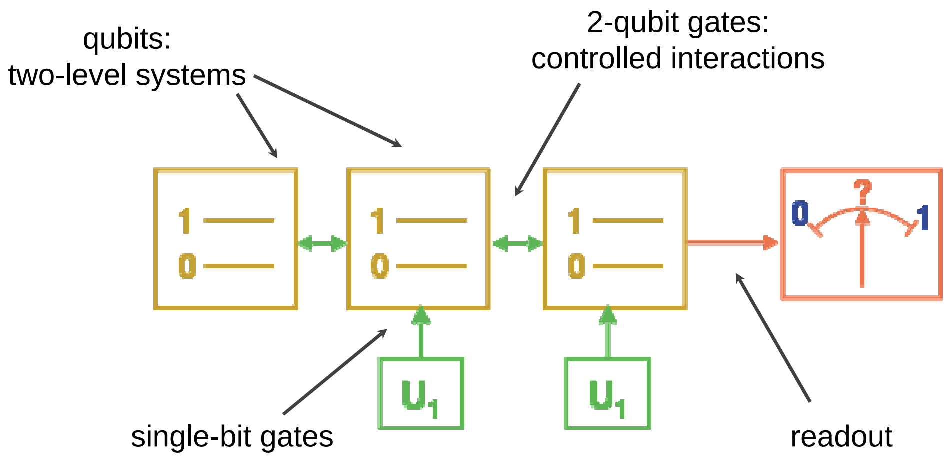
Generic Quantum Information Processor

The challenge:



- Quantum information processing requires excellent qubits, gates, ...
- Conflicting requirements: good isolation from environment while maintaining good addressability

M. Nielsen and I. Chuang,
Quantum Computation and Quantum Information (Cambridge, 2000)

The 5 (+2) DiVincenzo Criteria for Implementation of a Quantum Computer:

in the standard (circuit approach) to **quantum information processing** (QIP)

#1. A scalable physical system with well-characterized qubits.

#2. The ability to initialize the state of the qubits to a simple fiducial state.

#3. Long (relative) decoherence times, much longer than the gate-operation time.

#4. A universal set of quantum gates.

#5. A qubit-specific measurement capability.

#6. The ability to interconvert stationary and mobile (or flying) qubits.

#7. The ability to faithfully transmit flying qubits between specified locations.

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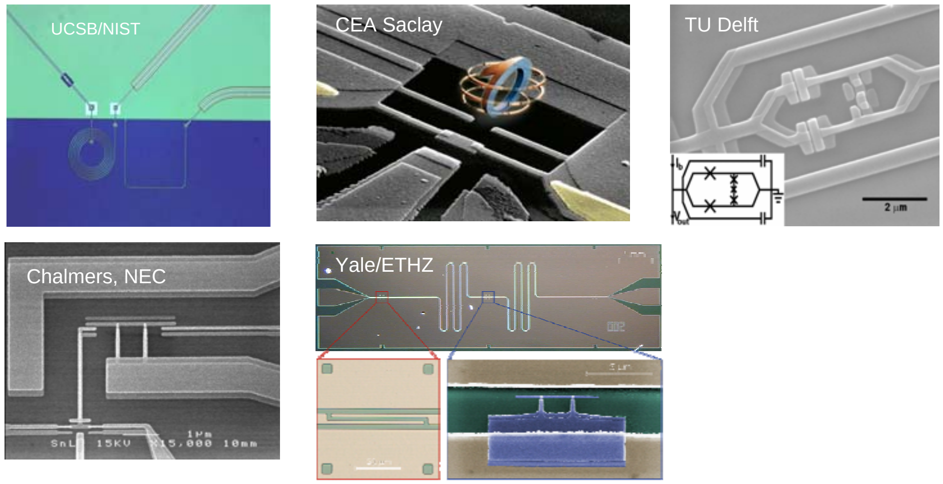
Quantum Information Processing with Superconducting Circuits



with material from
NIST, UCSB, Berkeley, NEC, NTT, CEA Saclay, Yale and ETHZ

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Outline

- realization of superconducting qubits
- harmonic oscillators
- the current biased phase qubit
- the charge qubit
- qubit read-out
- single qubit control
- decoherence
- two-qubit gates

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Some Basics ...

… on how to construct qubits
using superconducting circuit elements.

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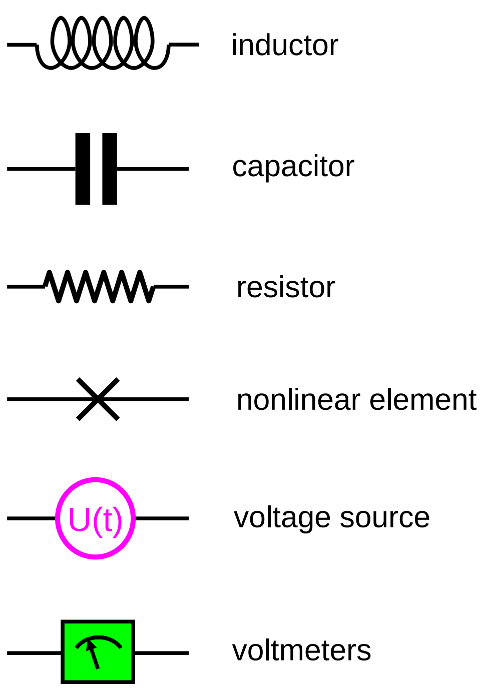
Building Quantum Electrical Circuits



requirements for quantum circuits:

- low dissipation
- non-linear (non-dissipative elements)
- low (thermal) noise

a solution:

- use superconductors
- use Josephson tunnel junctions
- operate at low temperatures

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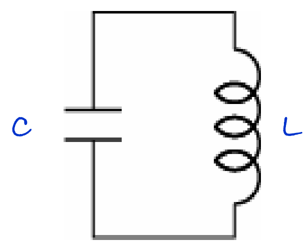
Superconducting Harmonic Oscillator

a simple electronic circuit:



- typical inductor: $L = 1$ nH
- a wire in vacuum has inductance ~ 1 nH/mm
- typical capacitor: $C = 1$ pF
- a capacitor with plate size 10 µm x 10 µm and dielectric AlOx ($\varepsilon = 10$) of thickness 10 nm has a capacitance $C \sim 1$ pF
- resonance frequency

$$\frac{1}{2\pi\sqrt{LC}} \sim 5\,\mathrm{GHz}$$

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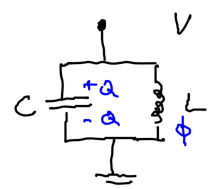
Quantization of the electrical LC harmonic oscillator:

parallel LC oscillator circuit:



voltage across the oscillator:

$$V = \frac{Q}{C} = -L\frac{\partial I}{\partial t}$$

total energy (Hamiltonian):

$$H = \frac{1}{2} C V^2 + \frac{1}{2} L I^2 = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}\frac{\phi^2}{L}$$

with the charge Q stored on the capacitor $\quad Q = VC$

a flux ϕ stored in the inductor $\quad \phi = LI$

properties of Hamiltonian written in variables Q and ϕ:

$$\frac{\partial H}{\partial Q} = \frac{Q}{C} = -L\frac{\partial I}{\partial t} = -\dot{\phi}$$

$$\frac{\partial H}{\partial \phi} = \frac{\phi}{L} = I = \dot{Q}$$

Q and ϕ are canonical variables

see e.g.: Goldstein, Classical Mechanics, Chapter 8, Hamilton Equations of Motion

Quantum version of Hamiltonian

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

with commutation relation

$$[\hat{\phi}, \hat{Q}] = i\hbar$$

compare with particle in a harmonic potential:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega^2 \hat{x}^2$$

analogy with electrical oscillator:
- charge Q corresponds to momentum p
- flux ϕ corresponds to position x

$$[\hat{x}, \hat{p}] = [\hat{x}, -i\hbar \frac{\partial}{\partial x}] = i\hbar$$

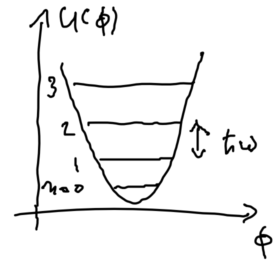
$$[\hat{\phi}, \hat{Q}] = [\hat{\phi}, -i\hbar \frac{\partial}{\partial \phi}] = i\hbar$$

Hamiltonian in terms of raising and lowering operators:

$$\hat{H} = \hbar\omega \left(a^\dagger a + \frac{1}{2}\right)$$

with oscillator resonance frequency:

$$\omega = \frac{1}{\sqrt{LC}}$$



Raising and lowering operators:

$$a^\dagger |n\rangle = \sqrt{n+1}\, |n+1\rangle \; ; \; \hat{a}|n\rangle = \sqrt{n}\, |n-1\rangle$$

$$a^\dagger a |n\rangle = n |n\rangle$$ number operator

in terms of Q and ϕ:

$$\hat{a} = \frac{1}{\sqrt{2\hbar Z_c}} \left(Z_c \hat{Q} + i \hat{\phi} \right)$$

with Z_c being the characteristic impedance of the oscillator

$$Z_c = \sqrt{\frac{L}{C}}$$

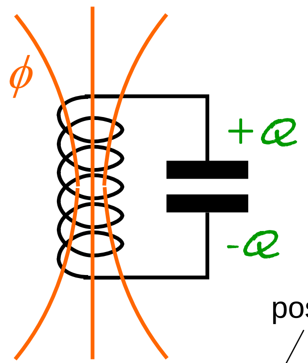
charge Q and flux ϕ operators can be expressed in terms of raising and lowering operators:

$$\hat{Q} = \sqrt{\frac{\hbar}{2 Z_c}} \left(a + a^\dagger \right)$$

$$\hat{\phi} = \sqrt{\frac{2 Z_c}{\hbar}} \left(a - a^\dagger \right)$$

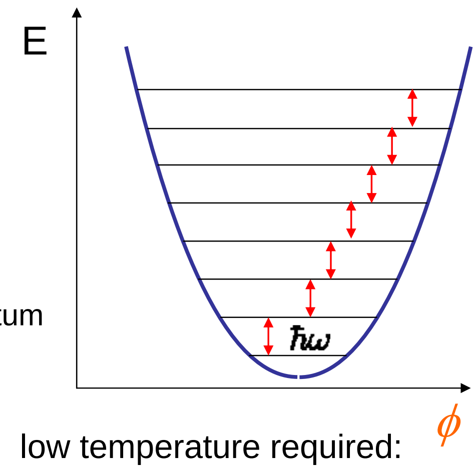
Exercise: Making use of the commutation relations for the charge and flux operators, show that the harmonic oscillator Hamiltonian in terms of the raising and lowering operators is identical to the one in terms of charge and flux operators.

Quantum LC Oscillator



$$[\phi, q] = i\hbar$$

position momentum



Hamiltonian $H = \frac{\phi^2}{2L} + \frac{q^2}{2C}$

$$\omega = 1/\sqrt{LC}$$

$$H = \hbar\omega \left(a^\dagger a + \frac{1}{2} \right)$$

low temperature required:

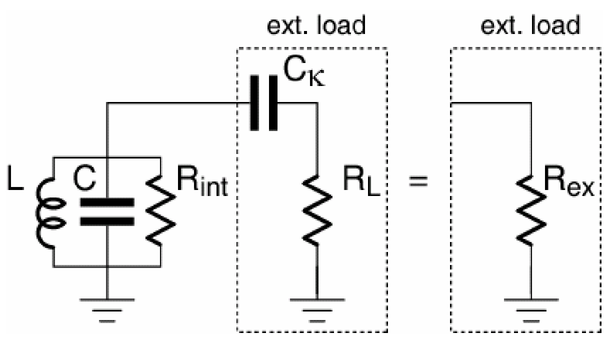
$$\hbar\omega \gg k_B T$$

10 GHz ~ 500 mK 20 mK

$$\langle n_{\text{th}} \rangle = \frac{1}{\exp\left(h\nu/k_B T\right) - 1} \sim 10^{-11}$$

problem 1: equally spaced energy levels (linearity)

Example: Dissipation in an LC Oscillator



internal losses: R_{int}
conductor, dielectric

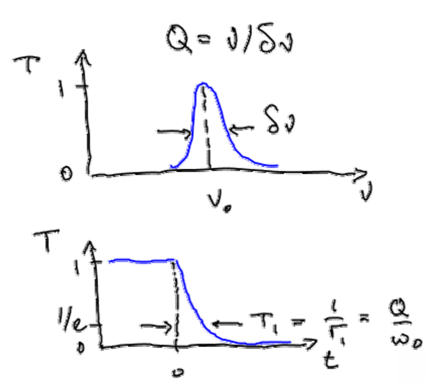
external losses: R_{ext}
radiation, coupling

total losses $\frac{1}{R} = \frac{1}{R_{\text{int}}} + \frac{1}{R_{\text{ext}}}$

impedance $Z = \sqrt{\frac{L}{C}}$

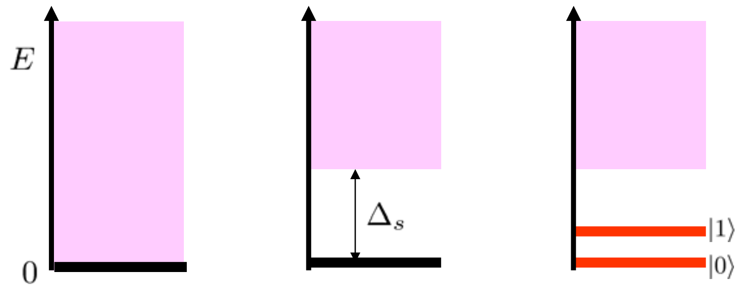
quality factor $Q = \frac{R}{Z} = \omega_0 RC$

excited state decay rate $\Gamma_1 = \frac{\omega_0}{Q} = \frac{1}{RC}$



problem 2: internal and external dissipation

Why Superconductors?



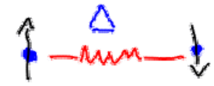
normal metal — superconductor — How to make qubit?

- single non-degenerate macroscopic ground state
- elimination of low-energy excitations

Superconducting materials (for electronics):

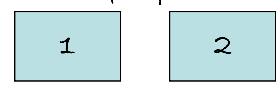
- Niobium (Nb): $2\Delta/h = 725$ GHz, $T_c = 9.2$ K
- Aluminum (Al): $2\Delta/h = 100$ GHz, $T_c = 1.2$ K

Cooper pairs:
bound electron pairs



are Bosons (S=0, L=0)

2 chunks of superconductors

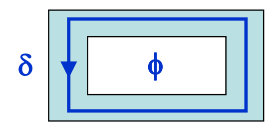


macroscopic wave function

$$\Psi_i = \sqrt{N_i^{CP}}\, e^{i\delta_i}$$

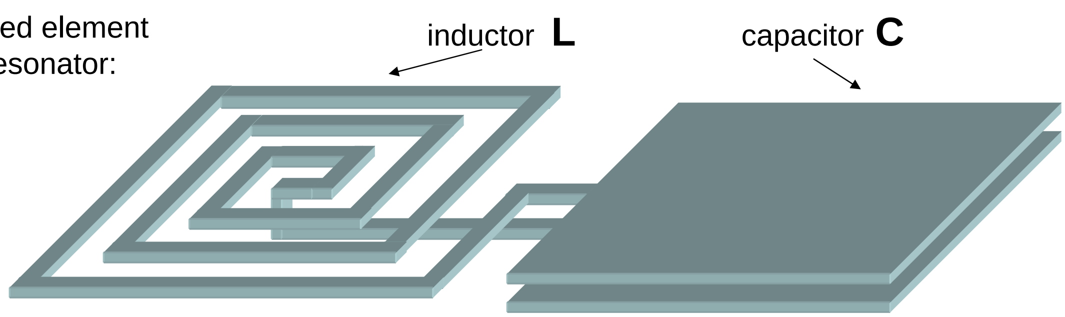
Cooper pair density n_i
and global phase δ_i

phase quantization: $\delta = n\, 2\pi$
flux quantization: $\phi = n\, \phi_0$

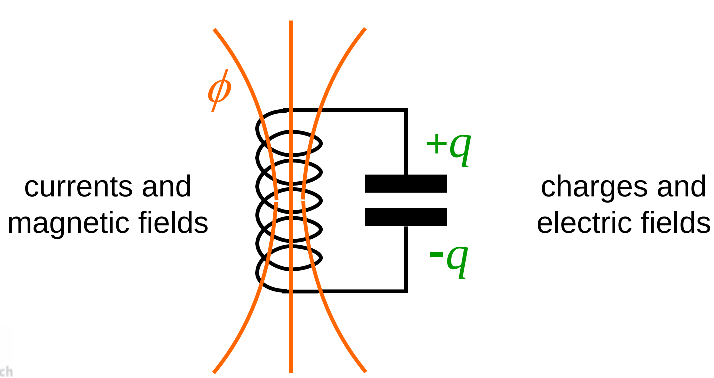


Can it be done?

lumped element
LC resonator:



a harmonic oscillator



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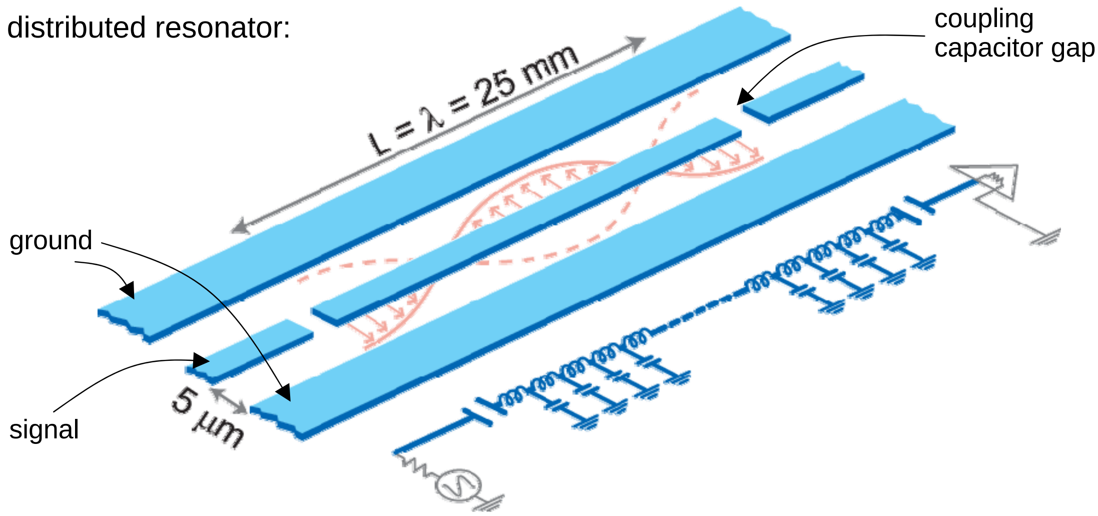
Transmission Line Resonator

distributed resonator:

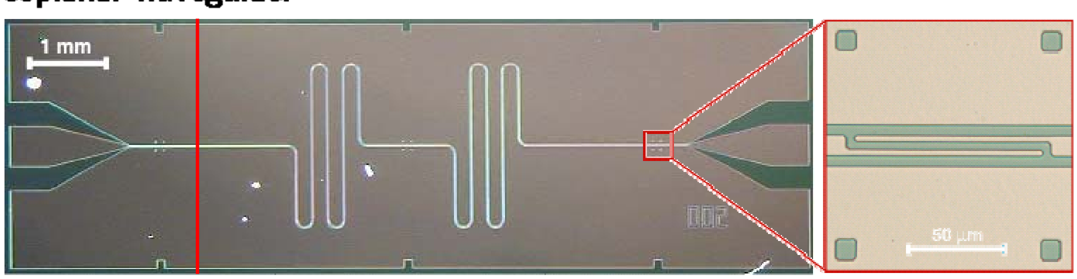


- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

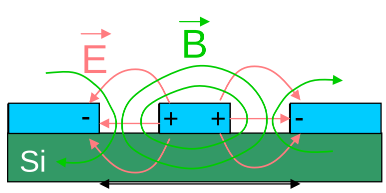
Transmission Line Resonator

$$H_r = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right)$$

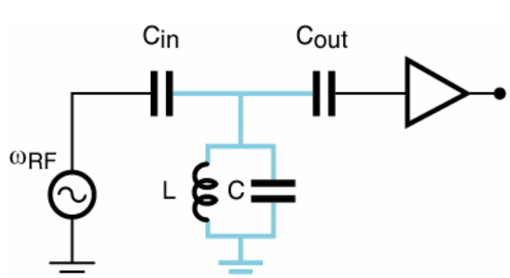
coplanar waveguide:



cross section:

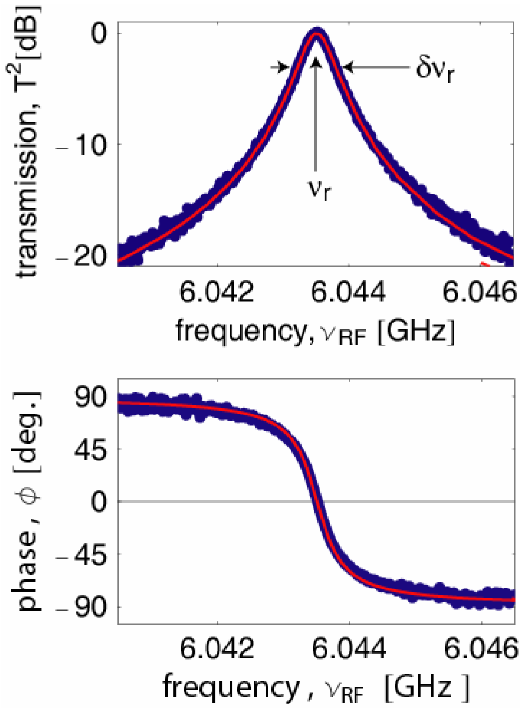


measuring the resonator:



photon lifetime (quality factor) controlled by coupling $C_{in/out}$

Resonator Quality Factor and Photon Lifetime



resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

photon decay rate:

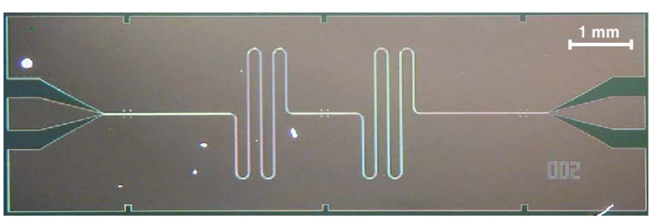
$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

photon lifetime:

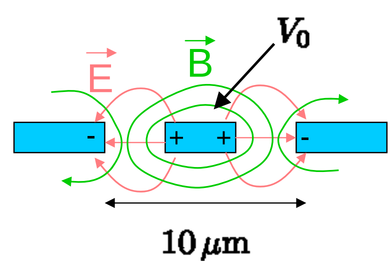
$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

Electric Field of a Single Photon in a Circuit

transmission line resonator:



cross-section of transm. line (TEM mode):



voltage across resonator in vacuum state ($n = 0$)

$$V_{0,\mathrm{rms}} = \sqrt{\frac{\hbar\omega_r}{2C}} \approx 1\,\mu\mathrm{V}$$

for $\omega_r/2\pi \approx 6\,\mathrm{GHz}$
($C \sim 1\,\mathrm{pF}$), $b \approx 5\,\mu\mathrm{m}$

$$E_0 = \frac{V_{0,\mathrm{rms}}}{b} \approx 0.2\,\mathrm{V/m}$$

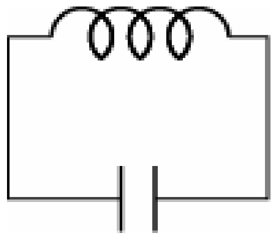
$\times 10^6$ larger than E_0 in 3D microwave cavity

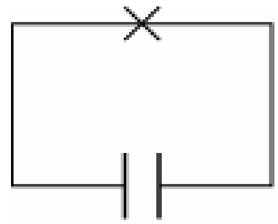
Superconducting Qubits

LC resonator

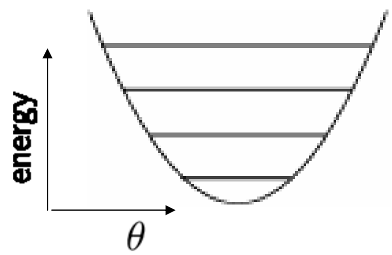
Josephson junction resonator

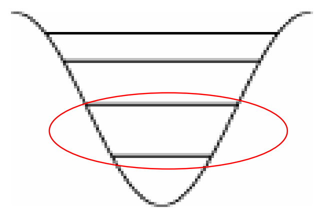
Josephson junction = nonlinear inductor





anharmonicity → effective two-level system

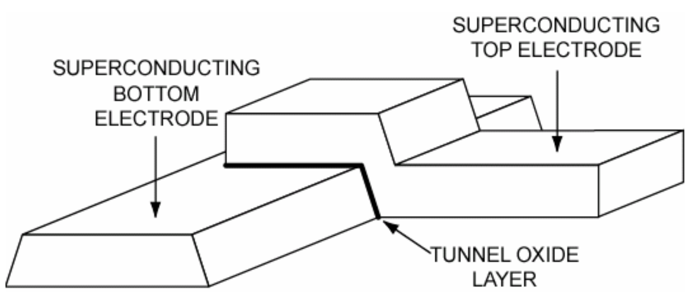




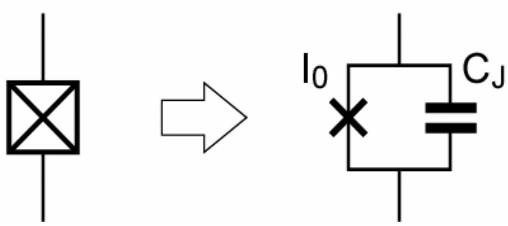
solution to problem 1

A Low-Loss Nonlinear Element

a (superconducting) Josephson junction



- superconductors: Nb, Al
- tunnel barrier: AlO_x



- critical current I_c
- junction capacitance C_J
- high internal resistance R_J

M. Tinkham, *Introduction to Superconductivity* (Krieger, Malabar, 1985).

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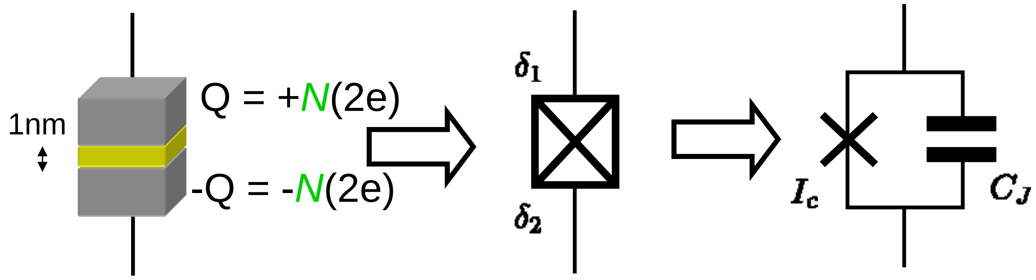
Josephson Tunnel Junction

the only non-linear LC resonator with no dissipation (BCS, $k_B T \ll \Delta$)



tunnel junction parameters:

- critical current I_c
- junction capacitance C_J
- high internal resistance R_J

Josephson relations:

$$I_0 = I_c \sin\delta$$

$$V = \phi_0 \frac{\partial \delta}{\partial t}$$

flux quantum: $\phi_0 = \frac{h}{2e}$

phase difference: $\delta = \delta_2 - \delta_1$

derivation of Josephson effect, see e.g.: chap. 21 in R. A. Feynman: Quantum mechanics, The Feynman Lectures on Physics. Vol. 3 (Addison-Wesley, 1965)

The Josephson junction as a non-linear inductor

induction law: $V = -L \frac{\partial I}{\partial t}$

Josephson effect:

dc-Josephson equation $I = I_c \sin\delta$

$$\frac{\partial I}{\partial t} = I_c \cos\delta \frac{\partial \delta}{\partial t}$$

ac-Josephson equation $V = \frac{\phi_0}{2\pi} \frac{\partial \delta}{\partial t} = \underbrace{\frac{\phi_0}{2\pi I_c} \frac{1}{\cos\delta}}_{L_J} \frac{\partial I}{\partial t}$

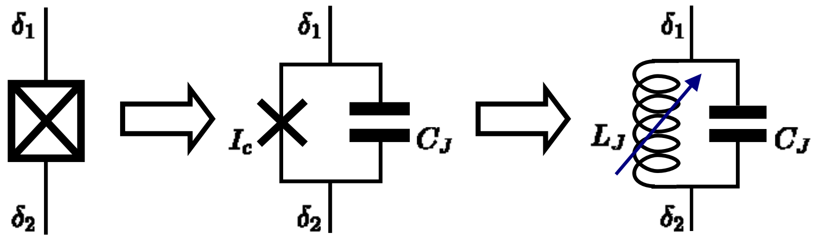
Josephson inductance $L_J = \underbrace{\frac{\phi_0}{2\pi I_c}}_{\text{specific Josephson inductance}} \frac{1}{\cos\delta}$ ← nonlinearity

A typical characteristic Josephson inductance for a tunnel junction with $I_c = 100\ nA$ is $L_{J0} \sim 3\ nH$.

review: M. H. Devoret *et al.*, Quantum tunneling in condensed media, *North-Holland,* (1992)

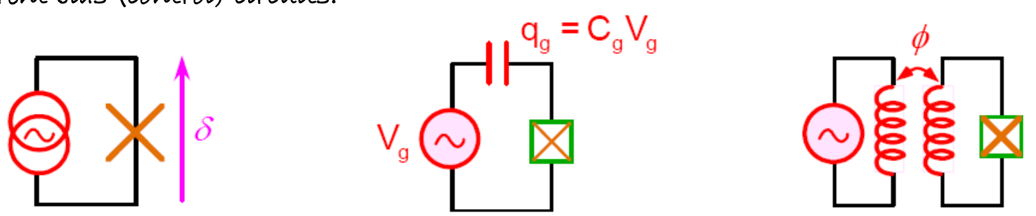
A Non-Linear Tunable Inductor w/o Dissipation

the Josephson junction as a circuit element:



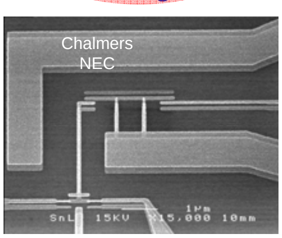
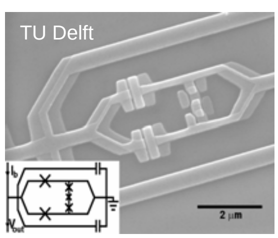
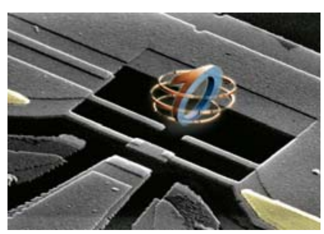
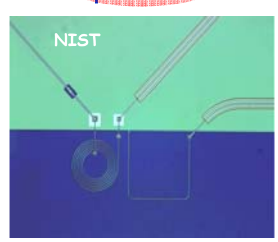
How to Make Use of the Josephson Junction in Qubits?

different bias (control) circuits:



current bias | charge bias | flux bias

Realizations of Superconducting Qubits

charge	flux	charge/phase	phase
			
NEC Chalmers Yale ...	Delft ...	Saclay Yale ETHZ ...	NIST Santa Barbara Maryland ...

... have demonstrated coherent control

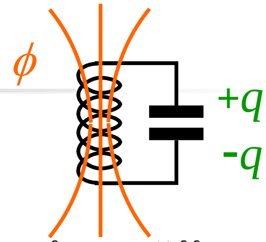
Nakamura, Pashkin, Tsai *et al. Nature* **398**, **421**, **425** (1999, 2003, 2003)

Chiorescu, van der Wal, Mooij, Orlando, S. Lloyd *et al. Science* **285**, **290, 299** (1999, 2000, 2003)

Vion, Esteve, Devoret *et al. Science* **296** (2002)

Martinis, Simmonds, Lang, Nam, Aumentado, Urbina *et al. Phys. Rev. Lett.* **89**, **93** (2002, 2004)

Flux/Charge Commutation Relation



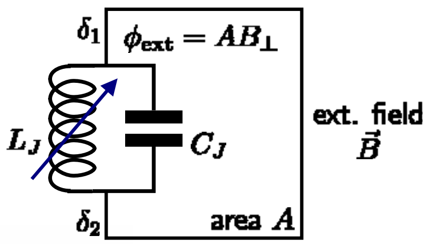
commutation relation: $[\hat{\phi}, \hat{Q}] = i\hbar$

with $\hat{\phi} = \phi_0 \frac{\hat{\delta}}{2\pi}$ flux ϕ in terms of phase difference δ

$\hat{Q} = 2e\hat{N}$ charge Q in terms of number of charges N

phase - number commutation relation: $[\hat{\delta}, \hat{N}] = i$

Flux Quantization



quantization condition for superconducting phase/flux:

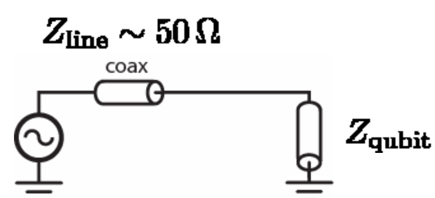
$$\delta + \frac{2\pi\phi_{ext}}{\phi_0} = n2\pi$$

$$\phi_0\frac{\delta}{2\pi} + \phi_{ext} = n\phi_0$$

Coupling to the Electromagnetic Environment

strong coupling to environment (bias wires):

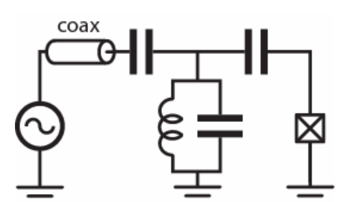
decoherence
from energy relaxation



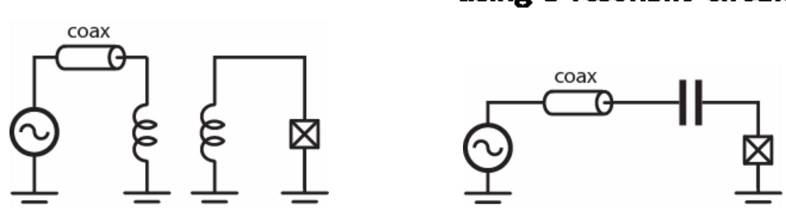
decoupling using impedance transformers:

control decoherence
by circuit design

solution to problem 2

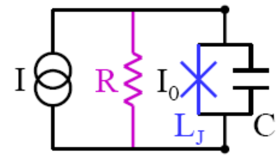


using a resonant circuit



using non-resonant impedance transformers

Current Biased Phase Qubit



The bias current I distributes into a Josephson current through an ideal Josephson junction with critical current I_c, through a resistor R and into a displacement current over the capacitor C.

Kirchhoff's law:

$$I_b = I_s + I_R + I_C$$
$$= I_c \sin\delta + \frac{V}{R} + C\dot{V}$$

$$I_C = \dot{Q}_C = C\dot{V}$$
$$I_R = V/R$$
$$I_s = I_c \sin\delta$$

use Josephson equations:

$$I_b = I_c \sin\delta + \frac{\Phi_0}{2\pi R}\dot{\delta} + \frac{\Phi_0 C}{2\pi}\ddot{\delta}$$

W.C. Stewart, Appl. Phys. Lett. **2**, 277, (1968)
D.E. McCumber, J. Appl. Phys. **39**, 3 113 (1968)

looks like equation of motion for a particle with mass m and coordinate δ in an external potential U:

$$m\ddot{\delta} + m\frac{1}{RC}\dot{\delta} + \frac{\partial U(\delta)}{\partial\delta} = 0$$

particle mass: $m = C\,(\Phi_0/2\pi)^2$

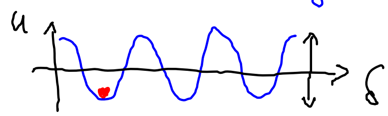
external potential: $U(\delta) = \frac{I_c\Phi_0}{2\pi}\left(-\frac{I_b}{I_c} - \cos\delta\right)$

Phase particle in a potential well

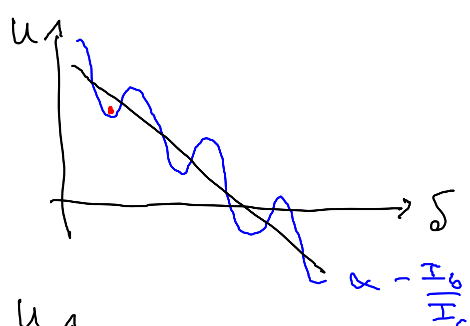
$$U(\delta) = \frac{I_c\Phi_0}{2\pi}\left(-\frac{I_b}{I_c}\delta - \cos\delta\right)$$

$$E_J = \frac{I_c\Phi_0}{2\pi}$$

cosine potential for $I_b = 0$:



'tilted washboard' potential for $I_b \neq 0$:



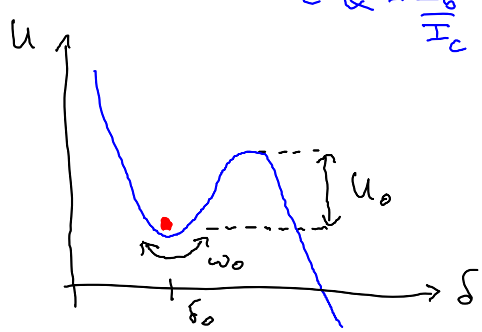
potential barrier:

$$U_0 = 2E_J\left[\sqrt{1-\gamma^2} - \gamma\arccos\gamma\right]$$

oscillation frequency:

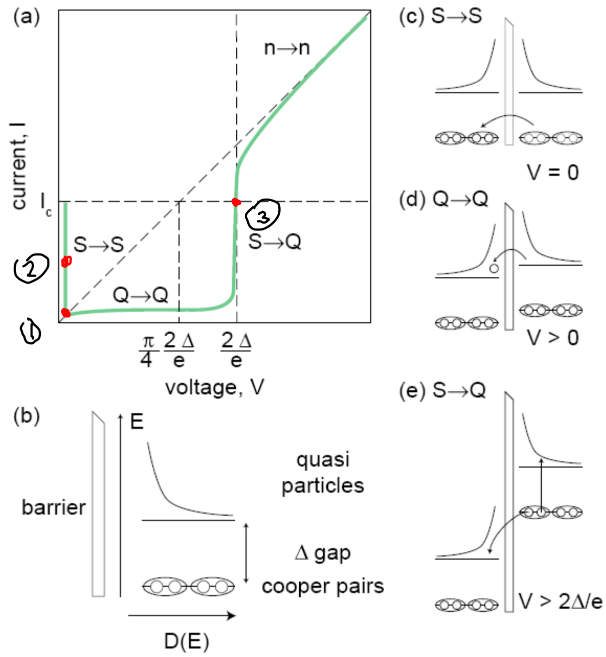
$$\omega_0 = \omega_p(1-\gamma^2)^{1/4} = \sqrt{\frac{U''(\delta_0)}{m}}$$

with: $\gamma = I_b/I_c$; $\omega_p = \sqrt{\frac{2\pi I_c}{\Phi_0 C}}$

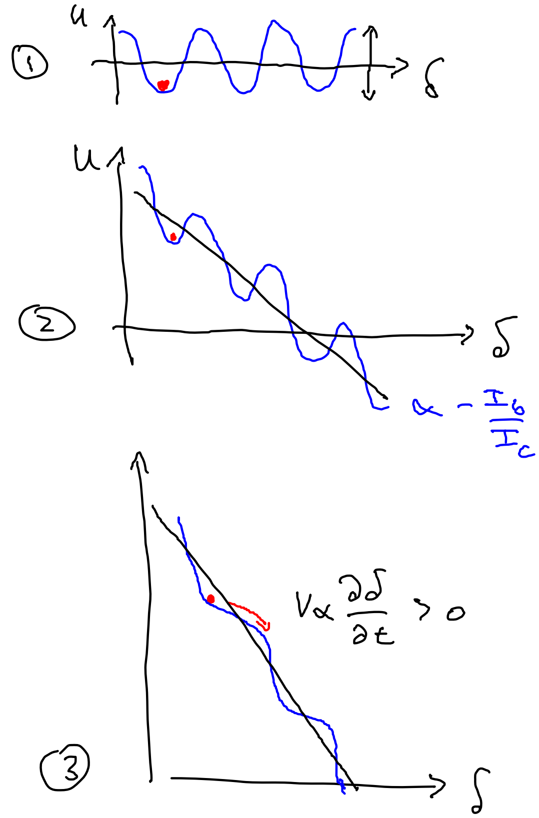


Current-voltage characteristics

typical I-V curve of underdamped Josephson junctions:



band diagram



Thermal Activation and Quantum Tunneling:

bias current dependence

$\omega_0(\gamma)$; $U_0(\gamma)$

thermal activation rate:

$$\Gamma_{th} = a_t \frac{\omega_0}{2\pi} \exp\left(-\frac{U_0}{k_B T}\right)$$

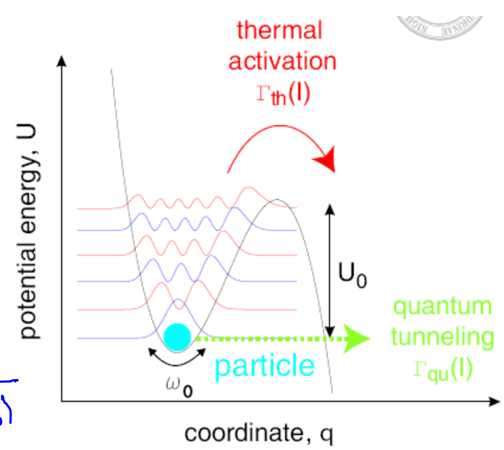
damping dependent prefactor

quantum tunneling rate:

$$\Gamma_{qu} = a_q \frac{\omega_0}{2\pi} \exp\left(-\frac{36}{5}\frac{U_0}{\hbar\omega_0}\right)$$

calculated using WKB method (exercise)

$$\Gamma_q = a_q\, \omega_0 \exp - \int_{\delta_1}^{\delta_2} \frac{1}{\hbar} \sqrt{2m\,(U(\delta) - E_0)}$$



energy level quantization:

$$E_n \approx \hbar\omega_0\left(n + \frac{1}{2}\right)$$

neglecting non-linearity

Quantum Mechanics of a Macroscopic Variable: The Phase Difference of a Josephson Junction
JOHN CLARKE, ANDREW N. CLELAND, MICHEL H. DEVORET, DANIEL ESTEVE, and JOHN M. MARTINIS
Science 26 February 1988 239: 992-997 [DOI: 10.1126/science.239.4843.992] (in Articles) Abstract » References » PDF »

Macroscopic quantum effects in the current-biased Josephson junction
M. H. Devoret, D. Esteve, C. Urbina, J. Martinis, A. Cleland, J. Clarke
in Quantum tunneling in condensed media, North-Holland (1992)

Early Results (1980's)

search for macroscopic quantum effects in superconducting circuits

theoretical predictions:

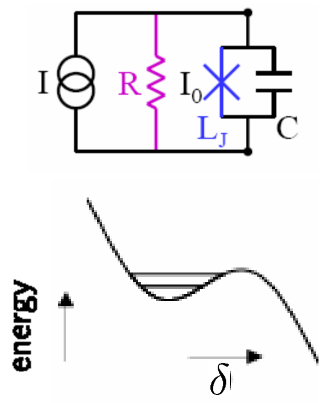
- tunneling ✓
- energy level quantization ✓
- coherence ✗

A.J. Leggett *et al.*, *Prog. Theor. Phys. Suppl.* **69**, 80 (1980), *Phys. Scr.* **T102**, 69 (2002).

short coherence times due to strong coupling to em environment

experimental verification:

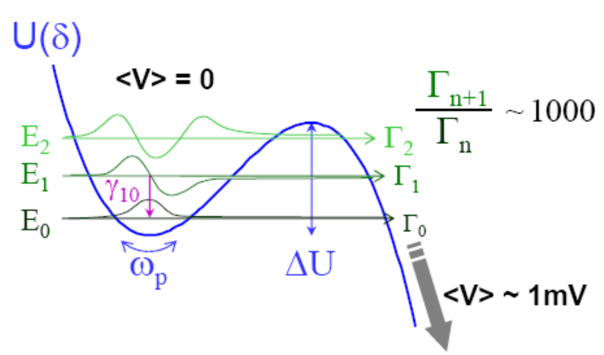
current biased JJ = phase qubit

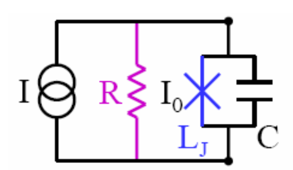


J. Clarke, J. Martinis, M. Devoret *et al.*, *Science* **239**, 992 (1988).

The Current Biased Phase Qubit

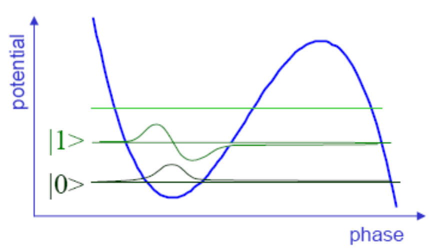
operating a current biased Josephson junction as a superconducting qubit:





<u>initialization:</u>

wait for |1> to decay to |0>, e.g. by spontaneous emission at rate γ_{10}

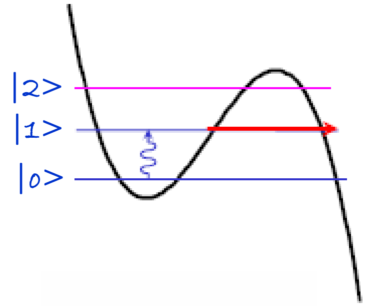


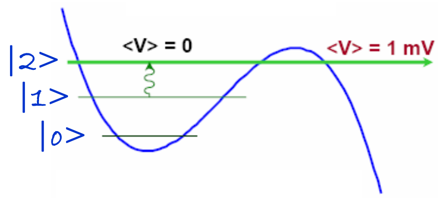
Read-Out

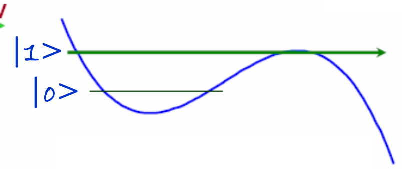
measuring the state of a current biased phase qubit

tunneling:

- prepare state |1> (pump)
- wait ($\Gamma_1 \sim 10^3\, \Gamma_0$)
- detect voltage
- |1> = voltage, |0> = no voltage







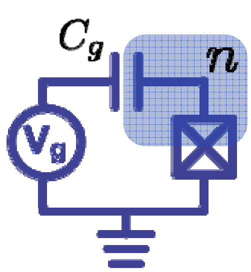
pump and probe pulses:

- prepare state |1> (pump)
- drive ω_{21} transition (probe)
- observe tunneling out of |2>

tipping pulse:

- prepare state |1>
- apply current pulse to suppress U_0
- observe tunneling out of |1>

A Charge Qubit: The Cooper Pair Box

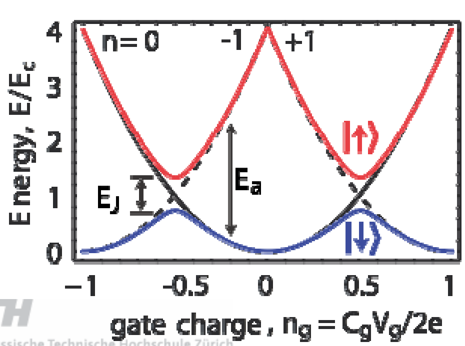


$$H = 4E_C n^2$$

$$H = 4E_C (n - n_g)^2 - E_J \cos\delta$$

$$[\delta, n] = i \quad \Rightarrow \quad e^{\pm i\delta}|n\rangle = |n \pm 1\rangle$$

$$H = \sum_n \left[4E_C (n - n_g)^2 |n\rangle\langle n| - \frac{E_J}{2} \left(|n\rangle\langle n+1| + |n+1\rangle\langle n| \right) \right]$$



Charging energy: $E_C = \dfrac{e^2}{2(C_g + C_J)}$

Gate charge: $n_g = \dfrac{C_g V_g}{2e}$

Josephson energy: $E_J = \dfrac{I_0 \Phi_0}{2\pi} = \dfrac{h\Delta}{8e^2 R_J}$

Bouchiat et al. Physica Scripta 176, 165 (1998)

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Cooper pair box Hamiltonian:

Hamiltonian: $\hat{H} = \underbrace{E_C(\hat{N}-N_g)^2}_{} - \underbrace{E_J \cos\hat{\delta}}_{}$ $\quad E_J\cos\hat\delta = \frac{E_J}{2}\left(e^{i\hat\delta}+e^{-i\hat\delta}\right)$ $\quad$ gate charge $N_g = \frac{C_g V_g}{2e}$

electrostatic $\qquad$ magnetic energy

charging energy $\qquad$ Josephson coupling Energy

$$E_C = \frac{(2e)^2}{2\,C_\Sigma} \qquad E_J = \frac{\phi I_c}{2\pi}$$

Hamiltonian in charge representation:

$$\hat{H} = E_C(N-N_g)^2\,|N\rangle\langle N| - \frac{E_J}{2}\sum_N \left(|N+1\rangle\langle N| + |N\rangle\langle N+1|\right)$$

easy to diagonalize numerically

$$\hat{H} = \begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & E_C(-1-N_g)^2 & -E_J/2 & 0 & \cdots \\ \cdots & -E_J/2 & E_C(0-N_g)^2 & -E_J/2 & \cdots \\ \cdots & 0 & -E_J/2 & E_C(1-N_g)^2 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

relation between **phase** and **number basis**:

$$|\delta\rangle = \frac{1}{\sqrt{2\pi}}\sum_N e^{i\hat{N}\delta}\,|N\rangle \qquad \text{with} \qquad e^{i\hat\delta}\,|N\rangle = |N+1\rangle$$

Phase representation of Cooper pair box Hamiltonian:

$$\hat{H} = E_C(\hat{N}-N_g)^2 - E_J\cos\hat\delta \qquad \text{with} \qquad \hat{N} = \frac{\hat{Q}}{2e} = -i\hbar\frac{1}{2e}\frac{\partial}{\partial\phi}$$

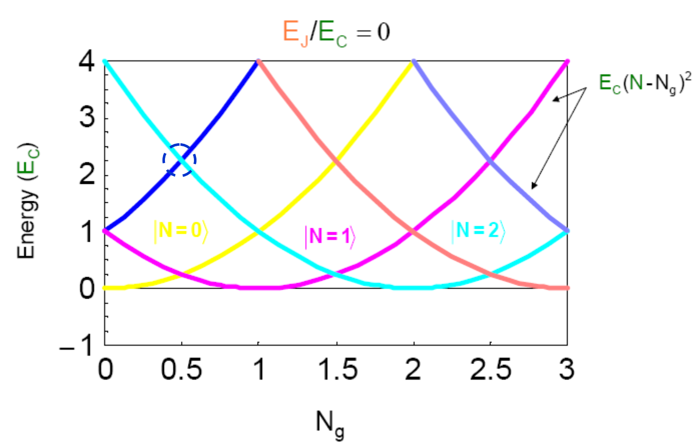
$$= E_C\left(-i\frac{\partial}{\partial\delta} - N_g\right)^2 - E_J\cos\hat\delta \qquad\qquad = -i\frac{\partial}{\partial\delta}$$

Equivalent solution to the Hamiltonian can be found in both representations, e.g. by numerically solving the Schrödinger equation for the charge (N) representation or analytically solving the Schrödinger equation for the phase (δ) representation.

$$\hat{H}\,|\psi\rangle = E\,|\psi\rangle$$

solutions for $E_J = 0$:

- crossing points are charge degeneracy points



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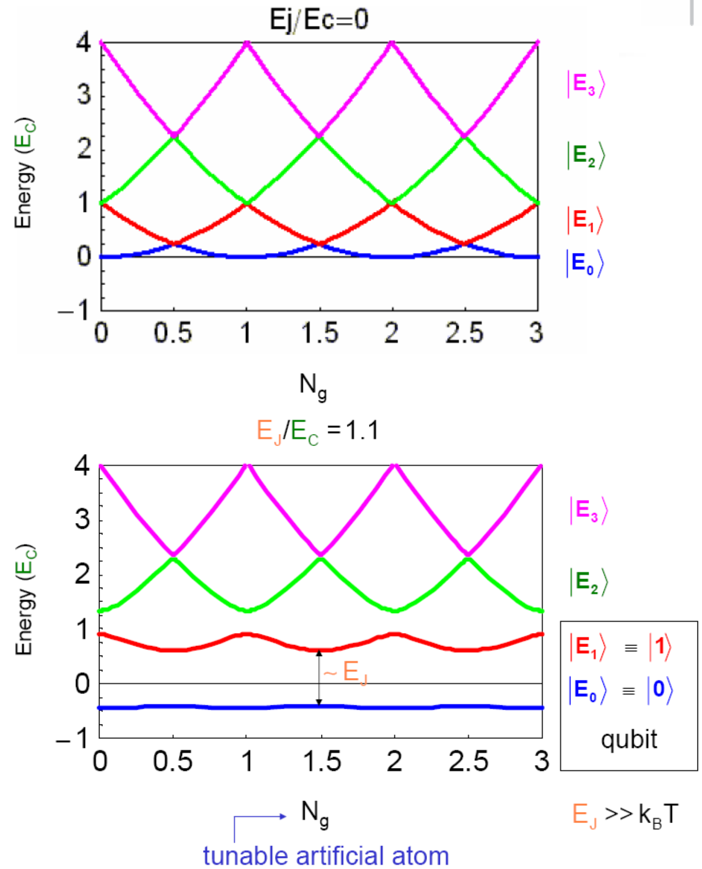
Energy Levels

energy level diagram for $E_J = 0$:

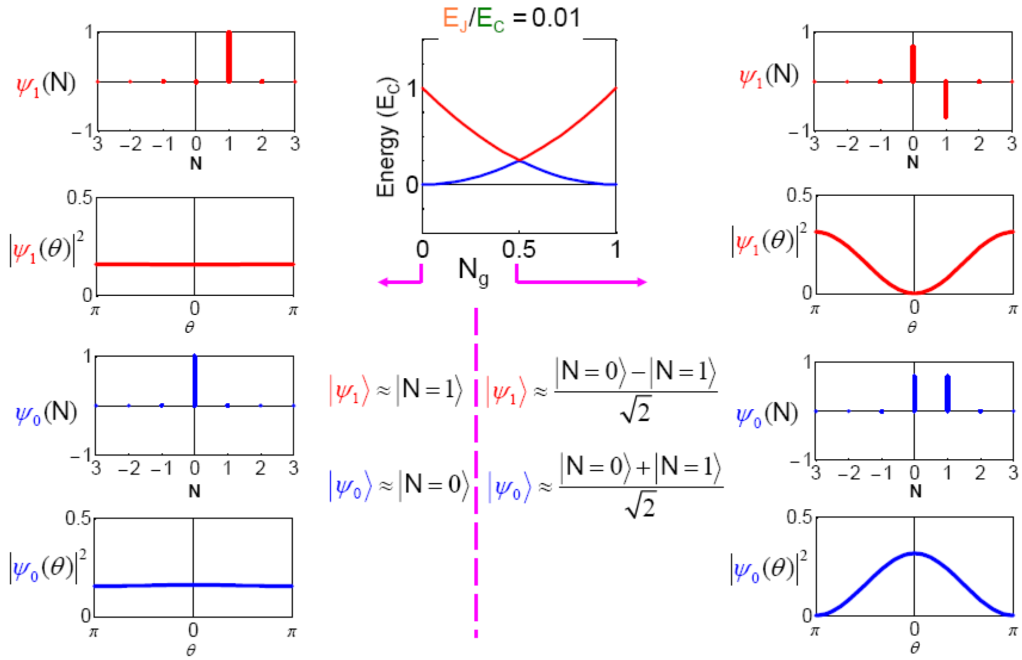
- energy bands are formed
- bands are periodic in N_g

energy bands for finite E_J

- Josephson coupling lifts degeneracy
- E_J scales level separation at charge degeneracy

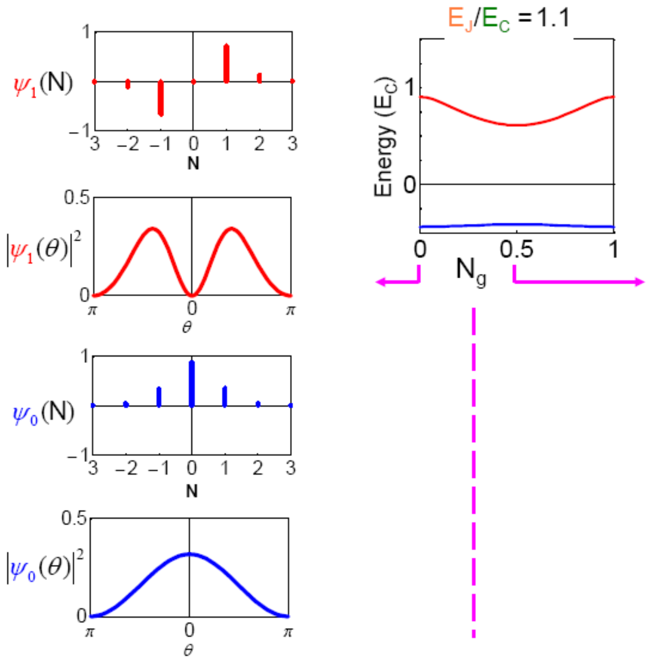


Charge and Phase Wave Functions ($E_j << E_c$)



courtesy Saclay

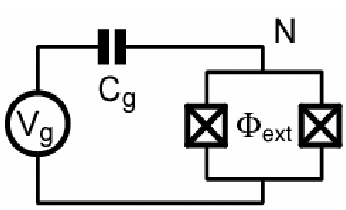
Charge and Phase Wave Functions ($E_j \sim E_c$)



courtesy Saclay

Tuning the Josephson Energy

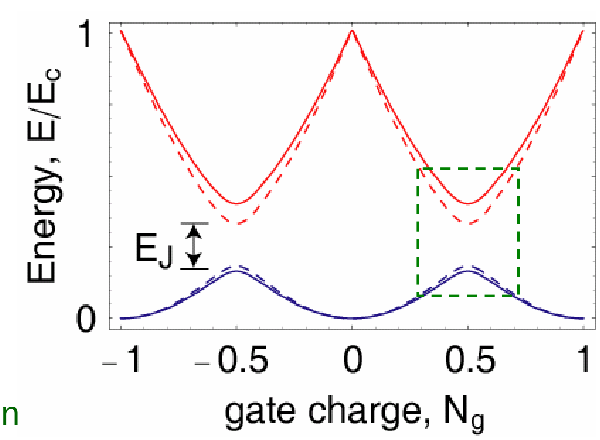
split Cooper pair box in perpendicular field



$$H = E_C\,(N - N_g)^2 - E_{J,\max}\cos\left(\pi\frac{\Phi_{\rm ext}}{\Phi_0}\right)$$

SQUID modulation of Josephson energy

$$E_J = E_{J,\max}\cos\left(\pi\frac{\Phi_{\rm ext}}{\Phi_0}\right)$$



consider two state approximation

J. Clarke, *Proc. IEEE* **77**, 1208 (1989)

Two State Approximation

$$H = 4E_C(n - n_g)^2 - E_J \cos\delta$$

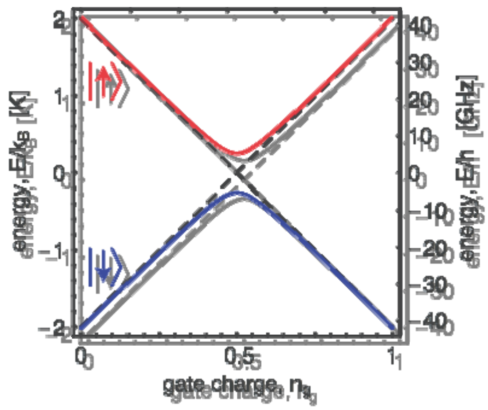
$$H = \sum_n \left[4E_C(n-n_g)^2 |n\rangle\langle n| - \frac{E_J}{2}\left(|n\rangle\langle n+1| + |n+1\rangle\langle n|\right)\right]$$

Restricting to a two-charge Hilbert space:

$$n = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1-\sigma_z}{2}$$

$$\cos\delta = \frac{\sigma_x}{2}$$

→ $$H = -\frac{E_{el}}{2}\sigma_z - \frac{E_J}{2}\sigma_x$$



Shnirman et al. Phys. Rev. Lett. 79, 2371 (1997)

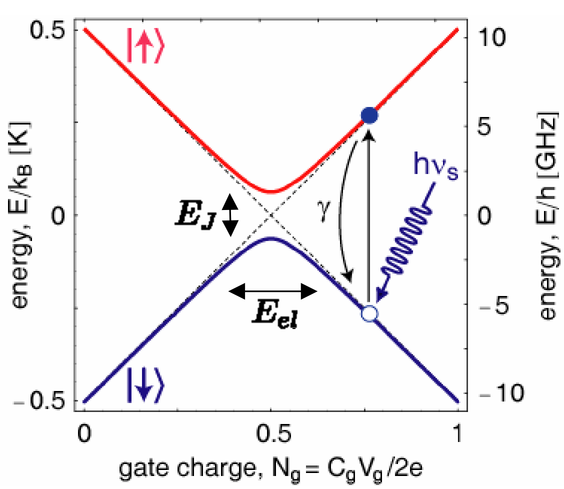
Control Parameters

2-state Hamiltonian and level separation:

$$H = -1/2\,(E_{el}\,\sigma_x + E_J\,\sigma_z)$$

$$E = \sqrt{E_{el}^2 + E_J^2}$$

control parameters: x and z rotations



in-situ controllable parameters:

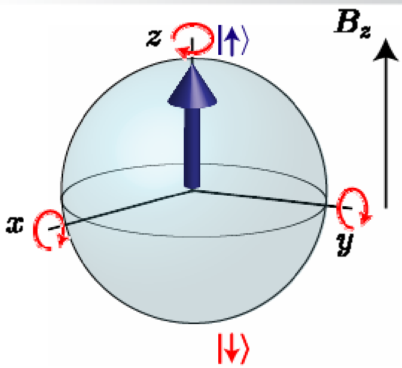
$$E_{el} = E_C\,(1/2 - N_g)$$

$$E_J = E_{J,max}\cos(\pi\Phi_{ext}/\Phi_0)$$

E_C, $E_{J,max}$ **engineerable in fabrication**

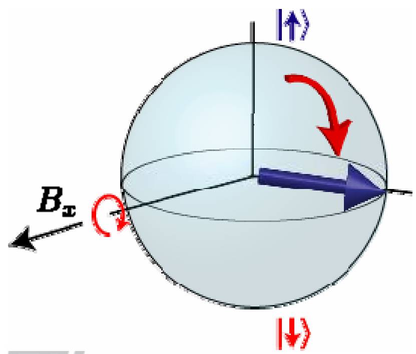
Nakamura, Pashkin, Tsai *et al.* *Nature* **398**, **421**, **425** (1999,2003, 2003)

Single Qubit Control



Bloch sphere representation of single qubit manipulation

rotations about x, y, z axes

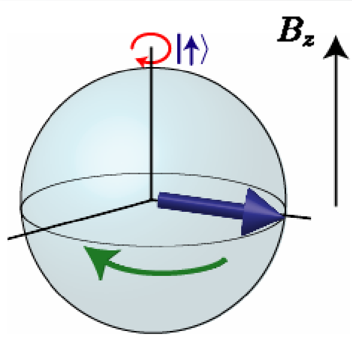


change qubit state with perpendicular field $B \perp B_z$

x-rotation

y-rotation is equivalent

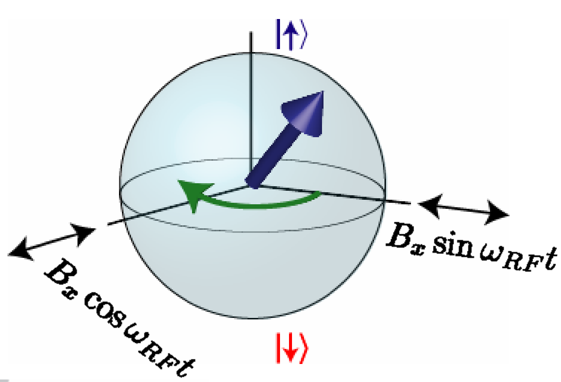
Larmor Precession



spin precession frequency

$$\hbar\omega_{01} = \mu B$$

$\equiv$ qubit transition frequency



rotating frame:

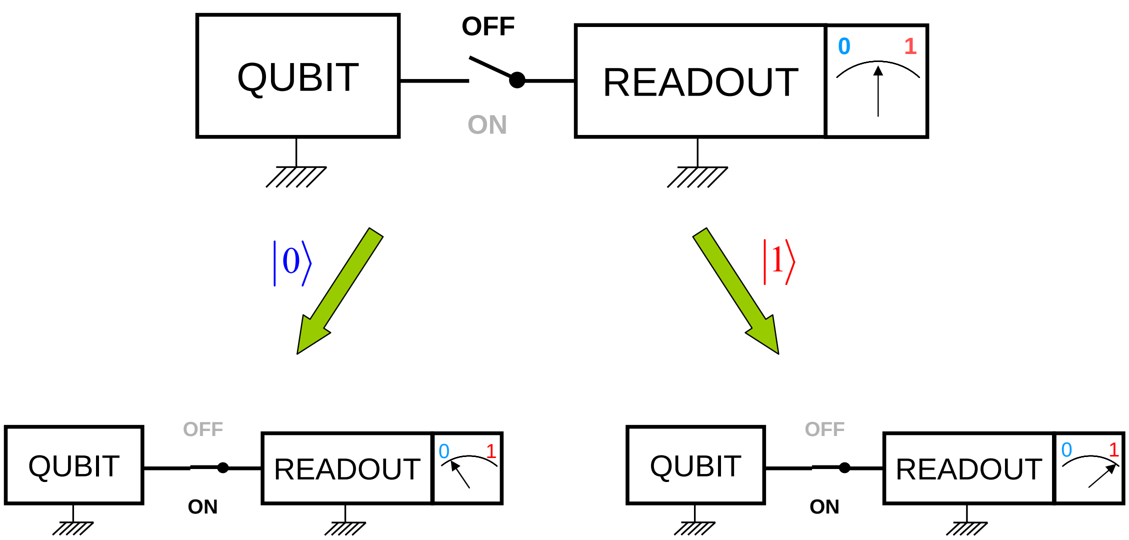
field oscillating at $\omega_{RF} \sim \omega_{01}$ drives transitions

at $\omega_{01} = \omega_{RF}$ driving field looks static in rotating frame

Read-Out ...

… of a superconducting charge qubit

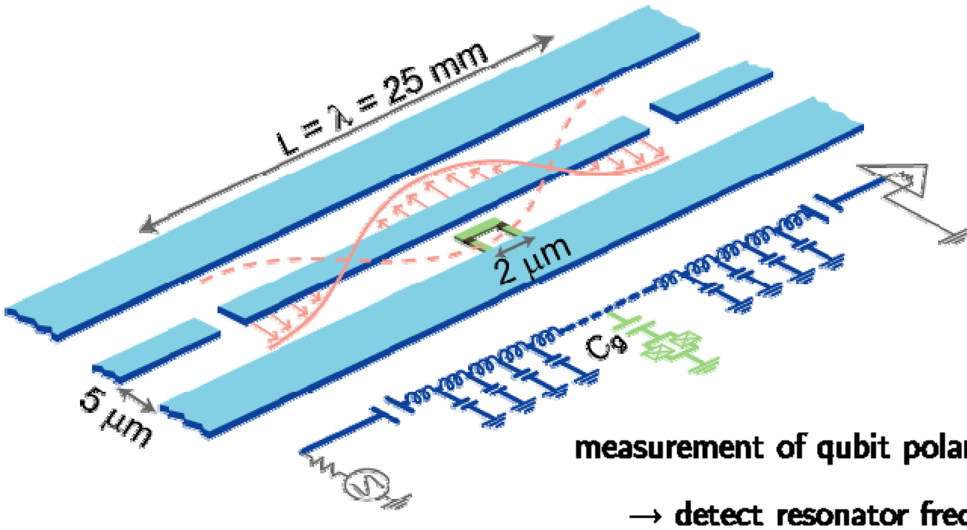
Qubit Read Out



desired: good on/off ratio
no relaxation in on state (QND)

Cooper Pair Box Embedded in a Resonator

realize projective readout of qubit:

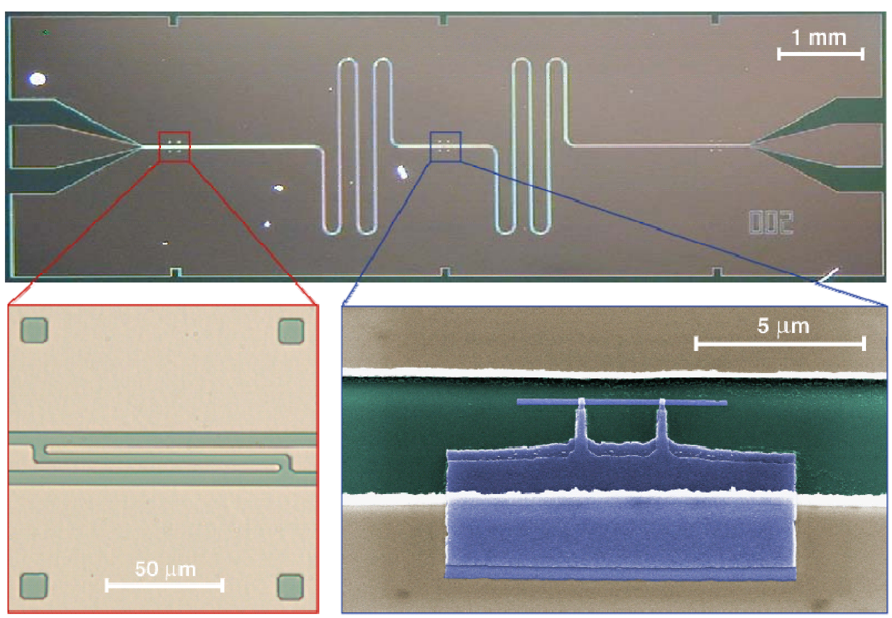


measurement of qubit polarizability

→ detect resonator frequency shift

A. Blais, R.-S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, *PRA* **69,** 062320 (2004)

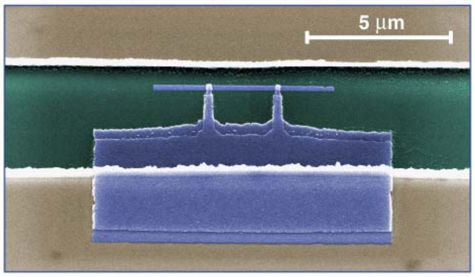
The Device

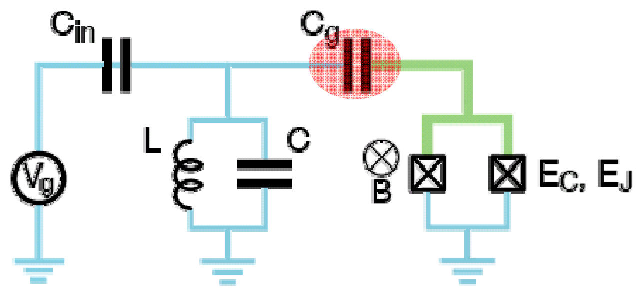


a qubit coupled to a harmonic oscillator for read-out

A. Wallraff, D. Schuster, ..., S. Girvin, and R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004)

Qubit/Photon Interaction Strength





coupling strength:

$$\hbar g = e V_{0,\mathrm{rms}} \frac{C_g}{C_\Sigma} \quad \Longrightarrow \nu_{\mathrm{vac}} = \frac{g}{\pi} \approx 1 \ldots 100\,\mathrm{MHz}$$

A. Blais *et al.*, *PRA* **69,** 062320 (2004)

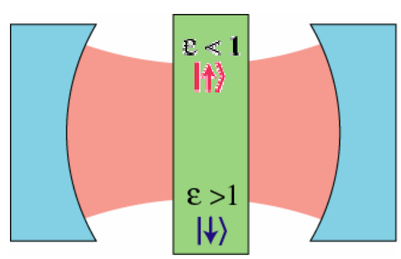
Non-Resonant Interaction: Qubit Readout

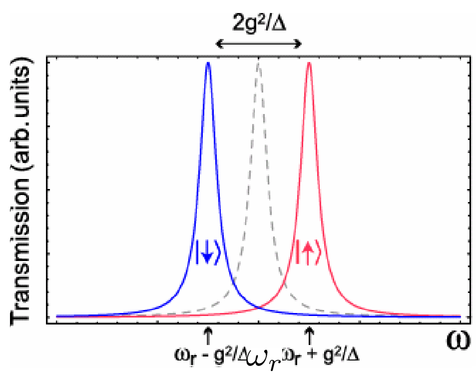
approximate diagonalization in the dispersive limit $|\Delta| = |\omega_a - \omega_r| \gg g$

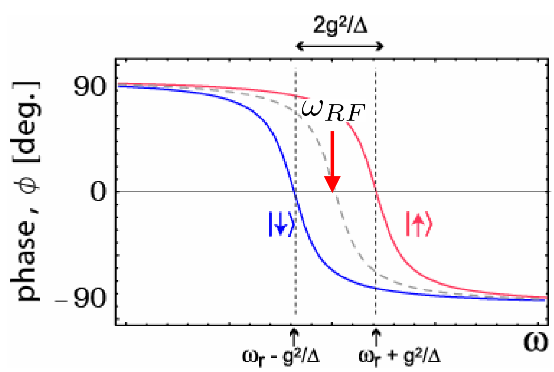
$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta} \sigma_z \right) a^\dagger a + \frac{1}{2} \hbar \left(\omega_a + \frac{g^2}{\Delta} \right) \sigma_z$$

cavity frequency shift
and qubit ac-Stark shift

Lamb shift







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A. Blais *et al.*, *PRA* **69,** 062320 (2004)

Qubit Spectroscopy with Dispersive Read-Out

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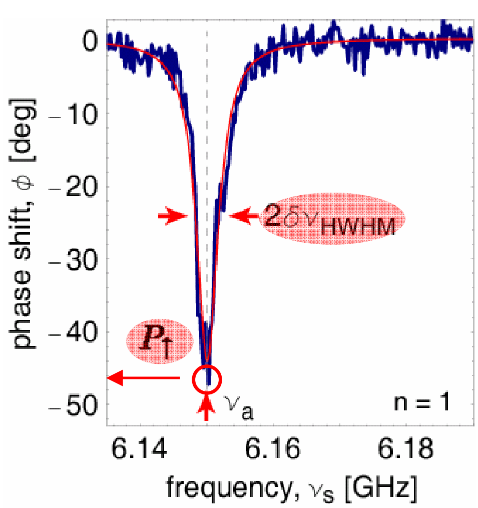
Line Shape

excited state population (steady-state Bloch equations):

$$P_\uparrow = 1 - P_\downarrow = \frac{1}{2}\frac{n_s \omega_{vac}^2 T_1 T_2}{1 + (T_2 \Delta_{s,a})^2 + n_s \omega_{vac}^2 T_1 T_2}$$



- fixed drive $P_s \propto n_s \omega_{vac}^2$
- varying $\Delta_{s,a} = \omega_s - \widetilde{\omega}_a$
- weak continuous measurement $(n \sim 1)$
- at charge degenracy $(n_g = 1)$

Abragam, Oxford University Press (1961)

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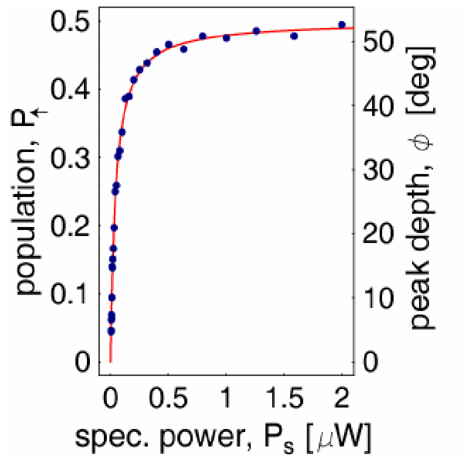
Excited State Population

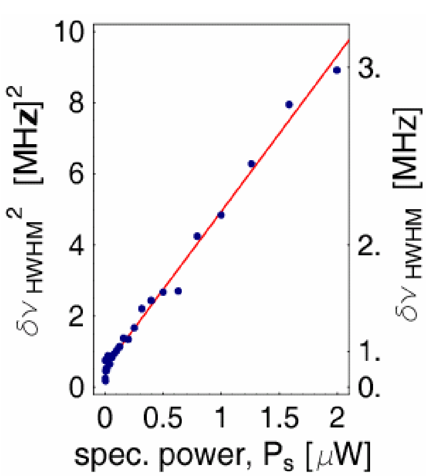


peak depth → population (saturation):

$$P_{\uparrow} = 1 - P_{\downarrow} = \frac{1}{2}\frac{n_s \omega_{\rm vac}^2 T_1 T_2}{1 + n_s \omega_{\rm vac}^2 T_1 T_2}$$

D. I. Schuster, A. Wallraff, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Girvin, and R. J. Schoelkopf, *Phys. Rev. Lett.* **94,** 123062 (2005)

Line Width



line width → coherence time:

$$2\pi\delta\nu_{\rm HWHM} = \frac{1}{T_2'} = \sqrt{\frac{1}{T_2^2} + n_s \omega_{\rm vac}^2 \frac{T_1}{T_2}}$$

$$\mathrm{Min}(\delta\nu_{\rm HWHM}) \sim 750\,\mathrm{kHz} \rightarrow \mathrm{T}_2 > 200\,\mathrm{ns}$$

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D. I. Schuster, A. Wallraff, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Girvin, and R. J. Schoelkopf, *Phys. Rev. Lett.* **94,** 123062 (2005)

AC-Stark Effect and Measurement Backaction

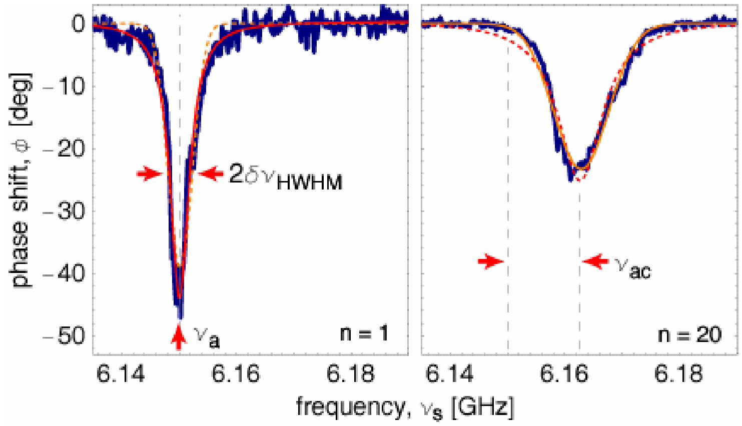
AC-Stark Effect

ac-Stark (light) shift

for $\Delta_{a,r} = \omega_a - \omega_r \gg g$

$$H \approx \hbar\omega_r a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta} + \frac{2g^2}{\Delta}a^\dagger a\right)\sigma_z$$

photon number dependence of line position and width



D. I. Schuster *et al., Phys. Rev. Lett.* **94,** 123062 (2005)

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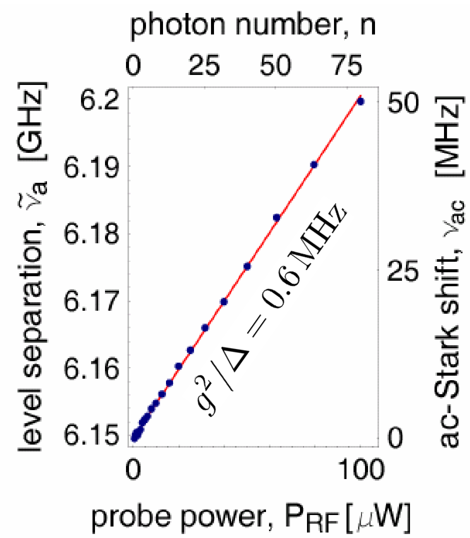
AC-Stark Effect: Line Shift



- ac-Stark (light) shift:

$$\nu_{ac} = \bar{n} \frac{g^2}{\pi \Delta_{a,r}}$$

- here $\nu_{ac}/\bar{n} = 0.6\,\mathrm{MHz}$
- use for photon number calibration

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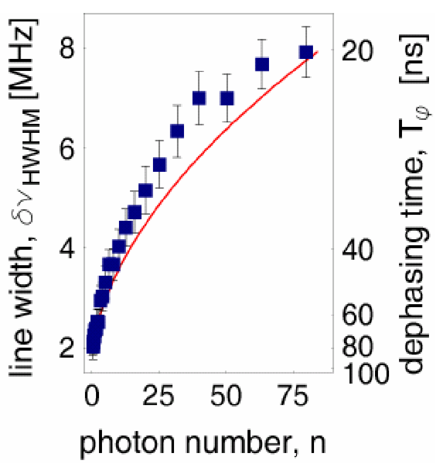
D. I. Schuster, A. Wallraff, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Girvin, and R. J. Schoelkopf, *Phys. Rev. Lett.* **94,** 123062 (2005)

AC-Stark Effect: Line Broadening

photon shot noise:

- quantum fluctuations δn in coherent field with n photons
- random fluctuations in qubit level separation (ac-Stark)



- for large n gaussian fluctuations in n:

$$\delta\nu_{HWHM} = \sqrt{2 \ln 2}\, \frac{g^2}{\pi \Delta_{a,r}} \sqrt{\bar{n}}$$

- characteristic measurement back-action

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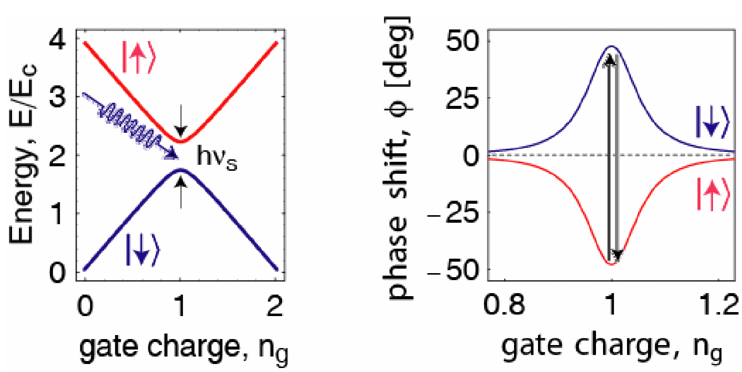
D. I. Schuster, A. Wallraff, …, S. Girvin, and R. J. Schoelkopf, *Phys. Rev. Lett.* **94,** 123062 (2005)

Coherent Control ...

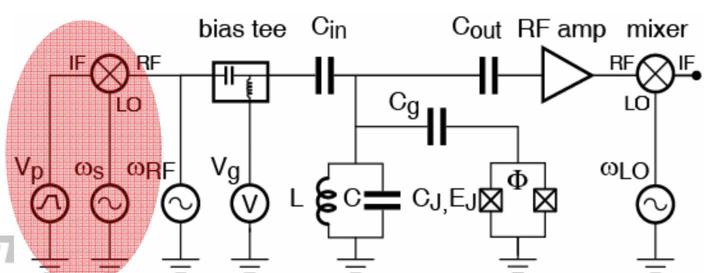
… of a superconducting charge qubit.

Coherent Control and Read-out in a Cavity



- apply resonant microwave pulse to qubit
- detect change of phase

realization:



- simultaneous control and measurement

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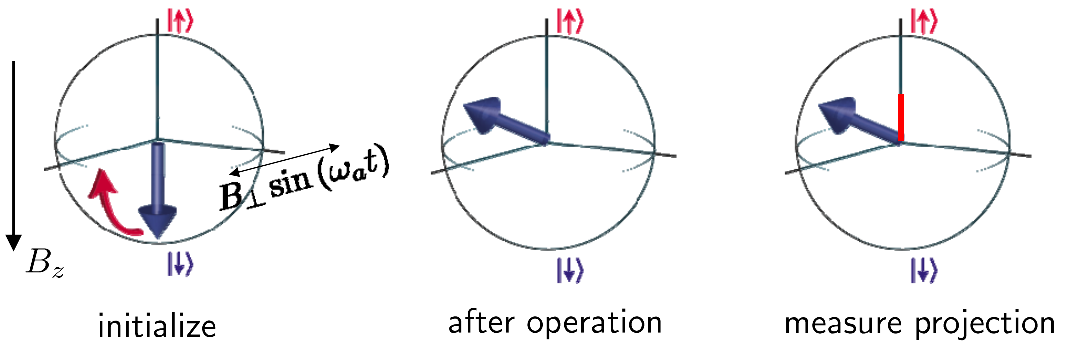
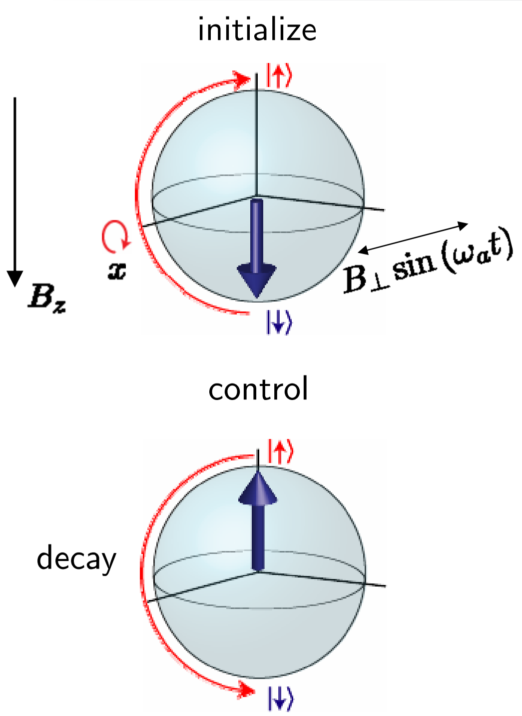
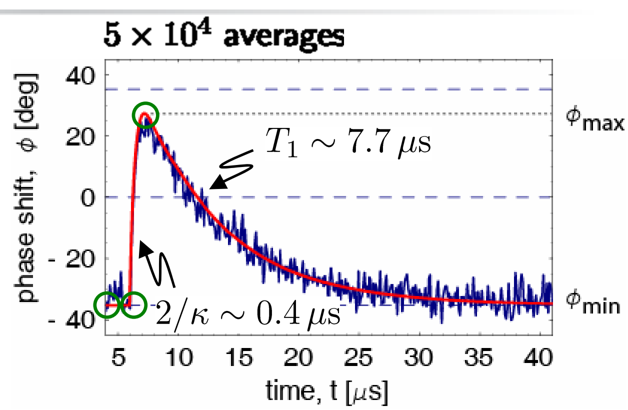
Coherent Control of a Qubit in a Cavity



- qubit state represented on a Bloch sphere
- NMR style operations
- vary length, amplitude and phase of pulse to control qubit state

Qubit Control and Readout





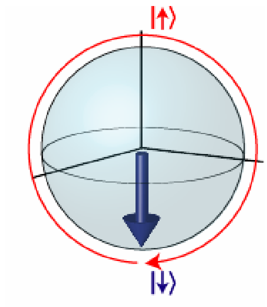
measurement properties:

- **continuous**
- **dispersive**
- **quantum non-demolition**
- **in good agreement with predictions**

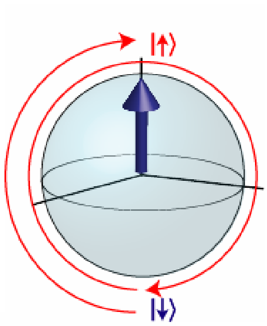
Wallraff, Schuster, Blais, ... Girvin, and Schoelkopf, *Phys. Rev. Lett.* **95**, 060501 (2005)

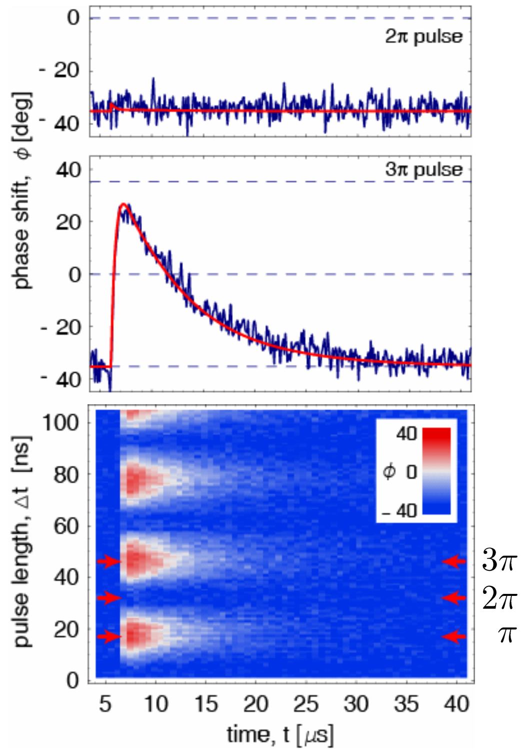
Varying the Control Pulse Length

2π rotation



3π rotation

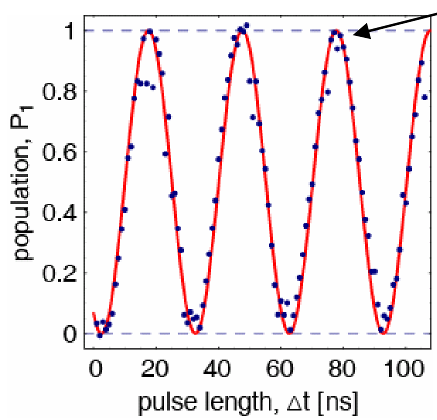




Wallraff, Schuster, Blais, ... Girvin, Schoelkopf, *PRL* **95**, 060501 (2005)

High Visibility Rabi Oscillations

Rabi oscillations:



visibility $95 \pm 5\%$

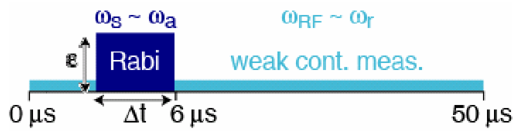
for superconducting qubits:

- **high visibility**
- **well characterized and understood measurement**
- **good control accuracy**

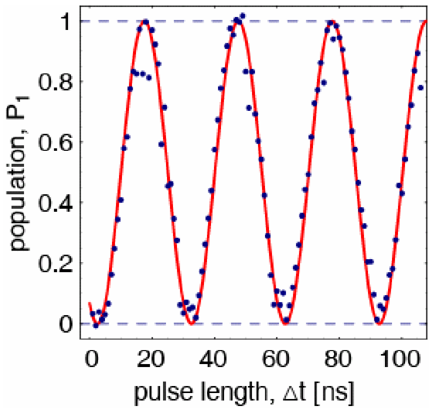
A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, J. Majer, S. M. Girvin, and R. J. Schoelkopf, *Phys. Rev. Lett.* **95**, 060501 (2005)

Rabi Frequency

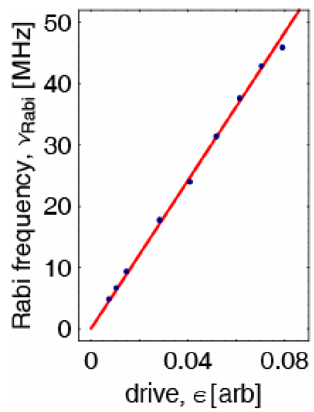
pulse scheme:



Rabi oscillations:



Rabi frequency:



- **linear dependence on drive amplitude**

Measurements of Coherence Time

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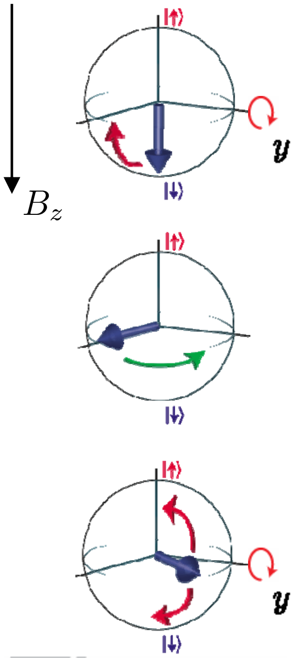
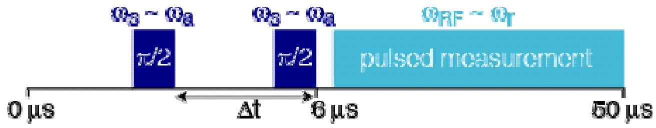
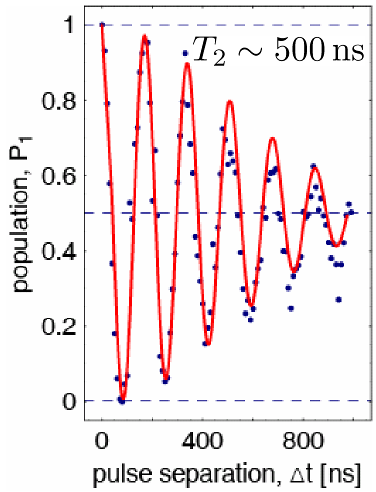
Ramsey Fringes: Coherence Time Measurement

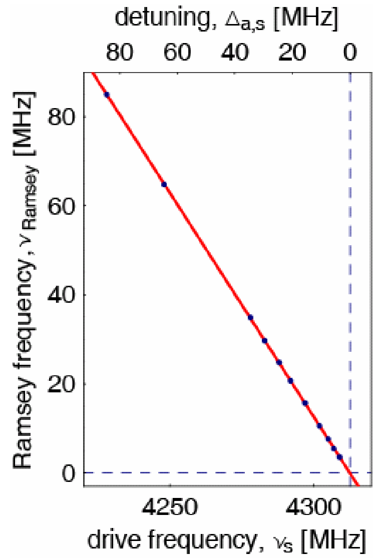
pulse scheme:





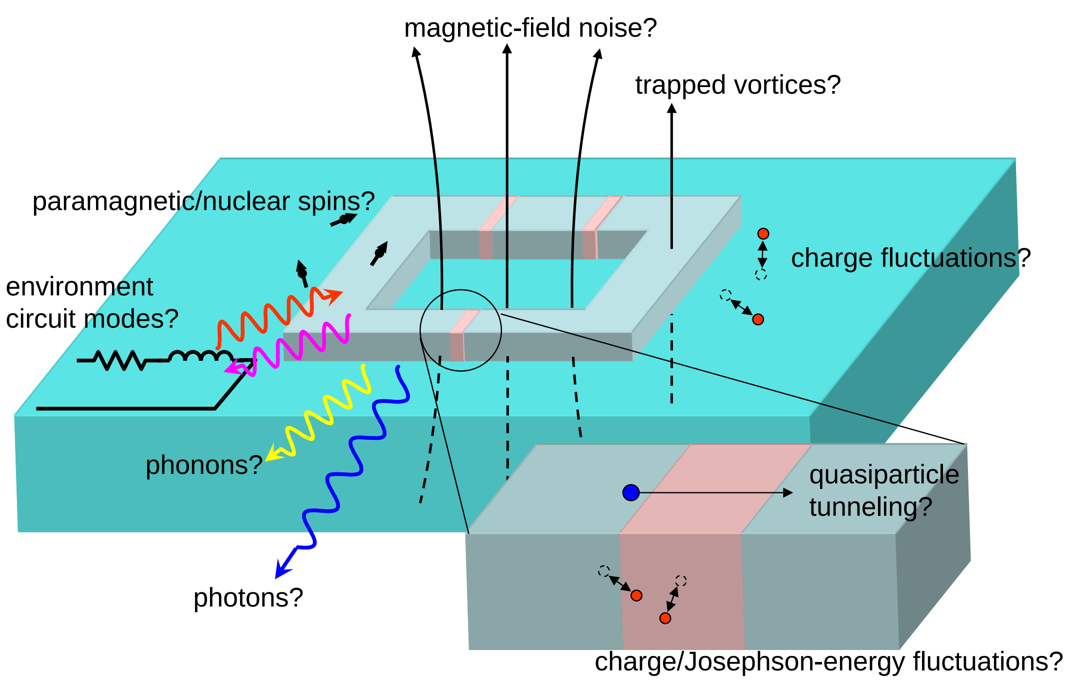
Ramsey fringes:





A. Wallraff *et al.*, *Phys. Rev. Lett.* **95**, 060501 (2005)

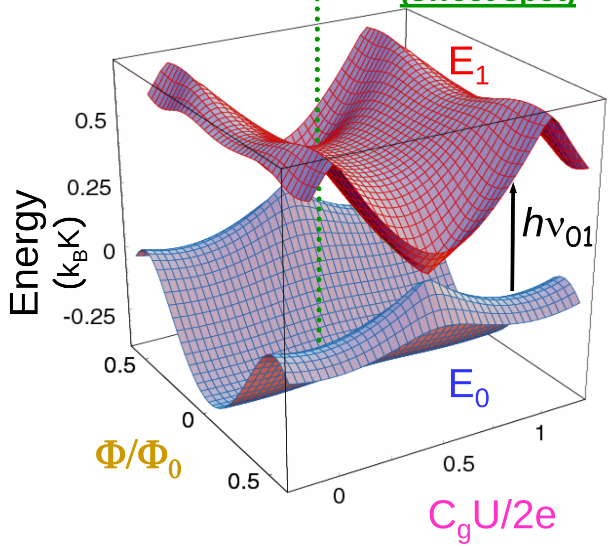
Sources of Decoherence



G. Ithier *et al.*, *Phys. Rev. B* **72**, 134519 (2005)

Reduce Decoherence

use of symmetries: optimum working point (sweet spot)



charge

$$Q_k = \frac{\partial E_k}{\partial U}$$

current

$$I_k = \frac{\partial E_k}{\partial \Phi}$$

D. Vion *et al.*, *Science* **296**, 886 (2002)

L. Ioffe *et al.*, *Nature* **415**, 503 (2002)

do matrials research

J. Martinis *et al.*, *Phys. Rev. Lett.* **95**, 210503 (2005)

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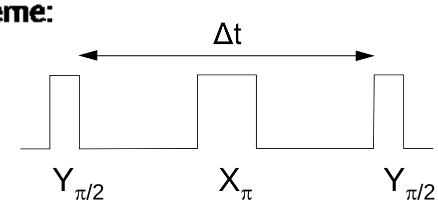
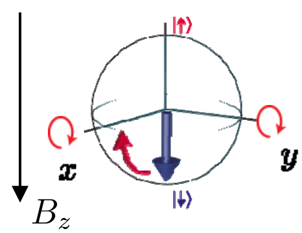
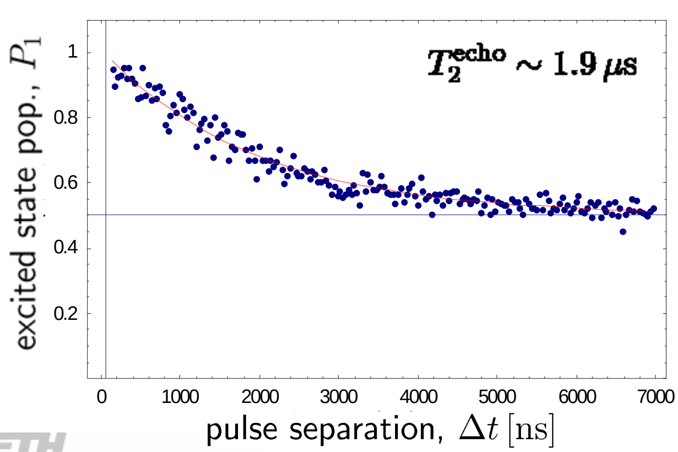
Spin Echo

pulse scheme:





result:



- refocusing
- elimination of low frequency fluctuations
- increased effective coherence time

L. Steffen *et al.* (2007)

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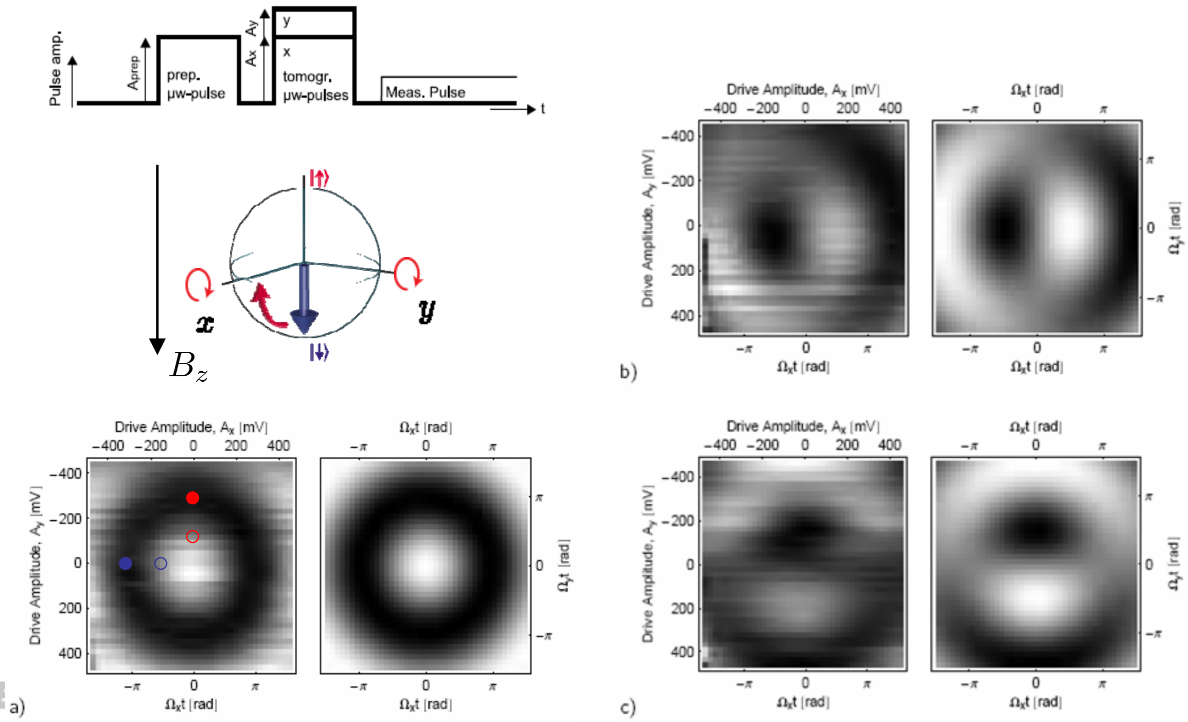
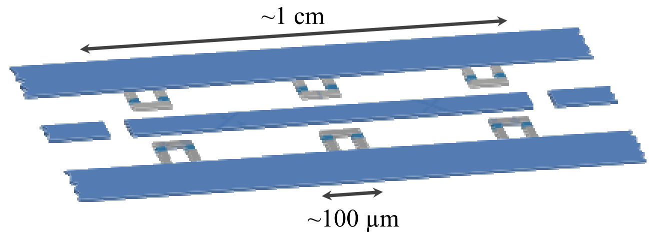
Quantum State Tomography



Coupled Superconducting Qubits

Multi-qubit Coupling



$$H = \omega_r a^\dagger a + \frac{\omega_a}{2}\sigma_z + g(a^\dagger \sigma_- + a\sigma_+)$$

$$H = \omega_r a^\dagger a + \sum_{j=1,2} \frac{\omega_{aj}}{2}\sigma_{z_j} + \sum_{j=1,2} g_j(a^\dagger \sigma_{-_j} + a\sigma_{+_j})$$

- Room for *many* qubits
- High-fidelity control of individual qubits by frequency selection
- Entanglement over "large" distances in a solid-state system
- Allows for parallel operations
- How to couple/decouple qubits without extra knobs?

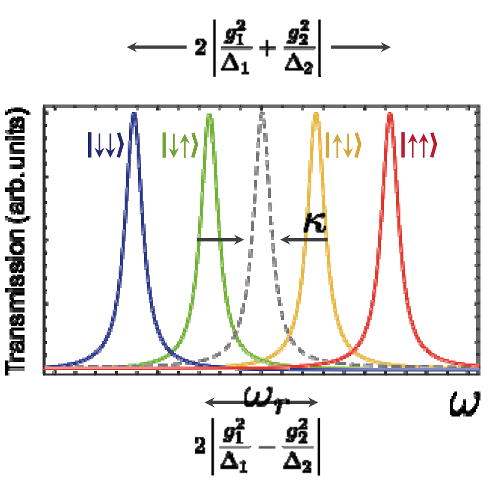
Blais, Gambetta, Wallraff, Schuster, Girvin, Devoret and Schoelkopf. Phys. Rev. A, 75, 032329 (2007)

Multi-Qubit Readout

Strong qubit-resonator detuning:

$$|\Delta_j| = |\omega_{aj} - \omega_r| \gg g_j$$

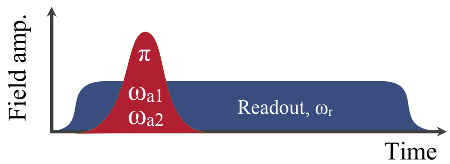
$$H_{\text{eff}} \approx \left[\omega_r + \sum_j \frac{g_j^2}{\Delta_j}\sigma_{z_j}\right] a^\dagger a + \sum_j \frac{1}{2}\left[\omega_{aj} + \frac{g_j^2}{\Delta_j}\right]\sigma_{z_j}$$

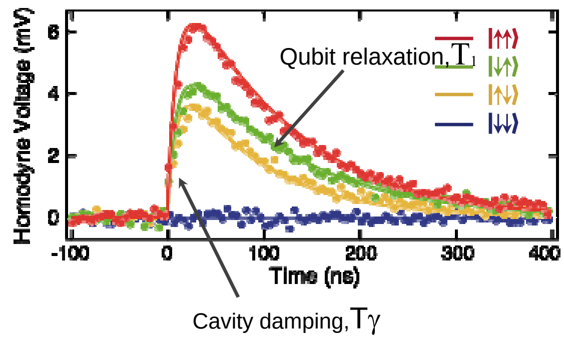


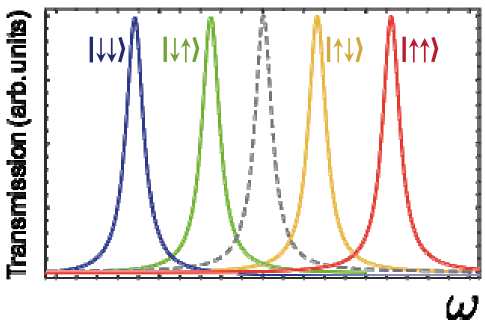
- Two-qubit state simultaneously measured by resolving pulled cavity frequencies
- Heterodyne measurement of the transmitted phase
- Two-qubit Quantum Non-Demolition measurement

Blais, Huang, Wallraff, Girvin & Schoelkopf, PRA **69**, 062320 (2004)

Multiplexed Control and Readout







- Independent control of the qubits
- 2 classical bits of info in one shot

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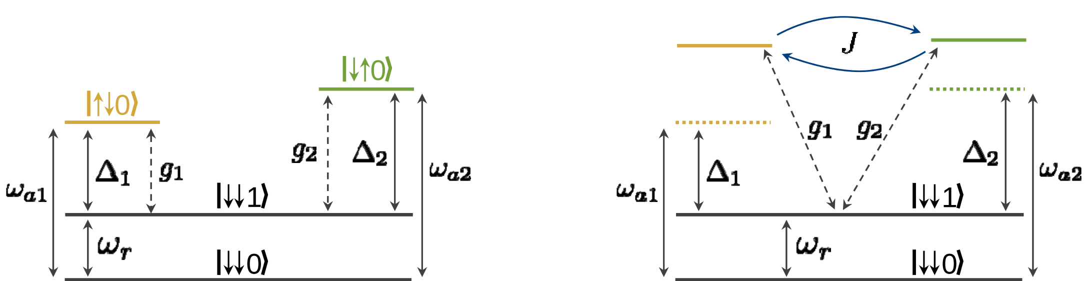
Majer, Chow, Gambetta, Koch, Johnson, Schreier, Frunzio, Schuster, Houck, Wallraff, Blais, Devoret, Girvin and Schoelkopf. Nature, *in press*.

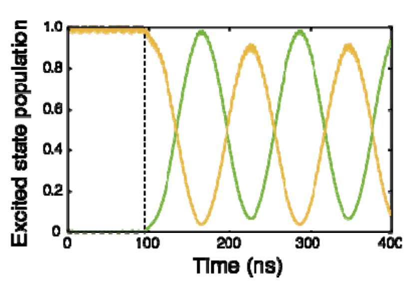
Entanglement Generation

$$H_{\text{eff}} \approx \omega_r a^\dagger a + \sum_j \frac{1}{2}\left[\omega_{aj} + 2\frac{g_j^2}{\Delta_j}\left(a^\dagger a + \frac{1}{2}\right)\right]\sigma_{z_j} + J\left(\sigma_{-_1}\sigma_{+_2} + \sigma_{+_1}\sigma_{-_2}\right)$$



- Interaction suppressed without control drive
- Interaction strength: $J = \frac{g_1 g_2}{2}\left(\frac{1}{\Delta_1} + \frac{1}{\Delta_2}\right)$

$\sim 1 - 50$ MHz



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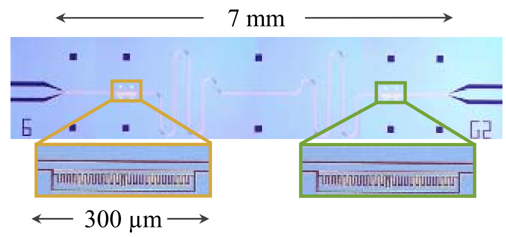
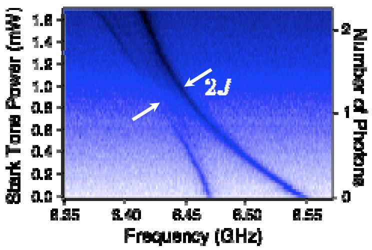
Blais, Gambetta, Wallraff, Schuster, Girvin, Devoret and Schoelkopf. Phys. Rev. A, 75, 032329 (2007)

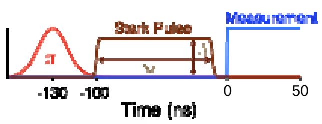
Experimental Realization



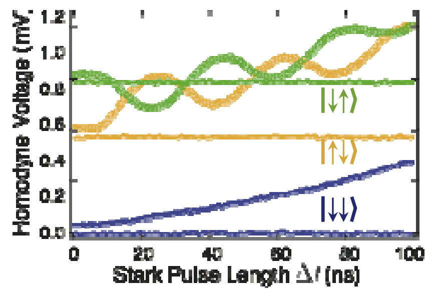
ac-Stark tuning:



The protocol:



The results:



Majer, Chow, Gambetta, Koch, Johnson, Schreier, Frunzio, Schuster, Houck, Wallraff, Blais, Devoret, Girvin and Schoelkopf. Nature, *in press*.

The 5 (+2) DiVincenzo Criteria for Implementation of a Quantum Computer:

in the standard (circuit approach) to **quantum information processing** (QIP)

#1. A scalable physical system with well-characterized qubits. ✓

#2. The ability to initialize the state of the qubits to a simple fiducial state. ✓

#3. Long (relative) decoherence times, much longer than the gate-operation time. ✓

#4. A universal set of quantum gates. ✓

#5. A qubit-specific measurement capability. ✓

#6. The ability to interconvert stationary and mobile (or flying) qubits. ✓

#7. The ability to faithfully transmit flying qubits between specified locations. ✓