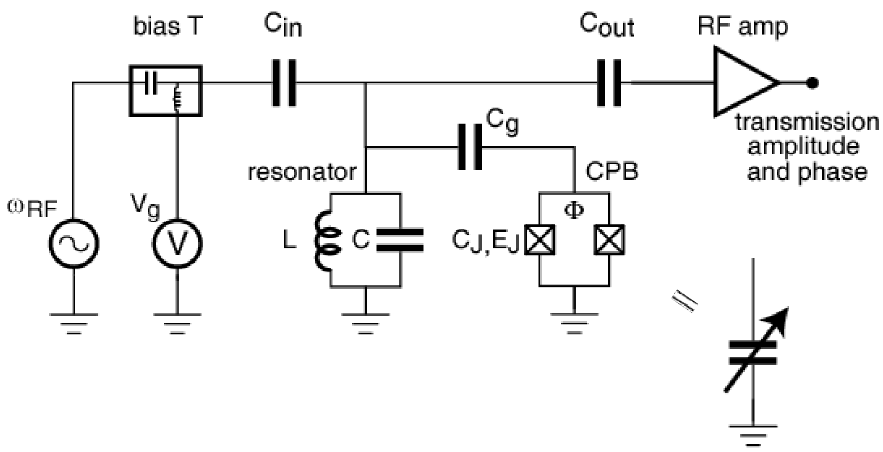
dispersive level diagram:





A. Blais, R.-S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, *PRA* **69,** 062320 (2004)

Measurement Technique

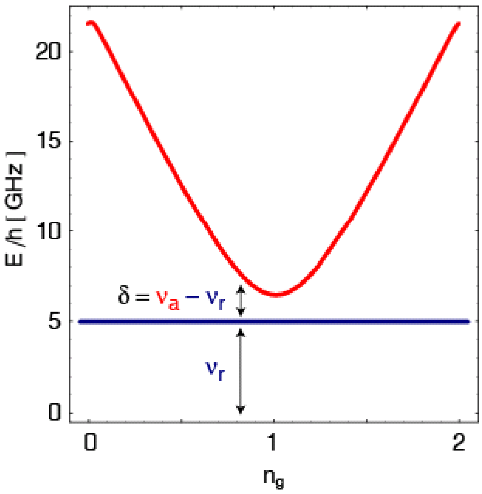


- measurement of microwave transmission amplitude T and phase ϕ
- intra-cavity photon number controllable from $n \sim 10^3$ to $n \ll 1$

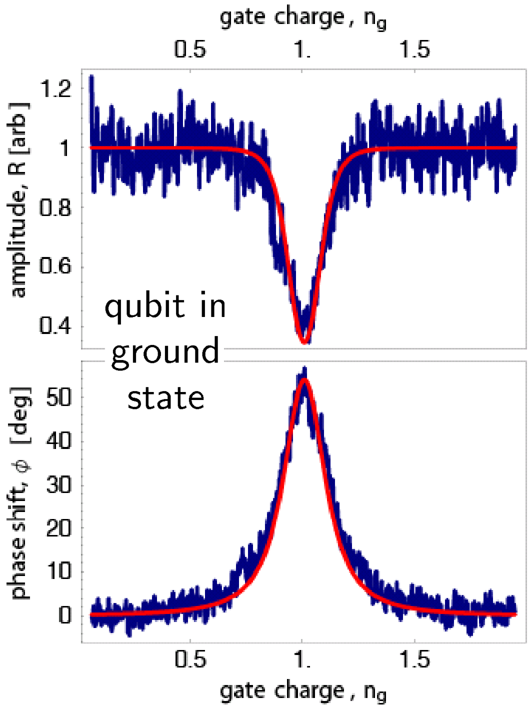
Dispersive Shift of Resonance Frequency

sketch of qubit level separation:

$$\Delta = 2\pi\delta > g$$

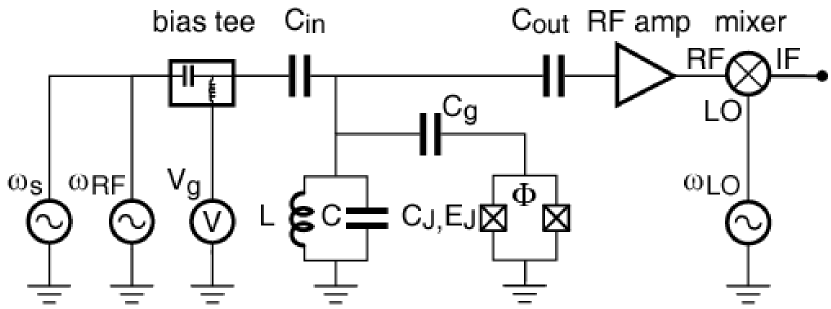


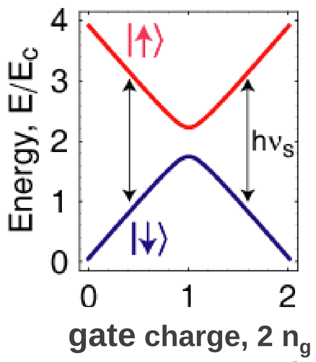
measured resonator transmission amplitude and phase:

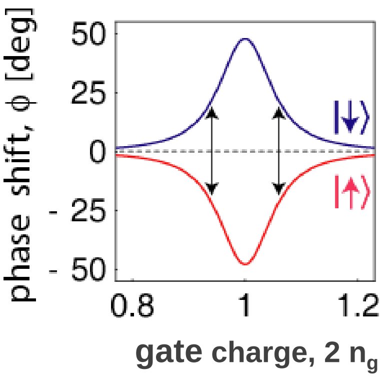


$$g/\pi = \nu_{vac} = 11\,\text{MHz}$$
$$\Delta(n_g = 1)/2\pi = 66\,\text{MHz}$$
$$n = 10$$

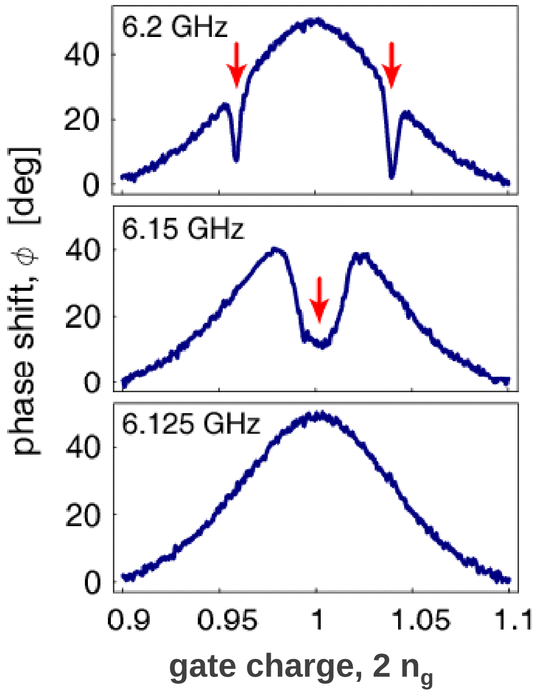
Realization of qubit spectroscopy

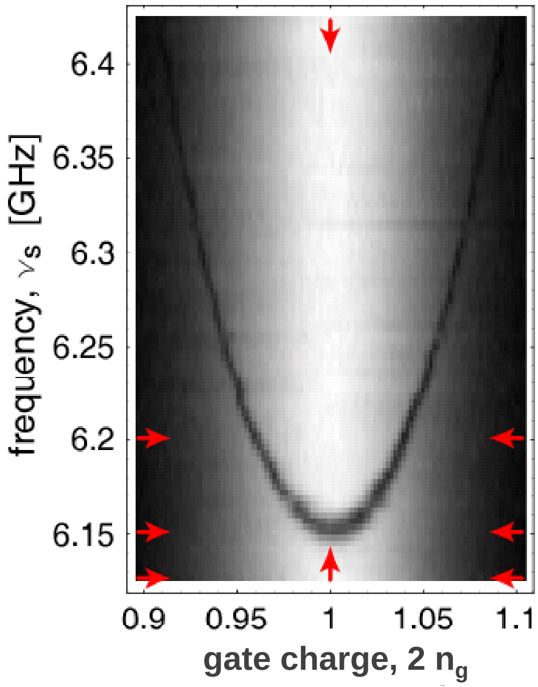






CW Spectroscopy of Cooper Pair Box





detuning $\Delta_{\mathrm{r,a}}/2\pi \sim 100\,\mathrm{MHz}$ extracted: $E_J = 6.2\,\mathrm{GHz},\ E_C = 4.8\,\mathrm{GHz}$

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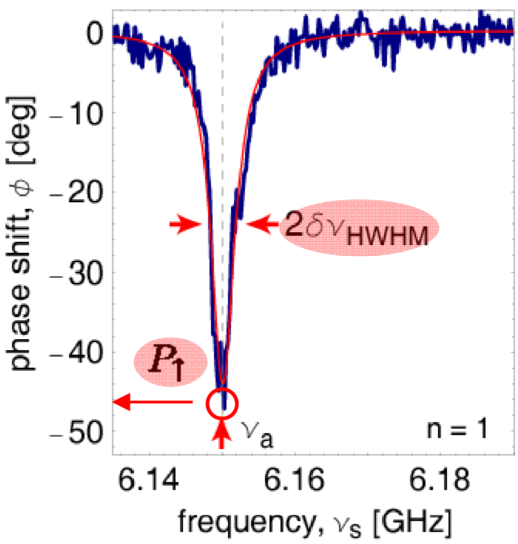
D. I. Schuster *et al.*, *Phys. Rev. Lett.* **94,** 123062 (2005)

Line Shape

excited state population (steady-state Bloch equations):

$$P_\uparrow = 1 - P_\downarrow = \frac{1}{2}\frac{n_s\omega_{\mathrm{vac}}^2 T_1 T_2}{1 + (T_2\Delta_{\mathrm{s,a}})^2 - n_s\omega_{\mathrm{vac}}^2 T_1 T_2}$$



- fixed drive $P_s \propto n_s\omega_{\mathrm{vac}}^2$
- varying $\Delta_{\mathrm{s,a}} = \omega_s - \widetilde{\omega}_a$
- weak continuous measurement $(n \sim 1)$
- at charge degenracy $(n_g = 1)$

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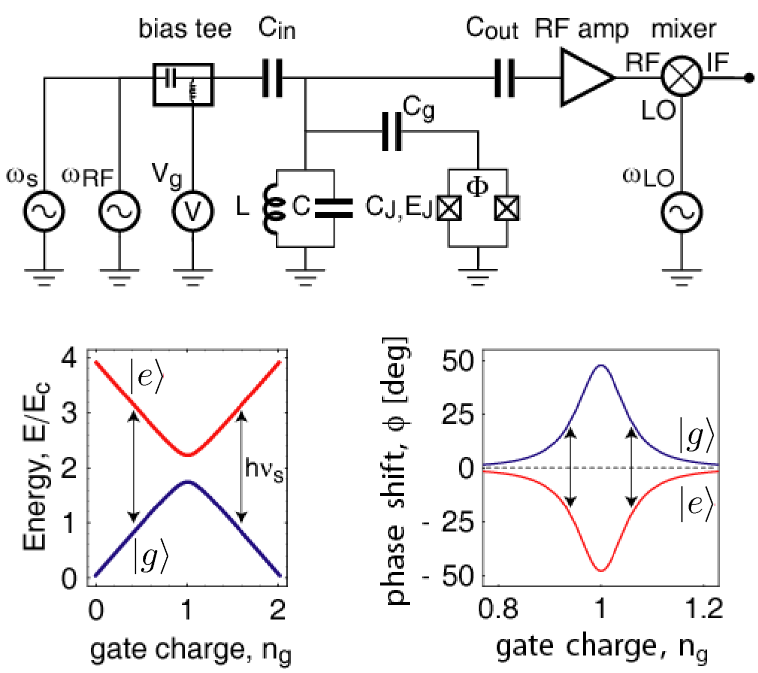
Abragam, Oxford University Press (1961)

Qubit Spectroscopy with Dispersive Read-Out

Realization



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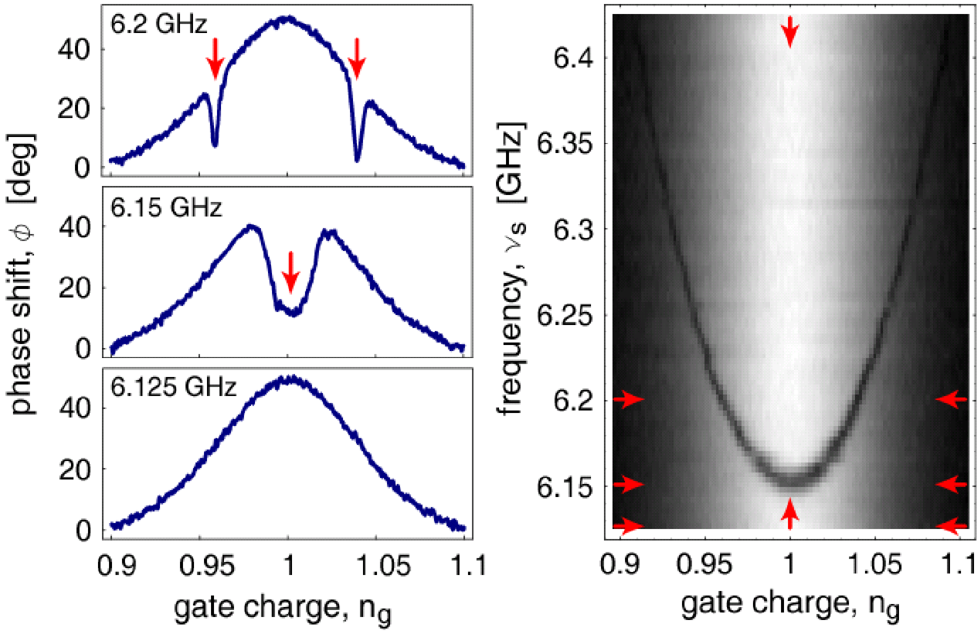

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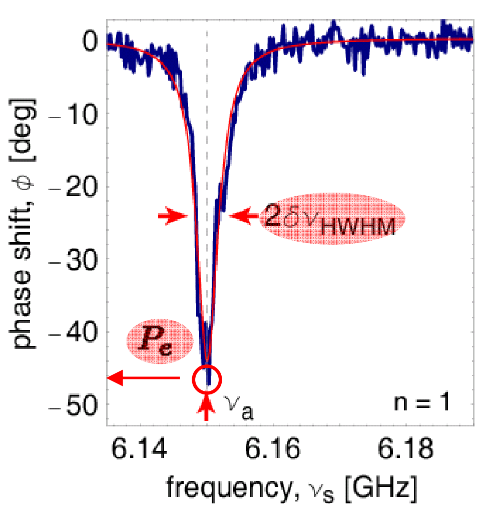

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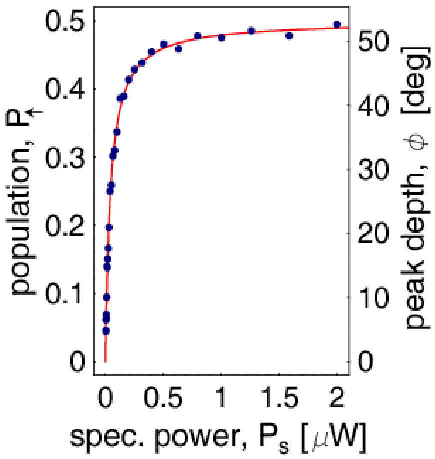
Abragam, Oxford University Press (1961)

Excited State Population

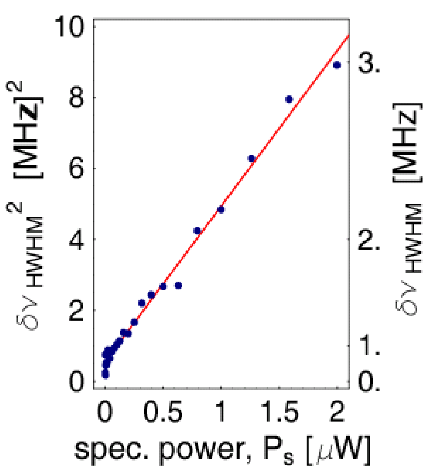


peak depth → population (saturation):

$$P_e = 1 - P_g = \frac{1}{2}\frac{\Omega_R^2 T_1 T_2}{1 + \Omega_R^2 T_1 T_2}$$

D. I. Schuster, A. Wallraff, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Girvin, and R. J. Schoelkopf, *Phys. Rev. Lett.* **94,** 123062 (2005)

Line Width



line width → coherence time:

$$2\pi\delta\nu_{\rm HWHM} = \frac{1}{T_2'} = \sqrt{\frac{1}{T_2^2} + \Omega_R^2\frac{T_1}{T_2}}$$

$$\mathrm{Min}(\delta\nu_{\rm HWHM}) \sim 750\,\mathrm{kHz} \rightarrow \mathrm{T}_2 > 200\,\mathrm{ns}$$

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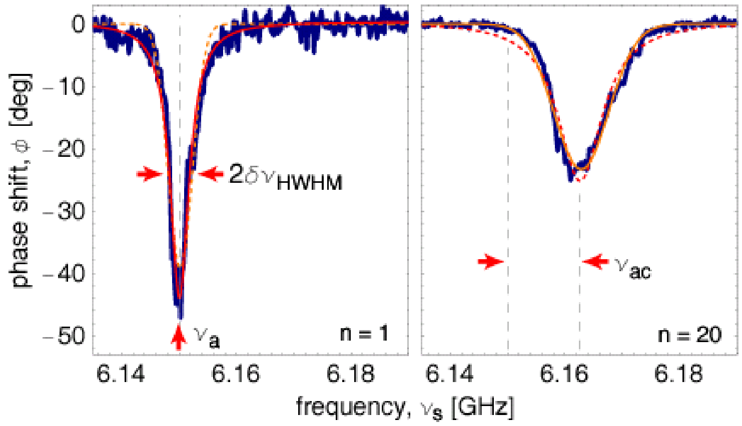
AC-Stark Effect & Measurement Back Action

for $\Delta_{a,r} = \omega_a - \omega_r \gg g$

ac-Stark (light) shift

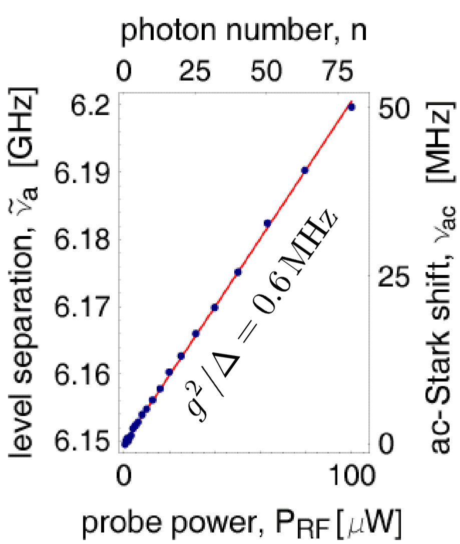
$$H \approx \hbar\omega_r a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta} + \frac{2g^2}{\Delta}a^\dagger a\right)\sigma_z$$

photon number dependence of line position and width



D. I. Schuster *et al., Phys. Rev. Lett.* **94,** 123062 (2005)

AC-Stark Effect: Line Shift



- **ac-Stark (light) shift:**

$$\nu_{ac} = \bar{n}\frac{g^2}{\pi\Delta_{a,r}}$$

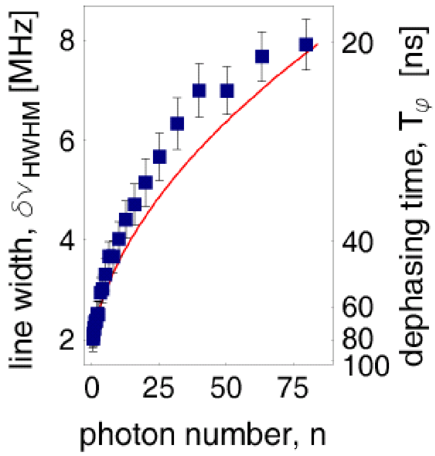
- **here $\nu_{ac}/\bar{n} = 0.6\,\mathrm{MHz}$**
- **use for photon number calibration**

D. I. Schuster, A. Wallraff, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Girvin, and R. J. Schoelkopf, *Phys. Rev. Lett.* **94,** 123062 (2005)

AC-Stark Effect: Line Broadening

photon shot noise:

- quantum fluctuations δn in coherent field with n photons
- random fluctuations in qubit level separation (ac-Stark)



- **for large n gaussian fluctuations in n:**

$$\delta\nu_{\mathrm{HWHM}} = \sqrt{2\ln 2}\,\frac{g^2}{\pi\Delta_{a,r}}\sqrt{\bar{n}}$$

- **characteristic measurement back-action**

D. I. Schuster, A. Wallraff, …, S. Girvin, and R. J. Schoelkopf,
Phys. Rev. Lett. **94,** 123062 (2005)

Coherent Control ...

… of a superconducting charge qubit.

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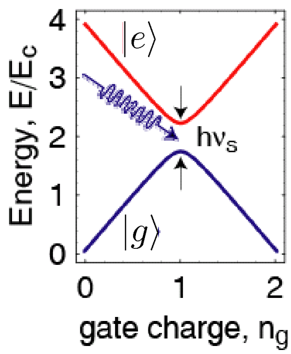
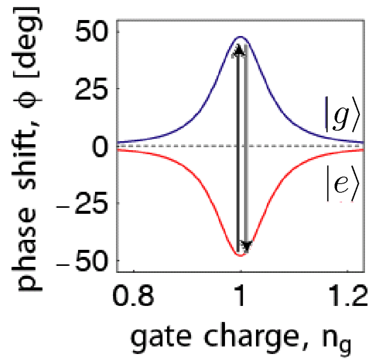
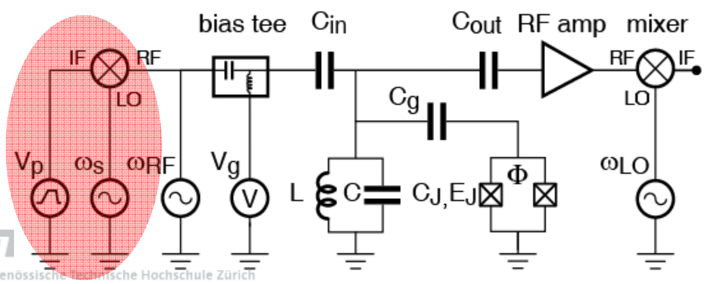
Coherent Control and Read-out in a Cavity





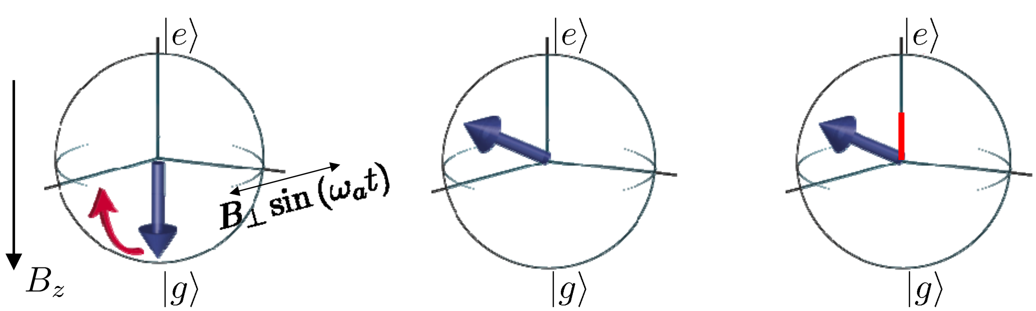
- apply resonant microwave pulse to qubit
- detect change of phase

realization:



- simultaneous control and measurement

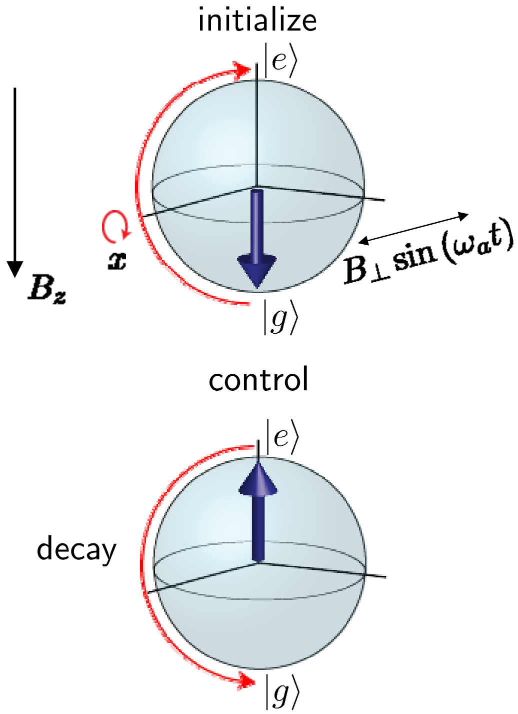
Coherent Control of a Qubit in a Cavity

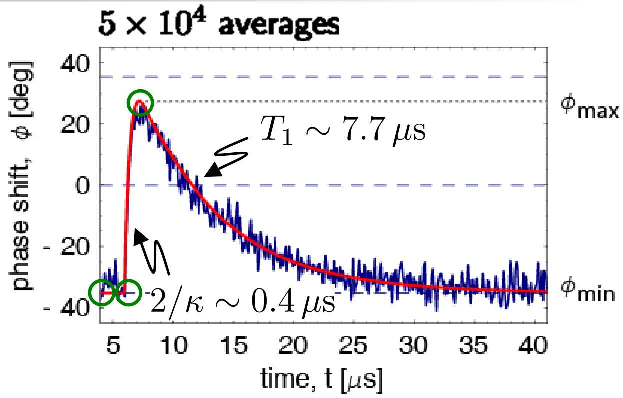


initialize | after operation | measure projection

- qubit state represented on a Bloch sphere
- NMR style operations
- vary length, amplitude and phase of pulse to control qubit state

Qubit Control and Readout



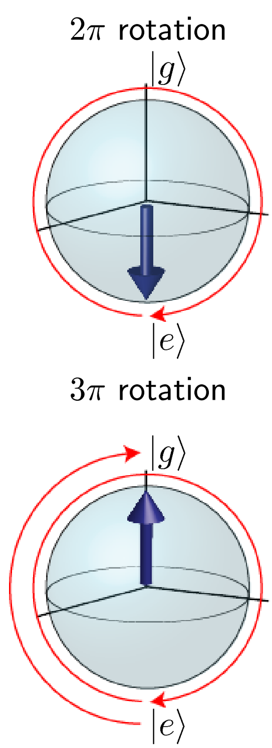


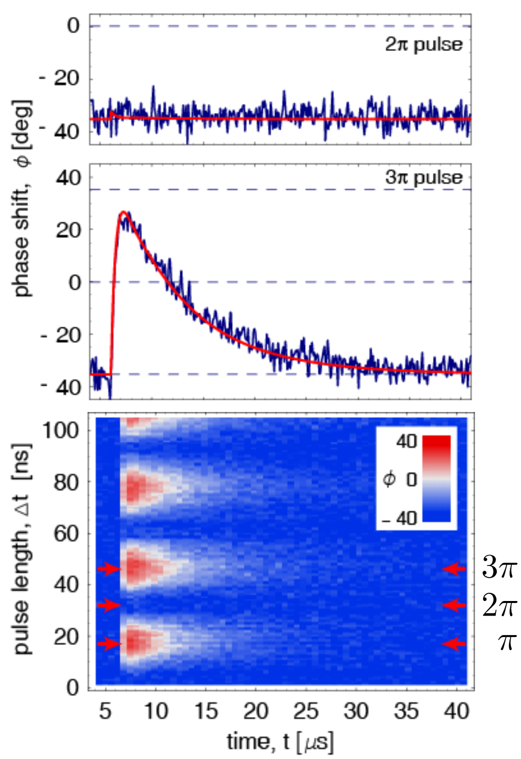
measurement properties:

- **continuous**
- **dispersive**
- **quantum non-demolition**
- **in good agreement with predictions**

Wallraff, Schuster, Blais, ... Girvin, and Schoelkopf, *Phys. Rev. Lett.* **95**, 060501 (2005)

Varying the Control Pulse Length



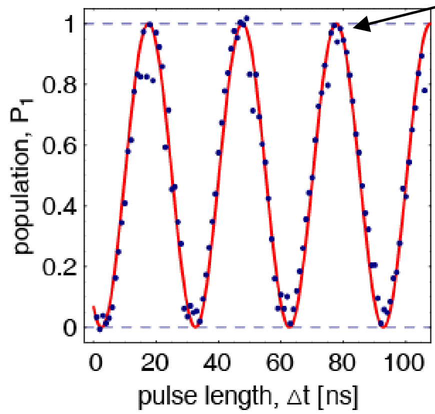


Wallraff, Schuster, Blais, ... Girvin, Schoelkopf, *PRL* **95**, 060501 (2005)

High Visibility Rabi Oscillations

Rabi oscillations:

visibility $95 \pm 5\%$



for superconducting qubits:

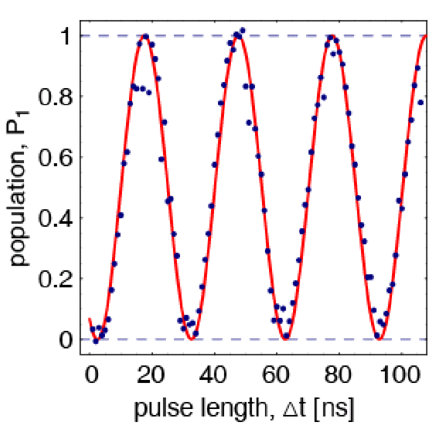
- **high visibility**
- **well characterized and understood measurement**
- **good control accuracy**

A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, J. Majer, S. M. Girvin, and R. J. Schoelkopf, *Phys. Rev. Lett.* **95**, 060501 (2005)

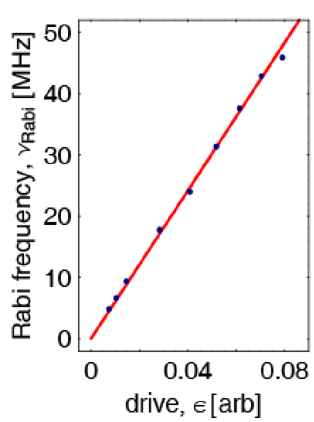
Rabi Frequency

pulse scheme:

Rabi oscillations:



Rabi frequency:



- **linear dependence on drive amplitude**

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Measurements of Coherence Time

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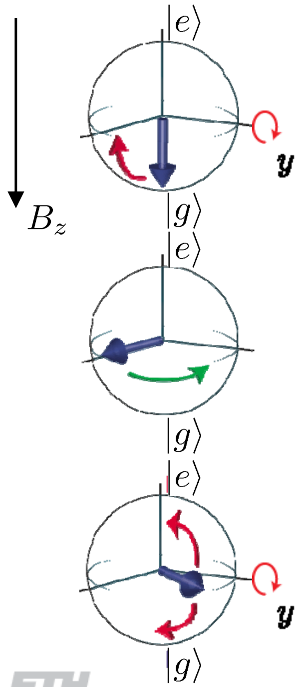
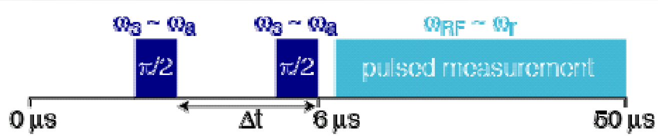
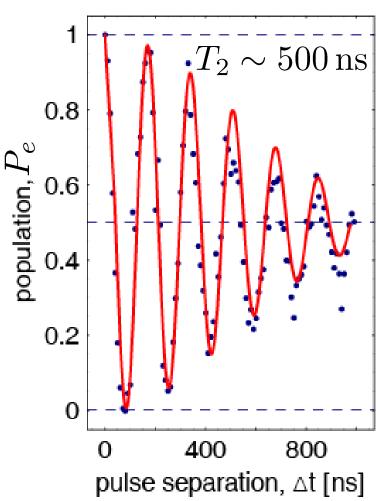
Ramsey Fringes: Coherence Time Measurement

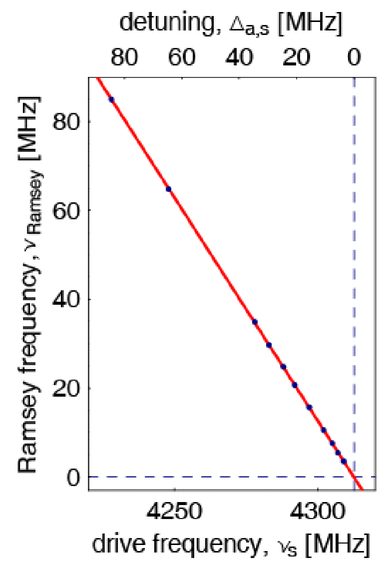
pulse scheme:





Ramsey fringes:





A. Wallraff *et al.*, *Phys. Rev. Lett.* **95**, 060501 (2005)

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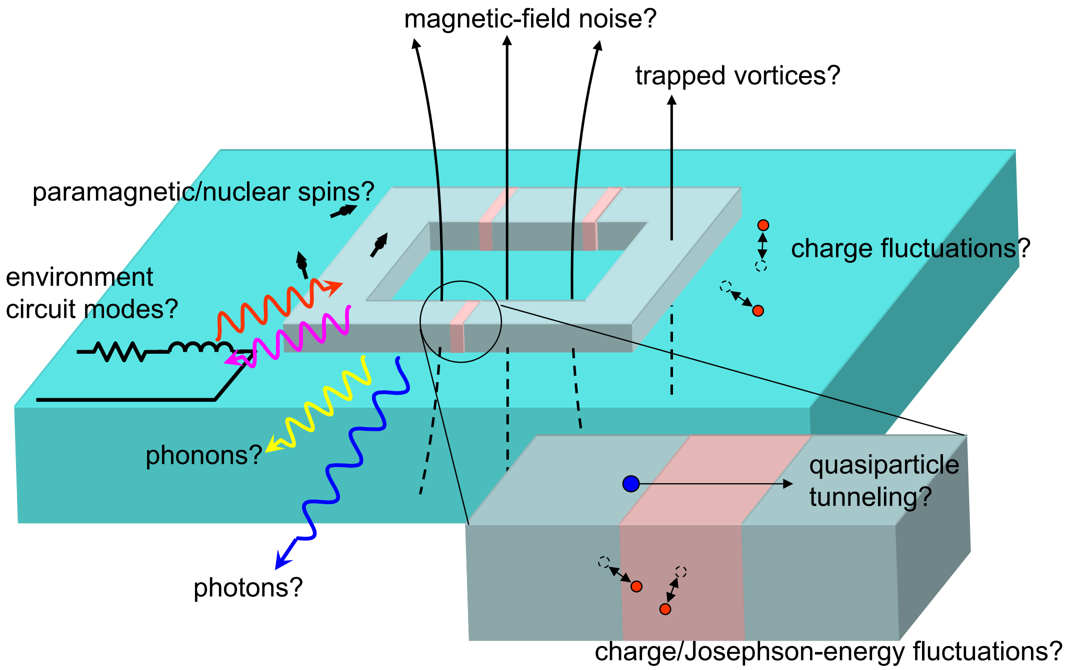
Sources of Decoherence

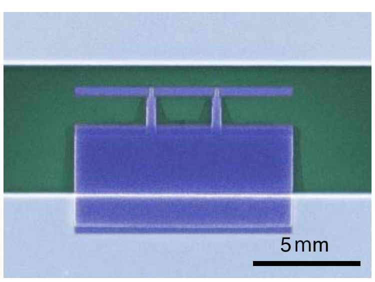


G. Ithier *et al.*, *Phys. Rev. B* **72**, 134519 (2005)

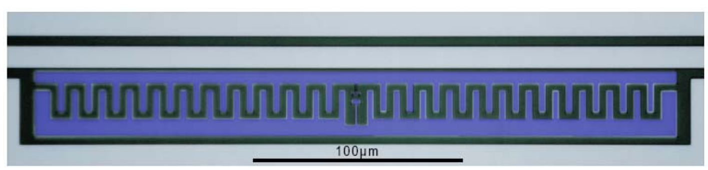
Reduce Decoherence using Symmetries

a Cooper pair box with a small charging energy

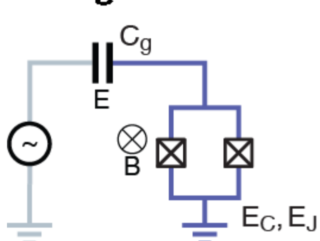
standard CPB:

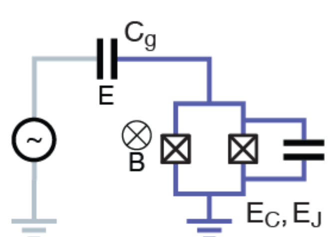


transmon:



circuit diagram:

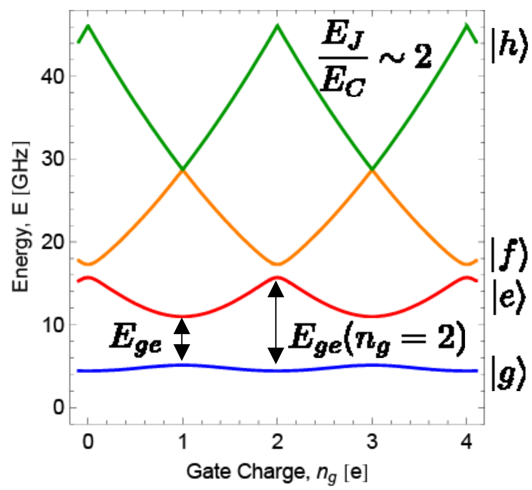




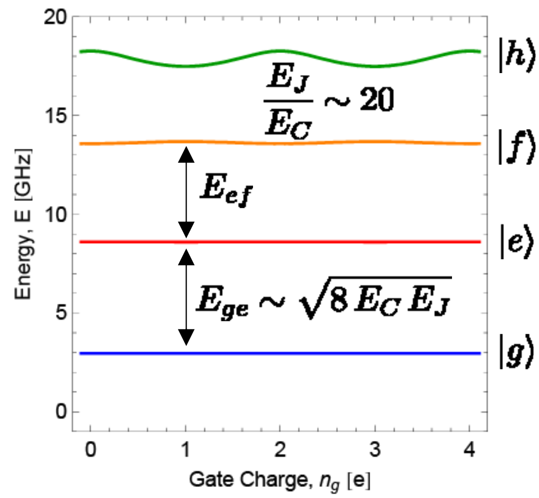
J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)
J. Schreier *et al.*, Phys. Rev. B **77**, 180502 (2008)

The Transmon: A Charge Noise Insensitive Qubit

Cooper pair box energy levels



Transmon energy levels



dispersion

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$

relative anharmonicity

$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$

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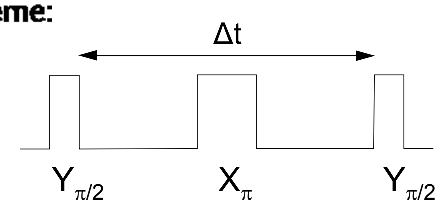
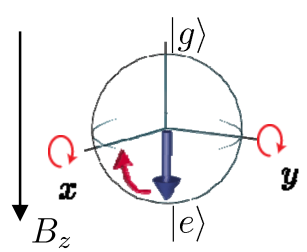
J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

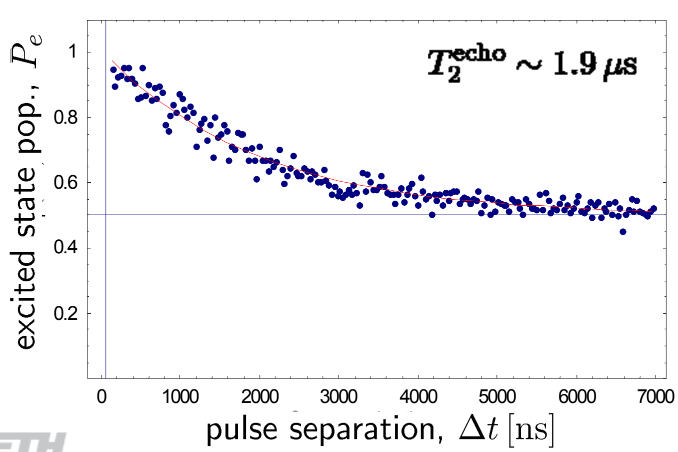
Reduce Decoherence Dynamically: Spin Echo

pulse scheme:





result:



- refocusing
- elimination of low frequency fluctuations
- increased effective coherence time

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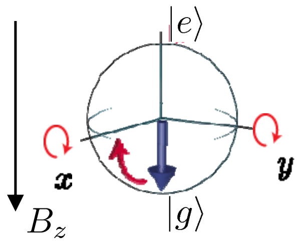
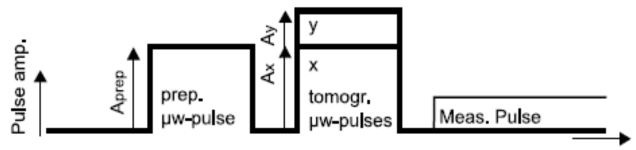
L. Steffen *et al.* (2007)

One-Qubit Tomography

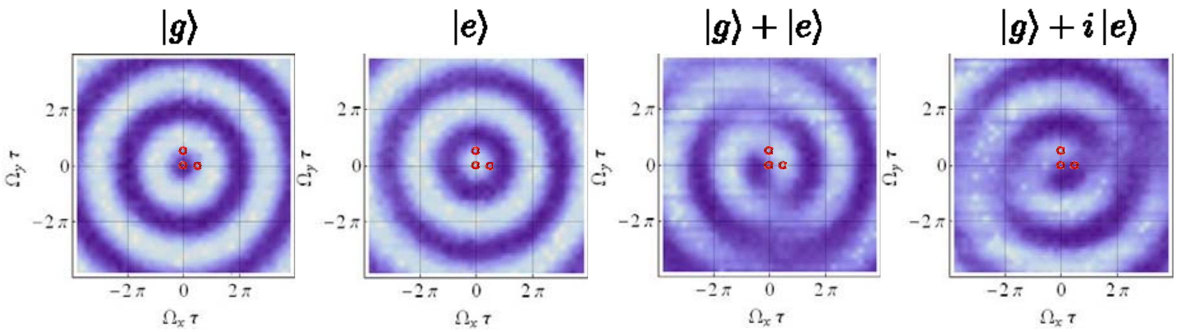
Bloch sphere:



pulse sequence:



initial states:

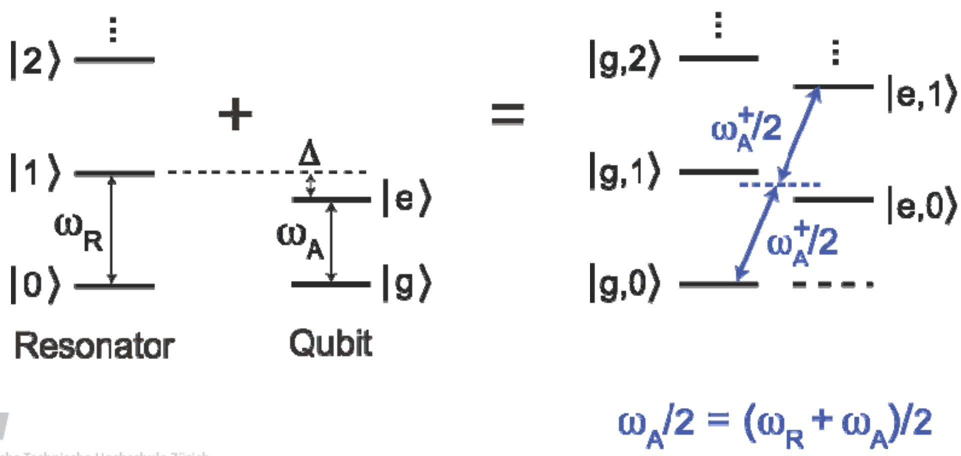


$\langle\sigma_z\rangle$ response vs. tomography pulse length along x and y simultaneously

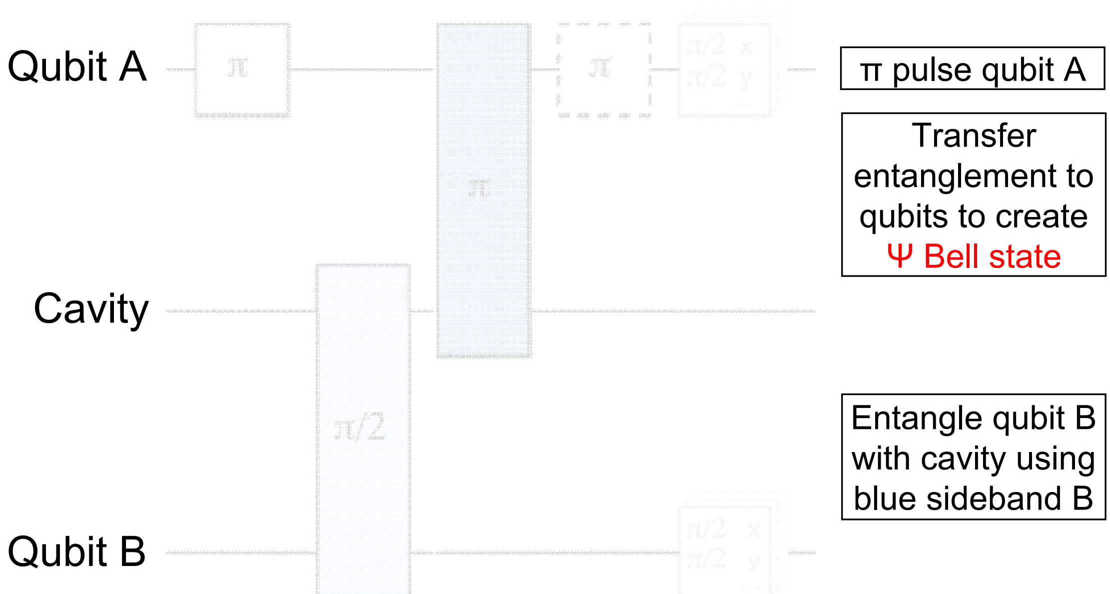
Coupling Superconducting Qubits and Generating Entanglement using Sideband Transitions

Sideband Transitions in Circuit QED

- System in dispersive limit (~uncoupled)
- Weak dispersive coupling still allows joint excitations to be driven
- Use sidebands to generate entanglement between qubit and resonator
- Sideband transitions forbidden to first order: use two photon transition

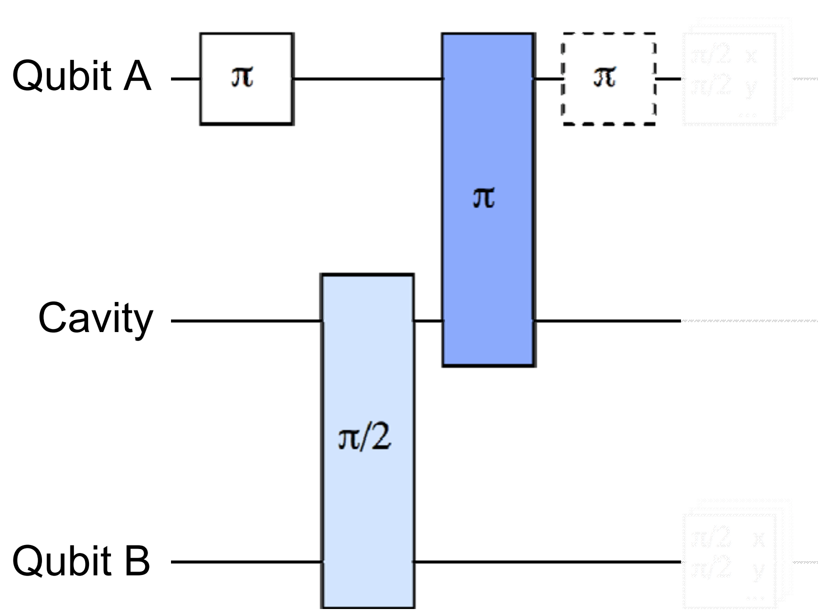


Bell State Preparation



$$|gg0\rangle \longrightarrow |eg0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|eg0\rangle + |ee1\rangle) \longrightarrow \boxed{\frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle)} \otimes |0\rangle$$

Bell State Preparation



π pulse qubit A to convert to Φ Bell state

π pulse qubit A

Transfer entanglement to qubit A to create Ψ Bell state

Entangle qubit B with cavity using blue sideband

Characterise the final state using tomography with joint msrmnt

$$\ldots \longrightarrow \frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle) \otimes |0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|gg\rangle + |ee\rangle) \otimes |0\rangle$$

Sidebands with 2 qubits and 0,1 photons

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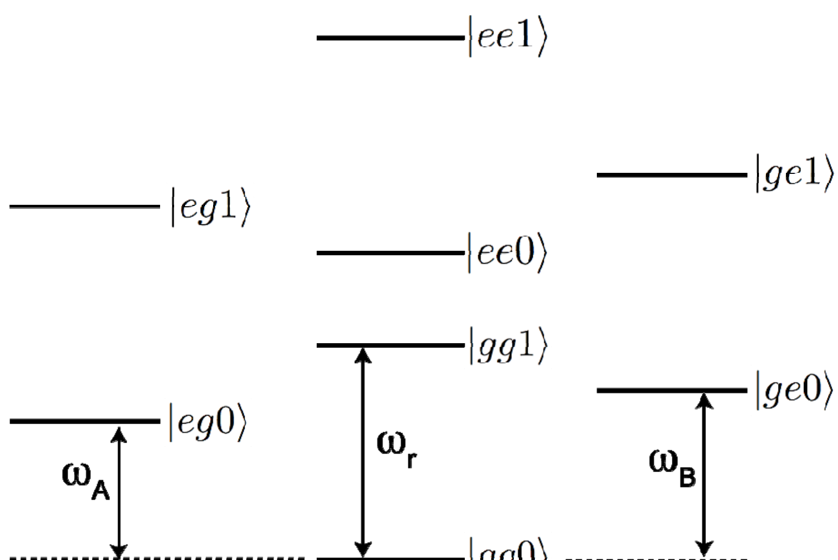
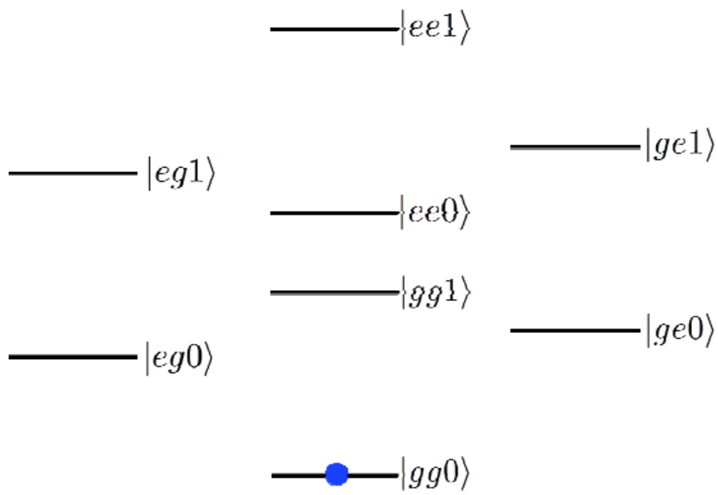
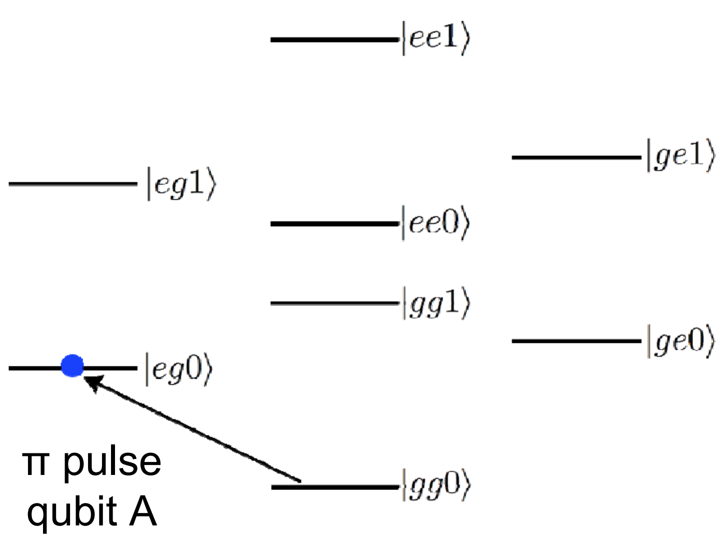


Bell state preparation sequence



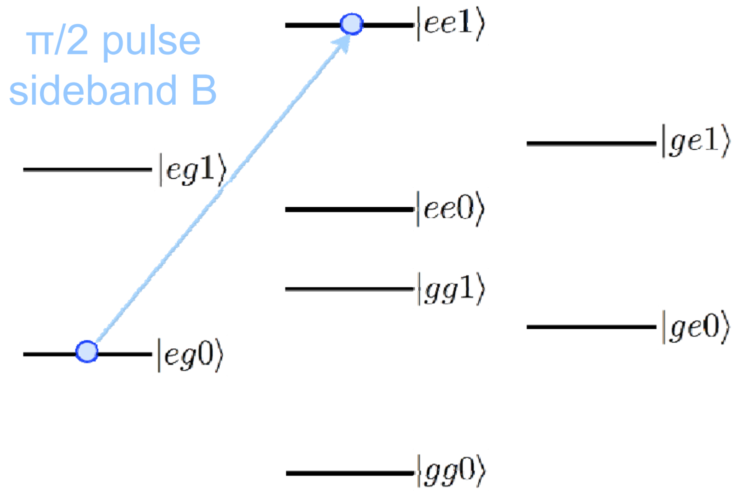
$$|gg0\rangle$$

Bell state preparation sequence



$$|gg0\rangle \longrightarrow |eg0\rangle$$

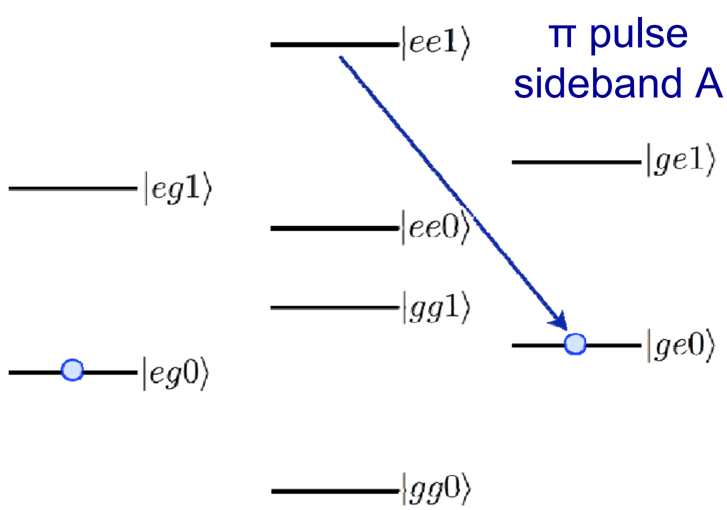
Bell state preparation sequence



Entangle qubit B with photon

$$|gg0\rangle \longrightarrow |eg0\rangle \longrightarrow \boxed{\frac{1}{\sqrt{2}}(|eg0\rangle + |ee1\rangle)}$$

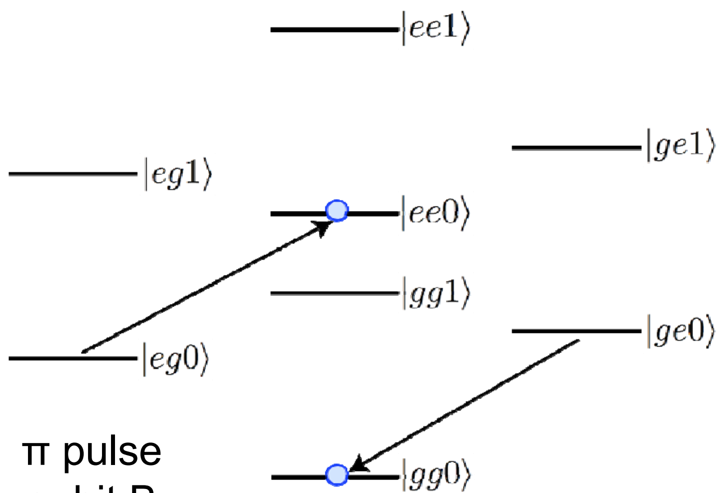
Bell state preparation sequence



Transfer entanglement to qubits to create
Ψ Bell state

$$|gg0\rangle \longrightarrow |eg0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|eg0\rangle + |ee1\rangle) \longrightarrow \boxed{\frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle)} \otimes |0\rangle$$

Bell state preparation sequence

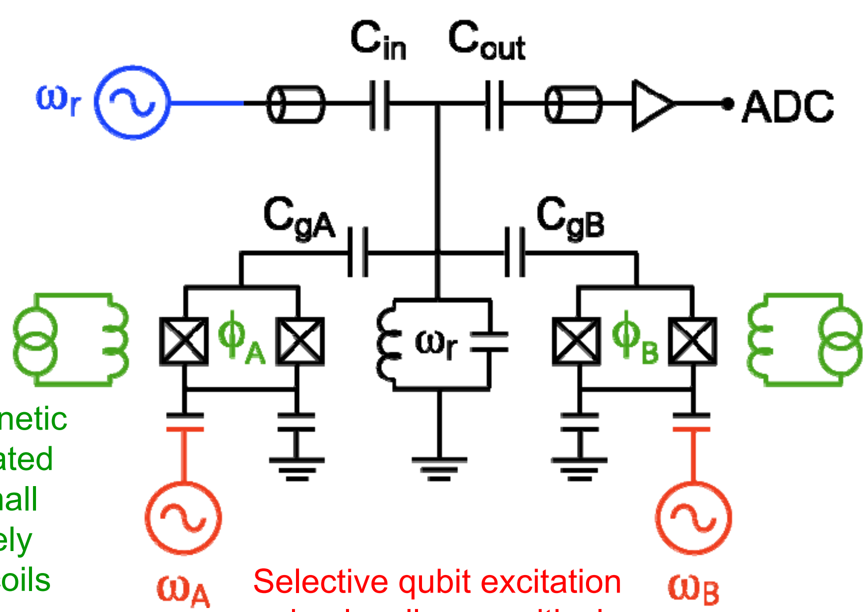


π pulse
qubit B

Transfer to
Φ Bell state

$$\ldots \longrightarrow \frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle) \otimes |0\rangle \longrightarrow \boxed{\frac{1}{\sqrt{2}}(|gg\rangle + |ee\rangle)} \otimes |0\rangle$$

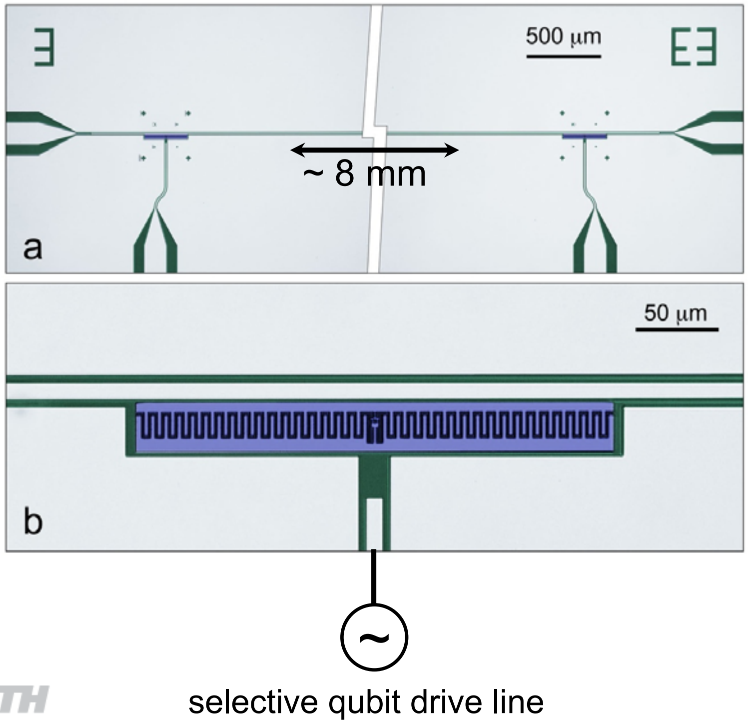
2-Qubit Circuit QED with Selective Control



Local magnetic fields created using small inductively coupled coils

Selective qubit excitation using locally capacitively coupled drive lines

2-Qubit Circuit QED Chip with Selective Control



- Two near identical superconducting qubits
- Local control of magnetic flux allows independent selection of qubit transition frequencies
- Local drive lines allow selective excitation of individual qubits

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Spectroscopy on selective drive lines

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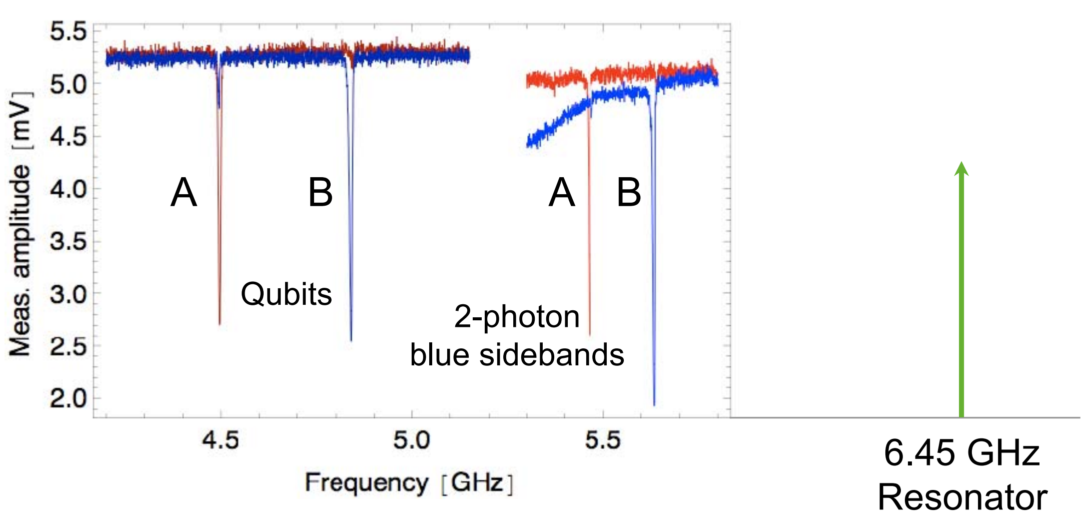
‣spectral lines observed halfway between qubits and resonator
=> 2-photon blue sidebands



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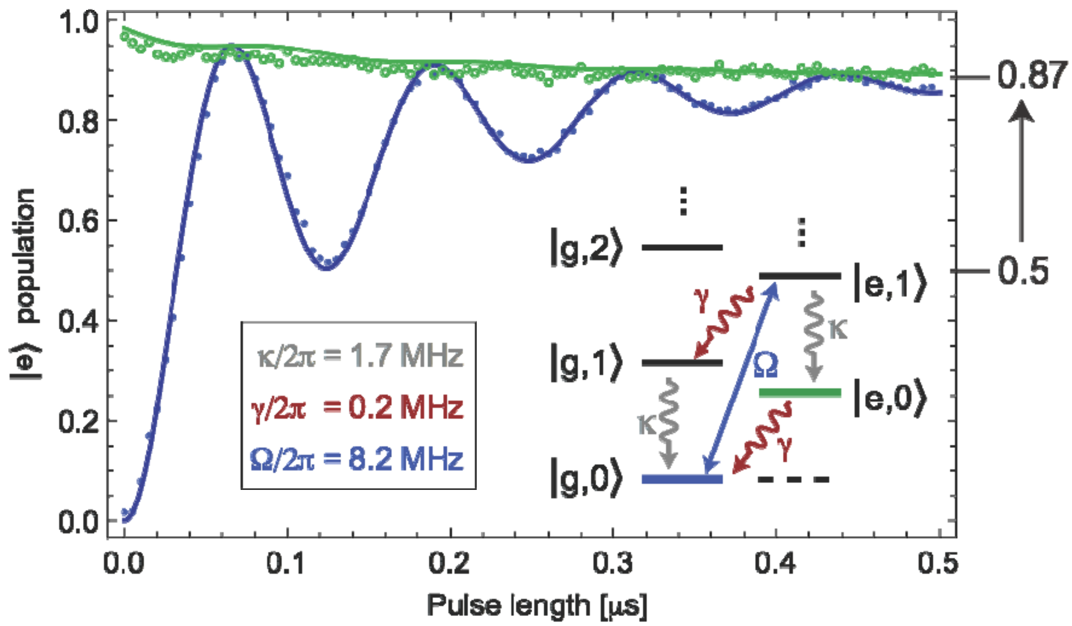
Blue Sideband Rabi Oscillations

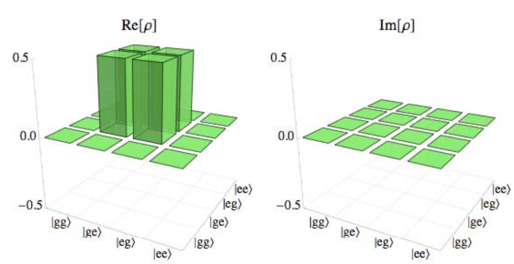


Full Two-Qubit Tomography

- Quantum state characterised with its density operator $\rho = |\Psi\rangle\langle\Psi|$
- Consider for example the Bell state $|\Psi_+\rangle = \frac{1}{\sqrt{2}}(|ge\rangle + |eg\rangle)$

$$\rho_{\Psi_+} = |\Psi_+\rangle\langle\Psi_+| = \frac{1}{2}(|ge\rangle\langle ge| + |ge\rangle\langle eg| + |eg\rangle\langle ge| + |eg\rangle\langle eg|)$$

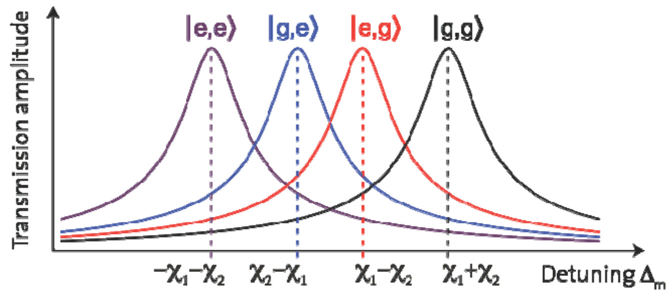
$$\rho_{\Psi_+} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



- Matrix is Hermitian, trace 1 => for 2 qubits, 15 independent parameters
- Full measurement of density matrix possible with repeated experiments and state tomography with 15 combinations of single qubit rotations

Joint Two-Qubit State Measurement

- Resonator Hamiltonian: $\hat{H} = \hbar(\Delta_m + \overbrace{\chi_1 \hat{\sigma}_z^1 + \chi_2 \hat{\sigma}_z^2}^{\hat{\chi}})\hat{a}^\dagger \hat{a}$
- Two-qubit state dependent resonator frequency shift:

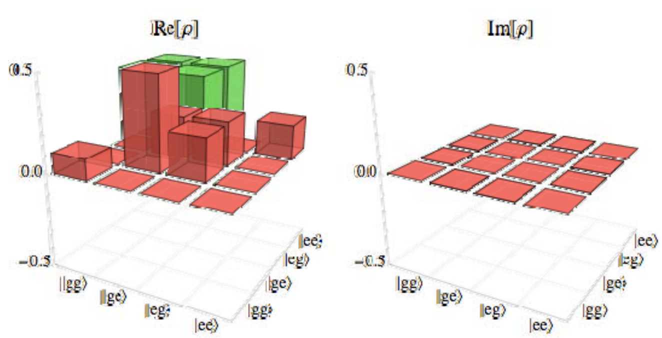


- Measured quantities are non-linear in the frequency shift

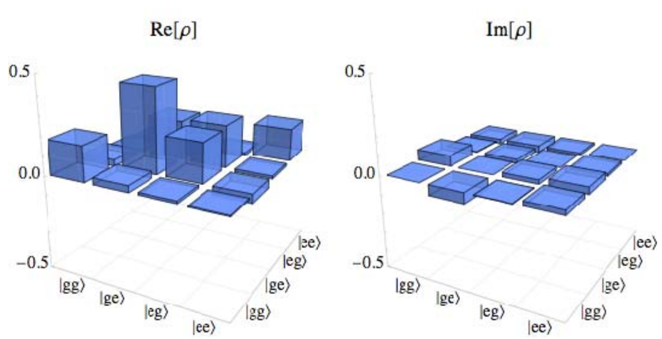
$$\hat{M}_I = \frac{2(\Delta_m + \hat{\chi})}{(\Delta_m + \hat{\chi}) + (\kappa/2)^2} \qquad \hat{M}_Q = \frac{i\kappa}{(\Delta_m + \hat{\chi}) + (\kappa/2)^2}$$

- => $\hat{\sigma}_z^1 \otimes \hat{\sigma}_z^2$ terms are present in the measurement operator, and two qubit correlations are intrinsically measurable

Bell State $|\Psi_+\rangle = \frac{1}{\sqrt{2}}(|ge\rangle + |eg\rangle)$

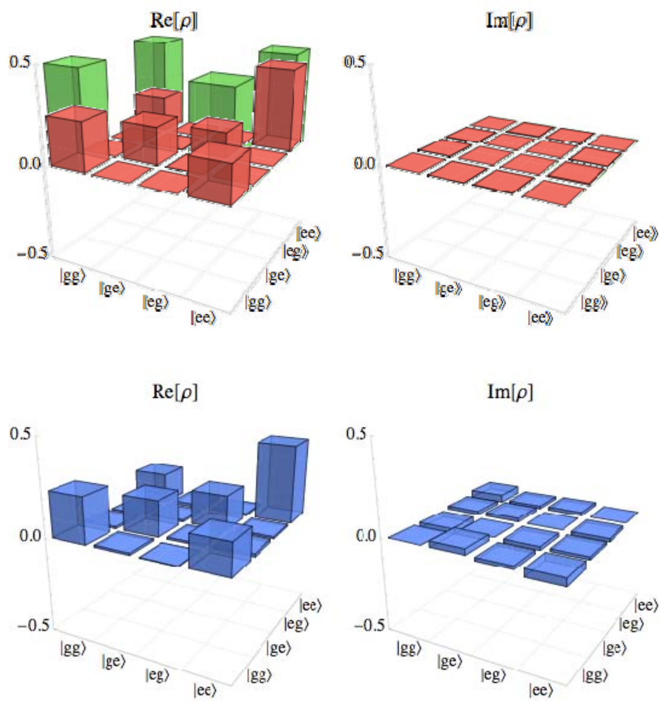


Simulation
F = 76%



Experimental
state fidelity
F = 73%

Bell State $|\Phi_+\rangle = \frac{1}{\sqrt{2}}(|gg\rangle + |ee\rangle)$



Simulation
F = 72%

Experiment
F = 72%

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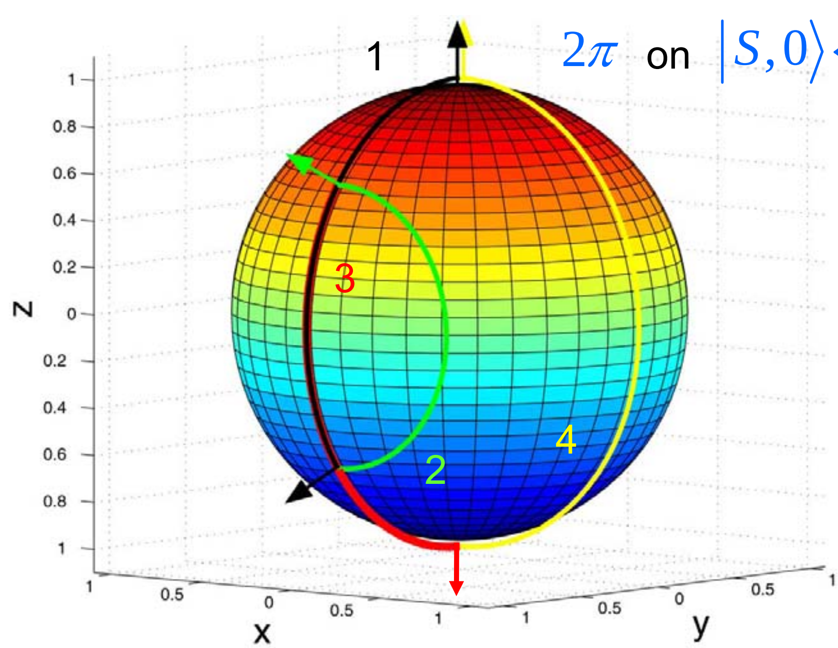
A phase gate with 4 pulses

$$R(\theta,\phi) = R_1^+(\pi,\pi/2)\,R_1^+\left(\pi/\sqrt{2},0\right)R_1^+(\pi,\pi/2)\,R_1^+\left(\pi/\sqrt{2},0\right)$$

2π on $|S,0\rangle \leftrightarrow |D,1\rangle$



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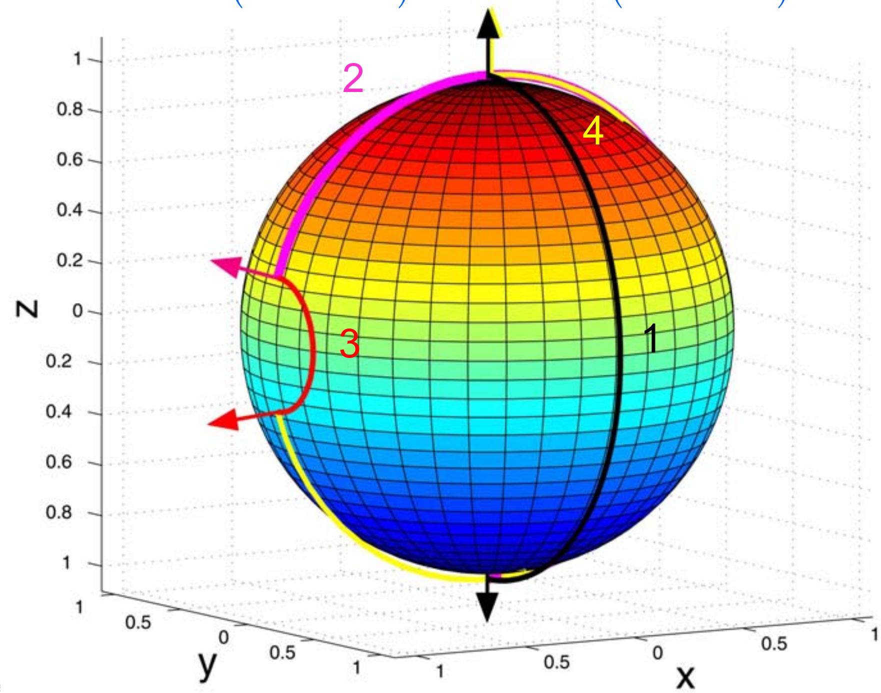
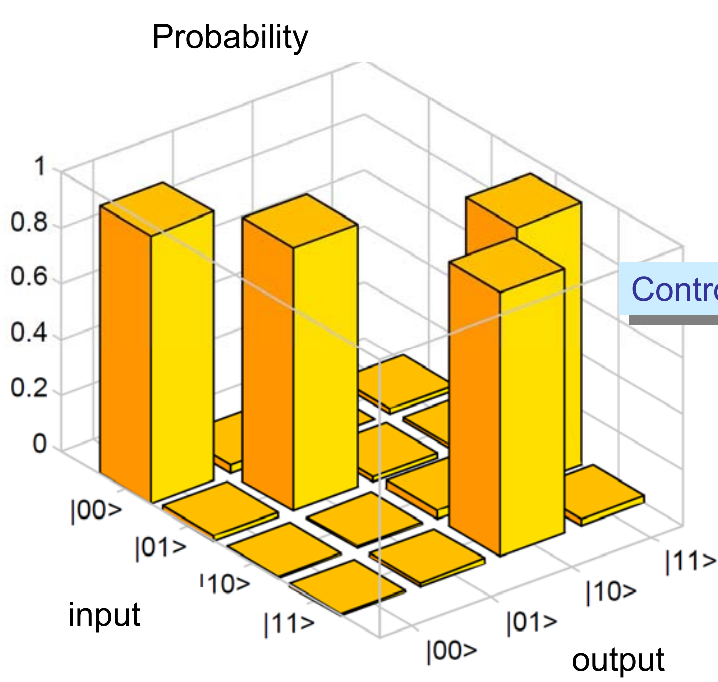
Population of |S,1> - |D,2> remains unaffected

$$R(\theta,\phi) = R_1^+\left(\pi\sqrt{2},\pi/2\right)R_1^+\left(\pi,0\right)R_1^+\left(\pi\sqrt{2},\pi/2\right)R_1^+\left(\pi,0\right)$$



Truth table of the CNOT



$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

Control bit   Target bit

The 5 (+2) DiVincenzo Criteria for Implementation of a Quantum Computer:

in the standard (circuit approach) to **quantum information processing** (QIP)

#1. A scalable physical system with well-characterized qubits. ✓

#2. The ability to initialize the state of the qubits to a simple fiducial state. ✓

#3. Long (relative) decoherence times, much longer than the gate-operation time. ✓

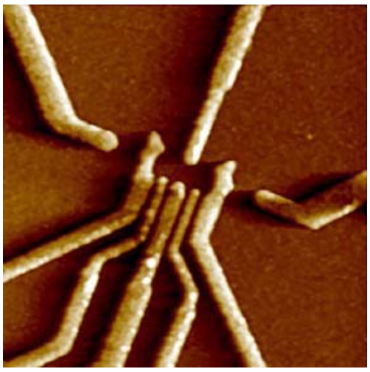
#4. A universal set of quantum gates. ✓

#5. A qubit-specific measurement capability. ✓

#6. The ability to interconvert stationary and mobile (or flying) qubits. ✓

#7. The ability to faithfully transmit flying qubits between specified locations. ✓

Quantum Information Processing with Semiconductor Quantum Dots



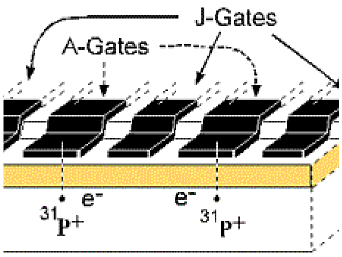
slides courtesy of Lieven Vandersypen, TU Delft

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

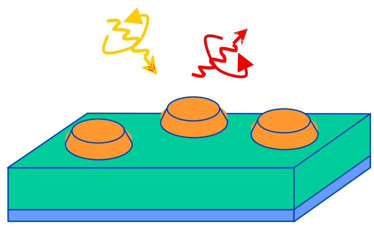
Can we access the quantum world

at the level of single-particles?

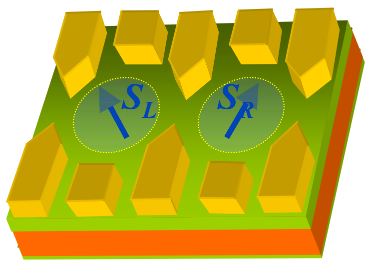
in a solid state environment?



Kane, Nature 1998



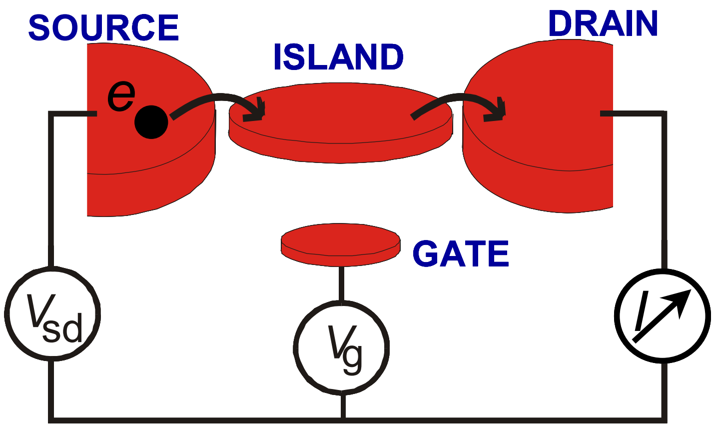
Imamoglu *et al*, PRL 1999



Loss & DiVincenzo PRA 1998

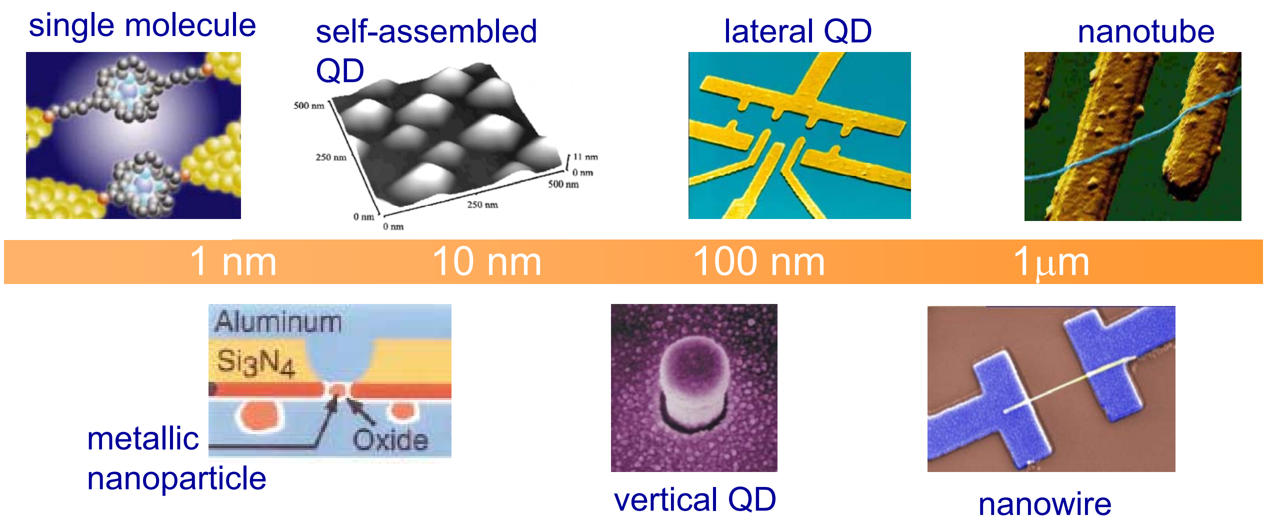
Electrically controlled and measured quantum dots

A small semiconducting (or metallic) island
where electrons are confined, giving a discrete level spectrum

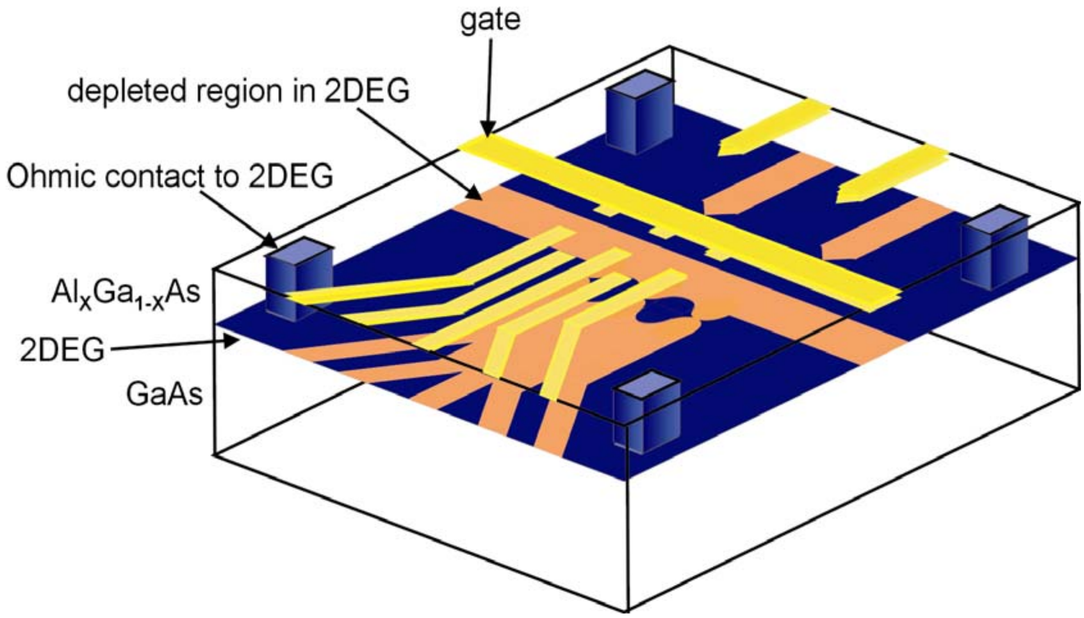


- Coupled via tunnel barriers to source and drain reservoirs
- Coupled capacitively to gate electrode, to control # of electrons

Examples of quantum dots

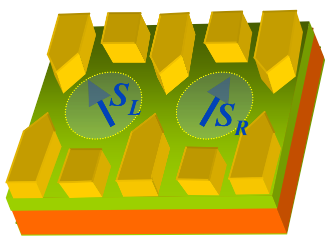


Electrostatically defined quantum dots



- Electrically measured (contact to 2DEG)
- Electrically controlled number of electrons
- Electrically controlled tunnel barriers

Spin qubits in quantum dots



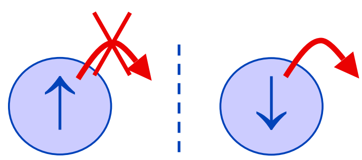
Loss & DiVincenzo, PRA 1998
Vandersypen et al., Proc. MQC02 (quant-ph/0207059)

Initialization 1-electron, low T, high B_0

$$H_0 \sim \Sigma\, \omega_i\, \sigma_{zi}$$

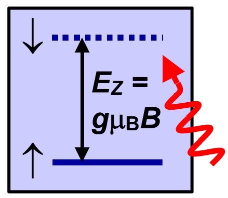
Read-out convert spin to charge

then measure charge



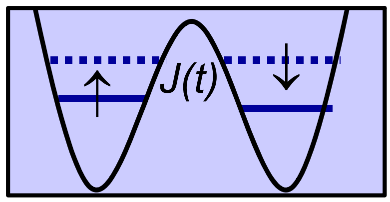
ESR pulsed microwave magnetic field

$$H_{RF} \sim \Sigma\, A_i(t) \cos(\omega_i t)\ \ \sigma_{xi}$$



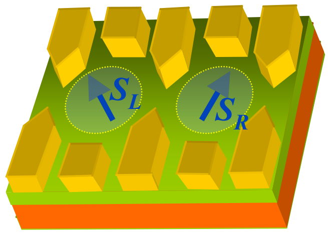
SWAP exchange interaction

$$H_J \sim \Sigma\, J_{ij}(t)\ \ \sigma_i \cdot \sigma_j$$



Coherence long relaxation time T_1
long coherence time T_2

Spin qubits in quantum dots



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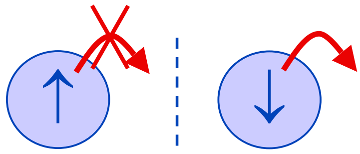
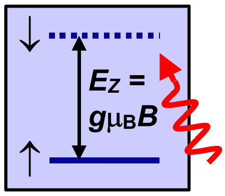
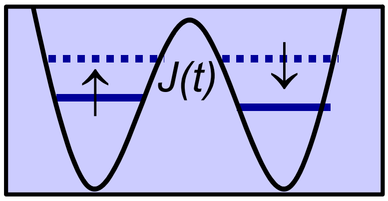
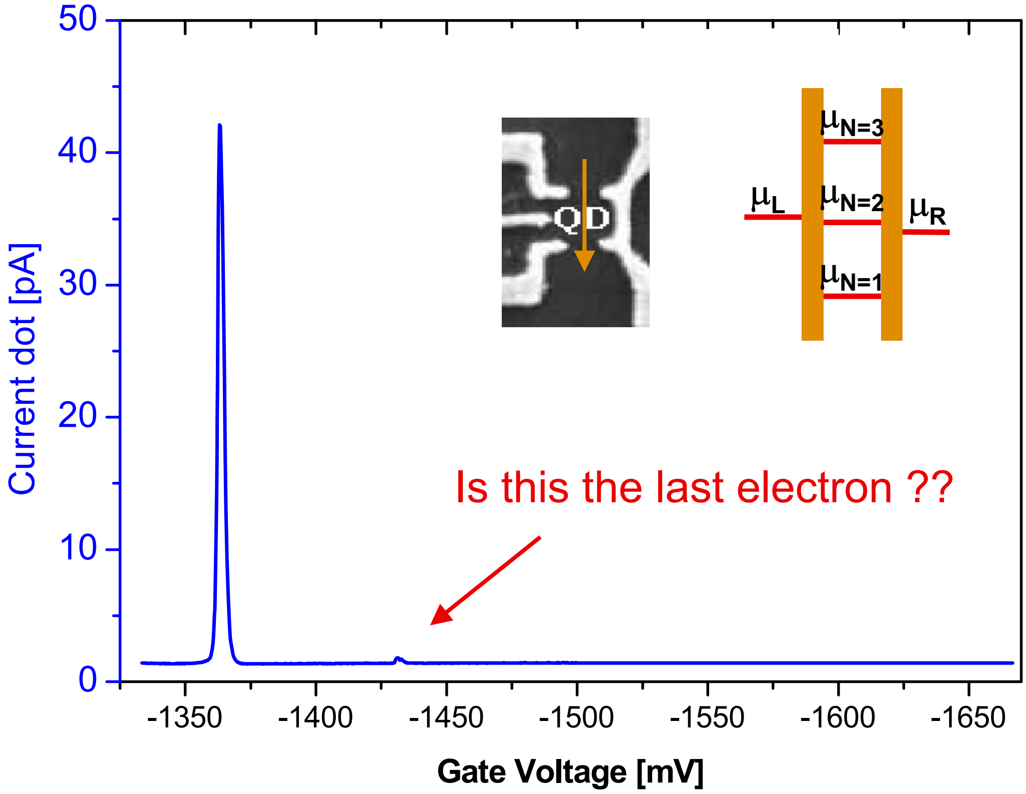




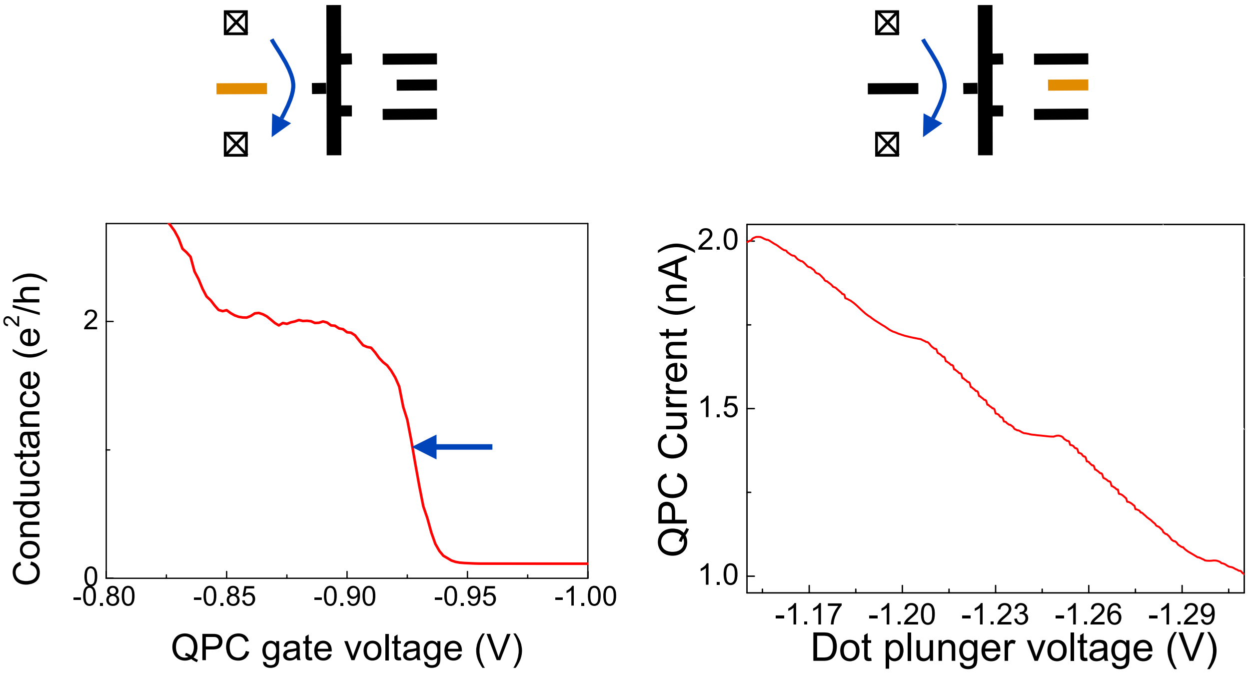


Transport through quantum dot - Coulomb blockade

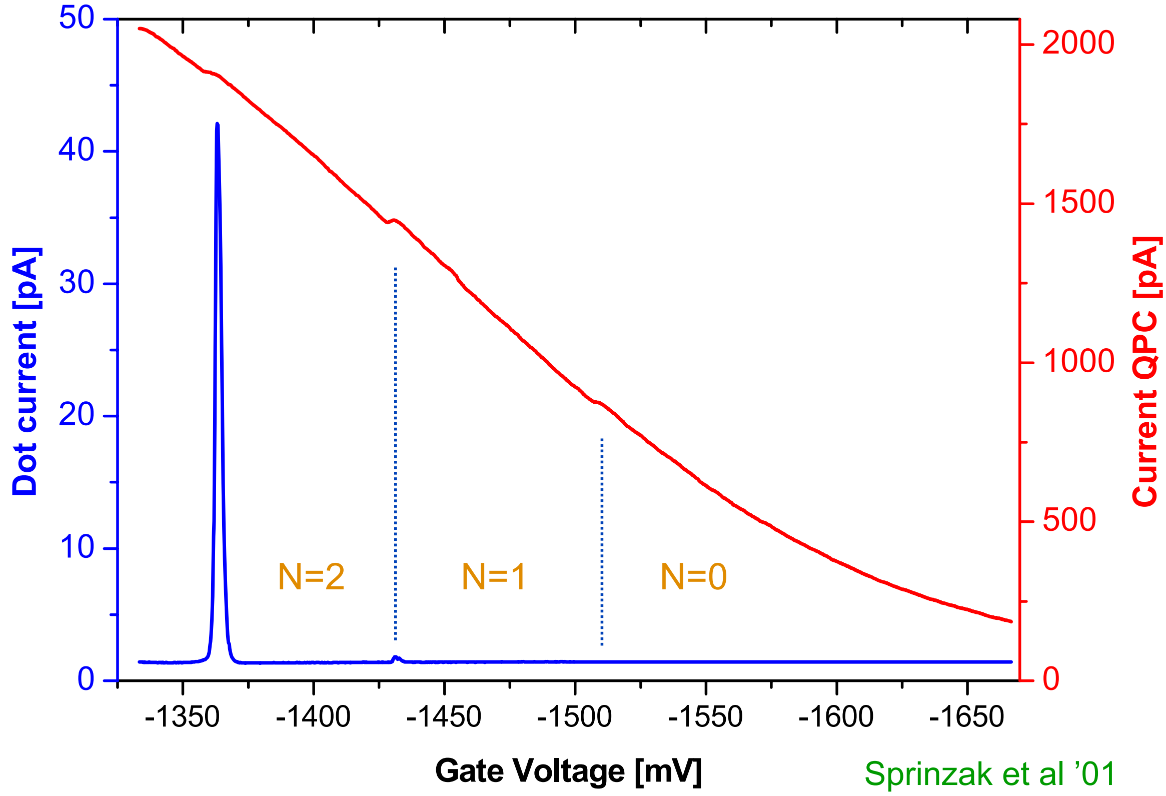


A quantum point contact (QPC) as a charge detector

Field *et al*, PRL 1993



The last electron!



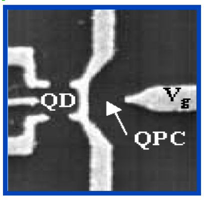
Sprinzak et al '01

Few-electron double dot design

Ciorga et al '99

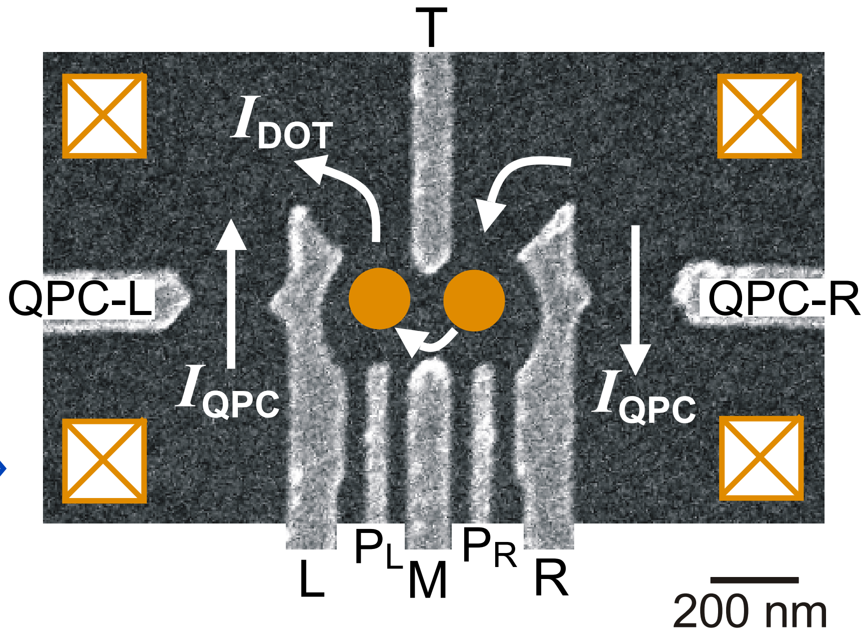
Open design

Field et al'93
Sprinzak et al '01



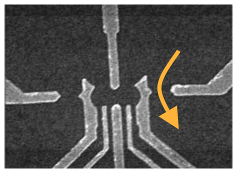
QPC for charge detection

Elzerman et al., PRB 2003

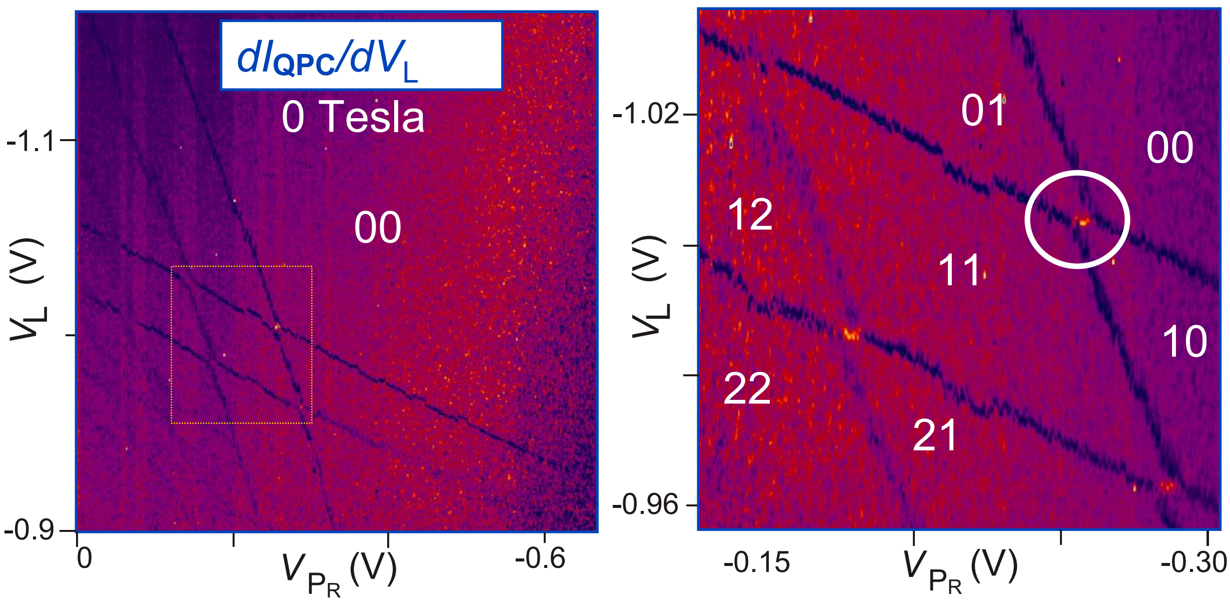


GaAs/AlGaAs wafers:
- NTT (T. Saku, Y. Hirayama)
- Sumitomo Electric
- Universität Regensburg (W. Wegscheider)

Few-electron double dot
Measured via QPC

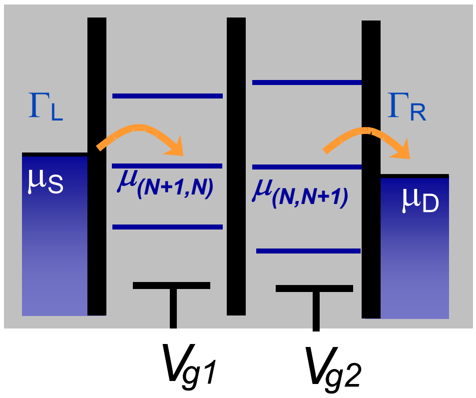


J.M. Elzerman et al., PRB **67**, R161308 (2003)



- Double dot can be emptied
- QPC can detect all charge transitions

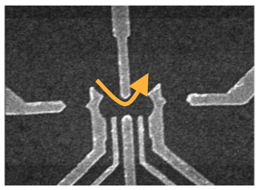
Single electron tunneling through two dots in series

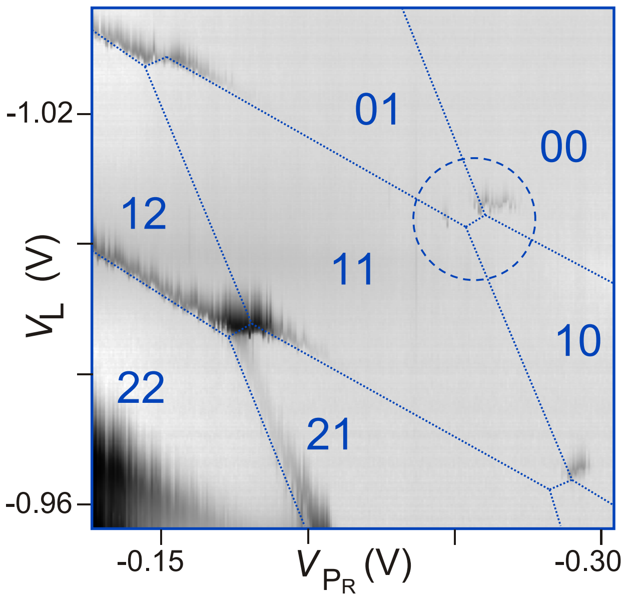


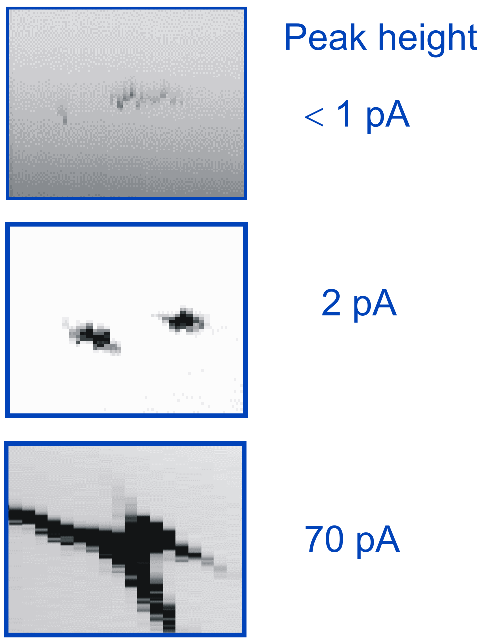
Few-electron double dot

Transport through dots

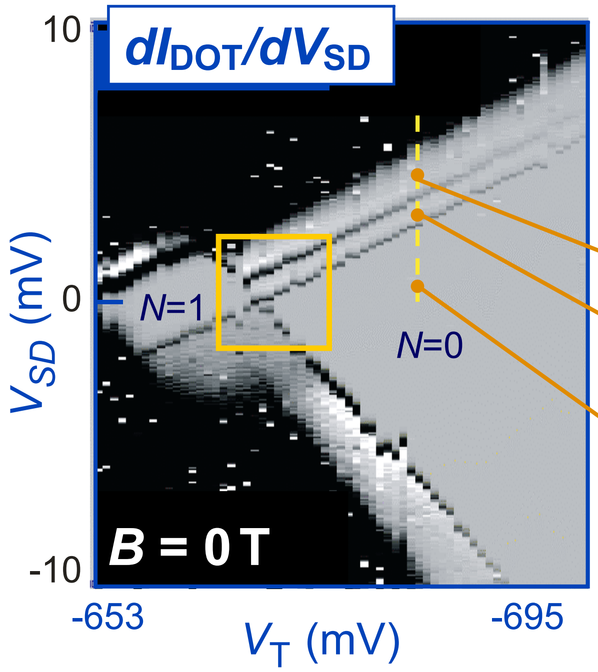
J. Elzerman et al., cond-mat/0212489

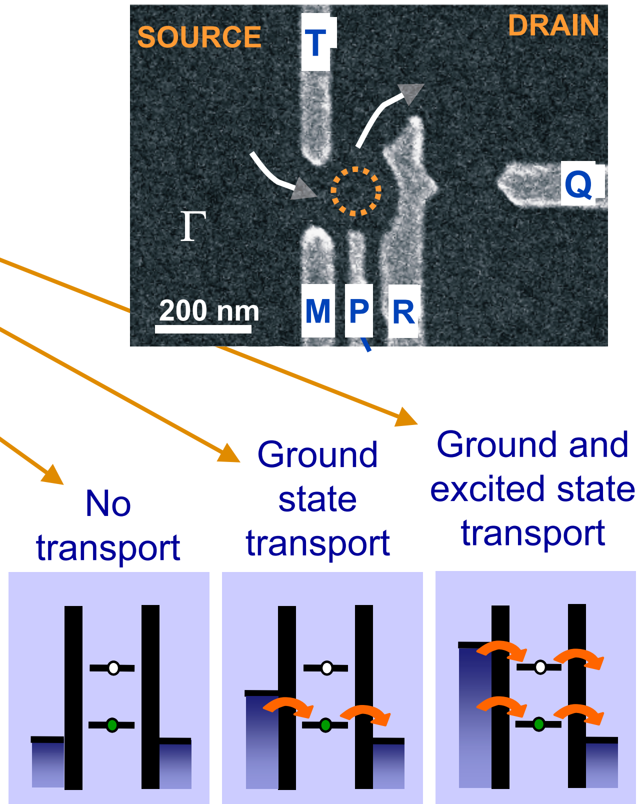






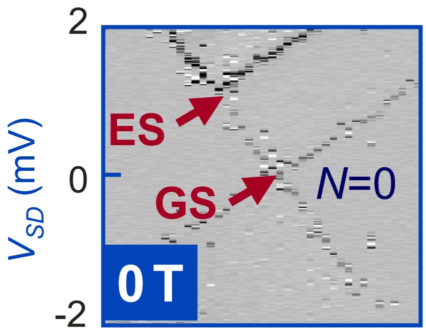
Energy level spectroscopy at $B = 0$



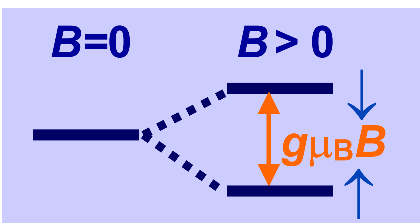


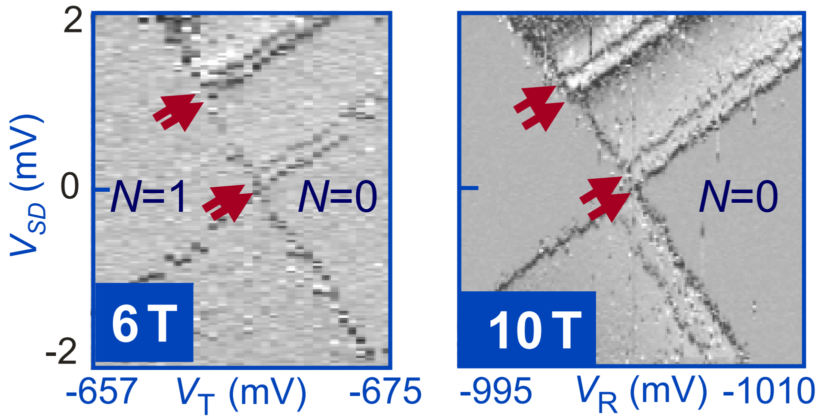
- ΔE ~ 1.1meV
- E_C ~ 2.5meV

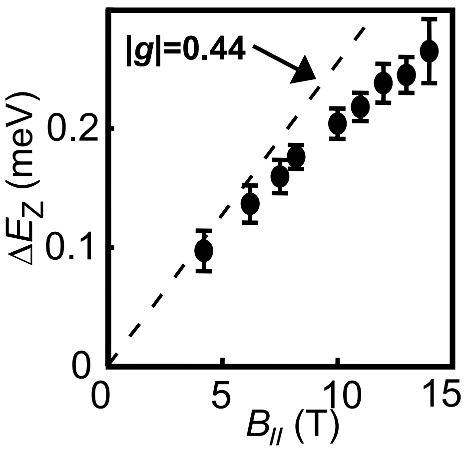
Single electron Zeeman splitting in $B_{//}$



Hanson et al, PRL 91, 196802 (2003)
Also: Potok et al, PRL 91, 016802 (2003)

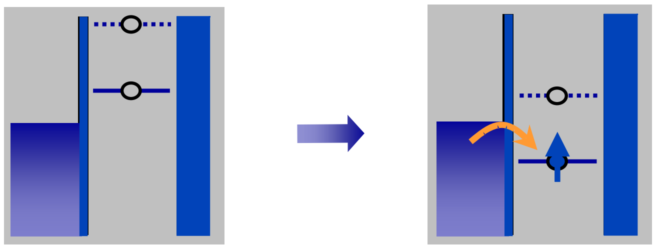




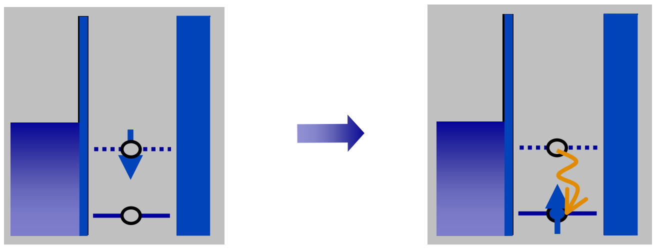


Initialization of a single electron spin

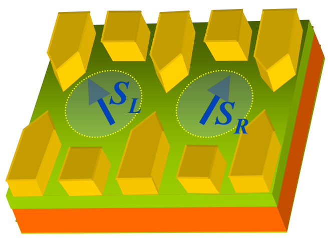
Method 1:
spin-selective
tunneling



Method 2:
relaxation to
ground state



Spin qubits in quantum dots



Loss & DiVincenzo, PRA 1998
Vandersypen et al., Proc. MQC02 (quant-ph/0207059)

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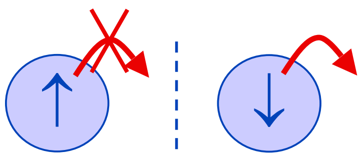
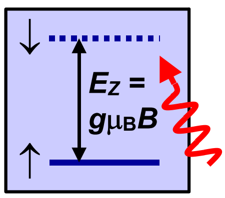
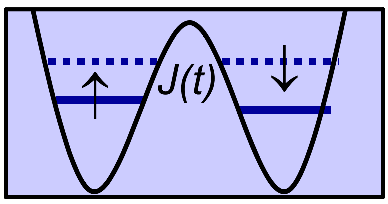
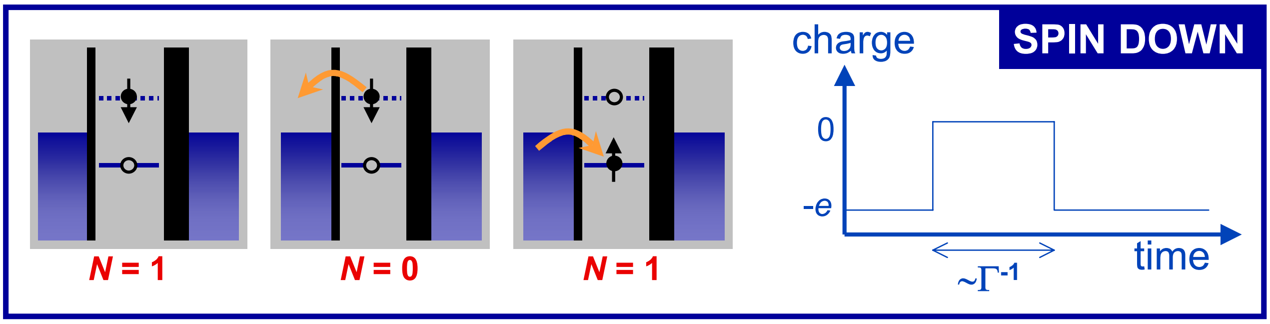




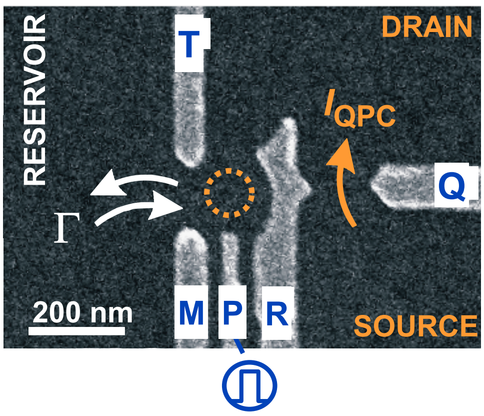


Spin read-out principle:
convert spin to charge

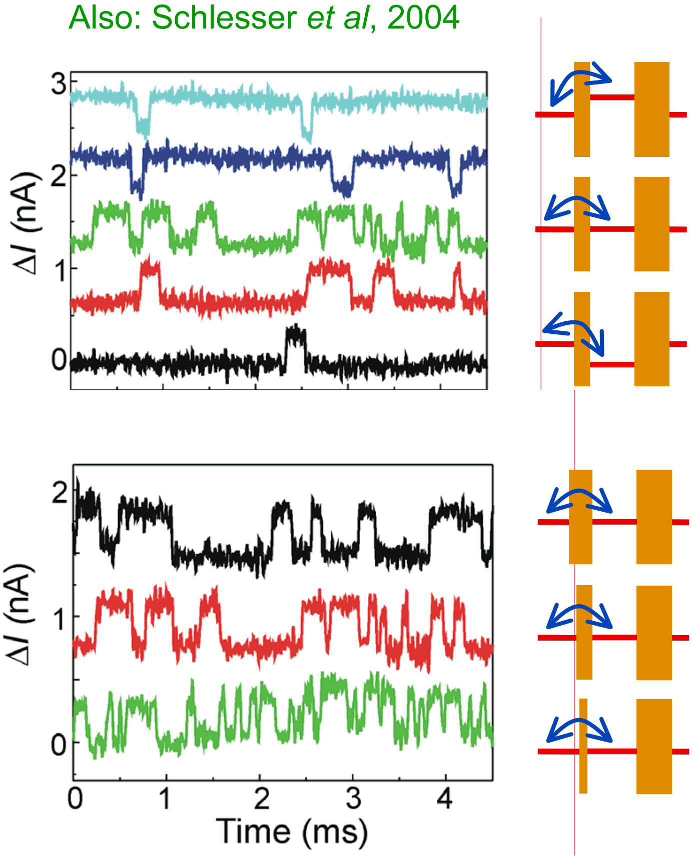


Observation of individual tunnel events

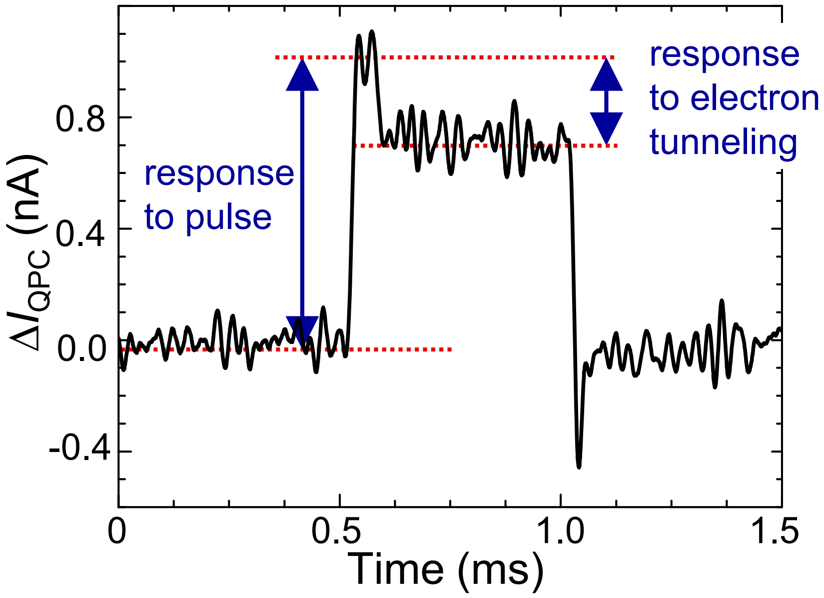
Vandersypen *et al*, APL 85, 4394, 2004
Also: Schlesser *et al*, 2004

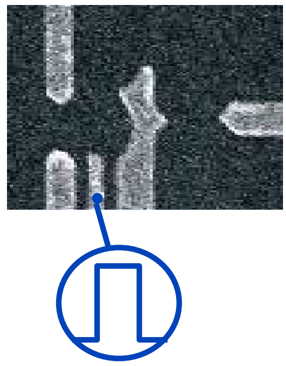


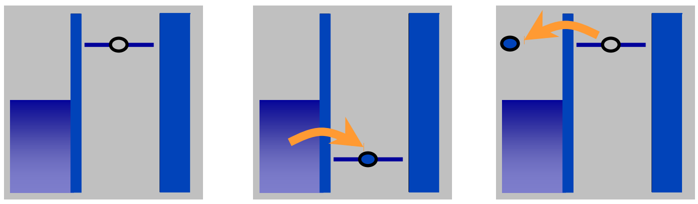
- V_{SD} = 1 mV
- I_{QPC} ~ 30 nA
- ΔI_{QPC} ~ 0.3 nA
- Shortest steps ~ 8 μs



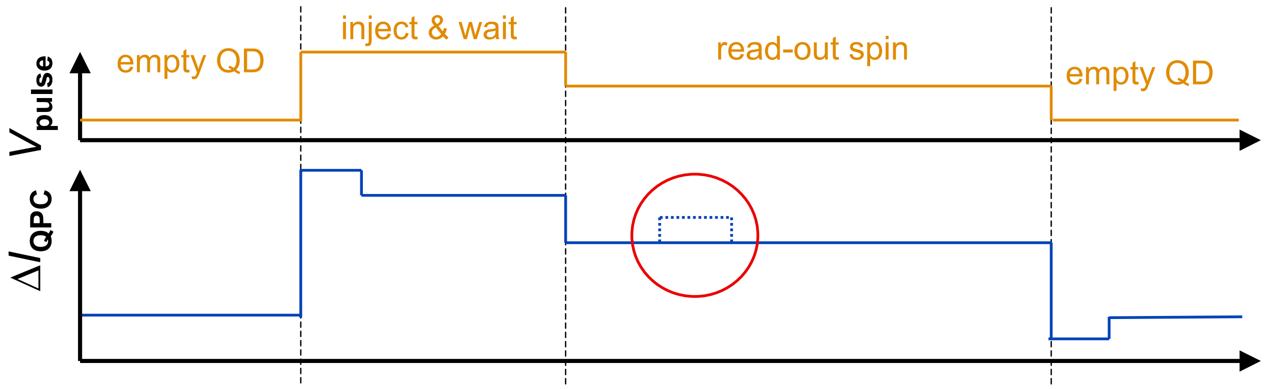
Pulse-induced tunneling

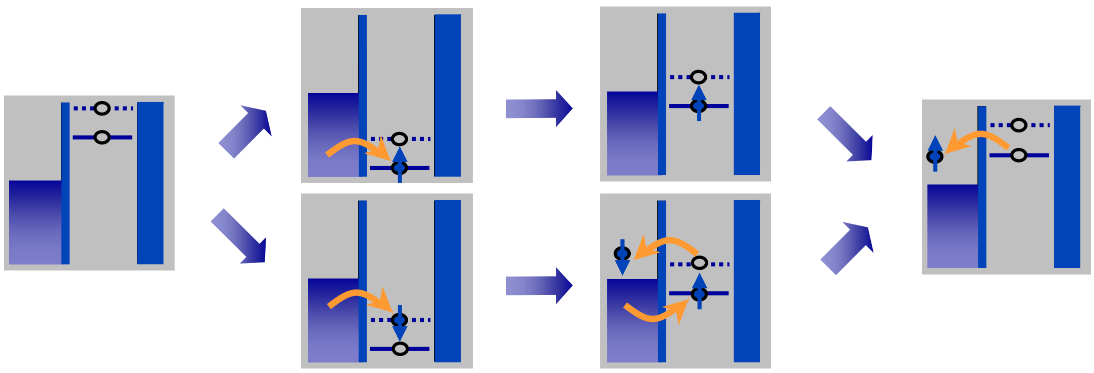






Spin read-out procedure

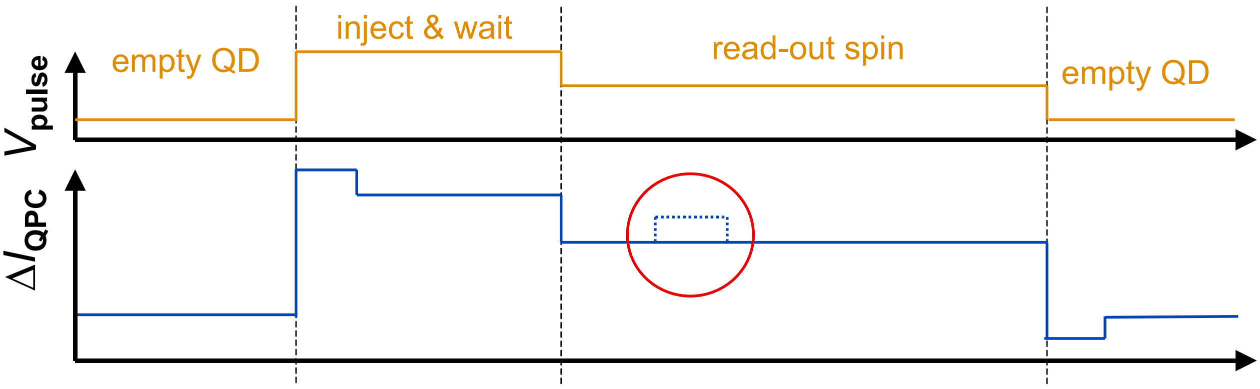


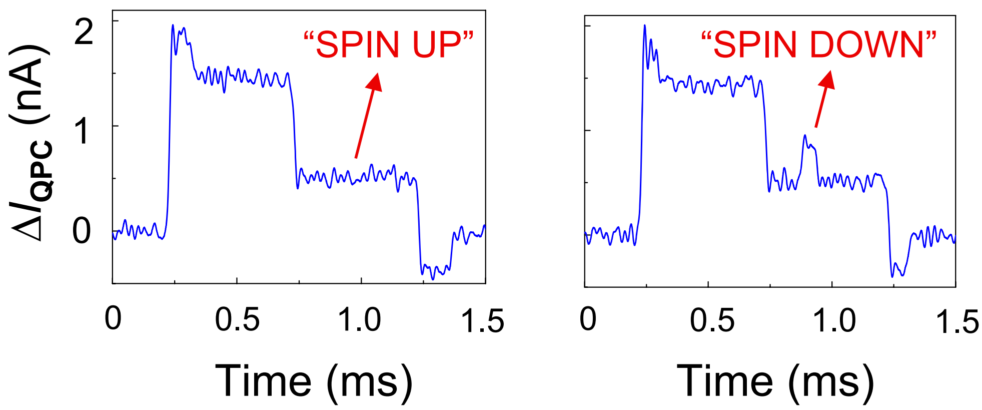


Inspiration: Fujisawa *et al.*, Nature **419**, 279, 2002

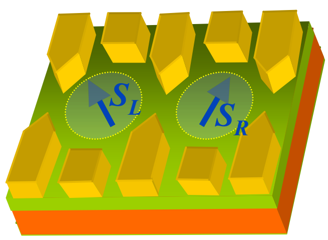
Spin read-out results

Elzerman *et al.*, Nature **430**, 431, 2004





Spin qubits in quantum dots



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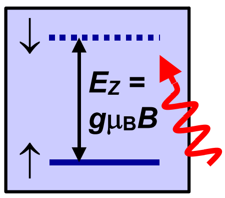

SWAP exchange interaction

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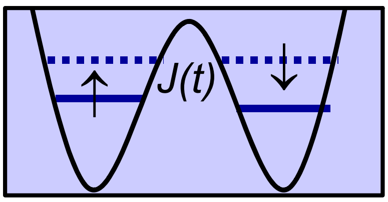
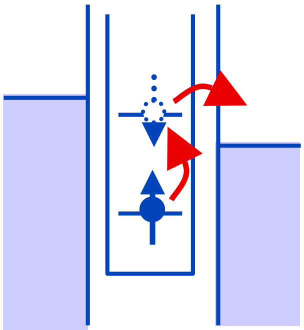
Coherence long relaxation time T_1
long coherence time T_2



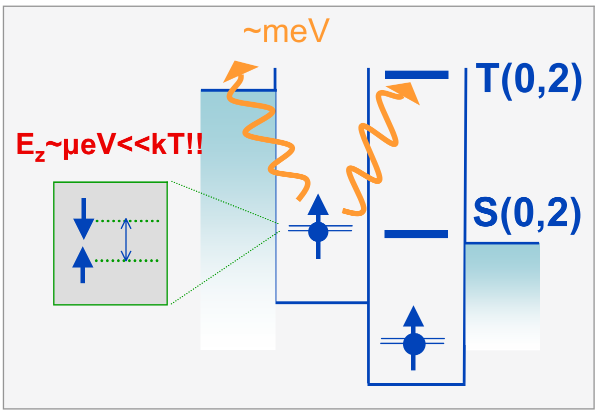
ESR detection in a single dot



ESR lifts Coulomb blockade

Engel & Loss, PRL 2001

Double dot in spin blockade for ESR detection



Advantage: ***interdot transition instead of dot-lead transition***

- Insensitive to temperature

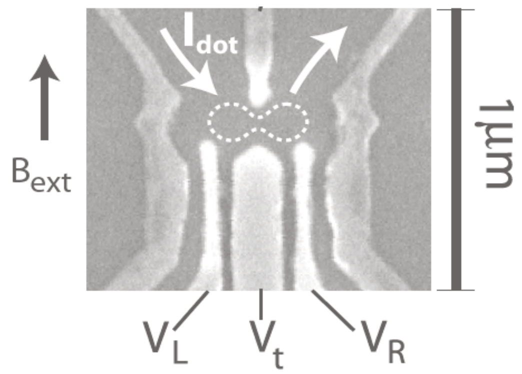
  ⇒ can use B < 100 mT, f < 500 MHz

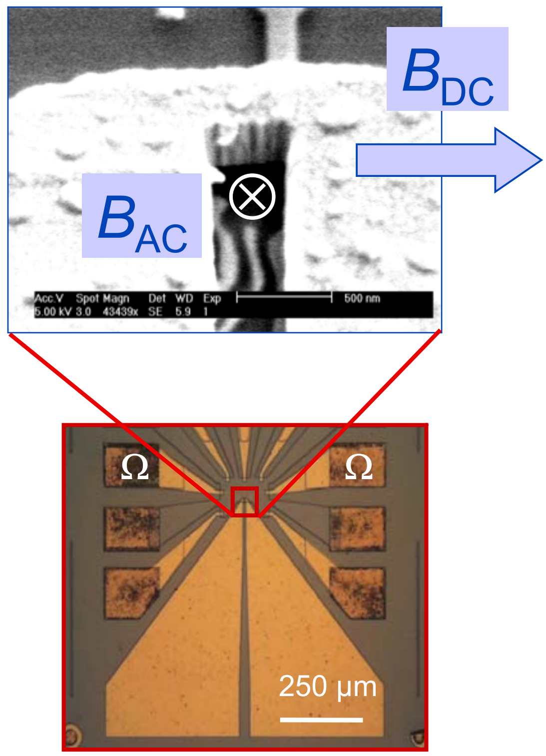
- Insensitive to electric fields

ESR flips spin, lifts spin blockade

Combine Engel & Loss (PRL 2001) ESR detection with Ono & Tarucha (Science 2002) spin blockade

ESR device design





Gates ~ 30 nm thick gold

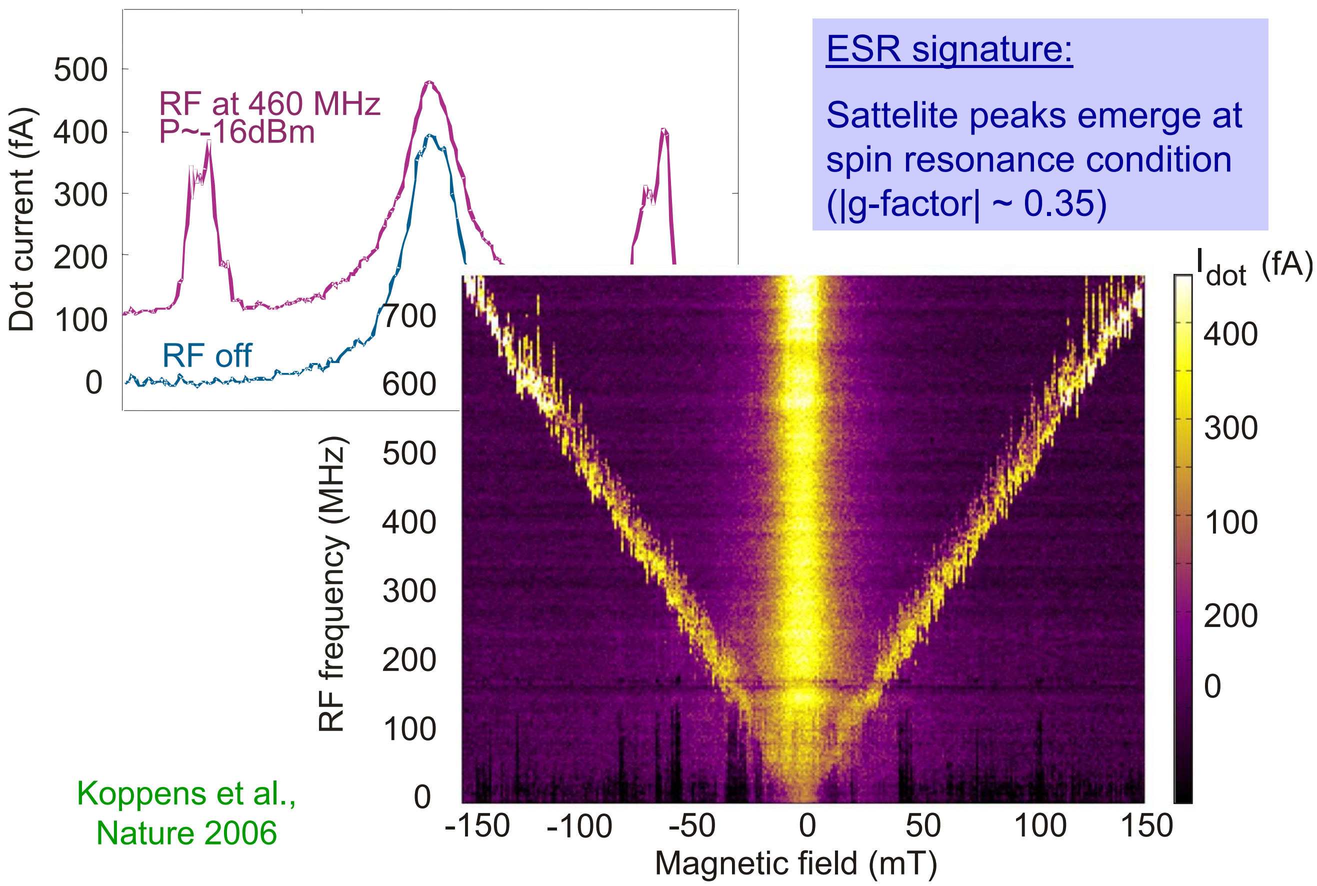
Dielectric ~ 100nm calixerene

Stripline ~ 400nm thick gold

Expected AC current ~ 1mA
Expected AC field ~ 1mT

Maximize B_1, minimize E_1

ESR spin state spectroscopy

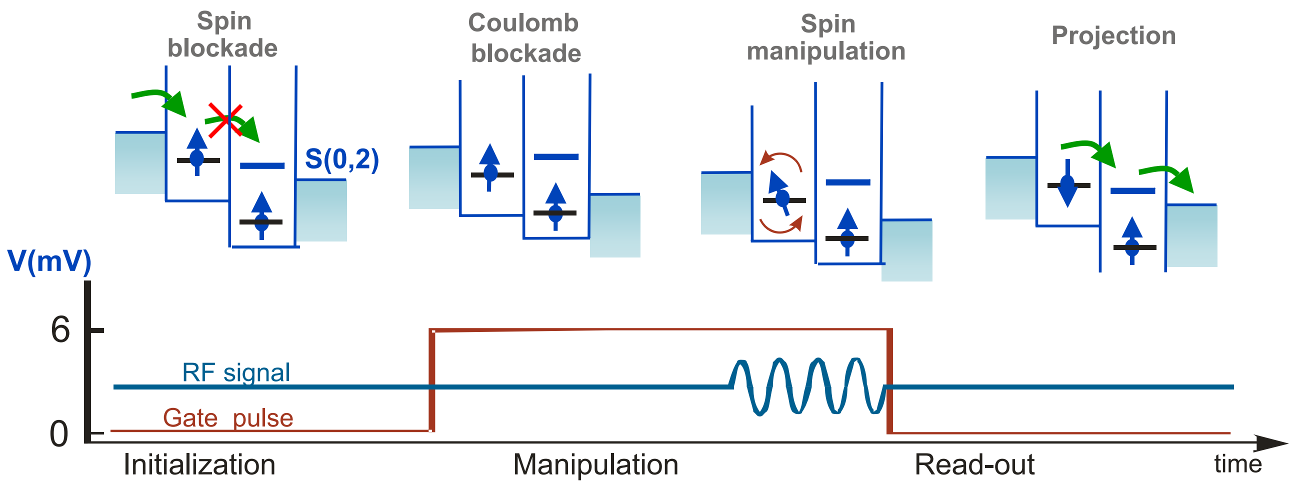


ESR signature:

Sattelite peaks emerge at spin resonance condition (|g-factor| ~ 0.35)

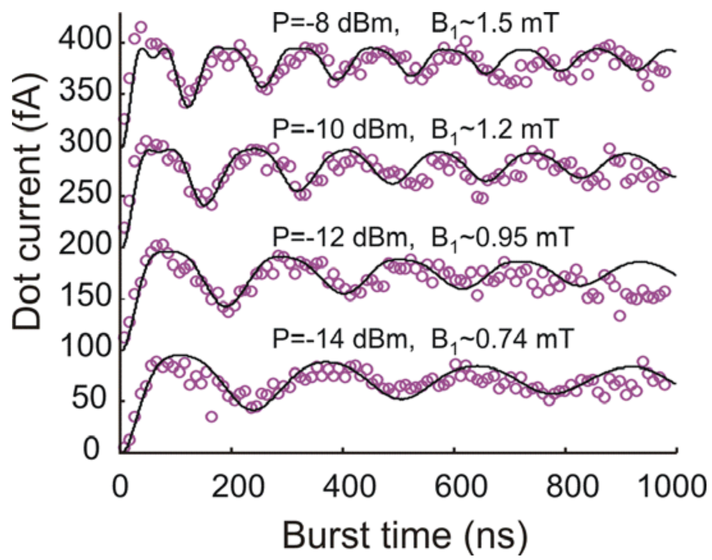
Koppens et al., Nature 2006

Coherent manipulation: pulse scheme



- Initialization in mixture of ↑↑ and ↓↓
- Measurement switched off (by pulsing to Coulomb blockade) during manipulation
- Read-out: projection on {↑↑,↓↓} vs. {↑↓,↓↑} basis

Coherent rotations of single electron spin!

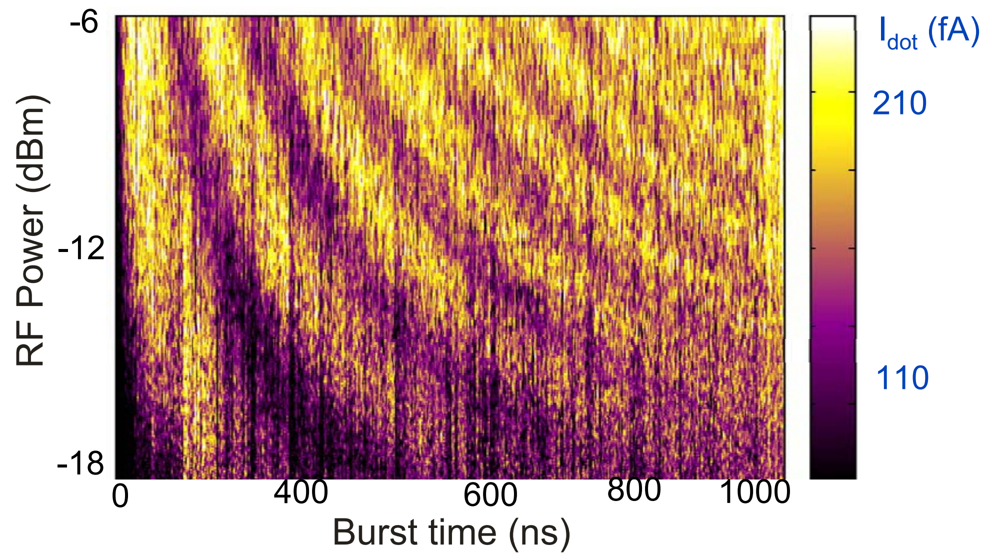


Koppens et al.
Nature 2006

- Oscillations visible up to 1μs
- Decay non exponential → slow nuclear dynamics (non-Markovian bath)
- Agreement with simple Hamiltonian

  taking into account different resonance conditions both dots

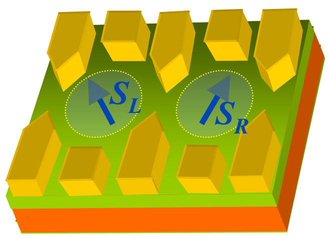
Driven coherent oscillations



- Oscillation frequency ~ B_{AC} → clear signature of Rabi oscillations
- π/2 pulse in 25ns
- max $B_1 = B_{AC}/2$ = 1.9 mT
  $B_{N,z}$ = 1.3 mT } estimated fidelity ~73%

Koppens et al.
Nature 2006

Spin qubits in quantum dots

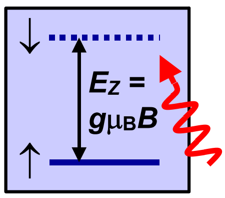


Loss & DiVincenzo, PRA 1998
Vandersypen et al., Proc. MQC02 (quant-ph/0207059)

Initialization 1-electron, low T, high B_0
$H_0 \sim \Sigma\, \omega_i\, \sigma_{zi}$

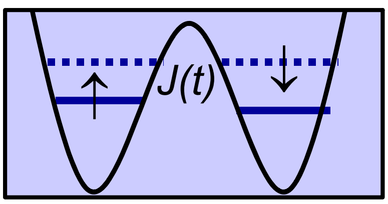
Read-out convert spin to charge
then measure charge

ESR pulsed microwave magnetic field
$H_{RF} \sim \Sigma\, A_i(t) \cos(\omega_i t)\ \sigma_{xi}$

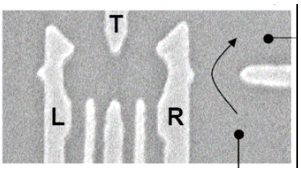


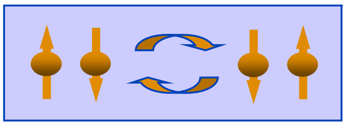
SWAP exchange interaction
$H_J \sim \Sigma\, J_{ij}(t)\ \sigma_i \cdot \sigma_j$

Coherence long relaxation time T_1
long coherence time T_2



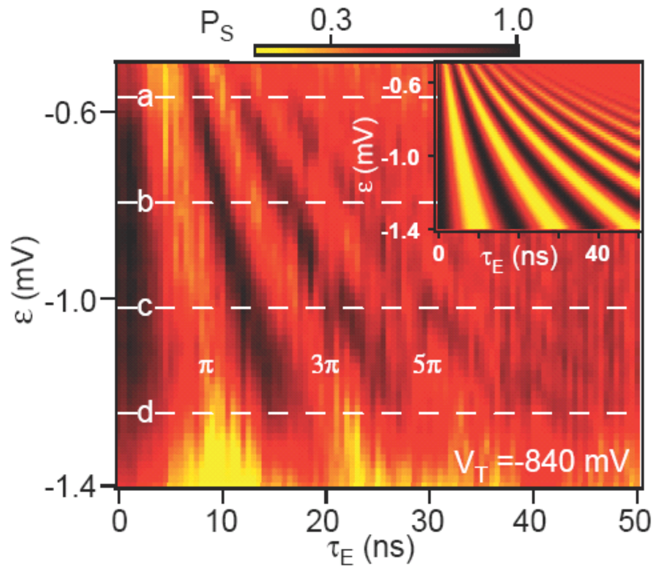
Coherent exchange of two spins

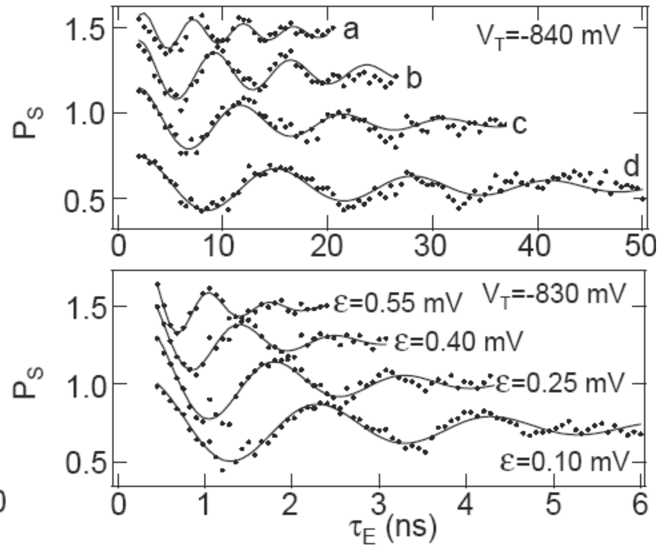




Petta *et al.*, Science 2005

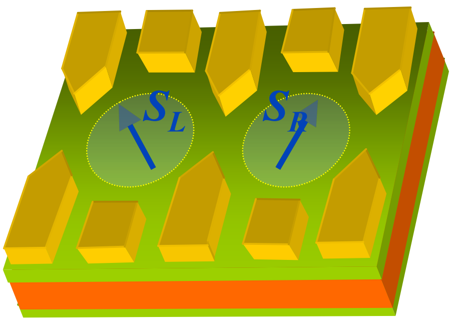
- free evolution under exchange Hamiltonian
- swap$^{1/2}$ in as little as 180 ps
- three oscillations visible, independent of J

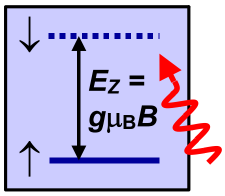


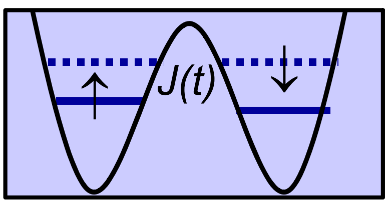


Spin qubits in quantum dots - present status

Initialization	1 electron, low T, high B_0 $H_0 \sim \Sigma\, \omega_i\, \sigma_{zi}$
Read-out	convert spin to charge then measure charge
ESR	pulsed microwave magnetic field $H_{RF} \sim \Sigma\, A_i(t) \cos(\omega_i t)\ \sigma_{xi}$
SWAP	exchange interaction $H_J \sim \Sigma\, J_{ij}(t)\ \sigma_i \cdot \sigma_j$
Coherence	measure coherence time $T_1 \sim 1$ ms; $T_2 > 1$ μs







Quantum computing with trapped ions

Hartmut Häffner

Institute for Quantum Optics and Quantum Information
Innsbruck, Austria

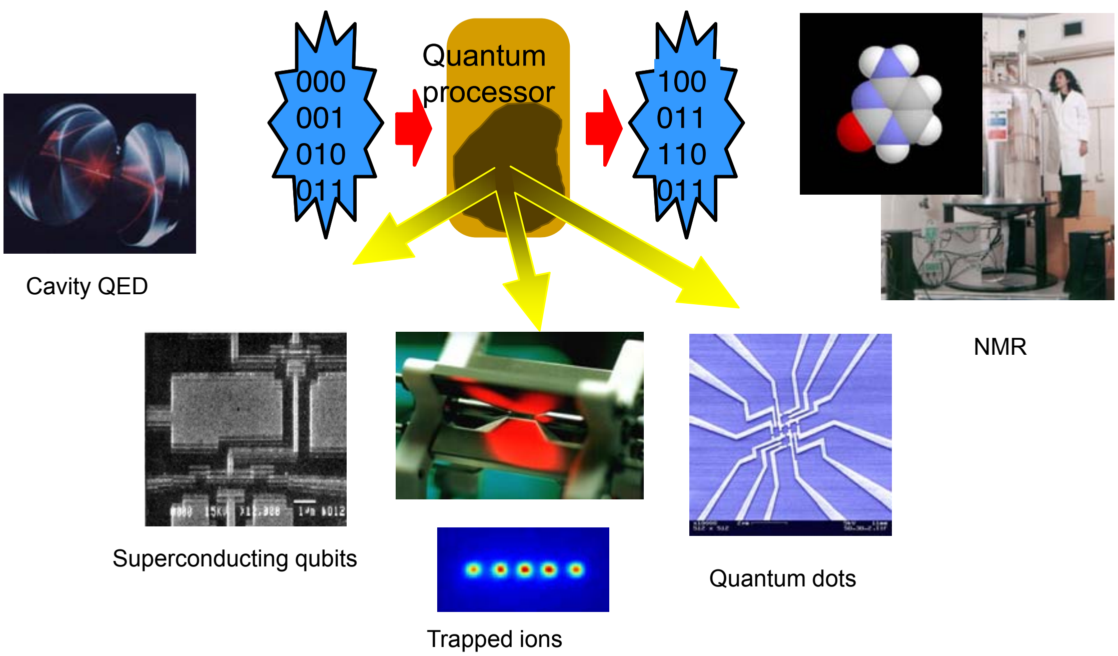
- Basics of ion trap quantum computing
- Measuring a density matrix
- Quantum gates

€ FWF SFB SCALA QGATES Industrie Tirol IQI GmbH

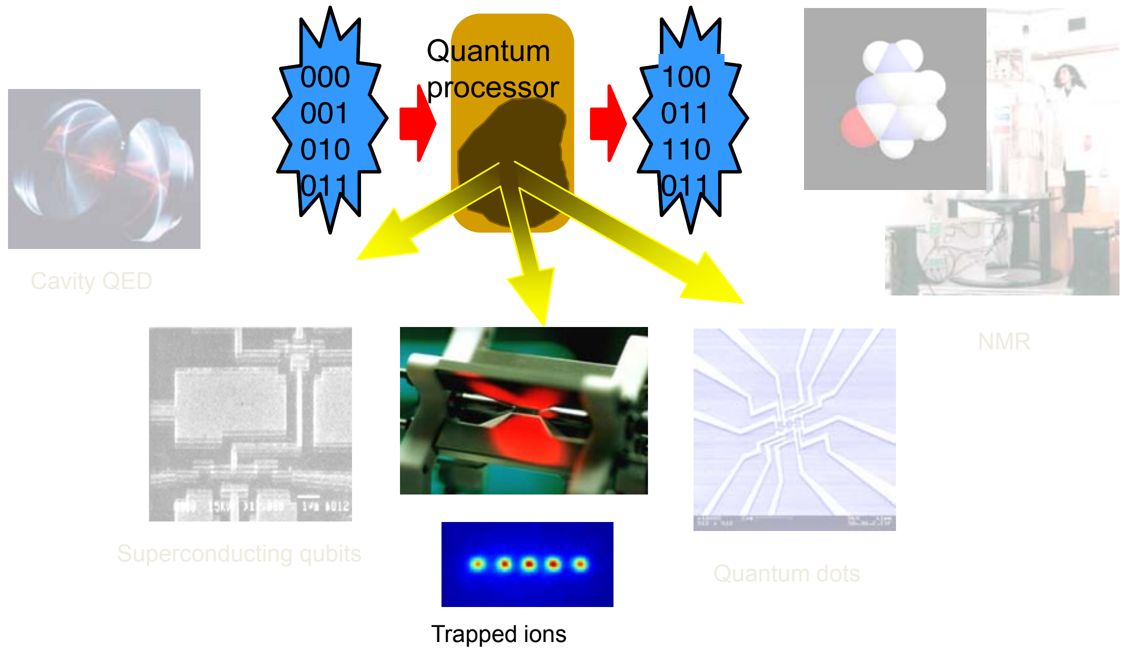
FWF bm:bwk $

Zürich, Dec 8th 2008

Which technology ?



Which technology ?



Why trapped ions ?

Good things about ion traps:

- Ions are excellent quantum memories; single qubit coherence times > 10 minutes have been demonstrated

- Ions can be controlled very well

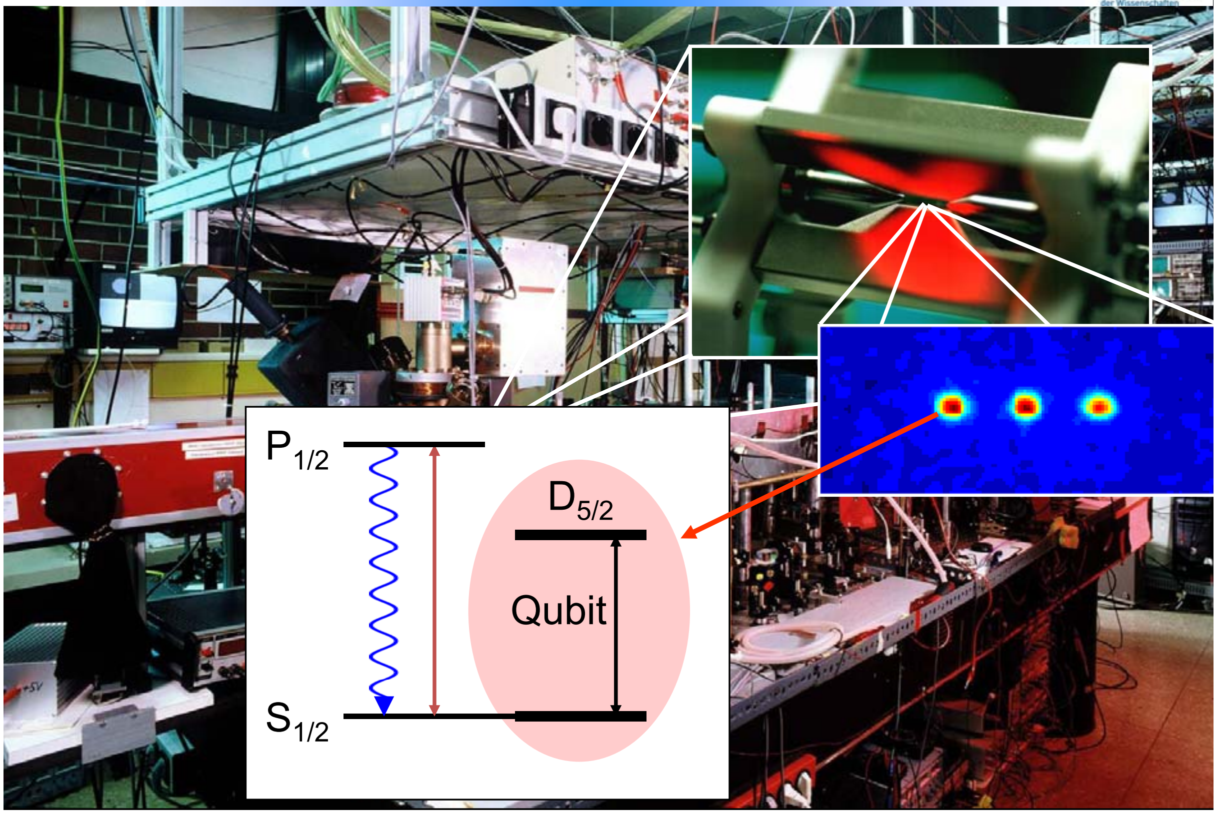
- Many ideas to scale ion traps

Bad things about ion traps:

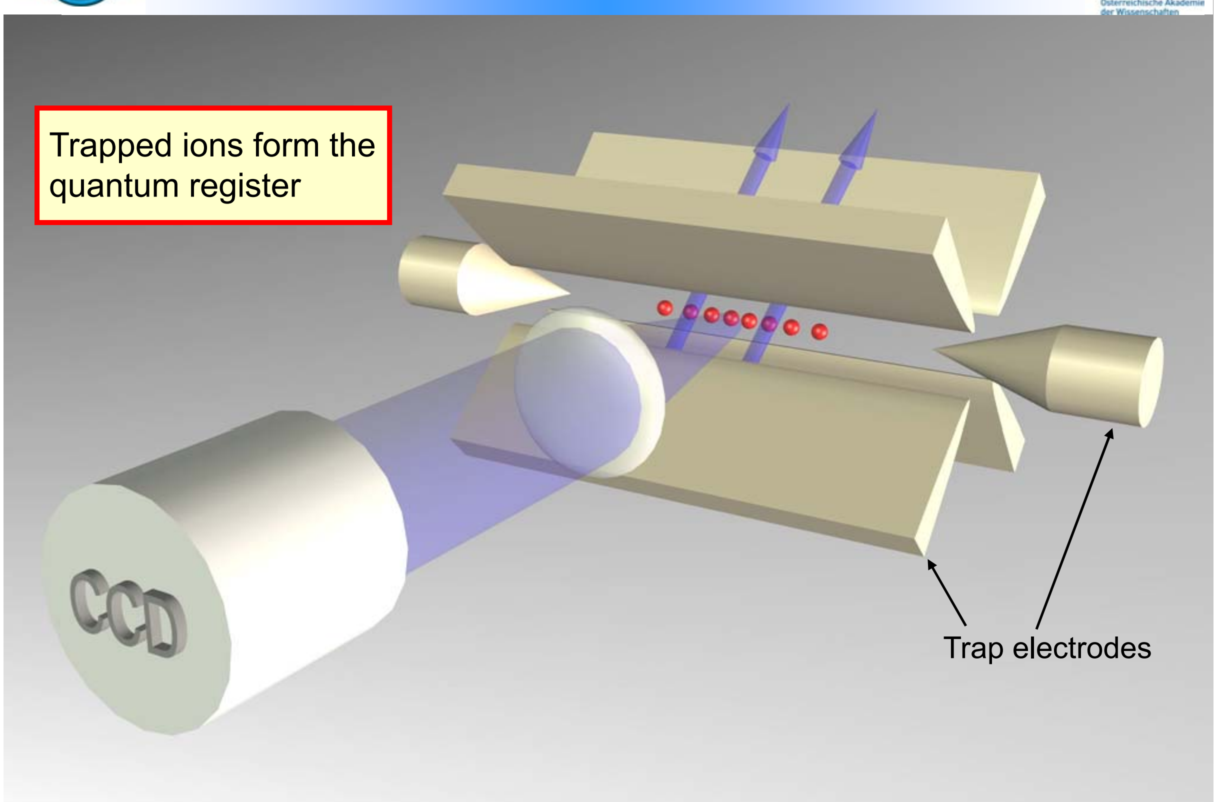
- Slow (~1 MHz)

- Technically demanding

The hardware



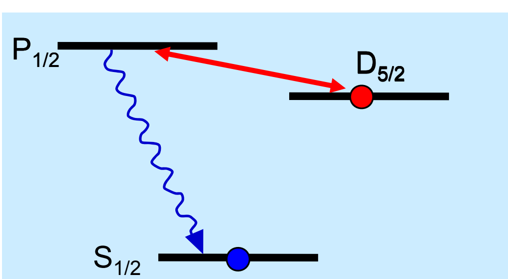
Innsbruck quantum processor



DiVincenzo criteria

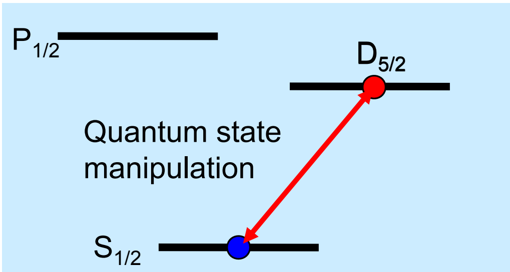
I. Scalable physical system, well characterized qubits
II. Ability to initialize the state of the qubits
III. Long relevant coherence times, much longer than gate operation time
IV. "Universal" set of quantum gates
V. Qubit-specific measurement capability

Experimental procedure



1. Initialization in a pure quantum state

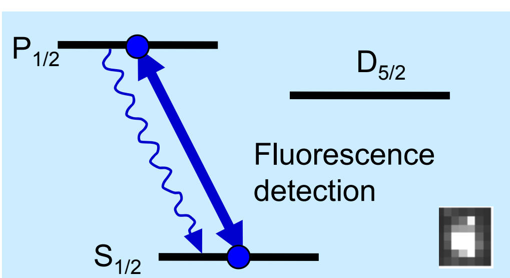
Experimental procedure



1. Initialization in a pure quantum state

2. Quantum state manipulation on $S_{1/2}$ – $D_{5/2}$ transition

Experimental procedure

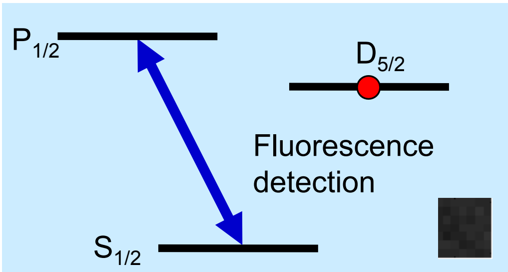


1. Initialization in a pure quantum state:

2. Quantum state manipulation on $S_{1/2}$ – $D_{5/2}$ transition

3. Quantum state measurement by fluorescence detection

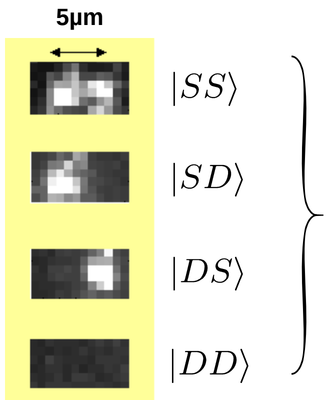
Experimental procedure



1. Initialization in a pure quantum state:
2. Quantum state manipulation on $S_{1/2} – D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

Two ions:

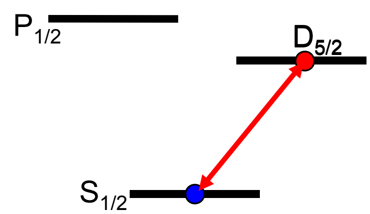
Spatially resolved detection with CCD camera

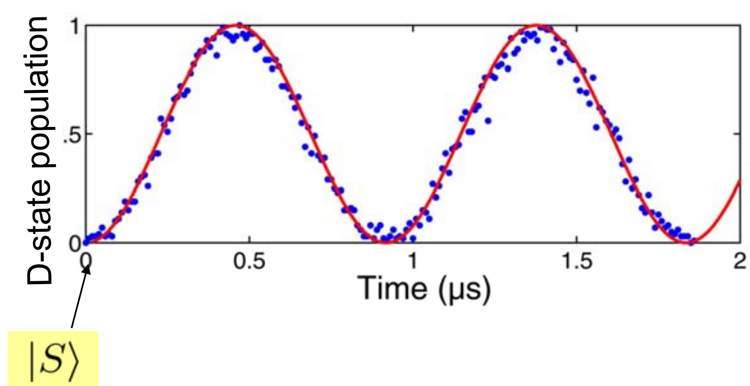


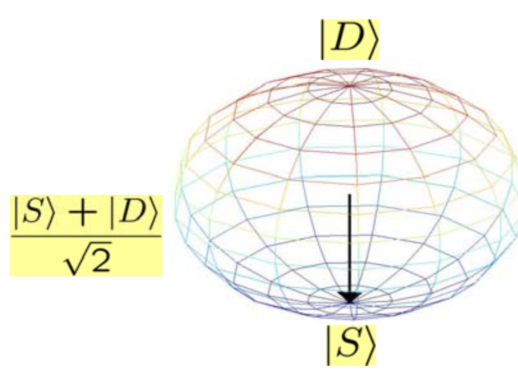
50 experiments / s

Repeat experiments 100-200 times

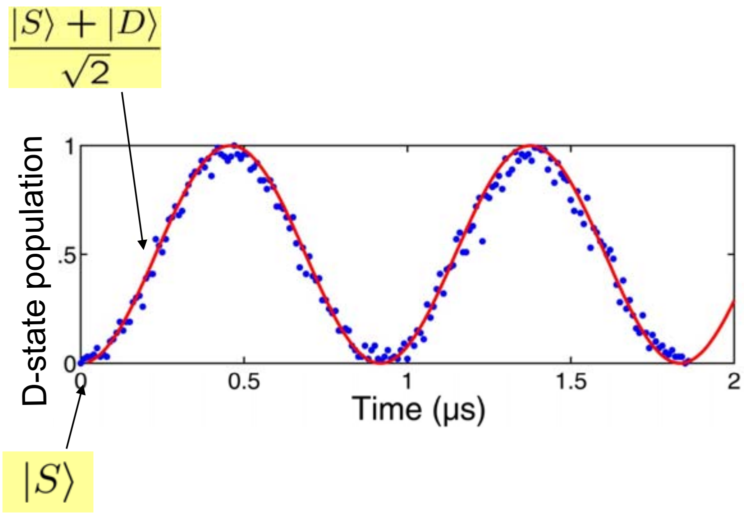
Rabi oscillations

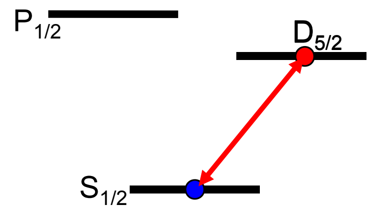


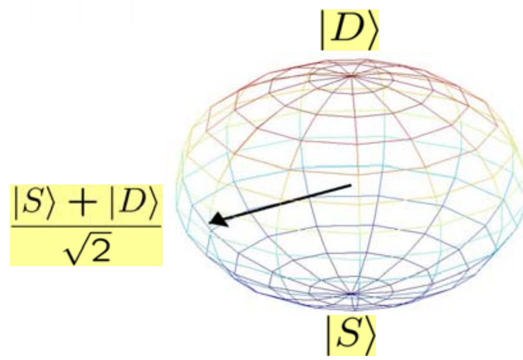




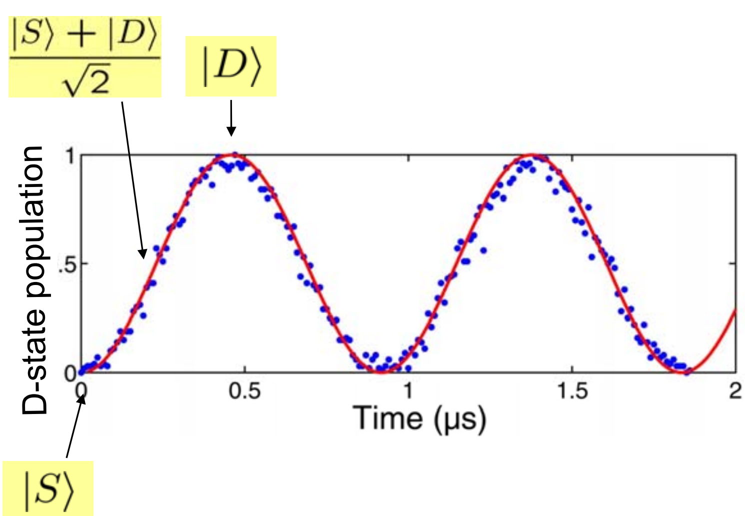
Rabi oscillations

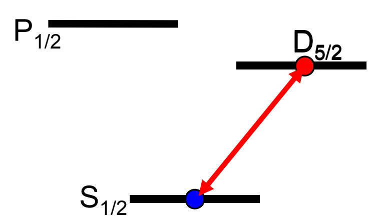


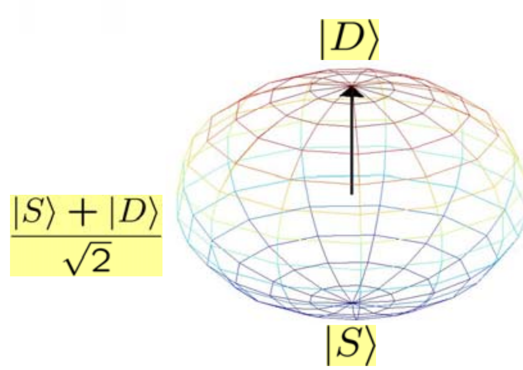




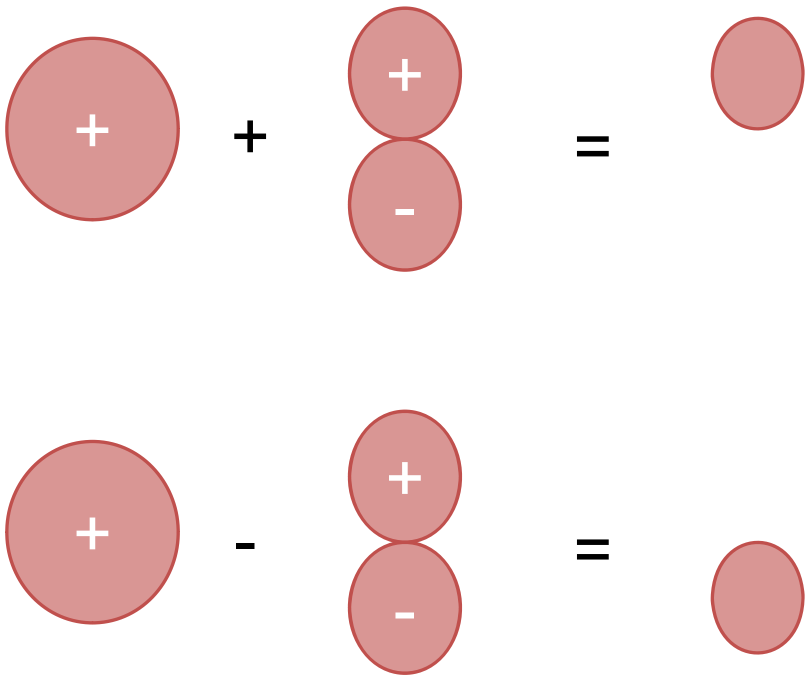
Rabi oscillations



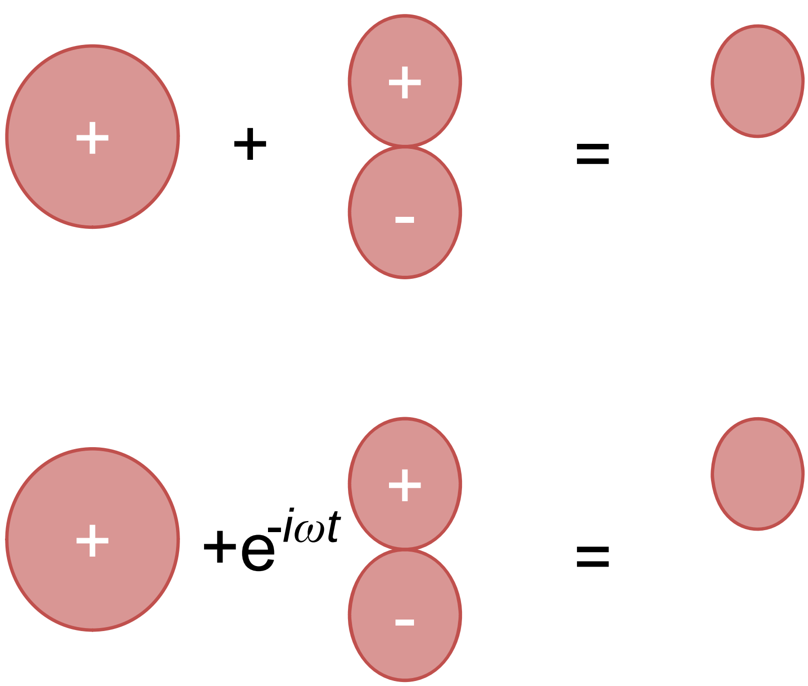




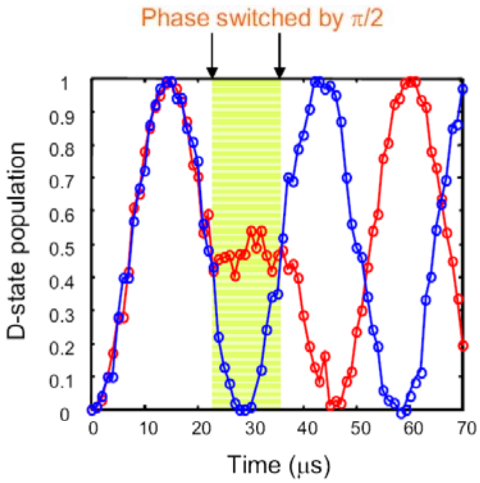
The phase ...



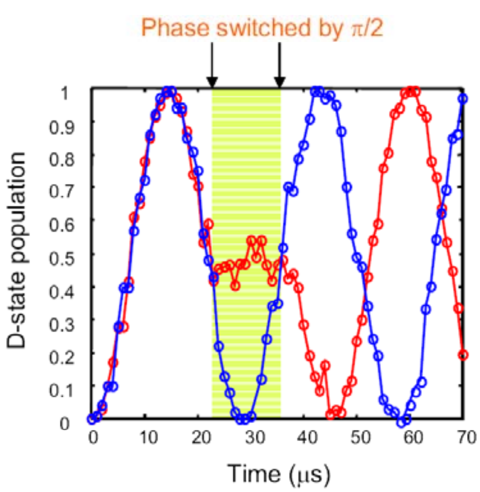
The phase ...

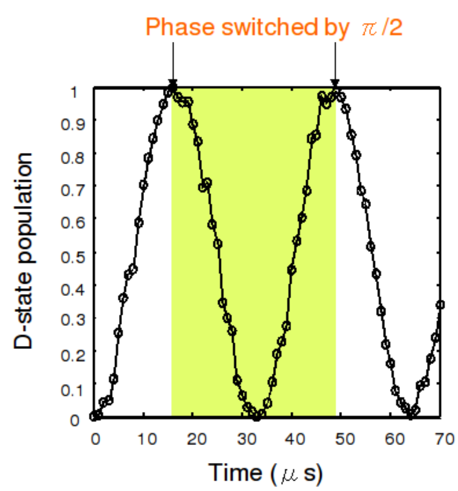


The phase ...

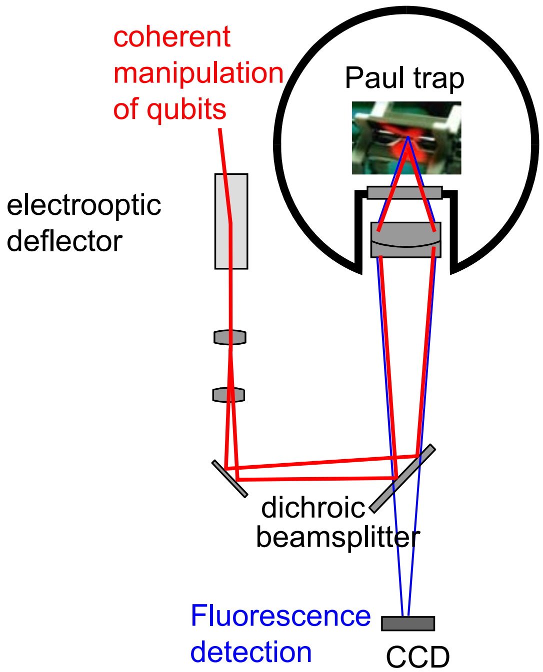


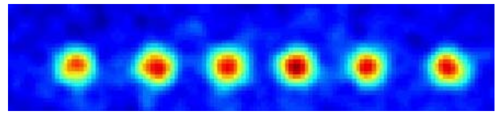
The phase ...

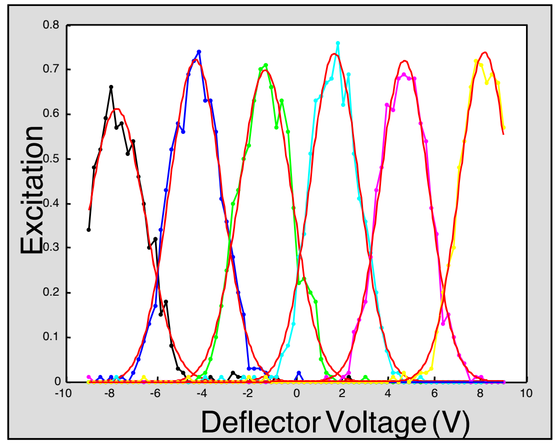




Addressing single qubits







- inter ion distance: ~ 4 µm

- addressing waist: ~ 2 µm

< 0.1% intensity on neighbouring ions

Decoherence mechanisms

Memory errors:

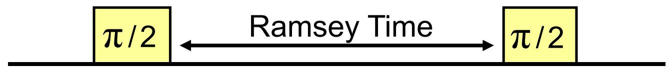
- Bit-flips
- Dephasing

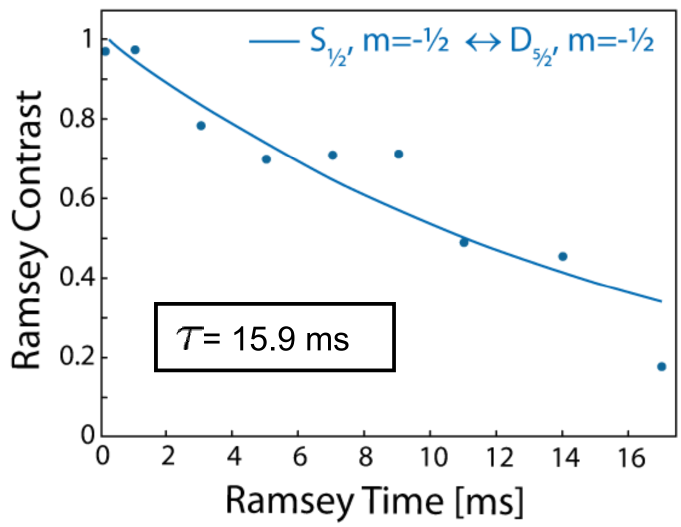
Operationial errors

- technical imperfections …

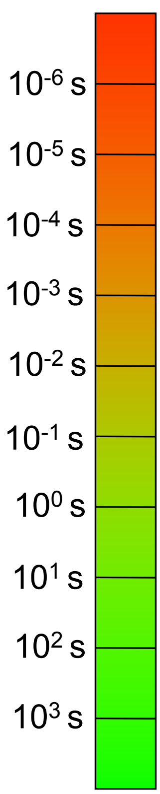
Dephasing of qubits

Ramsey Experiment





Realized time scales



Single qubit gates

Two qubit gates (Geometric phase gates)

Two qubit gates (Cirac-Zoller approach)

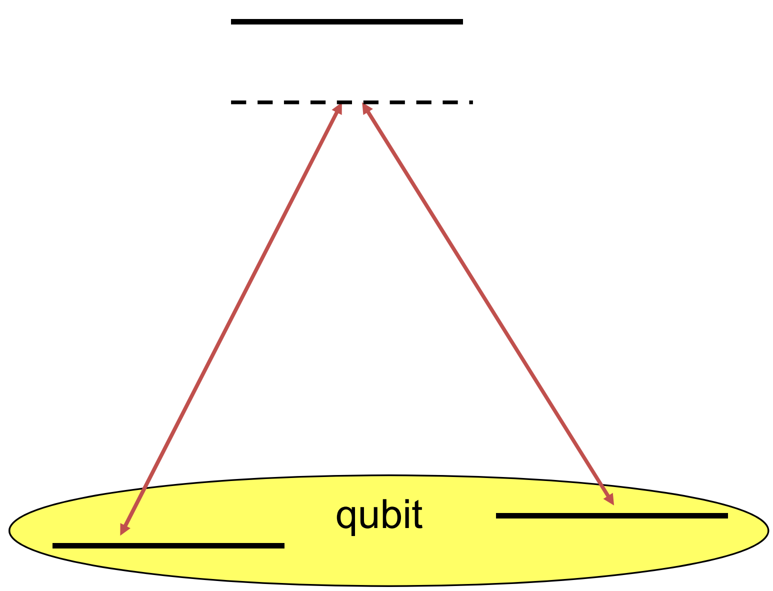
Single qubit coherence (magnetic field sensitive)

Coherence of the motion

Long lived qubits

Raman transitions:

Excited state

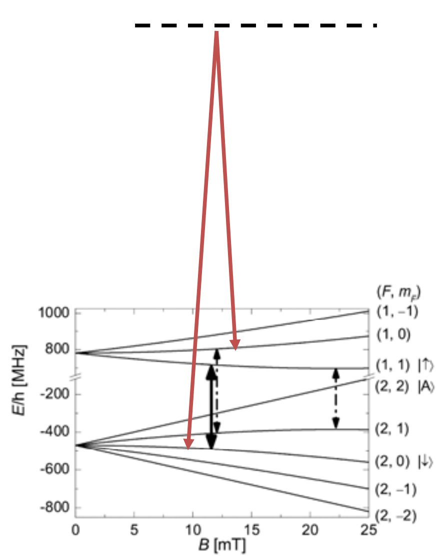


Ground state

Long lived qubits

Raman transitions:

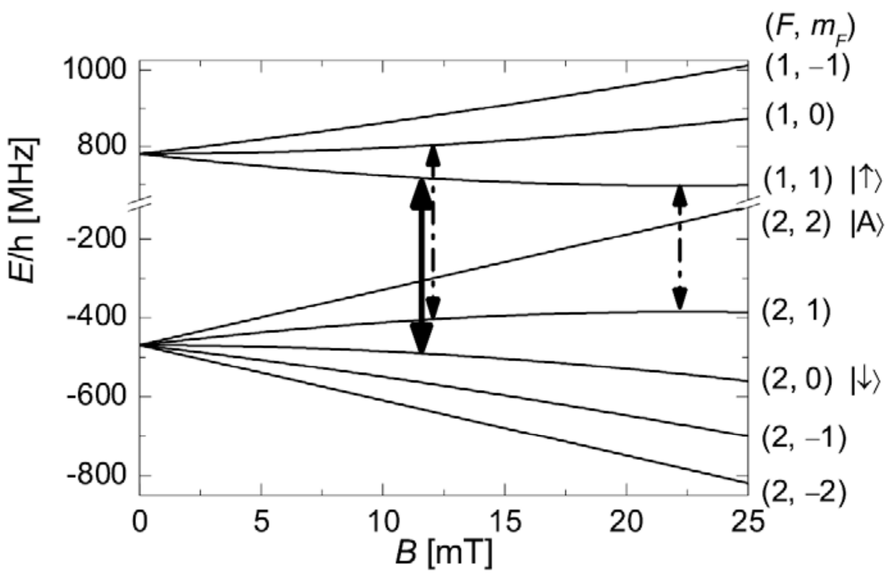
Excited state



Ground state

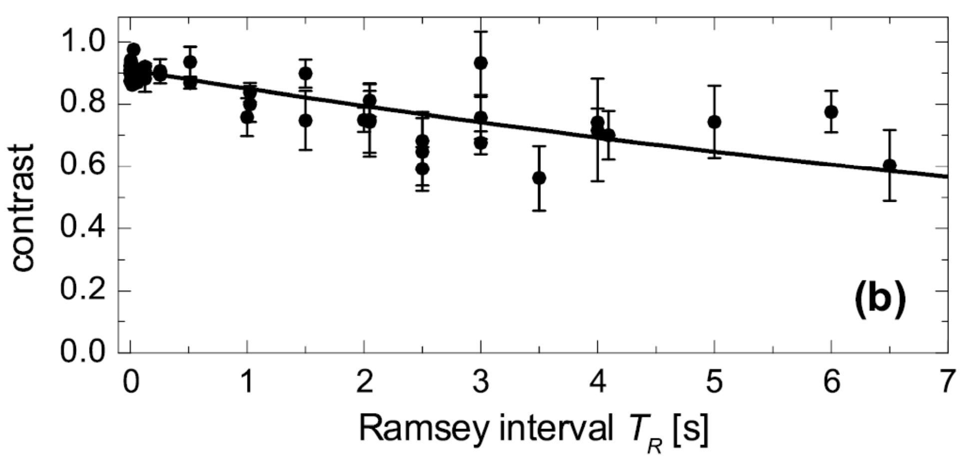
Long lived qubits

Level scheme of $^9Be^+$:



From: C. Langer *et al.*, PRL **95**, 060502 (2005), NIST

Long lived qubits

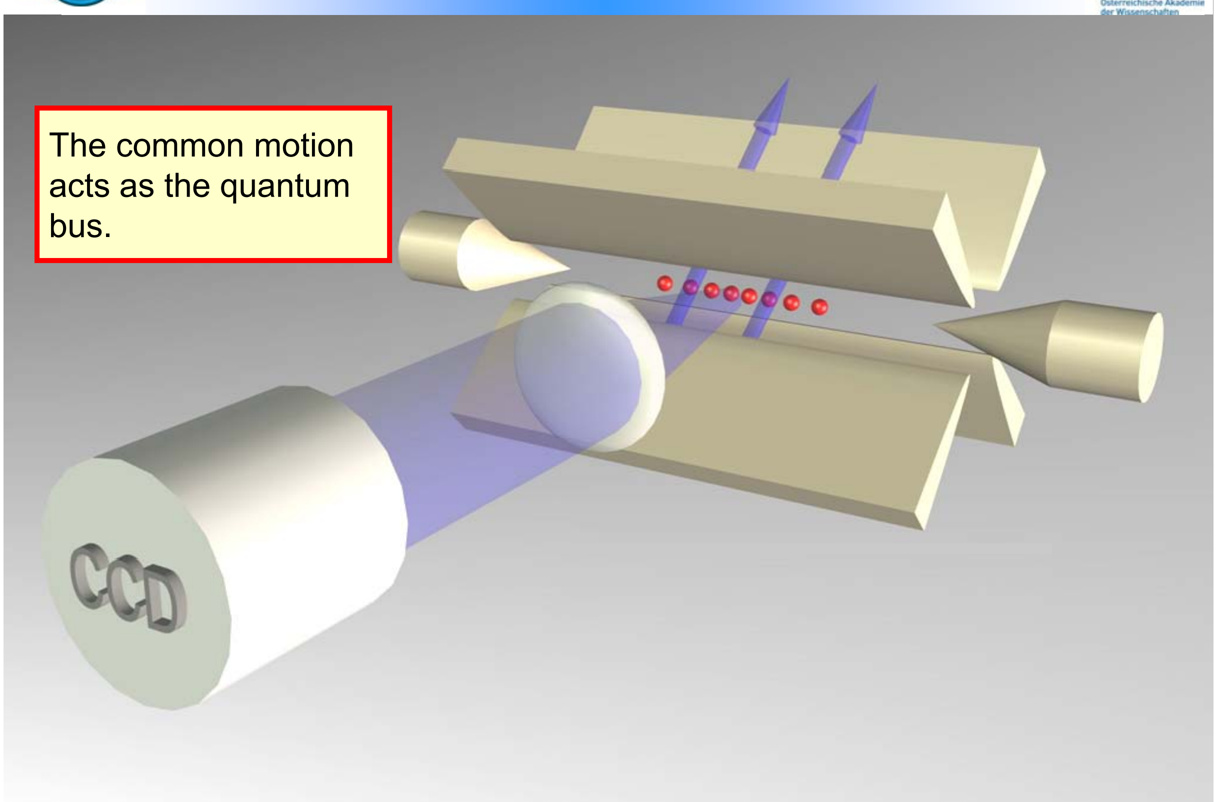


From: C. Langer *et al.*, PRL **95**, 060502 (2005), NIST

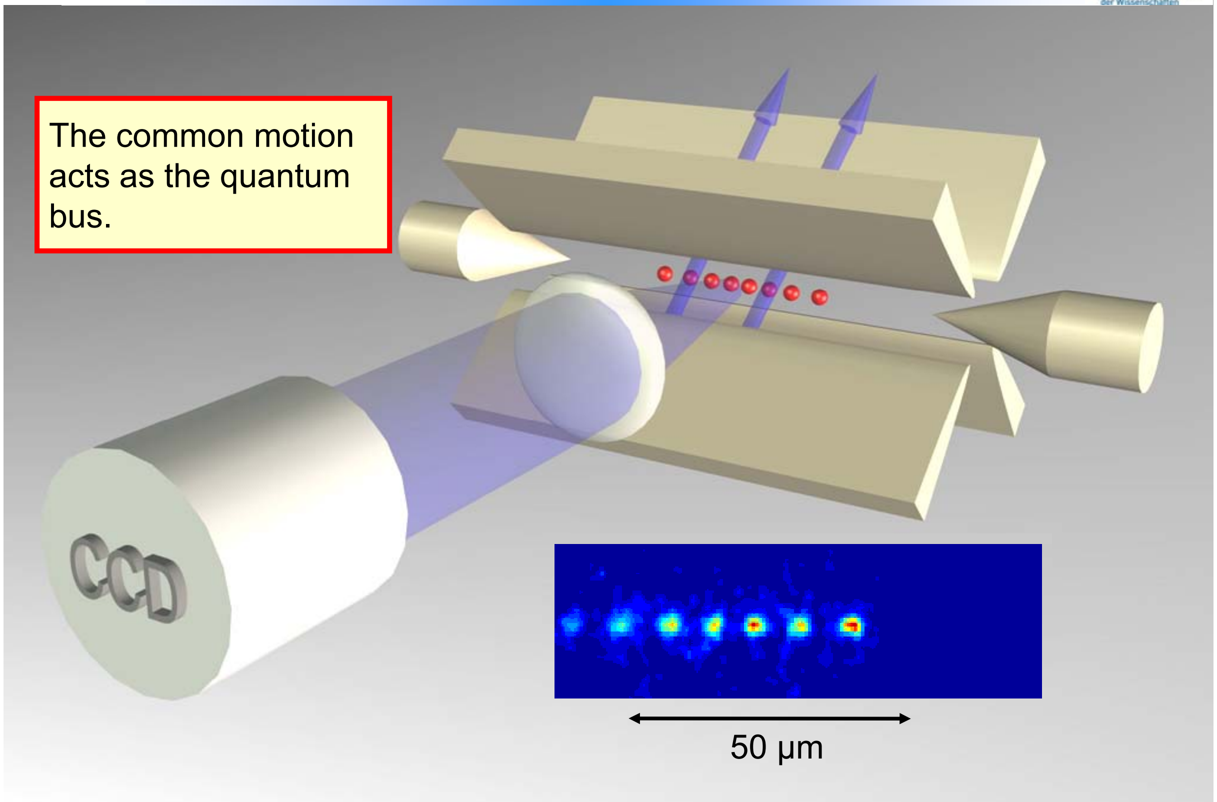
Realized time scales

Time	Process
	Single qubit gates
10^{-6} s	
	Two qubit gates (Geometric phase gates)
10^{-5} s	
10^{-4} s	
	Two qubit gates (Cirac-Zoller approach)
10^{-3} s	
10^{-2} s	Single qubit coherence (magnetic field sensitive)
10^{-1} s	Coherence of the motion
10^{0} s	
10^{1} s	Single qubit coherence (magnetic field insensitive)
10^{2} s	
10^{3} s	Single qubit coherence (magnetic field insensitive + RF drive)

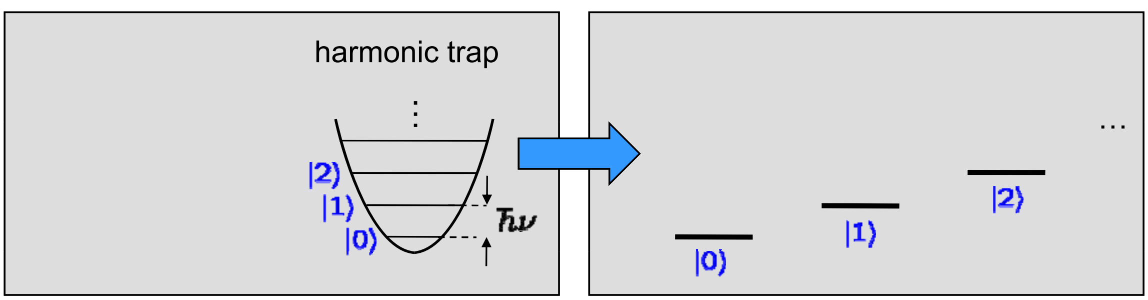
Having the qubits interact



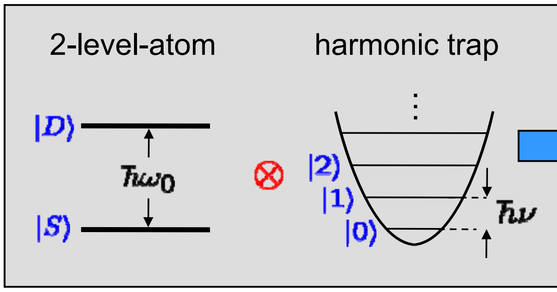
Having the qubits interact

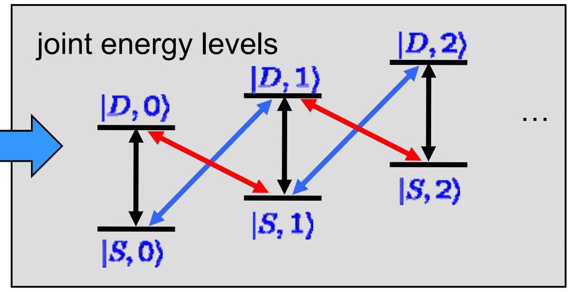


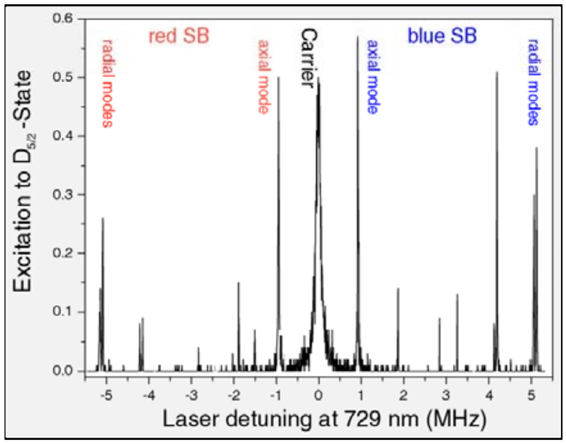
Ion motion



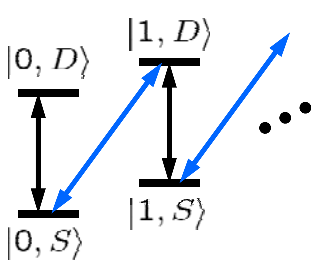
Ion motion



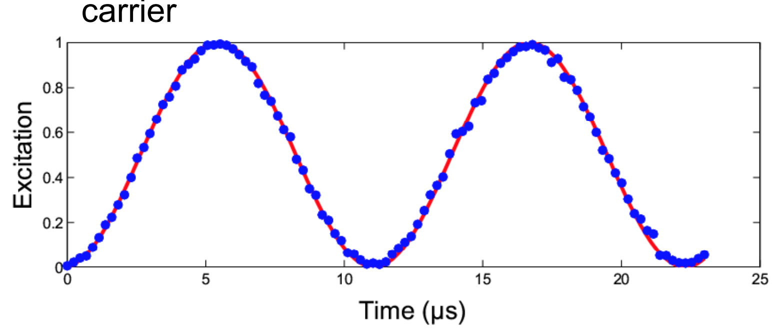




Coherent manipulation

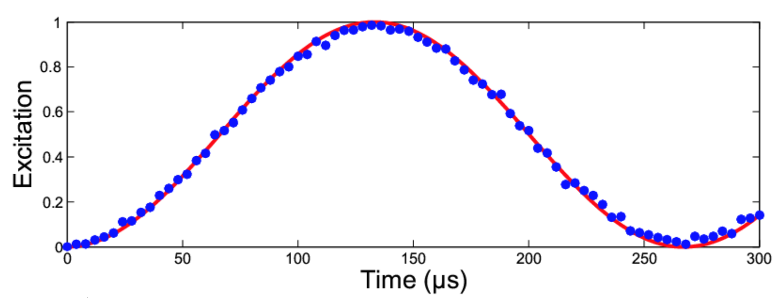


carrier



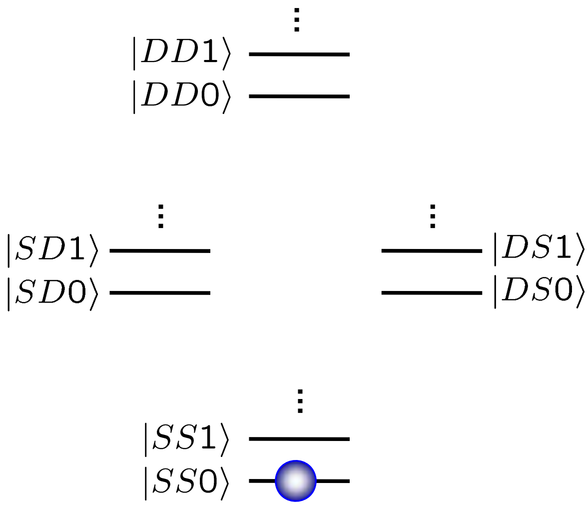
carrier and sideband
Rabi oscillations
with Rabi frequencies

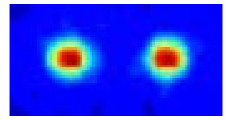
$$\Omega, \quad \eta\Omega$$



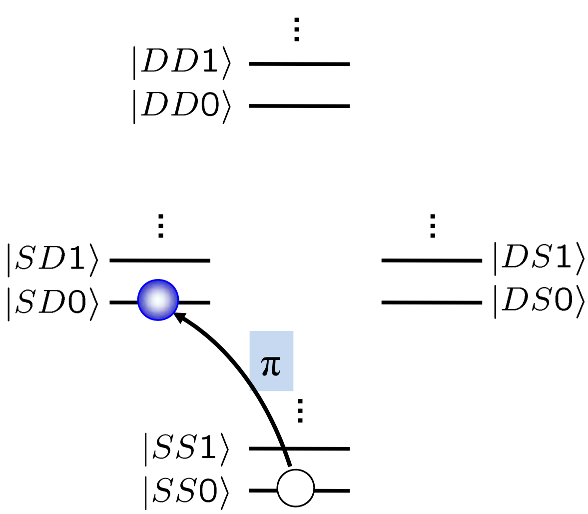
$\eta = kx_0$ Lamb-Dicke parameter

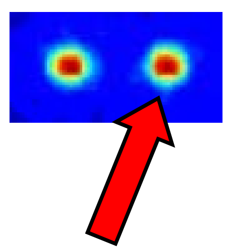
Generation of Bell states



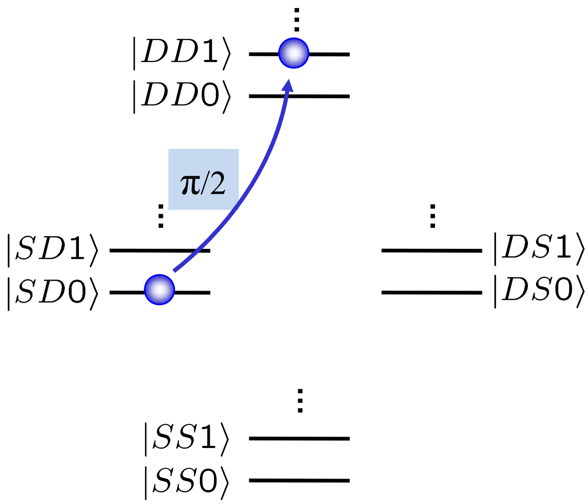


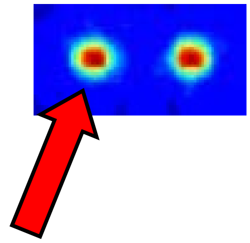
Generation of Bell states



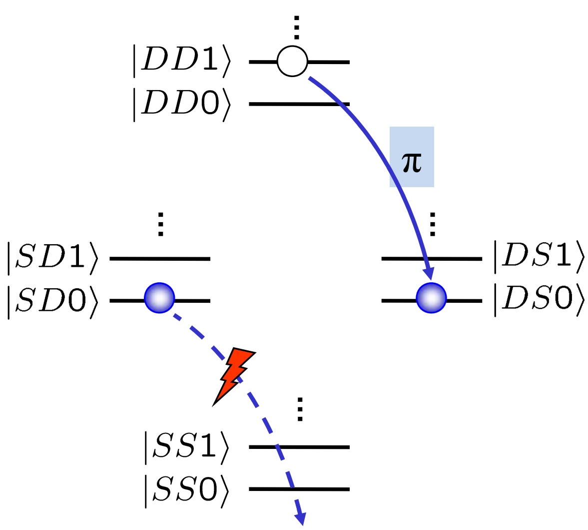


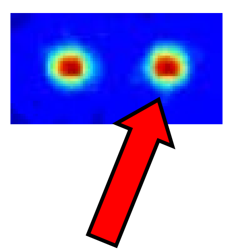
Generation of Bell states



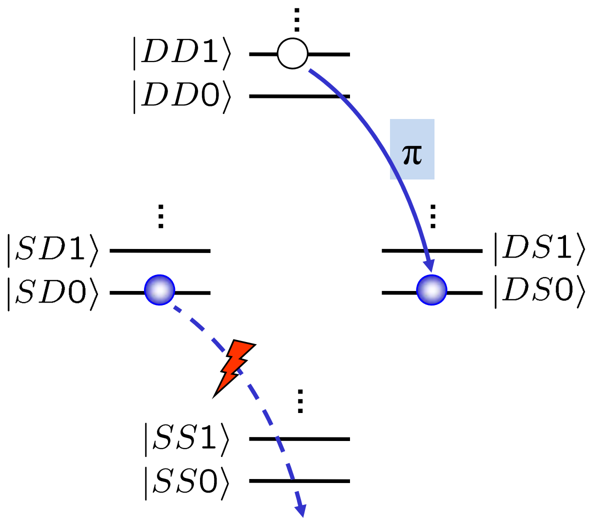


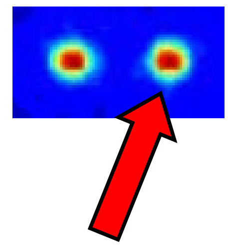
Generation of Bell states





Generation of Bell states





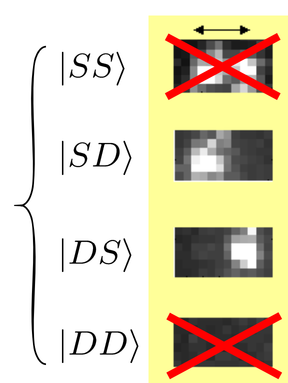
<u>Bell states with atoms</u>

- $^{9}Be^{+}$: NIST (fidelity: 97 %)
- $^{40}Ca^{+}$: Oxford (83%)
- $^{111}Cd^{+}$: Ann Arbor (79%)
- $^{25}Mg^{+}$: Munich
- $^{40}Ca^{+}$: Innsbruck (99%)

Analysis of Bell states

$$|SD\rangle + |DS\rangle$$

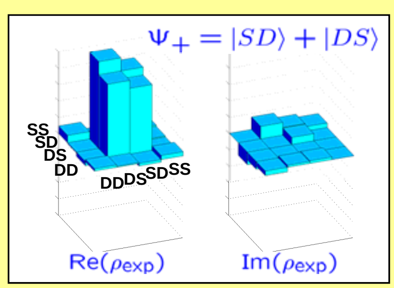
Fluorescence detection with CCD camera:



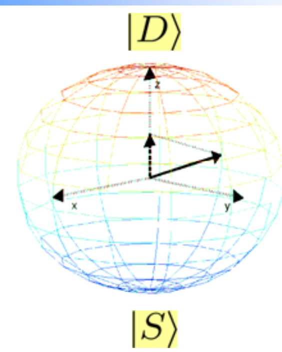
Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

➡ Measurement of the density matrix:



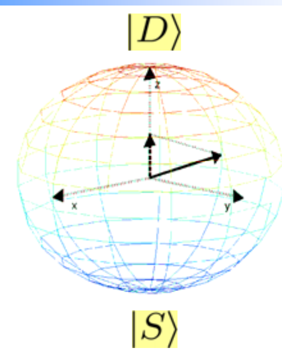
Measuring a density matrix



A measurement yields the *z*-component of the Bloch vector
=> Diagonal of the density matrix

$$\rho = \begin{pmatrix} P_S & C - iD \\ C + iD & P_D \end{pmatrix}$$

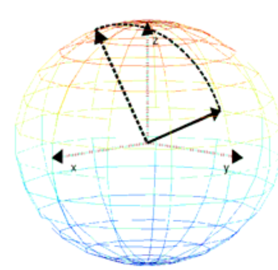
Measuring a density matrix



A measurement yields the *z*-component of the Bloch vector
=> Diagonal of the density matrix

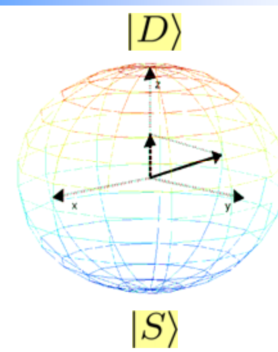
$$\rho = \begin{pmatrix} P_S & C - iD \\ C + iD & P_D \end{pmatrix}$$

Rotation around the *x*- or the *y*-axis prior to the measurement yields the phase information of the qubit.



Measuring a density matrix

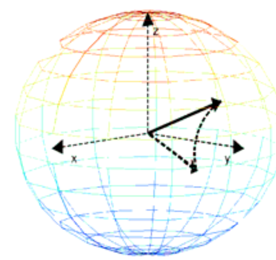
A measurement yields the *z*-component of the Bloch vector
=> Diagonal of the density matrix



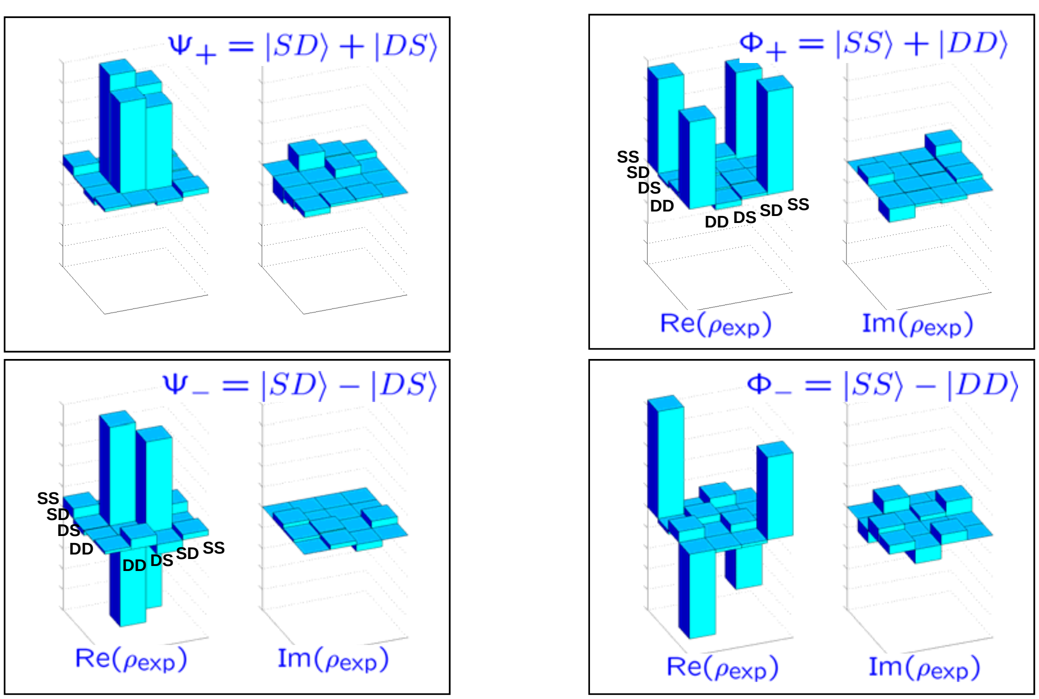
$$\rho = \begin{pmatrix} P_S & C - iD \\ C + iD & P_D \end{pmatrix}$$

Rotation around the *x*- or the *y*-axis prior to the measurement yields the phase information of the qubit.

=> coherences of the density matrix

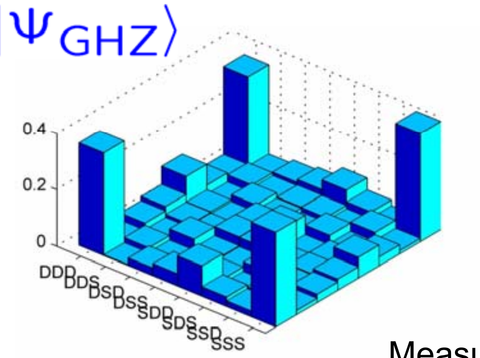


Decoherence properties of qubits

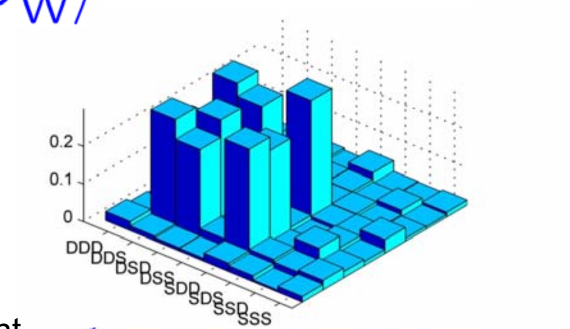


A “real” thought experiment

$|\Psi_{GHZ}\rangle$



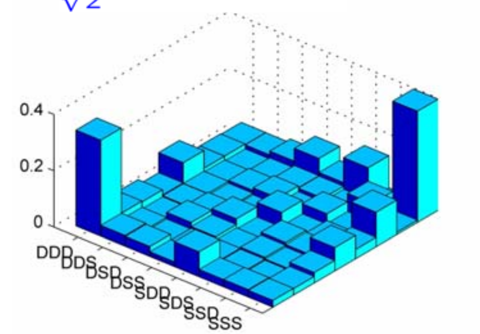
$|\Psi_{W}\rangle$

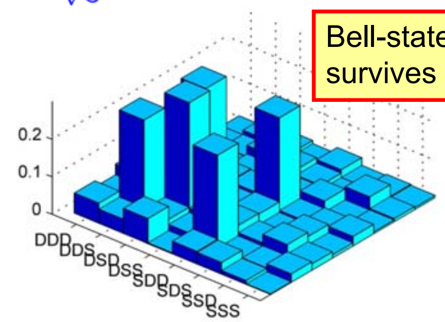


$\frac{1}{\sqrt{2}}(|SSS\rangle + |DDD\rangle)$

Measurement of the center ion

$\frac{1}{\sqrt{3}}(|SDD\rangle + |DSD\rangle + |DDS\rangle)$



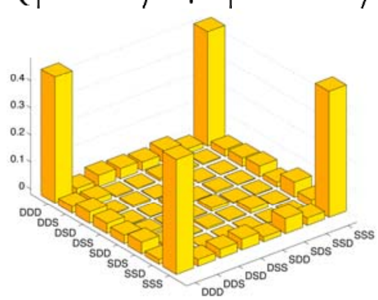


Bell-state survives

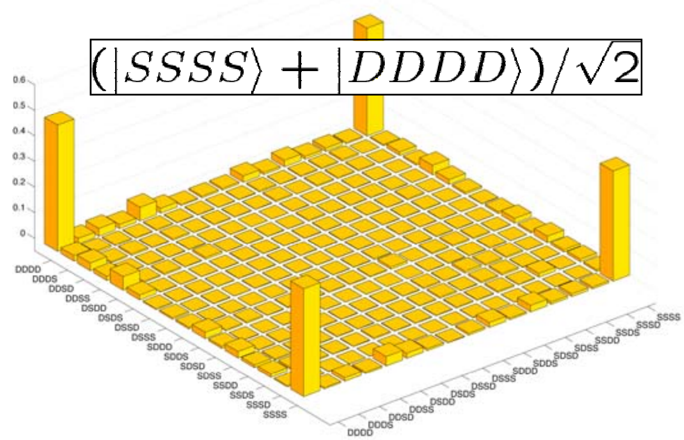
Roos et al., *Science* **304**, 1478 (20

Generalized Bell states

$(|SSS\rangle + |DDD\rangle)/\sqrt{2}$

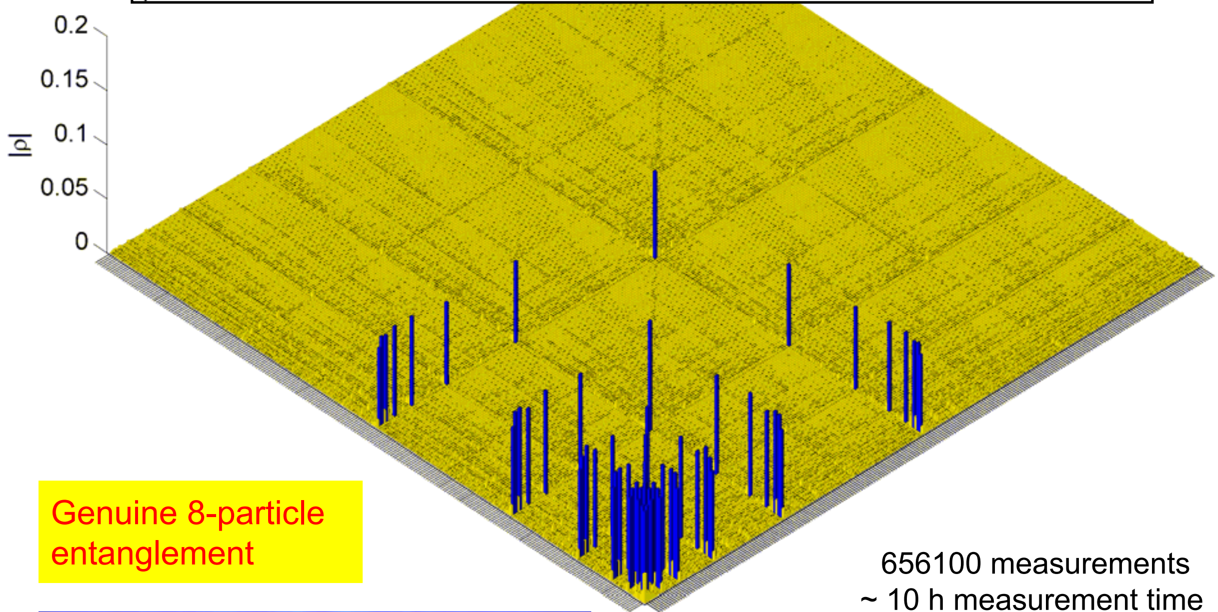


$(|SSSS\rangle + |DDDD\rangle)/\sqrt{2}$

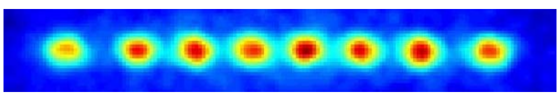


Generalized Bell states

$$\frac{1}{\sqrt{8}}(|DDDDDDDS\rangle + |DDDDDDSD\rangle + \ldots + |SDDDDDDD\rangle)$$



Genuine 8-particle entanglement



656100 measurements
~ 10 h measurement time

Häffner et al., Nature **438**, 643 (2005)

DiVincenzo criteria

I. Scalable physical system, well characterized qubits

II. Ability to initialize the state of the qubits

III. Long relevant coherence times, much longer than gate operation time

IV. “Universal” set of quantum gates

V. Qubit-specific measurement capability

Quantum gates …

Having the qubits interact

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

PACS numbers: 89.80.+h, 03.65.Bz, 12.20.Fv, 32.80.Pj

...allows the realization of a
universal quantum computer !

$$|D\rangle|D\rangle \rightarrow |D\rangle|D\rangle$$

$$|D\rangle|S\rangle \rightarrow |D\rangle|S\rangle$$

$$|S\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$

$$|S\rangle|S\rangle \rightarrow |S\rangle|D\rangle$$

control target

Having the qubits interact

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$$|S\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$
$$|S\rangle|S\rangle \rightarrow |S\rangle|D\rangle$$

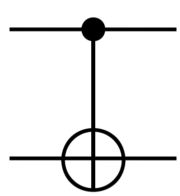
control target

Most popular gates:
- Cirac-Zoller gate (Schmidt-Kaler et al., Nature **422,** 408 (2003)).
- Geometric phase gate (Leibfried et al., Nature **422,** 412 (2003)).
- Mølmer-Sørensen gate (Sackett et al., Nature **404**, 256 (2000)).

A controlled-NOT operation

Control bit

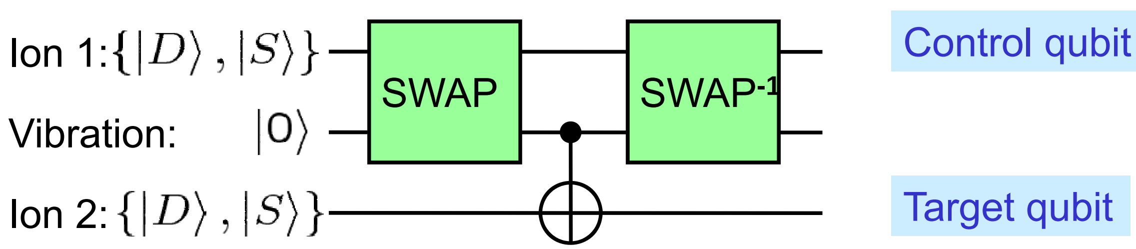
Target bit



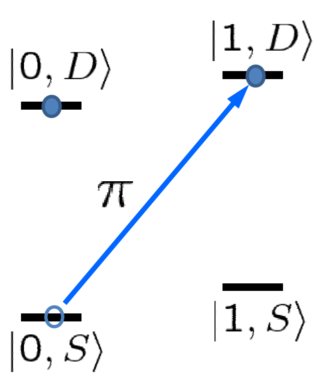
$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

Target

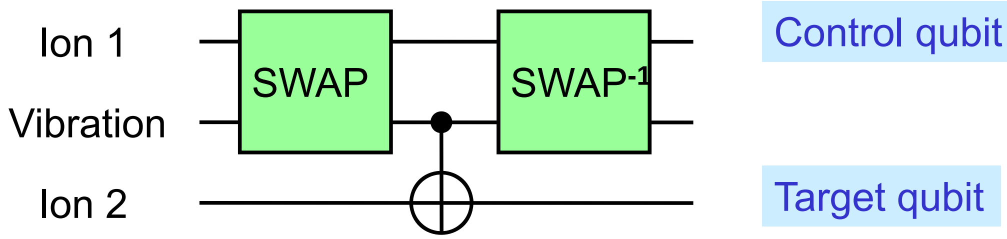
A controlled-NOT operation



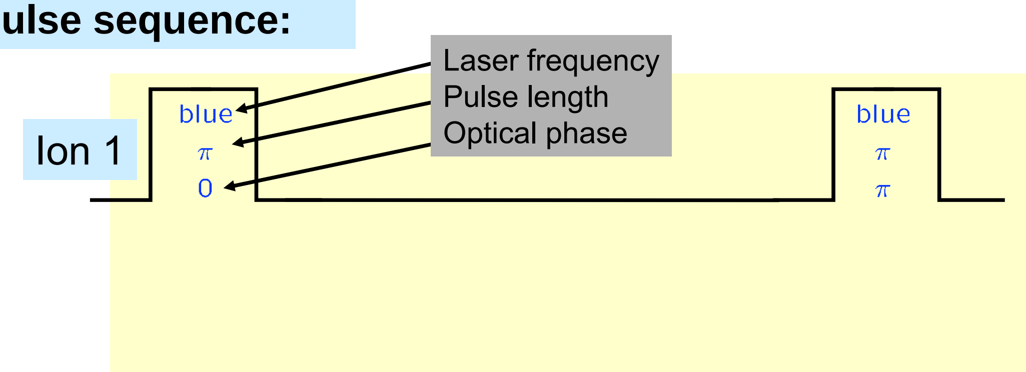
Mapping the qubit to the bus



A controlled-NOT operation



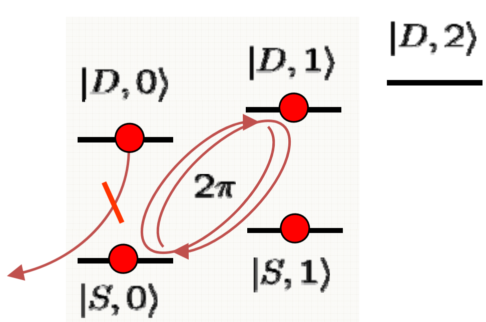
Pulse sequence:



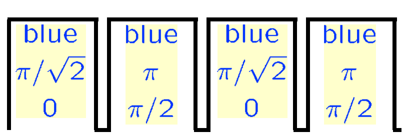
A phase gate

$$U_\Phi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

(columns: $|D,0\rangle$, $|S,0\rangle$, $|D,1\rangle$, $|S,1\rangle$)

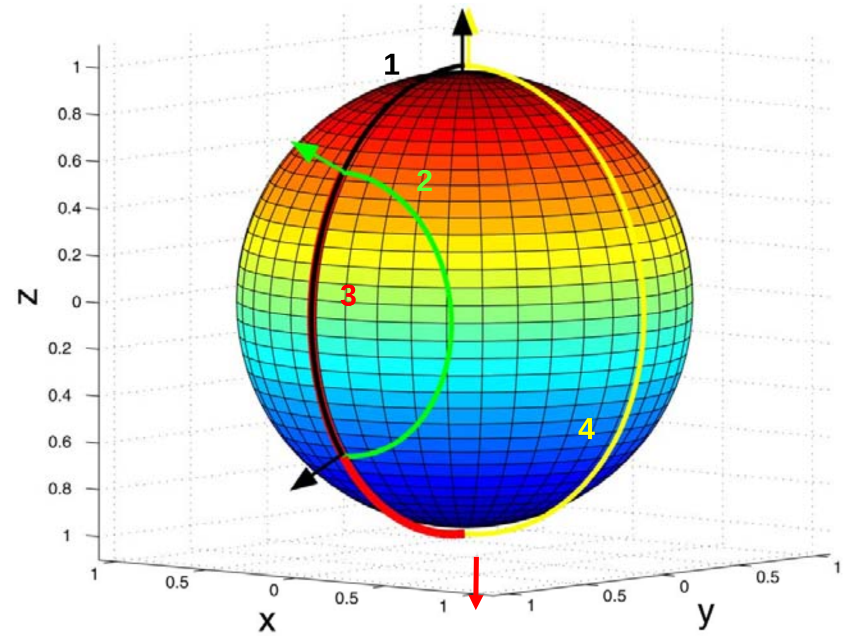


Composite 2π-rotation:

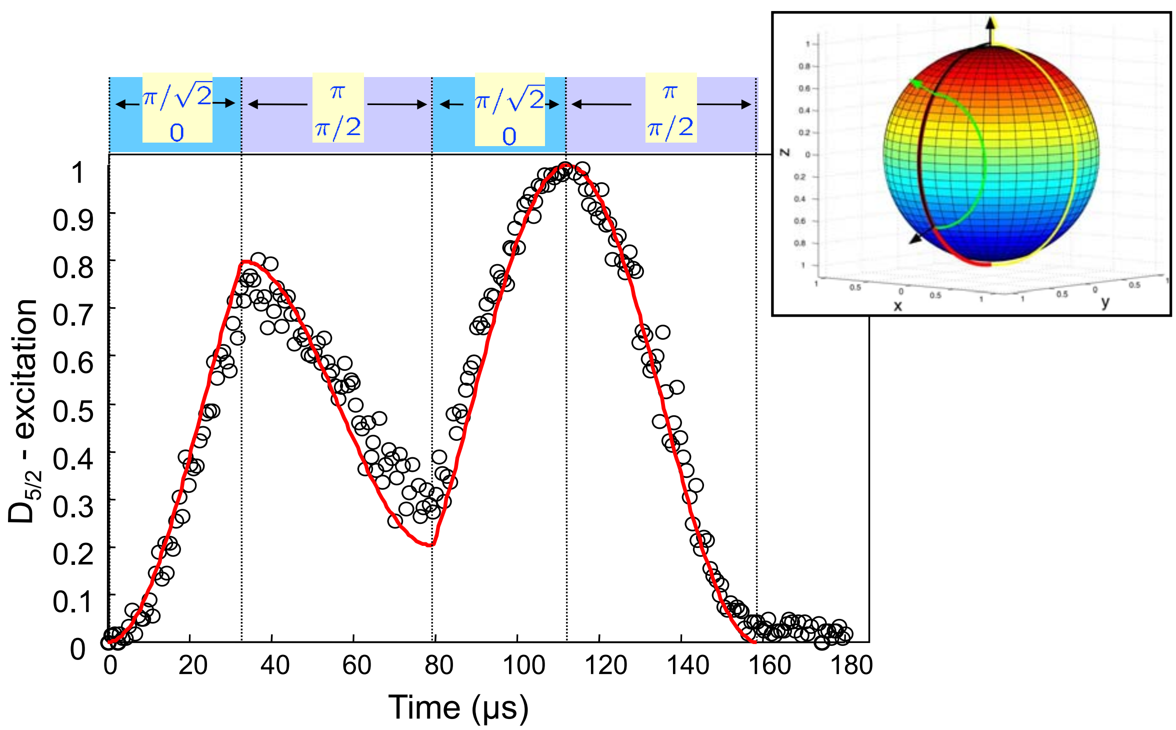


Composite phase gate

$$R^+(\pi,\pi/2)\,R^+\left(\pi/\sqrt{2},0\right)\,R^+(\pi,\pi/2)\,R^+\left(\pi/\sqrt{2},0\right)$$

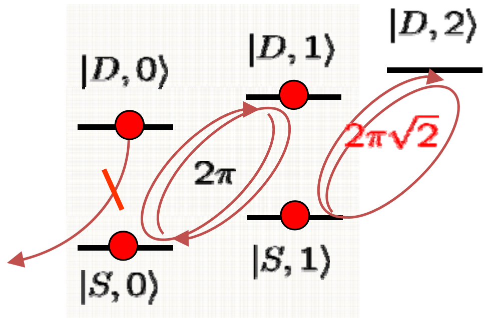


Composite phase gate



A phase gate

$$
\begin{array}{c} \\ U_\Phi = \end{array}
\begin{array}{c}
\begin{array}{cccc} |D,0\rangle & |S,0\rangle & |D,1\rangle & |S,1\rangle \end{array} \\
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & ? \end{pmatrix}
\end{array}
$$



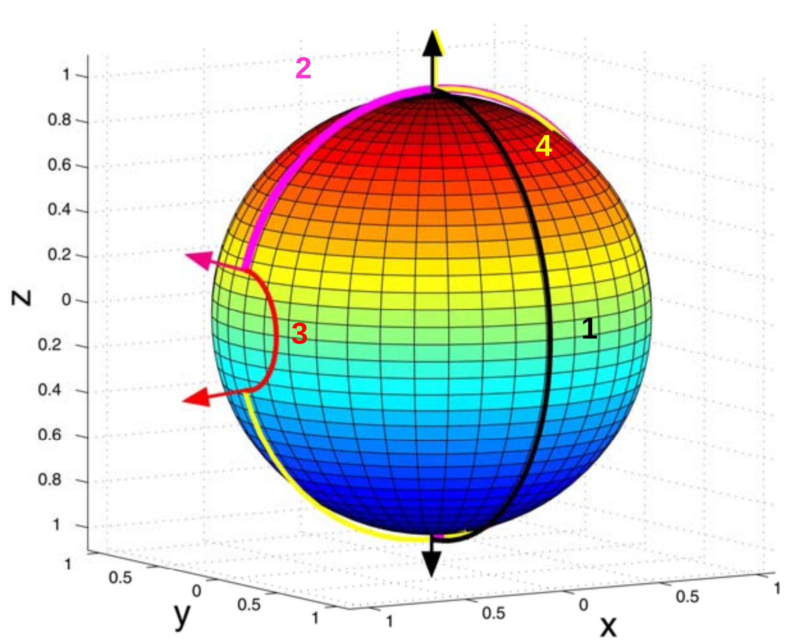
Composite 2π-rotation:

$$a^\dagger |n\rangle = \sqrt{n+1}\,|n+1\rangle$$

blue	blue	blue	blue
$\pi/\sqrt{2}$	π	$\pi/\sqrt{2}$	π
0	$\pi/2$	0	$\pi/2$

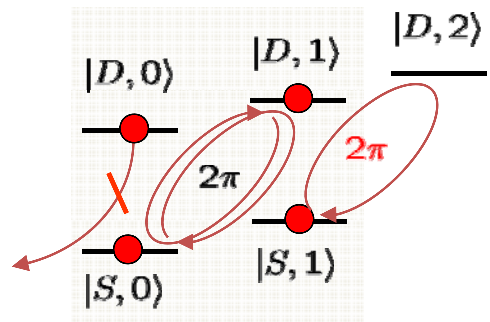
Action on the |1,S> ↔ |2,D> manifold

$$R^+(\pi,\pi/2)\, R^+\left(\pi/\sqrt{2},0\right)\, R^+(\pi,\pi/2)\, R^+\left(\pi/\sqrt{2},0\right)$$

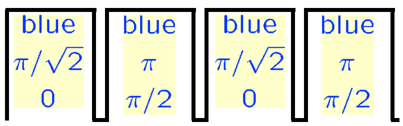


A phase gate

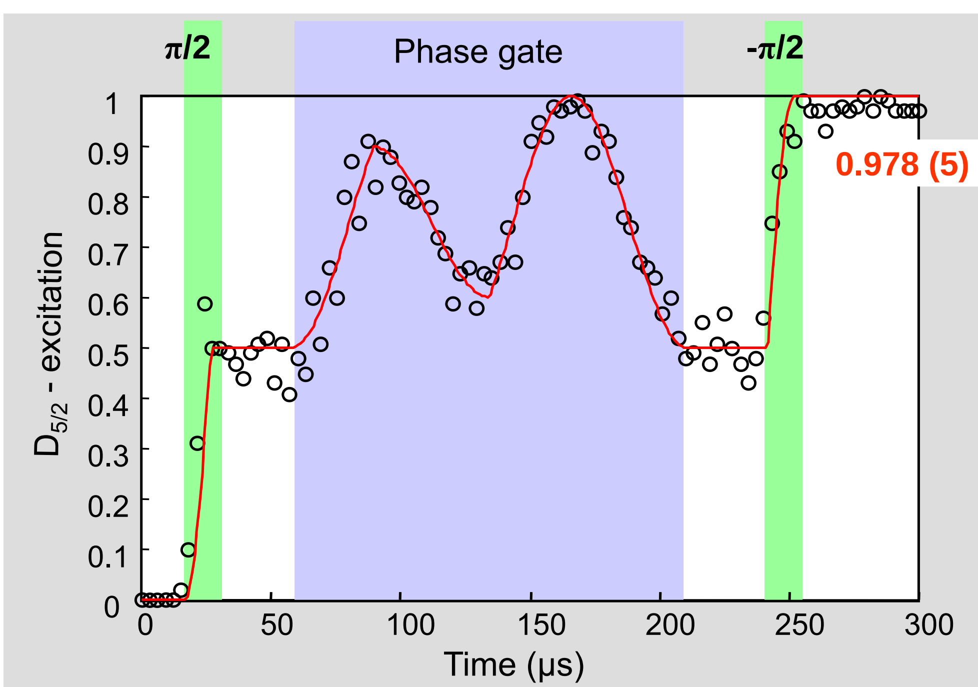
$$
\begin{array}{c} \\ U_{\Phi} = \end{array}
\begin{array}{c}
\begin{array}{cccc} |D,0\rangle & |S,0\rangle & |D,1\rangle & |S,1\rangle \end{array} \\
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}
\end{array}
$$



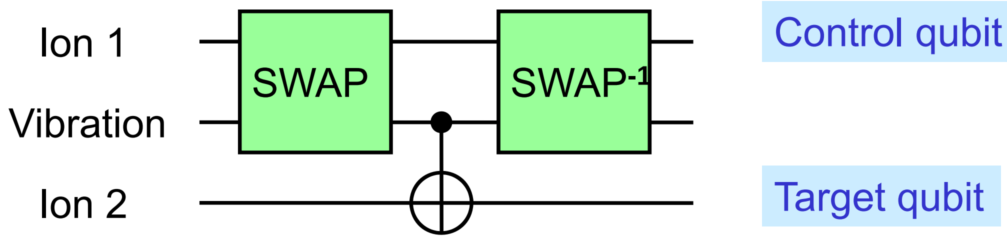
Composite 2π-rotation:



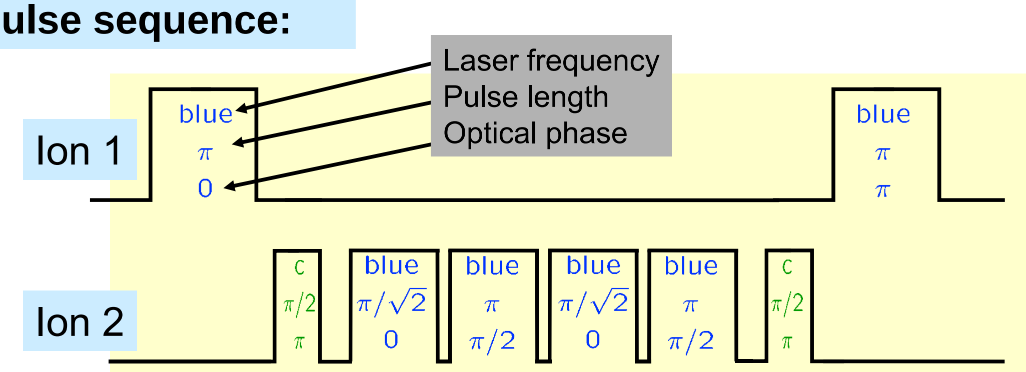
Single-ion CNOT



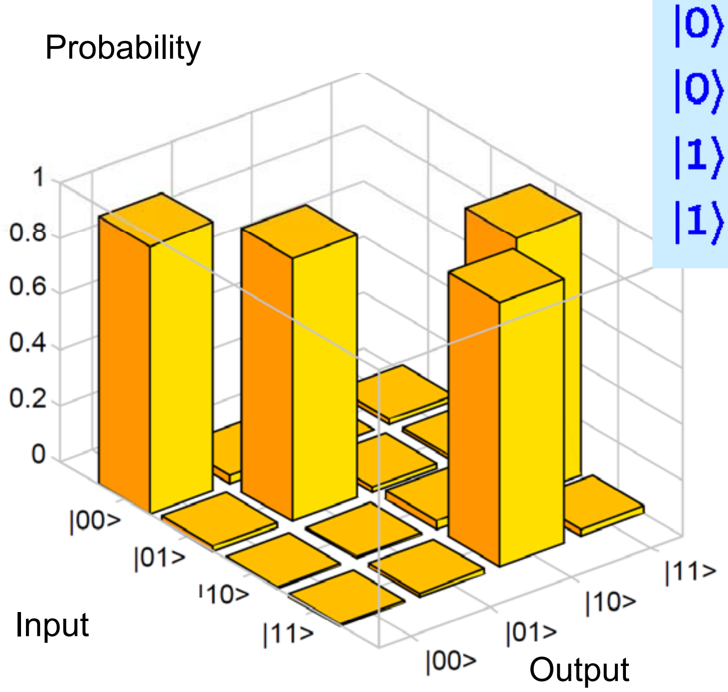
A controlled-NOT operation



Pulse sequence:



Truth table of the CNOT



$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$

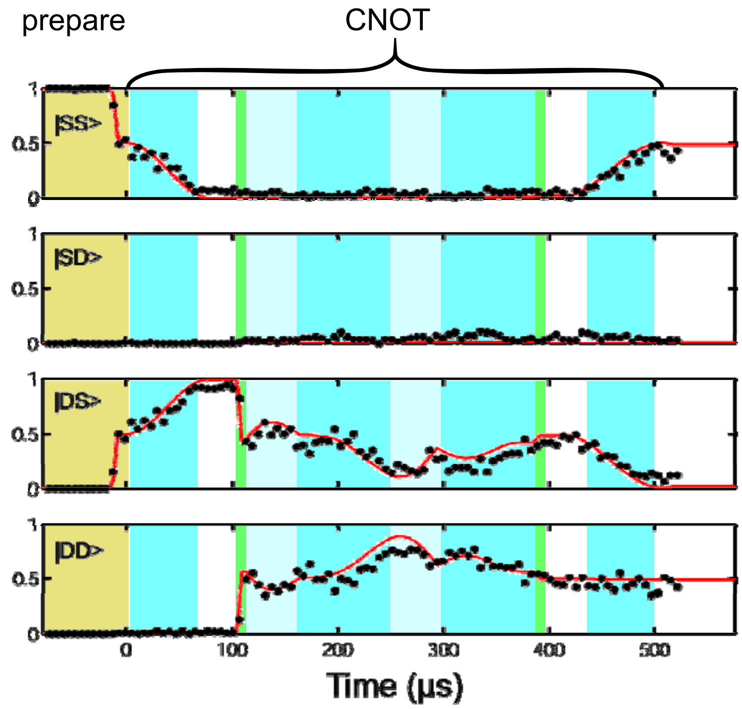
$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$

$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$

$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$

Using a CNOT to create a Bell state

$|SS\rangle \rightarrow |S+D\rangle|S\rangle \quad \longrightarrow \quad |SS\rangle + |DD\rangle$

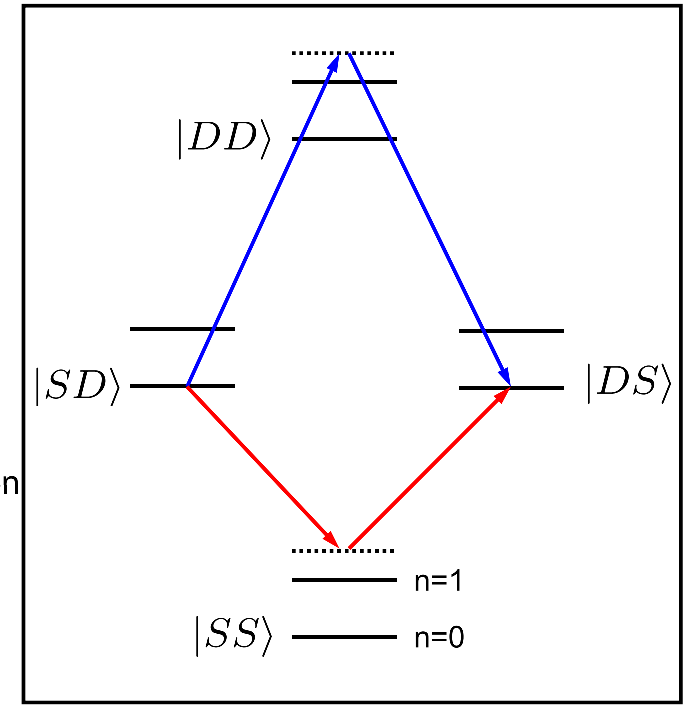


Cirac-Zoller gate

Draw backs of the Cirac-Zoller gate:

- slow (200 trap periods)
- single ion addressing required

Mølmer-Sørensen gate

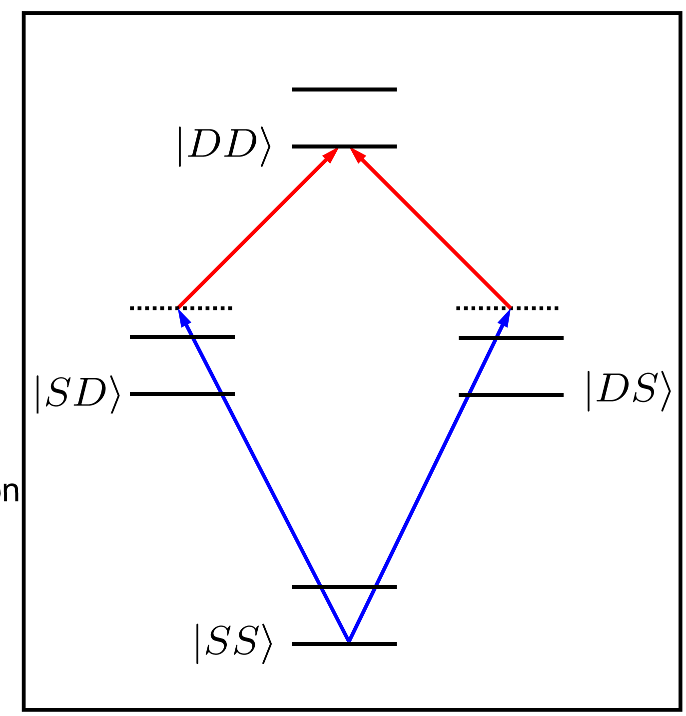


Raman transitions between

$$|SD\rangle \quad \Leftrightarrow \quad |DS\rangle$$

Interaction of two ions via common motion.

Mølmer-Sørensen gate

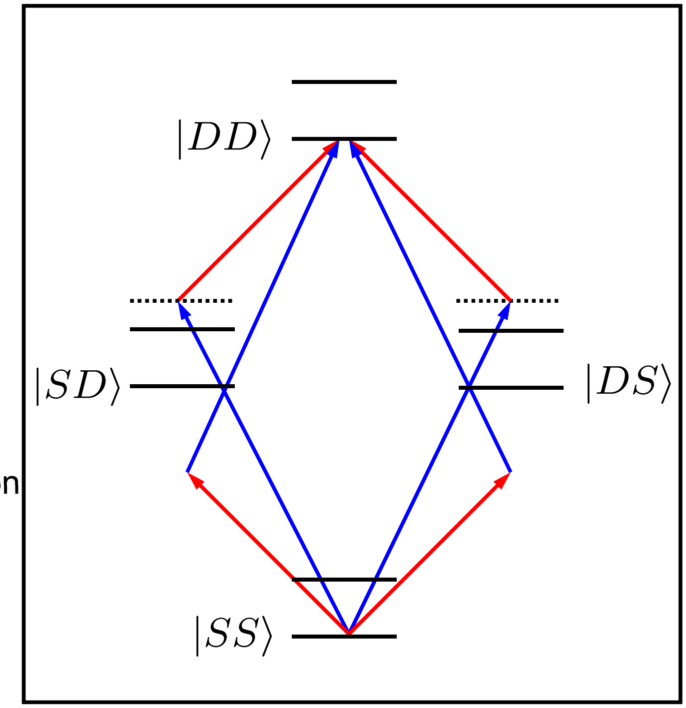


Raman transitions between

$$|SS\rangle \quad \Leftrightarrow \quad |DD\rangle$$

Interaction of two ions via common motion.

Mølmer-Sørensen gate



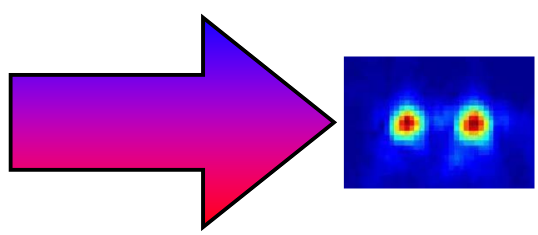
Raman transitions between

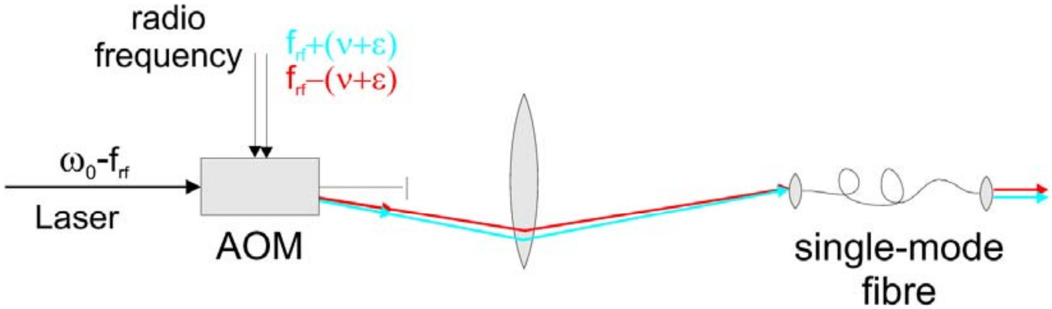
$$|SS\rangle \quad \Leftrightarrow \quad |DD\rangle$$

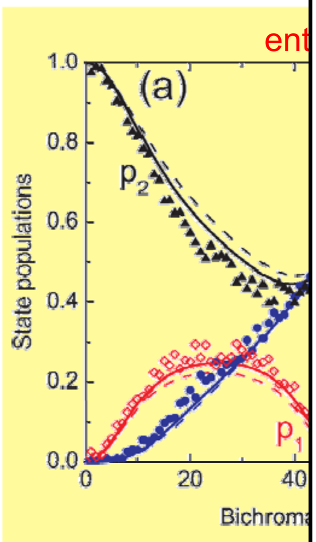
Interaction of two ions via common motion.

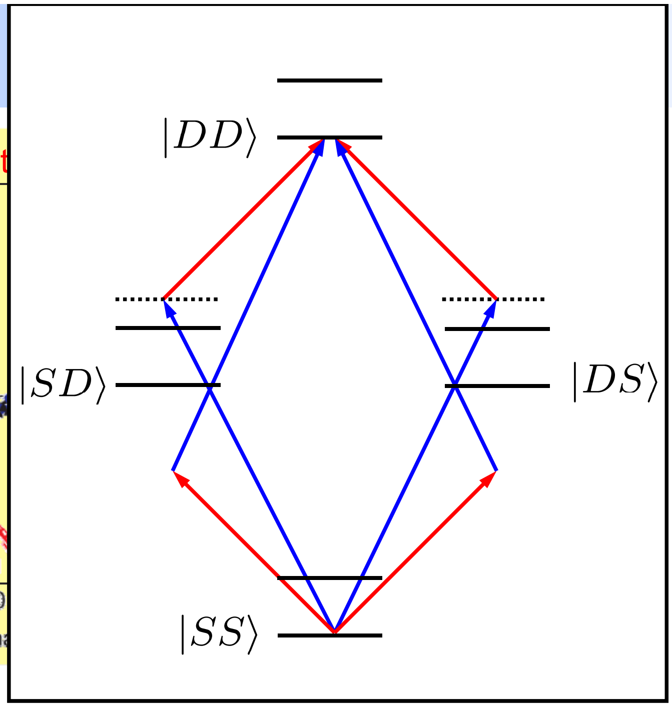
Technical realization

bicromatic beam
applied to both ions





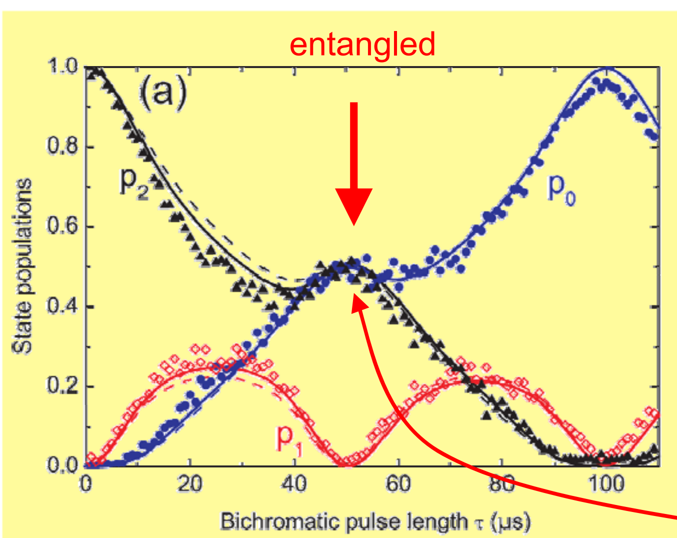




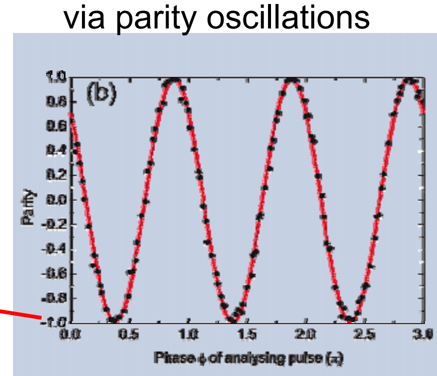
J. Benhelm et al., Nature Physics **4**, 463 (2008)

Theory: C. Roos, NJP **10,** 013002 (2008)

Entangling ions



measure entanglement
via parity oscillations



gate duration $51\mu s$

average fidelity: 99.3 (2) %

J. Benhelm et al., Nature Physics **4**, 463 (2008)

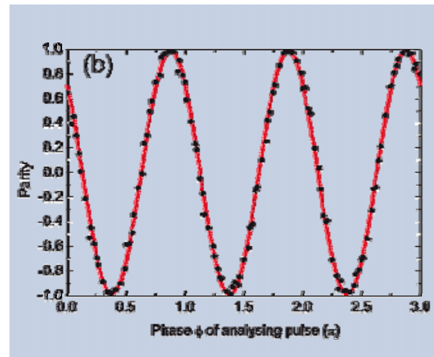
Theory: C. Roos, NJP **10,** 013002 (2008)

$$|00\rangle + |11\rangle \xrightarrow{R_2^C(\pi/2,\varphi),R_1^C(\pi/2,\varphi)}$$

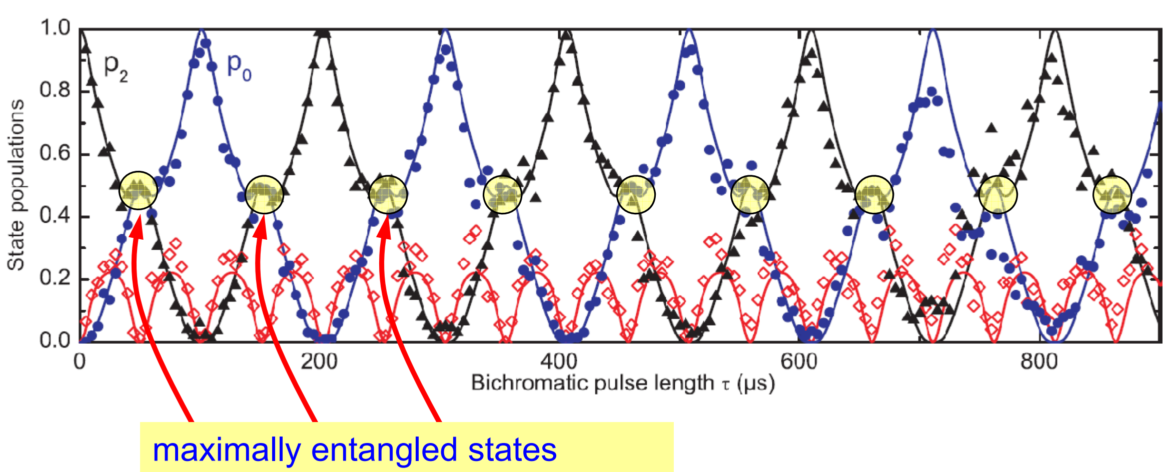
$$(|0\rangle + ie^{i\varphi}|1\rangle)(|0\rangle + ie^{i\varphi}|1\rangle) + (|1\rangle + ie^{-i\varphi}|0\rangle)(|1\rangle + ie^{-i\varphi}|0\rangle)$$

$$= (1 - e^{-2i\varphi})|00\rangle + ie^{i\varphi}(1 + e^{-2i\varphi})|01\rangle$$

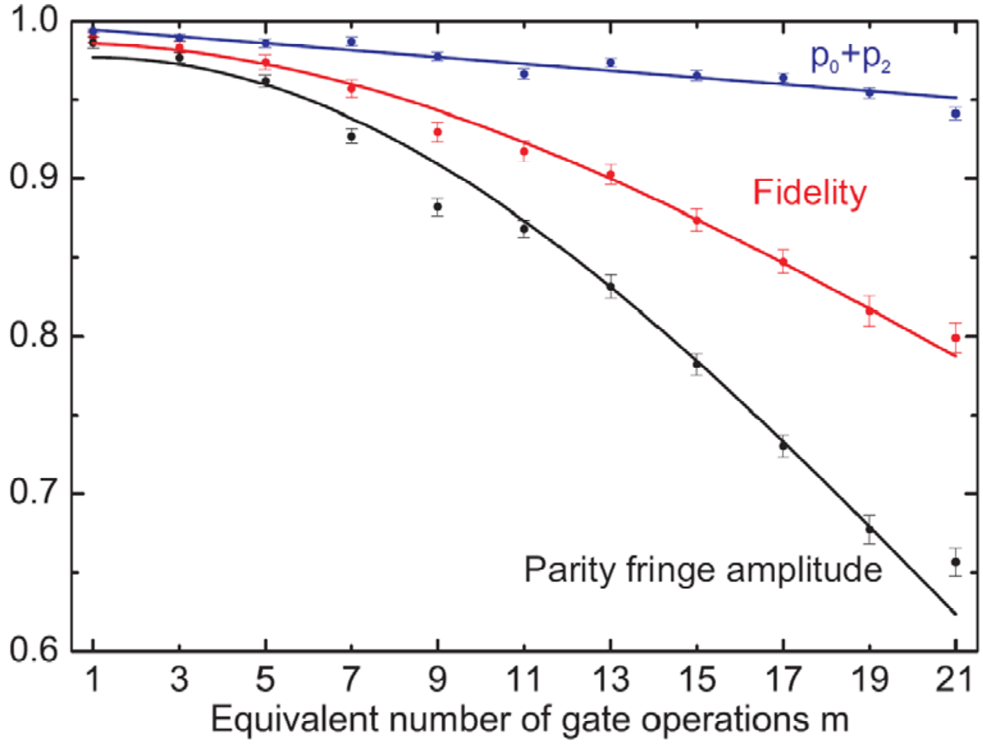
$$+ ie^{i\varphi}(1 + e^{-2i\varphi})|10\rangle + (1 - e^{-2i\varphi})|11\rangle ,$$



Gate concatenation



Gate performance



Scaling of this approach?

Problems :

- Coupling strength between internal and motional states of a N-ion string decreases as

$$\eta \propto \frac{1}{\sqrt{N}}$$

(momentum transfer from photon to ion string becomes more difficult)

-> Gate operation speed slows down

- More vibrational modes increase risk of spurious excitation of unwanted modes

- Distance between neighbouring ions decreases -> addressing more difficult

-> Use flexible trap potentials to split long ion string into smaller segments and perform operations on these smaller strings

DiVincenzo criteria

I. Scalable physical system, well characterized qubits ✓/ ?

II. Ability to initialize the state of the qubits ✓

III. Long relevant coherence times, much longer than gate operation time ✓

IV. “Universal” set of quantum gates ✓

V. Qubit-specific measurement capability ✓

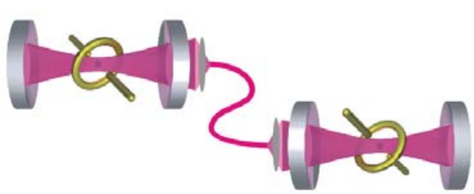
Often neglected:

- exceptional fidelity of operations
- low error rate also for large quantum systems
- all requirements have to met at the same time

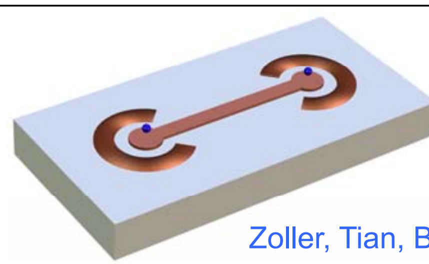
Scaling of ion trap quantum computers

Its easy to have thousands of coherent qubits …
but hard to control their interaction

Kielpinski, Monroe, Wineland

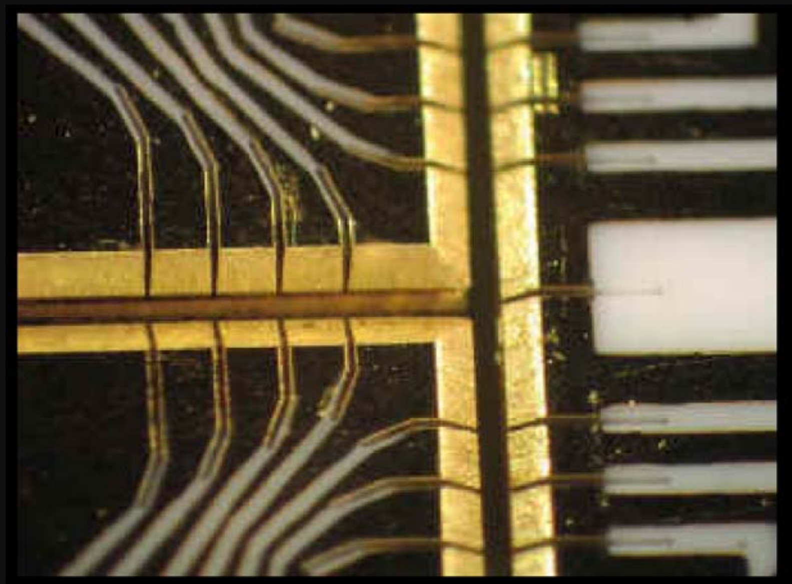


Cirac, Zoller, Kimble, Mabuchi



Zoller, Tian, Blatt

The Michigan T trap



An implementation of the Deutsch-algoritm …

Deutsch‘s problem: Introduction

Decide which class the coin is:

False (equal sides) or ***Fair***

A single measurement does NOT give the right answer

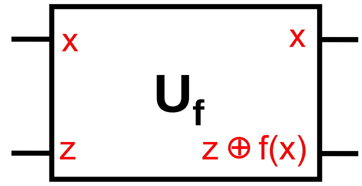
Deutsch‘s problem: Mathematical formulation

4 possible coins are representend by 4 functions

	Constant		**Balanced**	
	Case 1	**Case 2**	**Case 3**	**Case 4**
***f*(0)**	**0**	**1**	**0**	**1**
***f*(1)**	**0**	**1**	**1**	**0**

Deutsch's problem: Mathematical formulation

4 possible coins are representend by 4 functions

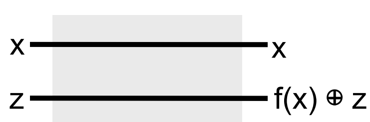
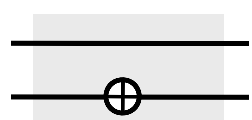
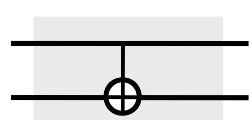
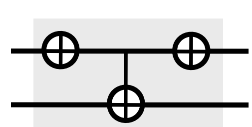


	Constant		Bal	
	Case 1	Case 2	Case 3	
$f(0)$	0	1	0	1
$f(1)$	0	1	1	0
$z \oplus f(x)$	ID	NOT	CNOT	Z-CNOT

$$U_{fn}\left|x,z\right\rangle = \left|x, f_n(x) \oplus z\right\rangle$$

Physically reversible process realized by a unitary transformation

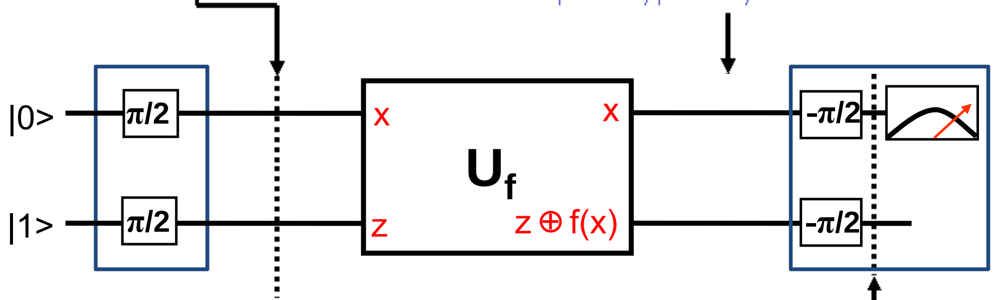
Deutsch Jozsa quantum circuit

Case	Logic	Quantum circuit	Matrix U_{fn}
f_1	*ID*		1000 0100 0010 0001
f_2	*NOT*		0100 1000 0001 0010
f_3	*CNOT*		1000 0100 0001 0010
f_4	*Z-CNOT*		0100 1000 0010 0001

Deutsch Jozsa quantum circuit

$$|\psi_1\rangle = |0+1\rangle|0-1\rangle$$

$$|\psi_2\rangle = \pm|0+1\rangle|0-1\rangle \text{ if } f(0) = f(1)$$
$$= \pm|0-1\rangle|0-1\rangle \text{ if } f(0) \neq f(1)$$

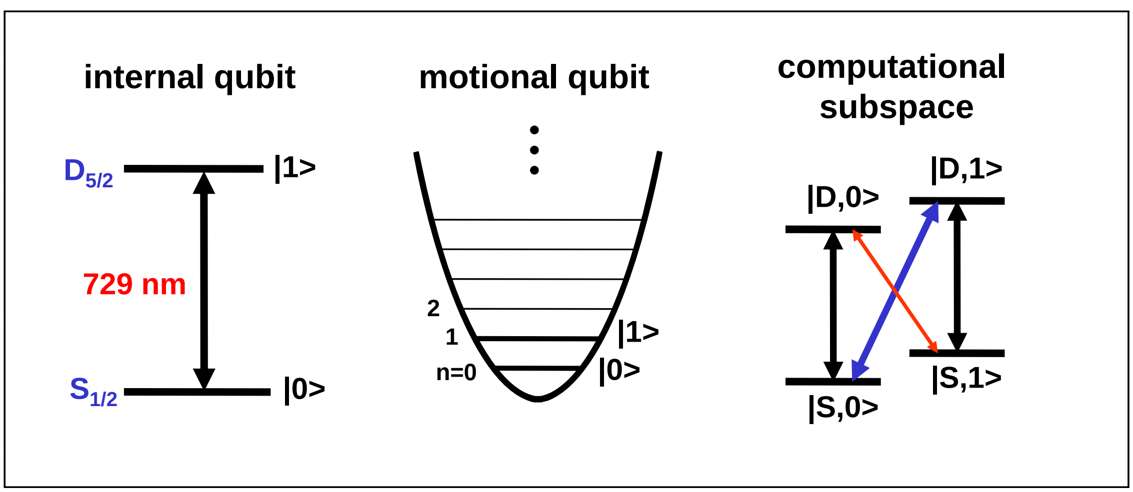


$$|\psi_3\rangle = \pm|0\rangle|1\rangle \text{ if } f(0) = f(1) \quad \textit{(Constant)}$$
$$= \pm|1\rangle|1\rangle \text{ if } f(0) \neq f(1) \quad \textit{(Balanced)}$$

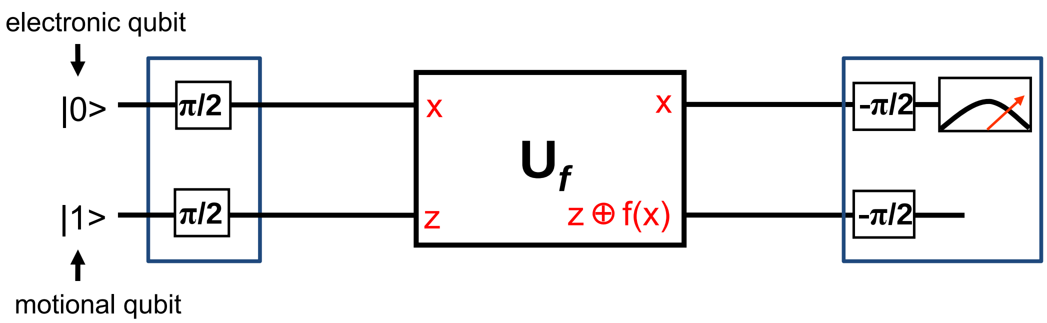
Quantum analysis gives the right answer after a single measurement!

- *D. Deutsch, R. Josza, Proc. R. Soc. London A439, 553 (1992)*
- *M. Nielsen, I. Chuang, QC and QI, Cambridge (2000)*

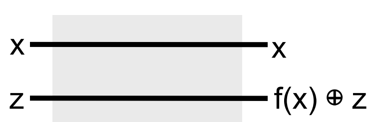
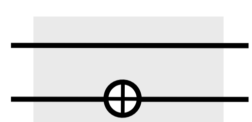
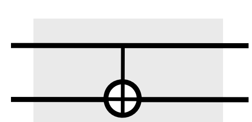
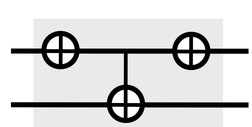
Qubits in $^{40}Ca^{+}$



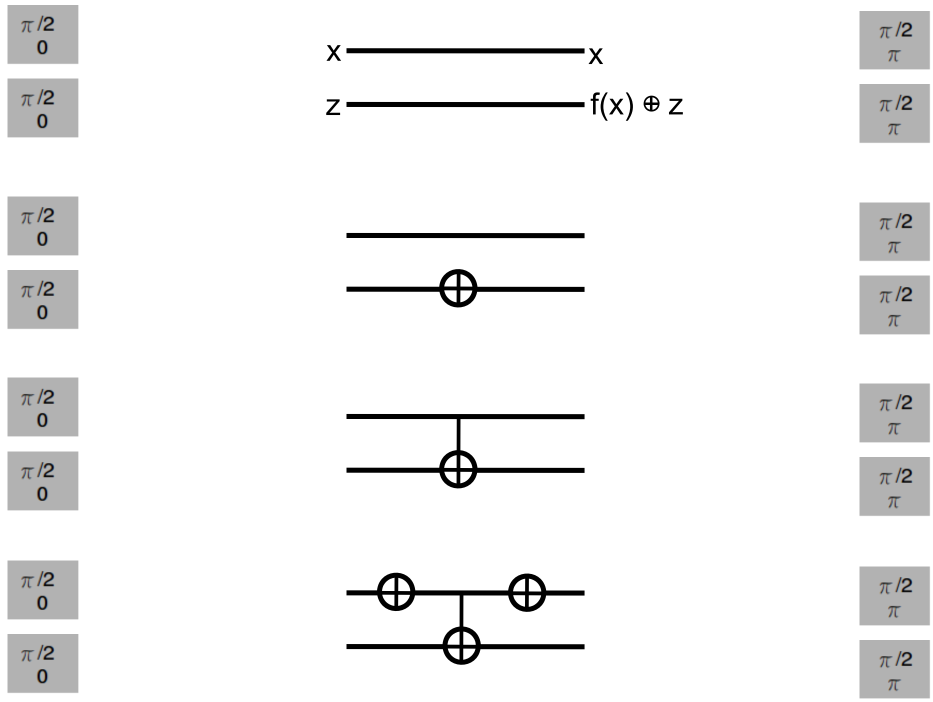
No information in the second qubit



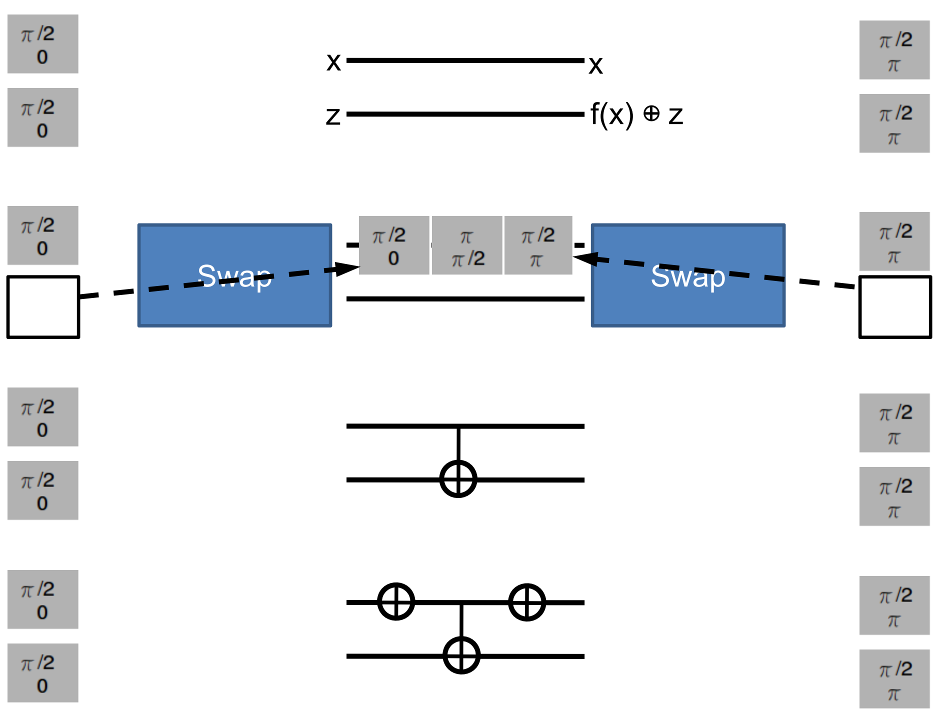
Deutsch Jozsa quantum circuit

Case	Logic	Quantum circuit	Matrix U_{fn}
f_1	*ID*		1000 0100 0010 0001
f_2	*NOT*		0100 1000 0001 0010
f_3	*CNOT*		1000 0100 0001 0010
f_4	*Z-CNOT*		0100 1000 0010 0001

Deutsch Jozsa: Realization

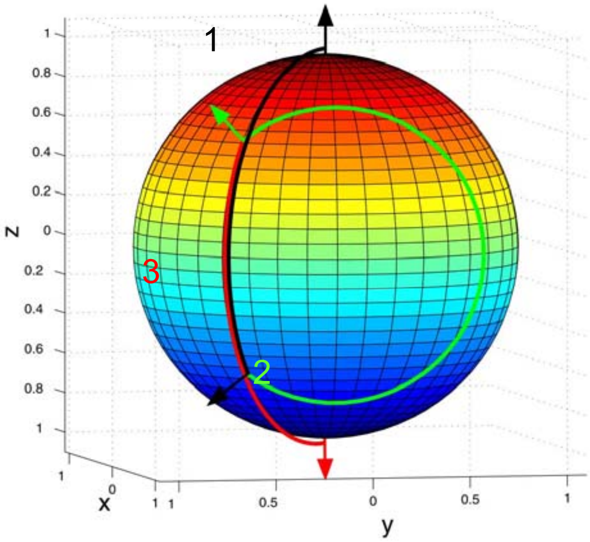


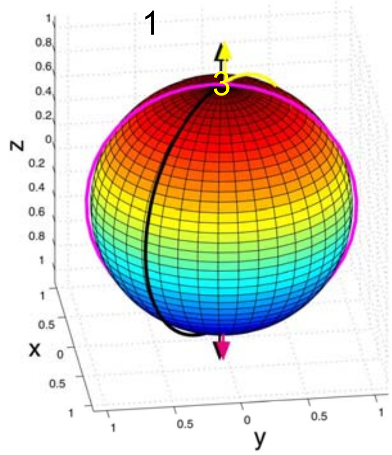
Deutsch Jozsa: Realization



3-step composite SWAP operation

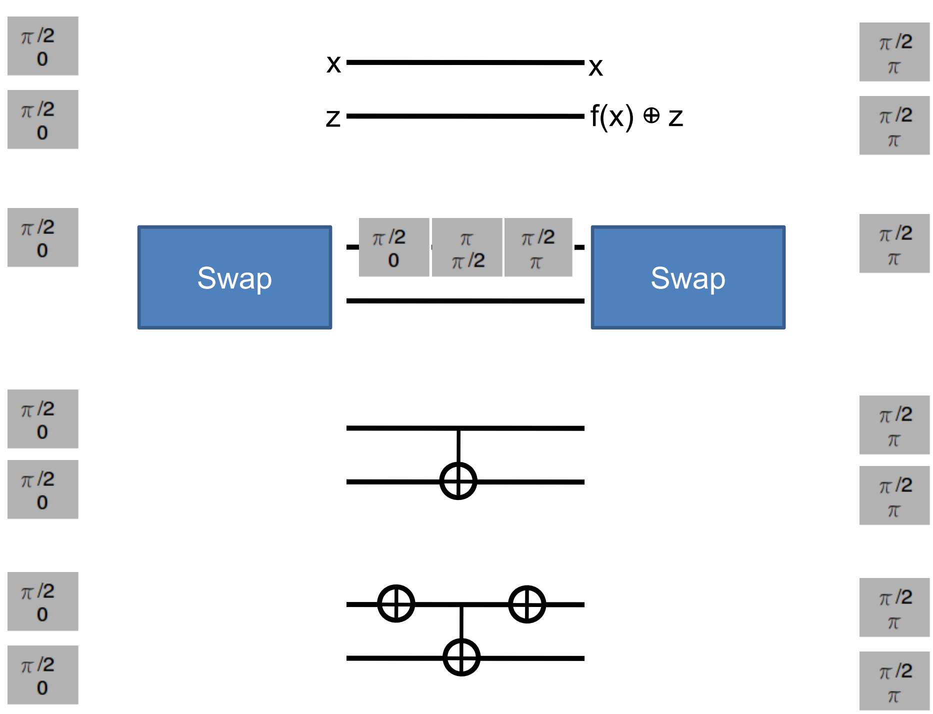
$$R^{+}\left(\frac{\pi}{\sqrt{2}},\pi\right)R^{+}\left(\frac{2\pi}{\sqrt{2}},\pi+\varphi_{\text{swap}}\right)R^{+}\left(\frac{\pi}{\sqrt{2}},\pi\right)$$



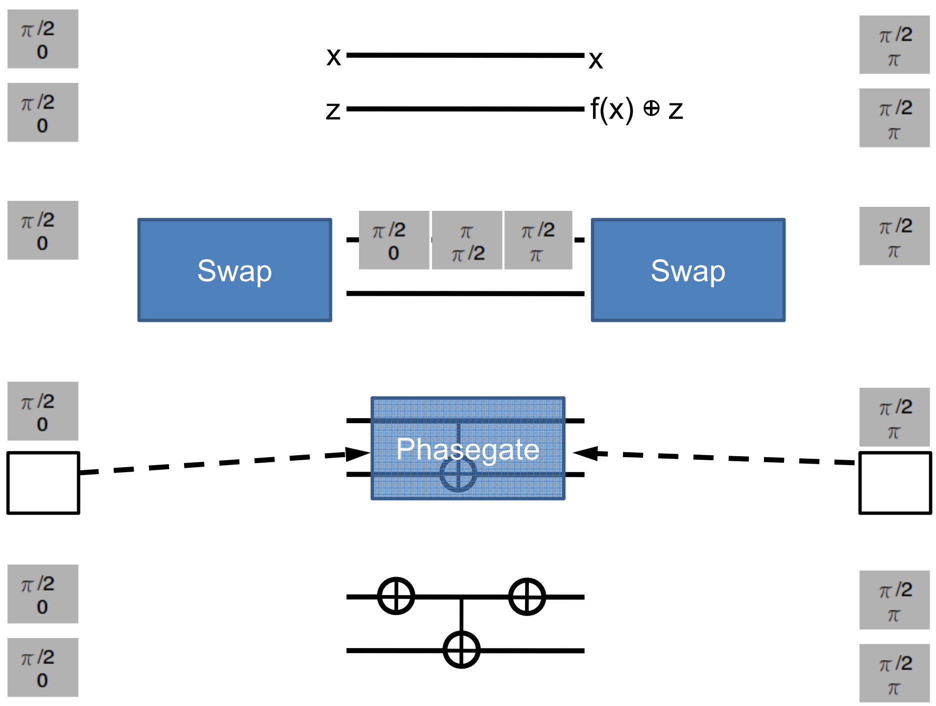


I. Chuang et al., Innsbruck (2002)

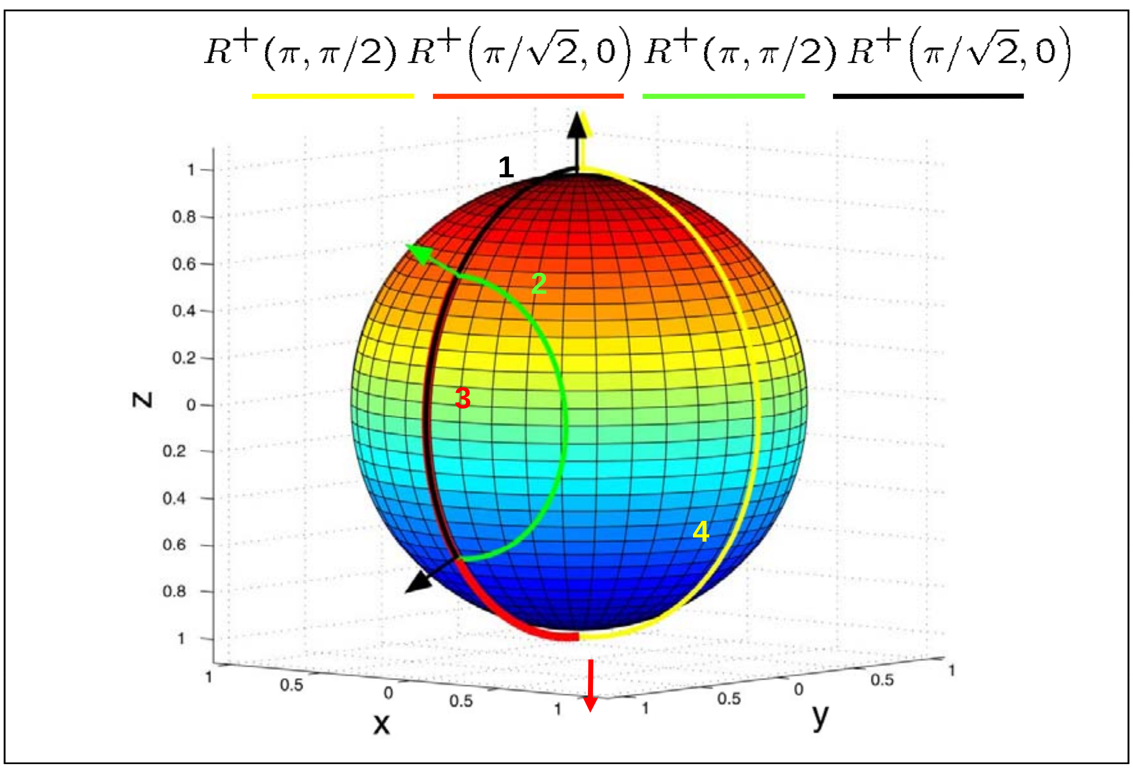
Deutsch Jozsa: Realization



Deutsch Jozsa: Realization

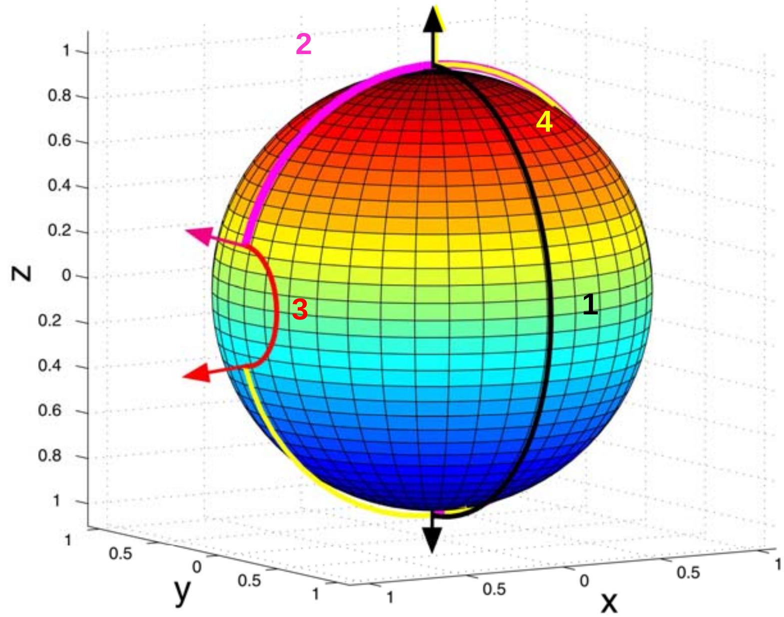


Composite phase gate (2π rotation)

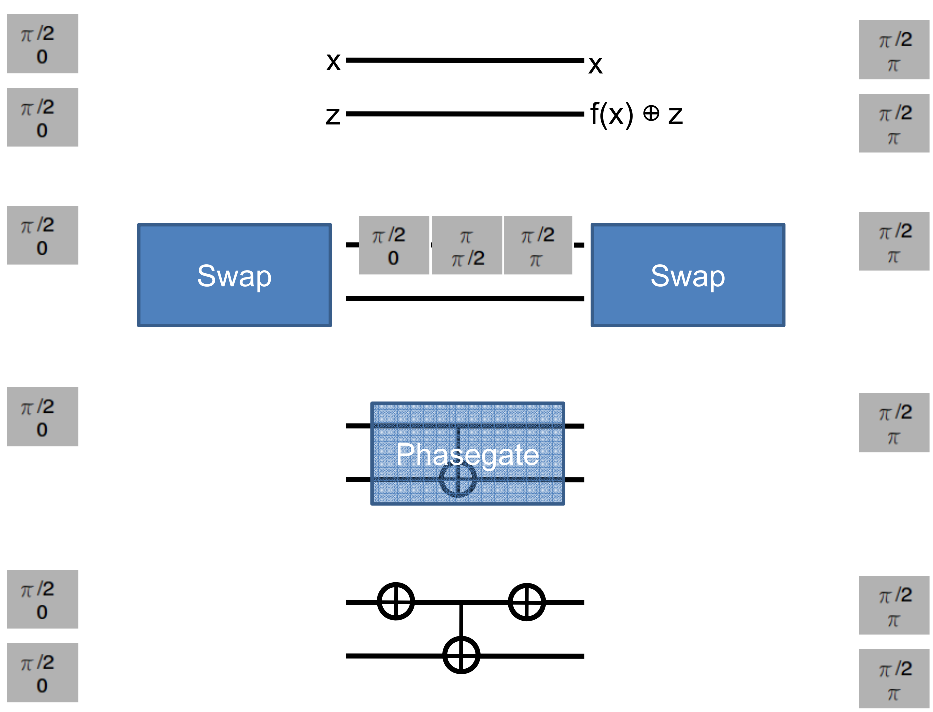


Action on |S,1> - |D,2>

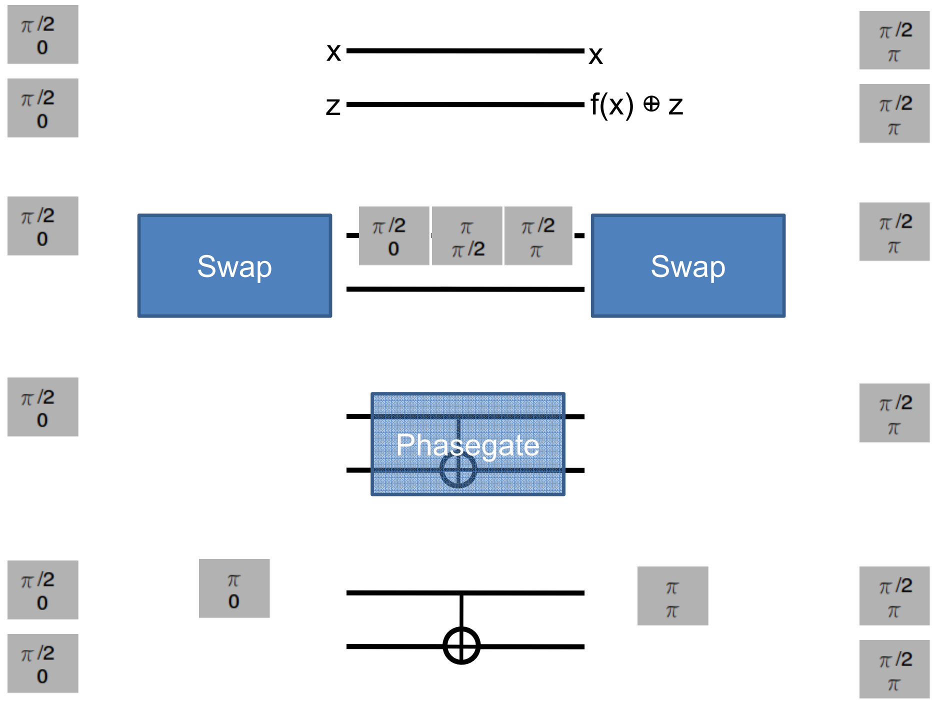
$$R^{+}(\pi,\pi/2)\,R^{+}\left(\pi/\sqrt{2},0\right)\,R^{+}(\pi,\pi/2)\,R^{+}\left(\pi/\sqrt{2},0\right)$$



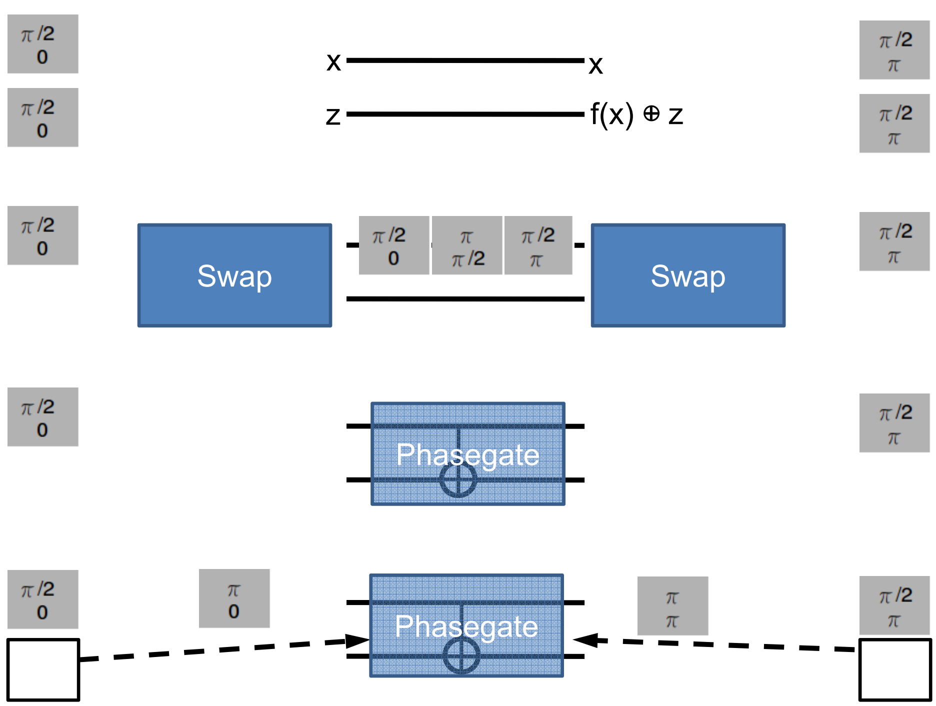
Deutsch Jozsa: Realization



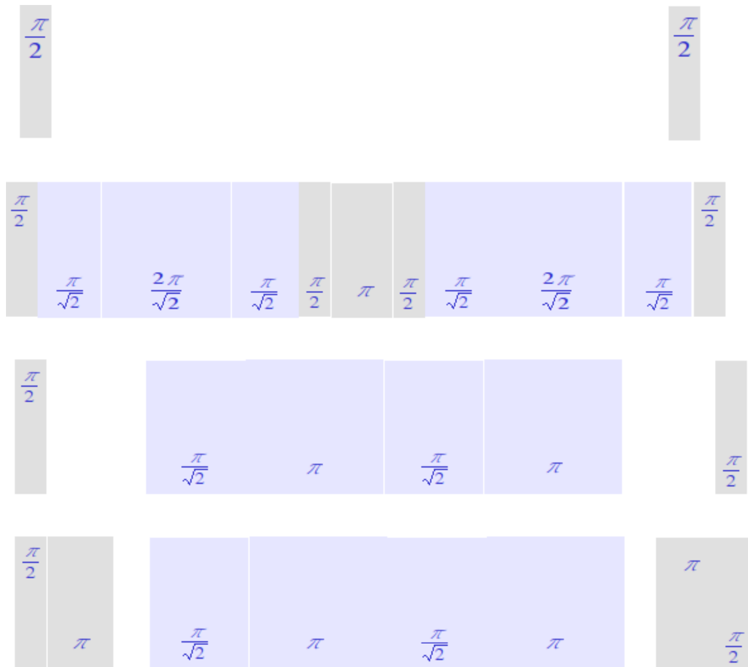
Deutsch Jozsa: Realization



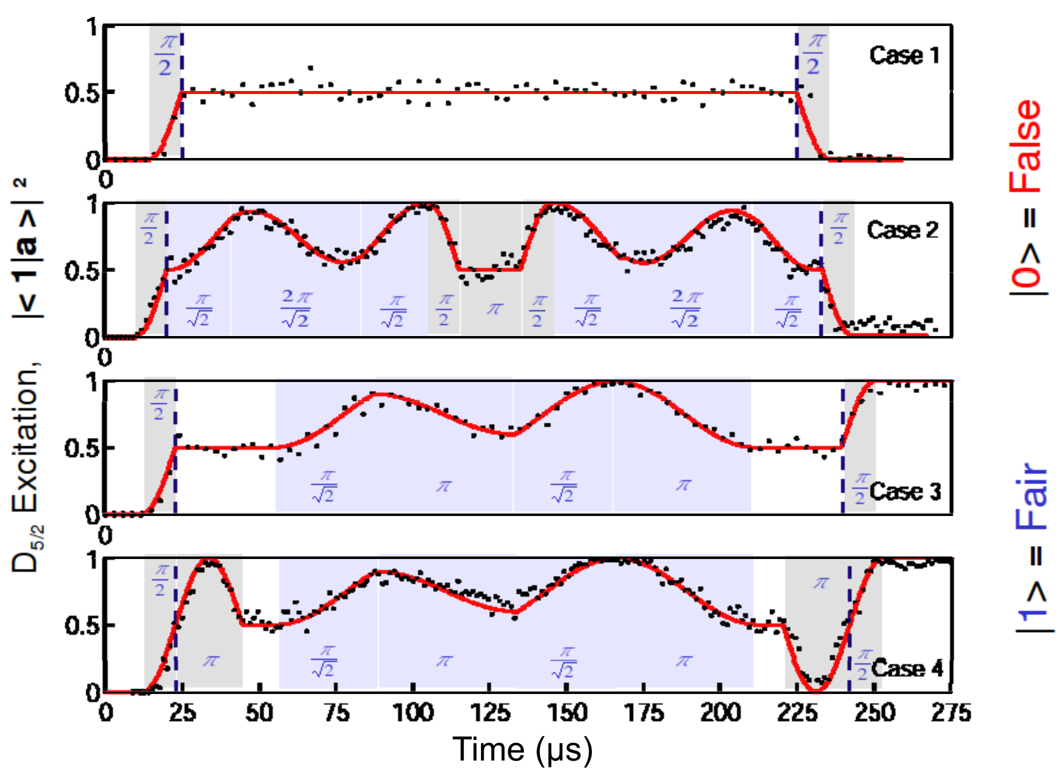
Deutsch Jozsa: Realization



Deutsch Jozsa: Realization



Deutsch Jozsa: Realization



Deutsch Jozsa: Result

	Constant		Balanced	
	Case 1	Case 2	Case 3	Case 4
expected $\|\langle 1\|a\rangle\|^2$	0	0	1	1
measured $\|\langle 1\|a\rangle\|^2$	0.019(6)	0.087(6)	0.975(4)	0.975(2)
expected $\|\langle 1\|w\rangle\|^2$	1	1	1	1
measured $\|\langle 1\|w\rangle\|^2$	--	0.90(1)	0.931(9)	0.986(4)

S. Gulde et al., Nature 412, 48 (2003)

IQI

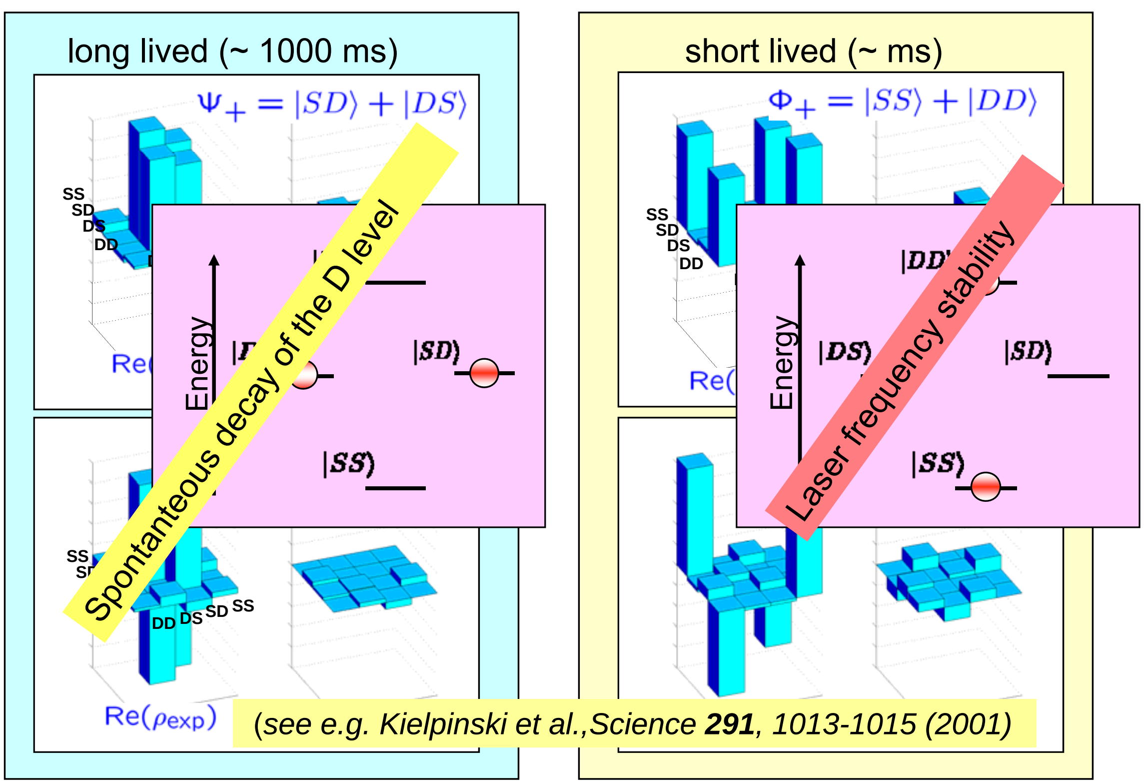
Conclusions

- Basics of ion trap quantum computing
- Measuring a density matrix
- Quantum gates
- Deutsch Algorithm

€ FWF SFB
SCALA QGATES
Industrie Tirol
IQI GmbH
FWF bm:bwk
$ IAEPA

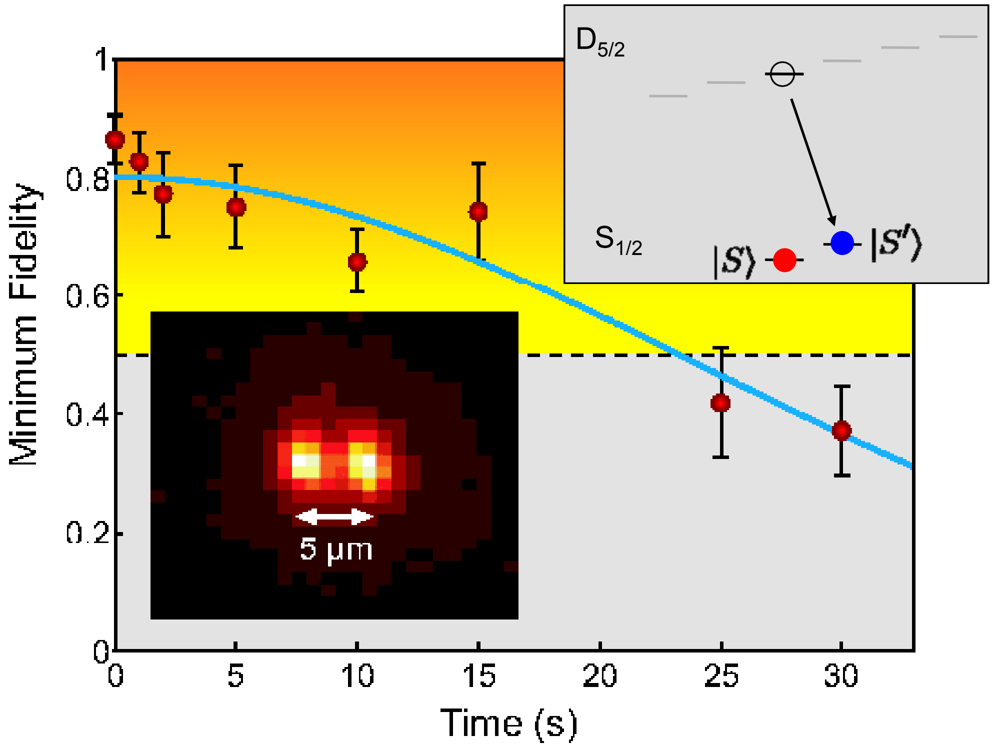
Berkeley, Nov 25th 2008

Decoherence properties of qubits

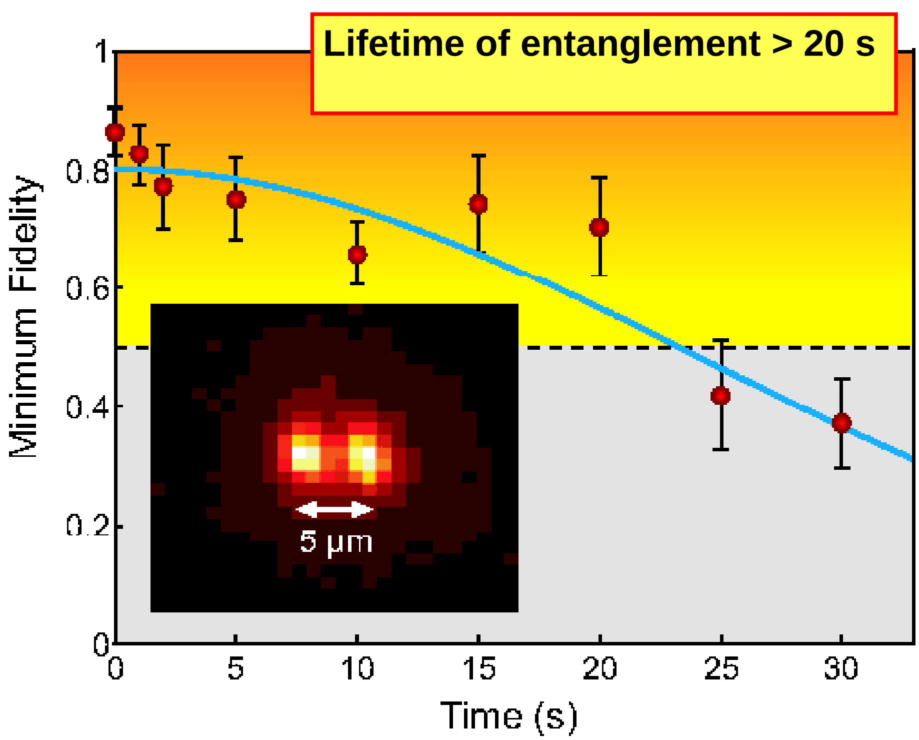


(see e.g. Kielpinski et al.,Science **291**, 1013-1015 (2001)

Ultra long lived entanglement

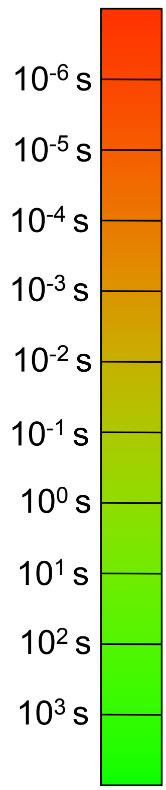


Ultra long lived entanglement



H. Häffner et al., App. Phys. B **81** 151 (2005).

Realized time scales



Single qubit gates

Two qubit gates (Geometric phase gates: Boulder, Innsbruck)

Two qubit gates (Cirac-Zoller approach)

Single qubit coherence (magnetic field sensitive)

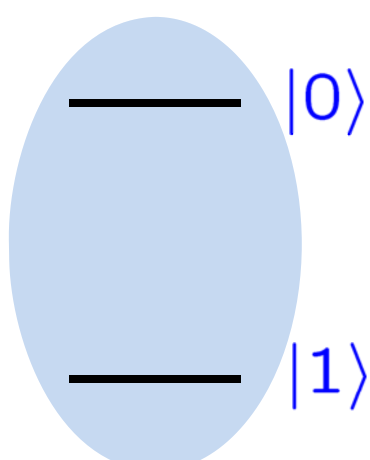
Coherence of the motion (Innsbruck)

Single qubit coherence (magnetic field insensitive, Boulder)
Decoherence free subspaces

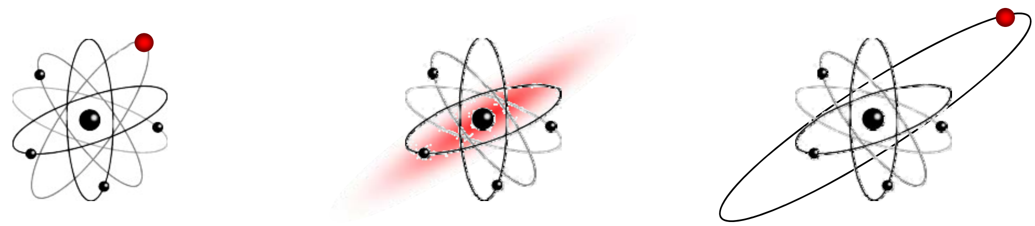
Well chosen single qubit coherence (Boulder)

A quantum bit

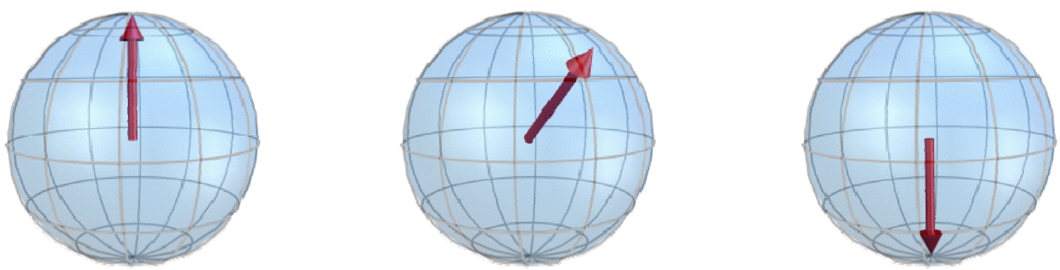
Two level system:



Qubits



$|0\rangle$ $\alpha|0\rangle + \beta|1\rangle$ $|1\rangle$



Possible qubit encodings

Physical Qubit $\longleftrightarrow$ **Logical Qubit**

$$|0\rangle_P = |D\rangle \qquad |0\rangle_L = |SD\rangle$$

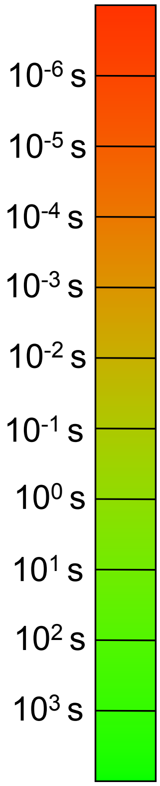
$$|1\rangle_P = |S\rangle \qquad |1\rangle_L = |DS\rangle$$

Effect of magnetic field or laser frequency fluctuations on qubits

$$\begin{array}{c} |D\rangle + |S\rangle \\ \Downarrow \\ e^{i\phi}|D\rangle + |S\rangle \end{array} \longleftrightarrow \begin{array}{c} |SD\rangle + |DS\rangle \\ \Downarrow \\ e^{i\phi}\left(|SD\rangle + |DS\rangle\right) \end{array}$$

Logical qubit experiences global phase only

Realized time scales



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Two qubit gates (Cirac-Zoller approach)

Single qubit coherence (magnetic field sensitive)

Coherence of the motion (Innsbruck)

Single qubit coherence (magnetic field insensitive)

Decoherence free subspaces

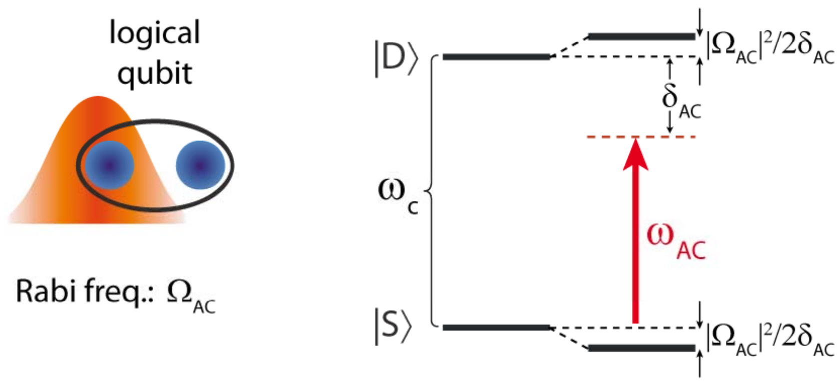
Well chosen single qubit (Boulder)

Universal set of gates in a DFS

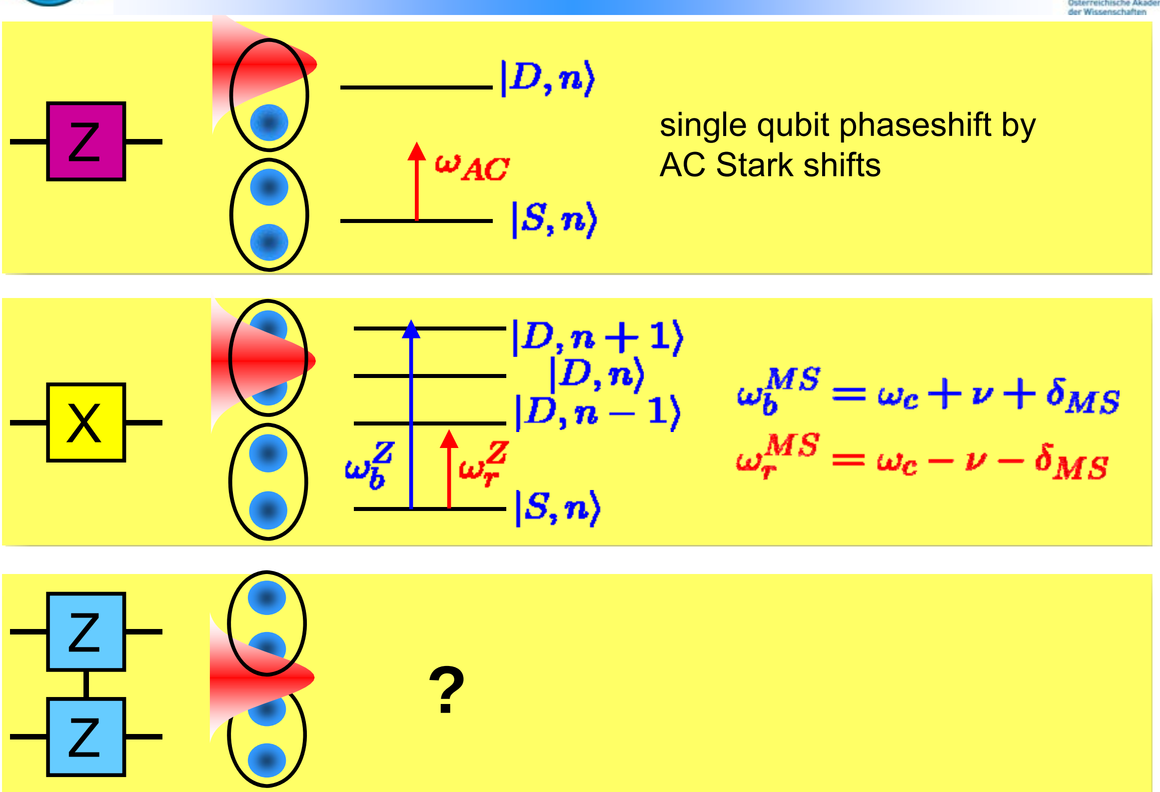
- single qubit operations
  - Z gates
  - X gates

- two –qubit operations
  - phase gate

Z gate

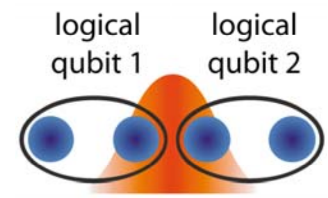


Universal set of gates in a DFS



Two logical qubit gate

Two body interactions preferred:



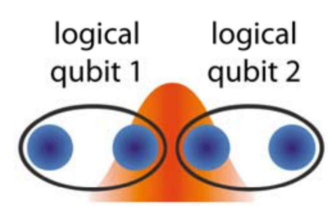
Most interactions cause the state to leave the decoherence free subspace.

Some solutions: L. Aolita et al., PRA **75** 052337 (2007)

Two logical qubit phase gate

Action of the phase gate on two physical qubits:

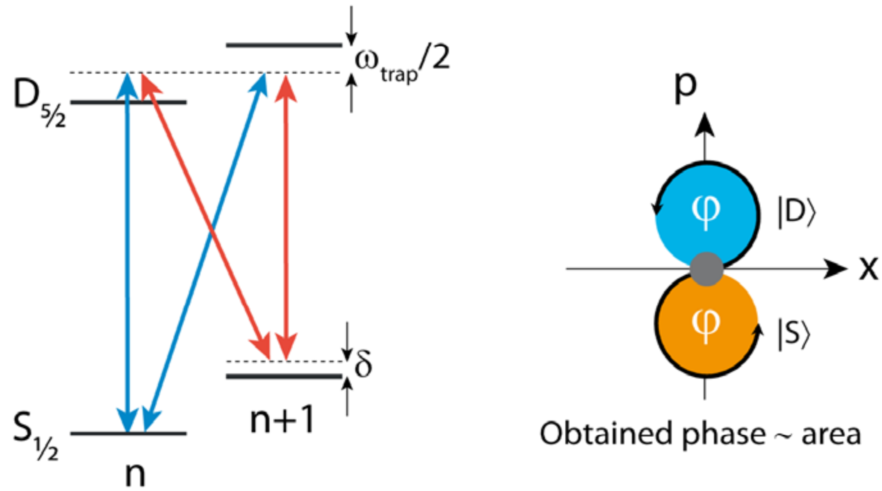
$$\begin{array}{c} |DD\rangle \\ |DS\rangle \\ |SD\rangle \\ |SS\rangle \end{array} \Rightarrow \begin{array}{c} e^{i\phi}|DD\rangle \\ |DS\rangle \\ |SD\rangle \\ e^{i\phi}|SS\rangle \end{array}$$



...and on the logical qubits:

$$\begin{array}{c} |00\rangle_l \\ |01\rangle_l \\ |10\rangle_l \\ |11\rangle_l \end{array} = \begin{array}{c} |S\rangle|DS\rangle|D\rangle \\ |S\rangle|DD\rangle|S\rangle \\ |D\rangle|SS\rangle|D\rangle \\ |D\rangle|SD\rangle|S\rangle \end{array} \Rightarrow \begin{array}{c} |S\rangle|DS\rangle|D\rangle \\ |S\rangle e^{i\phi}|DD\rangle|S\rangle \\ |D\rangle e^{i\phi}|SS\rangle|D\rangle \\ |D\rangle|SD\rangle|S\rangle \end{array} = \begin{array}{c} |00\rangle_l \\ e^{i\phi}|01\rangle_l \\ e^{i\phi}|10\rangle_l \\ |11\rangle_l \end{array}$$

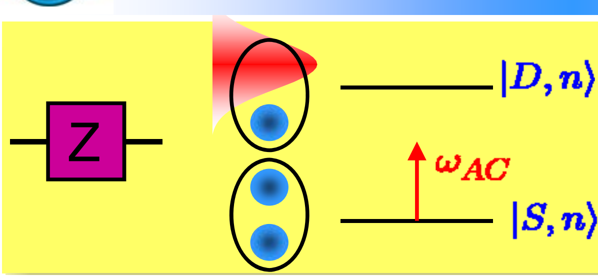
Two qubit phase gate



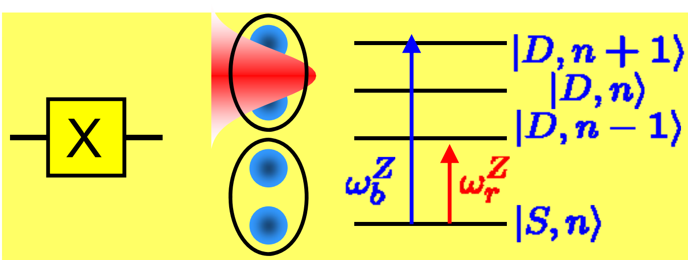
D. Leibfried, et al., Nature **422** 412 (2003)

K. Kim et. al., Phys. Rev. A**77**, 050303 (2008)

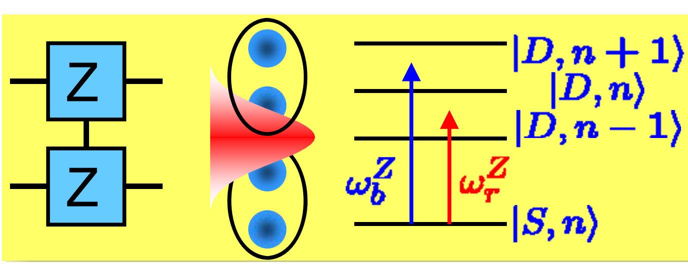
Universal set of gates in a DFS



single qubit phaseshift by AC Stark shifts

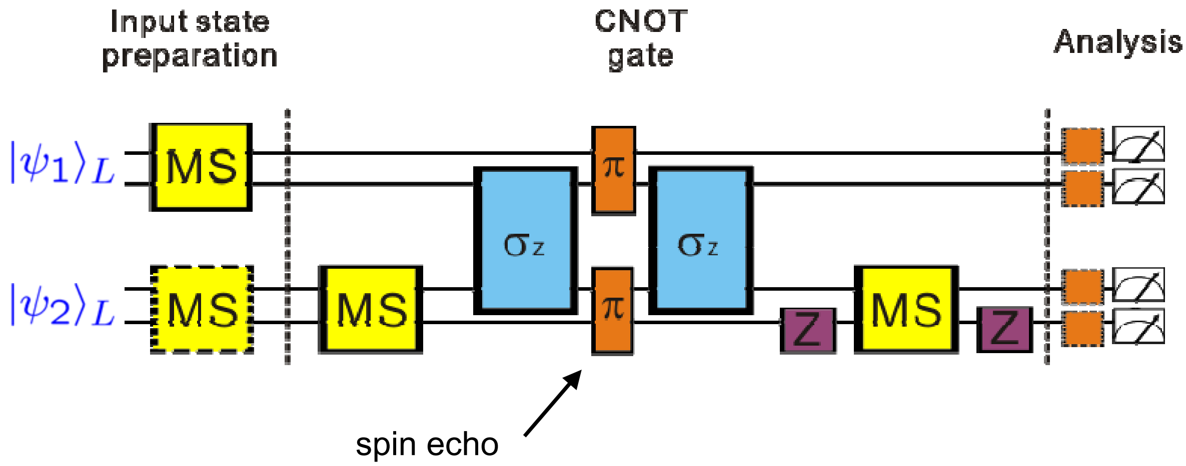


$$\omega_b^{MS} = \omega_c + \nu + \delta_{MS}$$
$$\omega_r^{MS} = \omega_c - \nu - \delta_{MS}$$

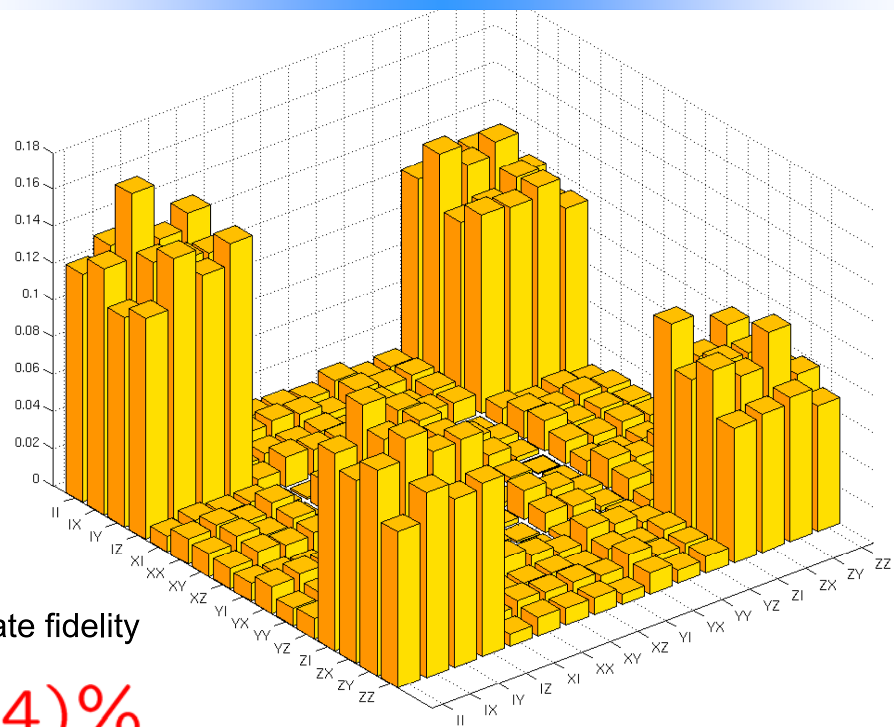


$$\omega_b^Z = \omega_c + \nu/2 + \delta_z$$
$$\omega_r^Z = \omega_c - \nu/2 - \delta_z$$

A CNOT in a DFS



Process tomography of the CNOT



mean gate fidelity

89(4)%

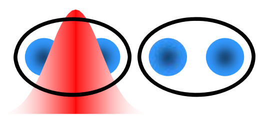
Discussion

mean gate fidelity: 89(4)%

(after DFS postselection)

Main limitations:

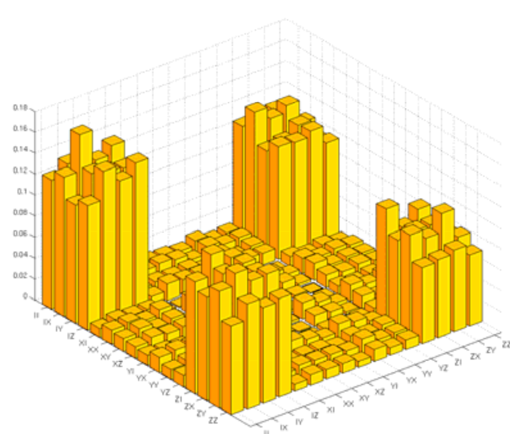
- spurious laser frequency components
- off-resonant coupling to other levels
- intensity stability on ions
- addressing errors



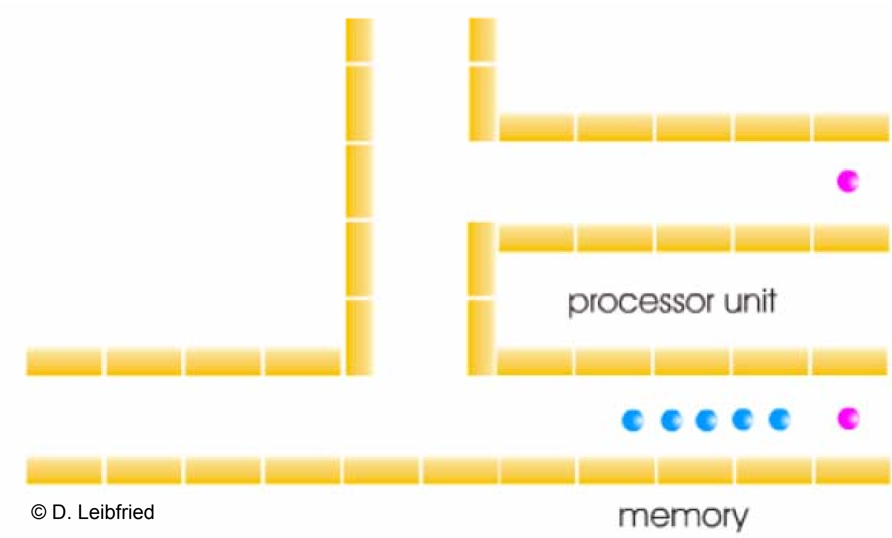
Discussion

Advantages:

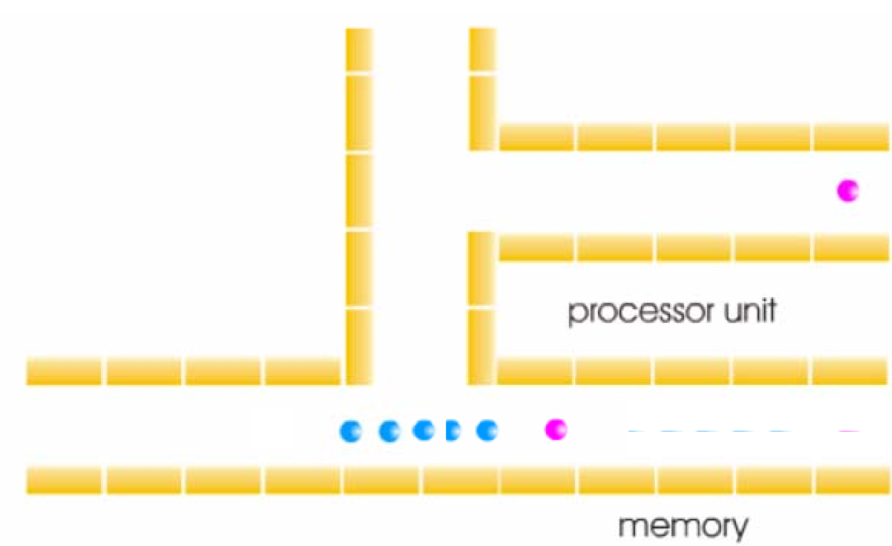
- lifetime limited coherence time
- insensitive to laser linewidth
- insensitive to AC-Stark shifts



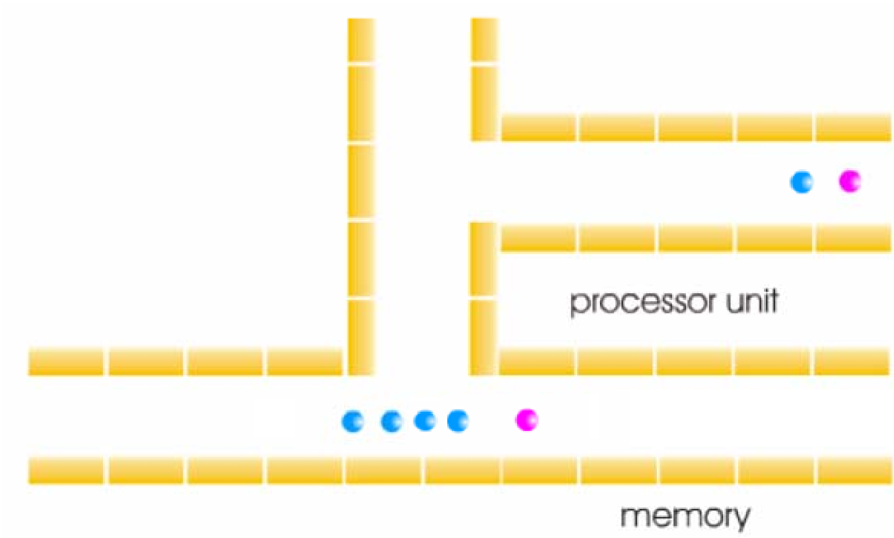
Scaling of ion trap quantum computers



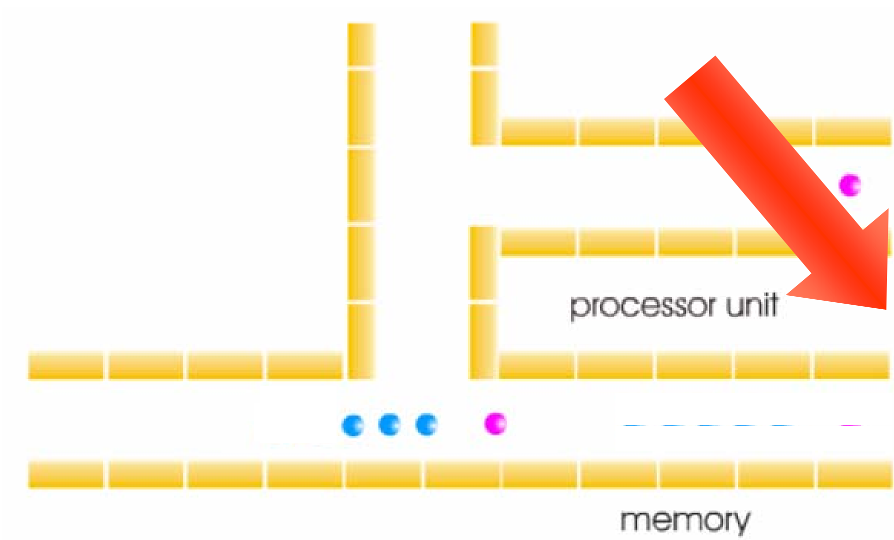
Scaling of ion trap quantum computers



Scaling of ion trap quantum computers

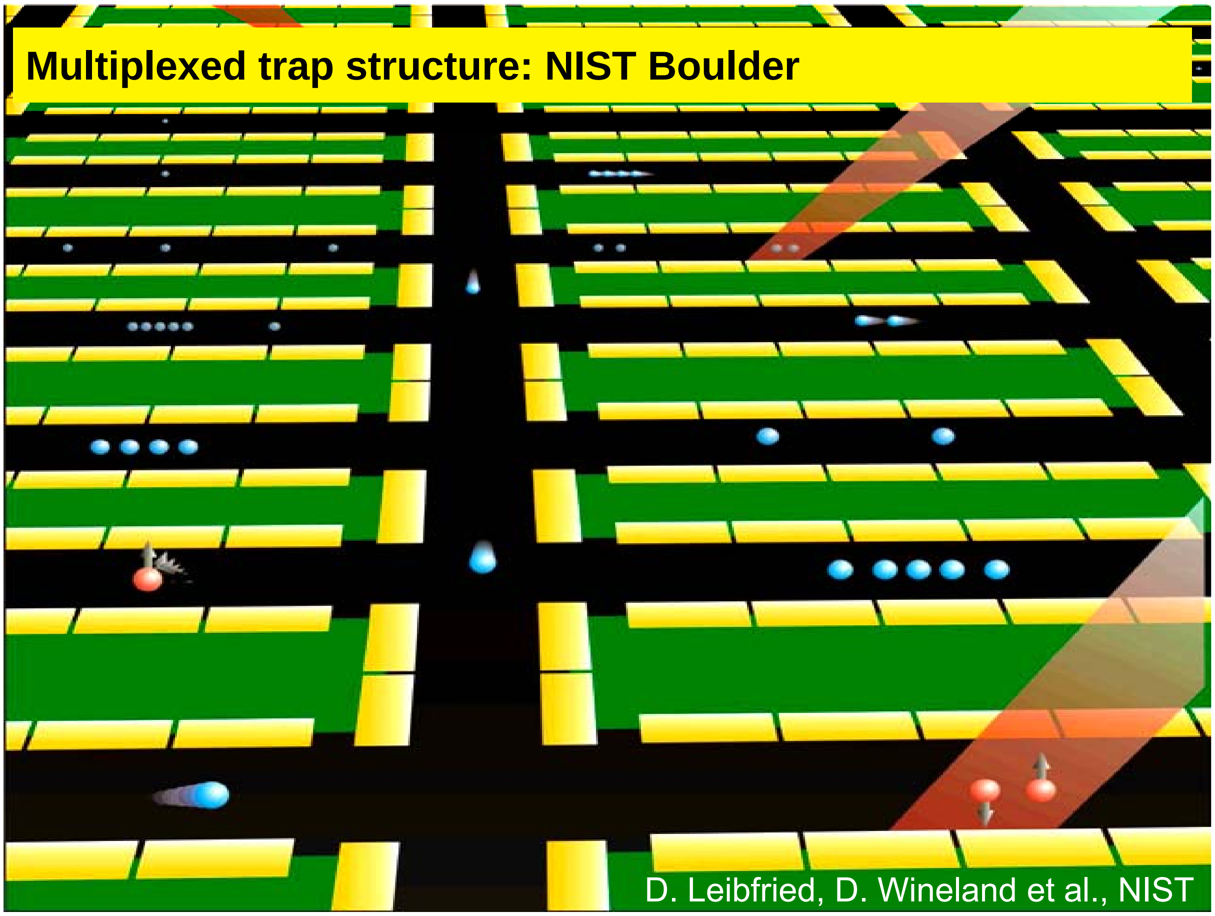


Scaling of ion trap quantum computers



„Architecture for a large-scale ion-trap quantum computer“, D. Kielpinski et al, Nature **417**, 709 (2002)

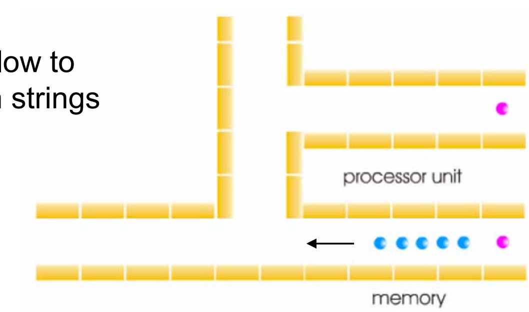
Multiplexed trap structure: NIST Boulder



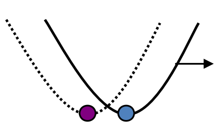
Segmented ion traps as scalable trap architecture

(ideas pioneered by D. Wineland, NIST)

Segmented trap electrode allow to transport ions and to split ion strings



State of the art:



<u>Transport of ions</u>

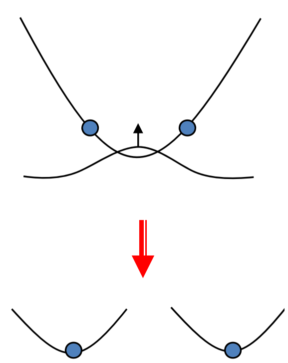
1 mm within 50 μs

no motional heating

<u>Splitting of two-ion crystal</u>

$t_{separation} \approx 200$ µs

small heating $n \approx 1$



„Architecture for a large-scale ion-trap quantum computer", D. Kielpinski et al, Nature **417**, 709 (2002)

„Transport of quantum states", M. Rowe et al, quant-ph/0205084

Scaling of this approach?

Problems :

- Coupling strength between internal and motional states of a N-ion string decreases as

$$\eta \propto \frac{1}{\sqrt{N}}$$

(momentum transfer from photon to ion string becomes more difficult)

-> Gate operation speed slows down

- More vibrational modes increase risk of spurious excitation of unwanted modes
- Distance between neighbouring ions decreases -> addressing more difficult

-> Use flexible trap potentials to split long ion string into smaller segments and perform operations on these smaller strings